NATIONAL CERTIFICATES (VOCATIONAL)

ASSESSMENT GUIDELINES

MATHEMATICS

NQF Level 3

IMPLEMENTATION: JANUARY 2012
MATHEMATICS – LEVEL 3

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SECTION A: PURPOSE OF THE SUBJECT ASSESSMENT GUIDELINES

This document provides the lecturer with guidelines to develop and implement a coherent, integrated assessment system for Mathematics in the National Certificates (Vocational). It must be read with the National Policy Regarding Further Education and Training Programmes: Approval of the Documents, Policy for the National Certificates (Vocational) Qualifications at Levels 2 to 4 on the National Qualifications Framework (NQF).

This document explains the requirements for the internal and external subject assessment. The lecturer must use this document with the Subject Guidelines: Mathematics Level 3 to prepare for and deliver the subject. Lecturers should use a variety of resources and apply a range of assessment skills in the setting, marking and recording of assessment tasks.

SECTION B: ASSESSMENT IN THE NATIONAL CERTIFICATES ( VOCATIONAL )

1 ASSESSMENT IN THE NATIONAL CERTIFICATES (VOCATIONAL)

Assessment in the National Certificates (Vocational) is underpinned by the objectives of the National Qualifications Framework (NQF). These objectives are to:

- Create an integrated national framework for learning achievements.
- Facilitate access to and progression within education, training and career paths.
- Enhance the quality of education and training.
- Redress unfair discrimination and past imbalances and thereby accelerate employment opportunities.
- Contribute to the holistic development of the student by addressing:
  - social adjustment and responsibility;
  - moral accountability and ethical work orientation;
  - economic participation; and
  - nation-building.

The principles that drive these objectives are:

- **Integration**
  To adopt a unified approach to education and training that will strengthen the human resources and develop the capacity of the nation.

- **Relevance**
  To be dynamic and responsive to national development needs.

- **Credibility**
  To demonstrate recognition of competencies and skills acquired, national and international added value and recognition of the acquired qualification

**Coherence**

To work within a consistent framework of principles and certification.

- **Flexibility**
  To allow for creativity and resourcefulness when achieving Learning Outcomes, to cater for different learning styles and use a range of assessment methods, instruments and techniques.

- **Participation**
  To enable stakeholders to participate in the setting of standards and the co-ordination of the achievements required for the qualification.

- **Access**
  To address barriers to learning experienced on different levels and to facilitate the students’ progress.
• **Progression**
To ensure the qualification framework permits individuals to move through the levels of the national qualification via different, appropriate combinations of the components of the delivery system.

• **Portability**
To enable students to transfer credits obtained within a qualification from one learning institution and/or employer to another institution or employer.

• **Articulation**
To allow for vertical and horizontal mobility in the educational system on condition that accredited pre-requisites have been successfully completed.

• **Recognition of Prior Learning**
To grant credits for a unit of learning following an assessment process or where a student possesses the capabilities as specified in the outcomes.

• **Validity of assessments**
To ensure assessment covers a broad range of knowledge, skills, values and attitudes (SKVAs) needed to demonstrate applied competency. This is achieved through:
  - clearly stating the outcome to be assessed;
  - selecting the appropriate or suitable evidence;
  - matching the evidence with a compatible or appropriate method of assessment; and
  - selecting and constructing an instrument(s) of assessment.
Topics should be assessed individually and then cumulatively with other topics. There should be a final summative internal assessment prior to the external assessment.

• **Reliability**
To assure assessment practices are consistent so that the same result or judgment is arrived at if the assessment is replicated in the same context. This demands consistency in the interpretation of evidence; therefore, careful monitoring of assessment is vital.
  - Cumulative and summative assessments must be weighted more than single topic tests for the internal mark.
  - There should be at least one standardised or norm test in each term
  - All standardised or norm tests must be moderated by a subject specialist.

• **Fairness and transparency**
To verify that assessment processes and/or method(s) used neither hinders nor unfairly advantage any student. The following could constitute unfairness in assessment:
  - Inequality of opportunities, resources or teaching and learning approaches
  - Bias based on ethnicity, race, gender, age, disability or social class
  - Lack of clarity regarding Learning Outcome being assessed
  - Comparison of students’ work with other students, based on learning styles and language
Assessment in Mathematics must take into consideration that the process or method carries more weight than the final answer.

• **Practicability and cost-effectiveness**
To integrate assessment tasks and/practices within an outcomes-based education and training system to strive for cost and time-effective assessment.
2 ASSESSMENT FRAMEWORK FOR VOCATIONAL QUALIFICATIONS

The assessment structure for the National Certificates (Vocational) qualification is as follows:

2.1 Internal continuous assessment (ICASS)

Knowledge, skills values, and attitudes (SKVAs) are assessed throughout the year using assessment instruments such as projects, tests, assignments, investigations, role-play and case studies. All internal continuous assessment (ICASS) evidence is kept in a Portfolio of Evidence (PoE) and must be readily available for monitoring, moderation and verification purposes. This component is moderated and quality assured both internally and externally.

2.2 External summative assessment (ESASS)

The external summative assessment comprises TWO papers set to meet the requirements of the Subject and Learning Outcomes. It is administered according to relevant assessment policies and requirements.

External summative assessments will be conducted annually between October and December, with provision made for supplementary sittings.

3 MODERATION OF ASSESSMENT

3.1 Internal moderation

Assessment must be moderated according to the internal moderation policy of the Further Education and Training (FET) College. Internal college moderation is a continuous process. The moderator’s involvement starts with the planning of assessment methods and instruments and follows with continuous collaboration with and support to the assessors. Internal moderation creates common understanding of Assessment Standards and maintains these across vocational programmes.

3.2 External moderation

External moderation is conducted according to relevant quality assurance bodies’ standards, policies, and requirements (currently the South African Qualifications Authority (SAQA) and Umalusi.)

The external moderator:
- monitors and evaluates the standard of all summative assessments;
- maintains standards by exercising appropriate influence and control over assessors;
- ensures proper procedures are followed;
- ensures summative integrated assessments are correctly administered;
- observes a minimum sample of ten (10) to twenty-five (25) percent of summative assessments;
- gives written feedback to the relevant quality assurer; and
- moderates in case of a dispute between an assessor and a student.

Policy on inclusive education requires that assessment procedures for students who experience barriers to learning be customised and supported to enable these students to achieve their maximum potential.

4 PERIOD OF VALIDITY OF INTERNAL CONTINUOUS ASSESSMENT (ICASS)

The period of validity of the internal continuous assessment mark is determined by the National Policy on the Conduct, Administration and Management of the Assessment of the National Certificates (Vocational).

The internal continuous assessment (ICASS) must be re-submitted with each examination enrolment for which it constitutes a component.

5 ASSESSOR REQUIREMENTS

Assessors must be subject specialists and a competent assessor.
6 TYPES OF ASSESSMENT

Assessment benefits the student and the lecturer. It informs students about their progress and helps lecturers make informed decisions at different stages of the learning process. Depending on the intended purpose, different types of assessment can be used.

6.1 Baseline assessment
At the beginning of a level or learning experience, baseline assessment establishes the knowledge, skills, values and attitudes (SKVAs) that students bring to the classroom. This knowledge assists lecturers to plan learning programmes and learning activities.

6.2 Diagnostic assessment
This assessment diagnoses the nature and causes of learning barriers experienced by specific students. It is followed by guidance, appropriate support and intervention strategies. This type of assessment is useful to make referrals for students requiring specialist help.

6.3 Formative assessment
This assessment monitors and supports teaching and learning. It determines student strengths and weaknesses and provides feedback on progress. It determines if a student is ready for summative assessment.

6.4 Summative assessment
This type of assessment gives an overall picture of student progress at a given time. It determines whether the student is sufficiently competent to progress to the next level.

7 PLANNING ASSESSMENT
An assessment plan should cover three main processes:

7.1 Collecting evidence
The assessment plan indicates which Subject Outcomes and Assessment Standards will be assessed, what assessment method or activity will be used and when this assessment will be conducted.

7.2 Recording
Recording refers to the assessment instruments or tools with which the assessment will be captured or recorded. Therefore, appropriate assessment instruments must be developed or adapted.

7.3 Reporting
All the evidence is put together in a report to deliver a decision for the subject.

8 METHODS OF ASSESSMENT
Methods of assessment refer to who carries out the assessment and includes lecturer assessment, self-assessment, peer assessment and group assessment.

<table>
<thead>
<tr>
<th>LECTURER ASSESSMENT</th>
<th>The lecturer assesses students’ performance against given criteria in different contexts, such as individual work, group work, etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SELF-ASSESSMENT</td>
<td>Students assess their own performance against given criteria in different contexts, such as individual work, group work, etc.</td>
</tr>
<tr>
<td>PEER ASSESSMENT</td>
<td>Students assess another student or group of students’ performance against given criteria in different contexts, such as individual work, group work, etc.</td>
</tr>
<tr>
<td>GROUP ASSESSMENT</td>
<td>Students assess the individual performance of other students within a group or the overall performance of a group of students against given criteria.</td>
</tr>
</tbody>
</table>
9 INSTRUMENTS AND TOOLS FOR COLLECTING EVIDENCE

All evidence collected for summative assessment purposes (ICASS) is kept or recorded in the student’s Portfolio of Evidence (PoE).

The following table summarises a variety of methods and instruments for collecting evidence. A method and instrument is chosen to give students ample opportunity to demonstrate the Subject Outcome has been attained. This will only be possible if the chosen methods and instruments are appropriate for the target group and the Specific Outcome being assessed.

<table>
<thead>
<tr>
<th>METHODS FOR COLLECTING EVIDENCE</th>
<th>Observation-based (Less structured)</th>
<th>Task-based (Structured)</th>
<th>Test-based (More structured)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assessment instruments</strong></td>
<td>• Observation</td>
<td>• Assignments or tasks</td>
<td>• Examinations</td>
</tr>
<tr>
<td></td>
<td>• Class questions</td>
<td>• Projects</td>
<td>• Class tests</td>
</tr>
<tr>
<td></td>
<td>• Lecturer, student, parent discussions</td>
<td>• Investigations or research</td>
<td>• Practical examinations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Case studies</td>
<td>• Oral tests</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Practical exercises</td>
<td>• Open-book tests</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Demonstrations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Role-play</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Interviews</td>
<td></td>
</tr>
<tr>
<td><strong>Assessment tools</strong></td>
<td>• Observation sheets</td>
<td>• Checklists</td>
<td>• Marks (e.g. %)</td>
</tr>
<tr>
<td></td>
<td>• Lecturer’s notes</td>
<td>• Rating scales</td>
<td>• Rating scales (1-7)</td>
</tr>
<tr>
<td></td>
<td>• Comments</td>
<td>• Rubrics</td>
<td></td>
</tr>
<tr>
<td><strong>Evidence</strong></td>
<td>• Focus on individual students</td>
<td>Open middle: Students produce the same evidence but in different ways.</td>
<td>Students answer the same questions in the same way, within the same time.</td>
</tr>
<tr>
<td></td>
<td>• Subjective evidence based on lecturer observations and impressions</td>
<td>Open end: Students use same process to achieve different results.</td>
<td></td>
</tr>
</tbody>
</table>

10 TOOLS FOR ASSESSING STUDENT PERFORMANCE

Rating scales are marking systems where a symbol (such as 1 to 7) or a mark (such as 5/10 or 50%) is defined in detail. The detail is as important as the coded score. Traditional marking, assessment and evaluation mostly used rating scales without details such as what was right or wrong, weak or strong, etc.

Task lists and checklists show the student what needs to be done. They consist of short statements describing the expected performance in a particular task. The statements on the checklist can be ticked off when the student has adequately achieved the criterion. Checklists and task lists are useful in peer or group assessment activities.

Rubrics are a hierarchy (graded levels) of criteria with benchmarks that describe the minimum level of acceptable performance or achievement for each criterion. It is a different way of assessment and cannot be compared to tests. Each criterion described in the rubric must be assessed separately. Mainly, two types of rubrics, namely holistic and analytical, are used.

11 SELECTING AND/OR DESIGNING RECORDING AND REPORTING SYSTEMS

The selection or design of recording and reporting systems depends on the purpose of recording and reporting student achievement. Why particular information is recorded and how it is recorded determine which instrument will be used.
Computer-based systems, for example spreadsheets, are cost and time effective. The recording system should be user-friendly and information should be easily accessed and retrieved.

12 COMPETENCE DESCRIPTIONS

All assessment should award marks to evaluate specific assessment tasks. However, marks should be awarded against the marking guidelines (memoranda/rubrics/checklists) and not simply be a total of ticks for right answers. Rubrics should explain the competence level descriptors for the skills, knowledge, values and attitudes (SKVAs) a student must demonstrate to achieve each level of the rating scale.

When lecturers or assessors prepare an assessment task or question, they must ensure that the task or question addresses an aspect of a Subject Outcome. The relevant Assessment Standard must be used to create the rubric to assess the task or question. The descriptions must clearly indicate the minimum level of attainment for each category on the rating scale.

13 STRATEGIES FOR COLLECTING EVIDENCE

A number of different assessment instruments may be used to collect and record evidence. Examples of instruments that can be (adapted and) used in the classroom include:

13.1 Record sheets

The lecturer observes students working in a group. These observations are recorded in a summary table at the end of each project. The lecturer can design a record sheet to observe students’ interactive and problem-solving skills, attitudes towards group work and involvement in a group activity.

13.2 Checklists

Checklists should have clear categories to ensure that the objectives are effectively met. The categories should describe how the activities are evaluated and against what criteria they are evaluated. Space for comments is essential.
ASSESSMENT IN MATHEMATICS

LEVEL 3
SECTION C: ASSESSMENT IN MATHEMATICS

1 ASSESSMENT SCHEDULE AND REQUIREMENTS

Internal and external assessments are conducted and the results of both are contributing to the final mark of a student in the subject.

The internal continuous assessment (ICASS) mark accounts for 25 percent and the external examination mark for 75 percent of the final mark. A student needs a minimum final mark of 30 percent to enable a pass in the subject.

1.1 Internal assessment

Lecturers must compile a detailed assessment plan/schedule of internal assessments to be undertaken during the year in the subject. (e.g. date, assessment task/or activity, rating code/marks allocated, assessor, moderator.)

Internal assessments are then conducted according to the plan/schedule using appropriate assessment instruments and tools for each assessment task (e.g. tests, assignments, practical tasks/projects and memorandum, rubric, checklist)

The marks allocated to both the practical and written assessment tasks conducted during the internal continuous assessment (ICASS) are kept and recorded in the Portfolio of Evidence (PoE) which is subjected to internal and external moderation.

A year mark out of 100 is calculated from the ICASS marks contained in the PoE and submitted to the Department on the due date towards the end of the year.

The following internal assessment units GUIDE the assessment of Mathematics Level 3.

<table>
<thead>
<tr>
<th>NUMBER OF UNITS</th>
<th>ASSESSMENT</th>
<th>Time and Mark Allocation</th>
<th>Weight %</th>
<th>COVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Formal written tests</td>
<td>1 hour each 30 -35 marks</td>
<td>20% (10% for each test)</td>
<td>One or more completed topics</td>
</tr>
<tr>
<td>1</td>
<td>Formal written test</td>
<td>2 hours 70 marks</td>
<td>20%</td>
<td>All completed topics</td>
</tr>
<tr>
<td>1</td>
<td>Internal written examination</td>
<td>1 paper 3 hours 100 marks OR 2 papers 2 hours each 70 marks each</td>
<td>30% OR 15% for each paper</td>
<td>All completed topics</td>
</tr>
</tbody>
</table>
# Mathematics Level 3 (January 2012) National Certificates (Vocational)

## Assignments

Approximately 2 hours per assignment  
20% (10% for each assignment)  
One or more completed topics  
Open book tests and/or group work completed over a period of 1 to 5 days may also be used.

## Practical assessment

Determined by type of practical task  
10%

Any related Subject Outcomes, for example:

1. Draw up a budget for a social club and determine where savings can be best invested.
2. Use advertisements dealing with investments or money lenders to motivate the choice of a specific loan.
3. Work with loan repayments that involve time lines.
4. Construct the net of a hexagonal pyramid (on paper) with a side length of 3 cm and a vertical height of 5 cm showing the flaps. Build the three dimensional shape calculating the area of the paper used and the volume of the pyramid.
5. Use the results of a test given to learners breaking the scores into classes with a class width of 5 and calculate the mean, median and mode. (Combine the marks of 2 classes to ensure the sample is approximately 50)

## RECORDING AND REPORTING

Mathematics is assessed according to seven levels of competence. The level descriptions are explained in the following table.

### Scale of achievement for the Fundamental component

<table>
<thead>
<tr>
<th>RATING CODE</th>
<th>RATING</th>
<th>MARKS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Outstanding</td>
<td>80 – 100</td>
</tr>
<tr>
<td>6</td>
<td>Meritorious</td>
<td>70 – 79</td>
</tr>
<tr>
<td>5</td>
<td>Substantial</td>
<td>60 – 69</td>
</tr>
<tr>
<td>4</td>
<td>Adequate</td>
<td>50 – 59</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
<td>40 – 49</td>
</tr>
<tr>
<td>2</td>
<td>Elementary</td>
<td>30 – 39</td>
</tr>
<tr>
<td>1</td>
<td>Not achieved</td>
<td>0 – 29</td>
</tr>
</tbody>
</table>
The planned/scheduled assessments should be recorded in the Lecturer’s Portfolio of Assessment (PoA) for each subject. The minimum requirements for the Lecturer’s Portfolio of Assessment should be as follows:

- Lecturer information
- A contents page
- Subject and Assessment Guidelines
- Year plans /Work schemes/Pace Setters
- A formal schedule of assessment
- Instrument(s) (tests, assignments, practical) and tools (memorandum, rubric, checklist) for each assessment task
- A mark/result sheet for assessment tasks

The college could standardise these documents.

The minimum requirements for the student’s Portfolio of Evidence (PoE) should be as follows:

- Student information/identification
- A contents page/list of content (for accessibility)
- A record/summary/ of results showing all the marks achieved per assessment for the subject
- The evidence of marked assessment tasks and feedback according to the assessment schedule
- Where tasks cannot be contained as evidence in the Portfolio of Evidence (PoE), its exact location must be recorded and it must be readily available for moderation purposes.
### 3  INTERNAL ASSESSMENT OF OUTCOMES IN MATHEMATICS - LEVEL 3

**Topic 1: Complex Numbers**

(Minimum of 12 hours face to face teaching which excludes time for revision, test series and internal and external examination)

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Represent complex numbers in a form appropriate to the context.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imaginary numbers are written to its simplest form.</td>
<td>Write imaginary numbers to its simplest form where $i^2 = -1$.</td>
</tr>
<tr>
<td>Negative roots are simplified into imaginary numbers</td>
<td>Simplify negative roots into imaginary numbers.</td>
</tr>
<tr>
<td>Imaginary numbers are simplified to perform addition, subtraction, multiplication and division on the numbers.</td>
<td>Simplify and perform addition, subtraction, multiplication and division on imaginary numbers.</td>
</tr>
<tr>
<td>Argand diagrams are constructed to represent the modulus and argument.</td>
<td>Construct Argand diagrams to find and represent the modulus and positive argument.</td>
</tr>
<tr>
<td>Complex numbers are represented in polar form with positive argument</td>
<td>Represent complex numbers in polar form with positive argument.</td>
</tr>
</tbody>
</table>

**ASSESSMENT TASKS OR ACTIVITIES**

- Assignments
- Test
- Examination

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 Perform operations on complex numbers.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operations are performed on complex numbers both in standard and polar form where applicable. <strong>RANGE:</strong> addition, subtraction, multiplication and division. <strong>Note:</strong> Combinations of polar and rectangular form will not be assessed in the same problem.</td>
<td>Perform addition, subtraction and multiplication on complex numbers in standard/rectangular form. Perform division on complex numbers in standard form introducing the concept of conjugate. Perform multiplication and division on complex numbers in polar form.</td>
</tr>
</tbody>
</table>

**ASSESSMENT TASKS OR ACTIVITIES**
### Topic 2: Functions and Algebra

(Minimum of 42 hours face to face teaching which excludes time for revision, test series and internal and external examination)

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.1 Use a variety of techniques to sketch and interpret information from graphs of functions.</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>ASSESSMENT STANDARD</strong></th>
<th><strong>LEARNING OUTCOME</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A variety of techniques are used to sketch and interpret information from graphs of functions. (Sketching of graphs using point by point plotting is an option)</td>
<td>Use a variety of techniques to sketch and interpret information from graphs of functions. (Sketching of graphs using point by point plotting is an option)</td>
</tr>
<tr>
<td>$y = a(x + p)^2 + q$ as well as $y = ax^2 + bx + c$</td>
<td>$y = a(x + p)^2 + q$ as well as $y = ax^2 + bx + c$</td>
</tr>
<tr>
<td>$y = \frac{a}{x + p} + q$</td>
<td>$y = \frac{a}{x + p} + q$</td>
</tr>
<tr>
<td>$y = ab^{x+p} + q ; b &gt; 0$</td>
<td>$y = ab^{x+p} + q ; b &gt; 0$</td>
</tr>
<tr>
<td>$y = a \sin(kx)$</td>
<td>$y = a \sin(kx)$</td>
</tr>
<tr>
<td>$y = a \cos(kx)$</td>
<td>$y = a \cos(kx)$</td>
</tr>
<tr>
<td>$y = a \tan(kx)$</td>
<td>$y = a \tan(kx)$</td>
</tr>
<tr>
<td>$y = a \sin(x + p)$</td>
<td>$y = a \sin(x + p)$</td>
</tr>
<tr>
<td>$y = a \cos(x + p)$</td>
<td>$y = a \cos(x + p)$</td>
</tr>
<tr>
<td>$y = a \tan(x + p)$</td>
<td>$y = a \tan(x + p)$</td>
</tr>
</tbody>
</table>

*Note: Cubic functions will only be done in differential calculus in level 4.*

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| The effects of $k$, $p$, $a$, $b$, $c$ and $q$ in the functions below are generalised: | Investigate and generalize the impact of $k$, $p$, $a$, $b$, $c$ and $q$ in the following functions.

*Note: Cubic functions will only be done in differential calculus in level 4.*
\( y = a(x + p)^2 + q \) as well as \( y = ax^2 + bx + c \)

\[
y = \frac{a}{(x + p)} + q
\]

\( y = ab^{x+p} + q \); \( b > 0 \)

\( y = a \sin(kx) \)

\( y = a \cos(kx) \)

\( y = a \tan(kx) \)

\( y = a \sin(x + p) \)

\( y = a \cos(x + p) \)

\( y = a \tan(x + p) \)

Range: A maximum of two variations per function only.

- The following characteristics of functions are identified:
  - Domain and range.
  - Intercepts with axes.
  - Turning points, minima and maxima.
  - Asymptotes
  - Shape and symmetry.
  - Periodicity and amplitude
  - Functions or non functions.
  - Continuous or discontinuous.
  - Intervals in which a function increases/decreases.

- The equation of the following graphs is found.

\( y = a(x + p)^2 + q \) as well as \( y = ax^2 + bx + c \)

\[
y = \frac{a}{(x + p)} + q
\]

\( y = ab^{x+p} + q \); \( b > 0 \)

\( y = a \sin(kx) \)

\( y = a \cos(kx) \)

\( y = a \tan(kx) \)

\( y = a \sin(x + p) \)

\( y = a \cos(x + p) \)

\( y = a \tan(x + p) \)

- Identify the following characteristics of functions.
  - Domain and range.
  - Intercepts with axes.
  - Turning points, minima and maxima.
  - Asymptotes
  - Shape and symmetry.
  - Periodicity and amplitude
  - Functions or non functions.
  - Continuous or discontinuous.
  - Intervals in which a function increases/decreases.

- Find the equation of the following graphs by calculations or using the method of inspection (investigating the transformation of the graph).

\( y = a(x + p)^2 + q \) as well as \( y = ax^2 + bx + c \)

\[
y = \frac{a}{(x + p)} + q
\]

\( y = ab^{x+p} + q \); \( b > 0 \)

\( y = a \sin(kx) \)

\( y = a \cos(kx) \)

\( y = a \tan(kx) \)

\( y = a \sin(x + p) \)

\( y = a \cos(x + p) \)

\( y = a \tan(x + p) \)
SUBJECT OUTCOMES

2.2 Manipulate and simplify algebraic expressions.

ASSESSMENT STANDARD

• Algebraic fractions with binomial denominators are simplified.
  
  \[ \text{Range: Monomial, binomial and trinomial denominators include linear expressions, difference of squares, quadratic expressions.} \]
  
  \[ \text{Note: Fractions where grouping is required in the numerator/denominator are excluded.} \]

• Algebraic expressions are manipulated and simplified by completing the square.

LEARNING OUTCOME

• Simplify algebraic fractions with monomial, binomial and trinomial denominators where two or more fractions are added, subtracted, divided or multiplied to each other.

• Manipulate and simplify algebraic expressions by completing the square.

ASSESSMENT TASKS OR ACTIVITIES

• Assignments
• Test
• Examination

SUBJECT OUTCOMES

2.3 Solve algebraic equations and inequalities

ASSESSMENT STANDARD

• Quadratic equations are solved
  
  \[ \text{RANGE: Factorizing, completing the square, quadratic formula.} \]
  
  \[ \text{Note: Solutions of complicated examples for completing the square e.g.} \ 6x^2 - 2px - 3p^2 = 0 \ \text{are excluded.} \]

• Simultaneous equations with two unknowns are solved algebraically and graphically; where the one equation is linear and the other equation is quadratic.

• Quadratic inequalities in one variable are solved and the solution is represented in the following ways:
  
  \[ \text{o In set builder notation} \]
  
  \[ \text{o Interval notation} \]
  
  \[ \text{o On the number line.} \]
  
  \[ \text{Note: Fractions with the variable in the denominator are} \]

LEARNING OUTCOME

• Solve quadratic equations by means of:
  
  \[ \text{o Factorization} \]
  
  \[ \text{o Completing the square} \]
  
  \[ \text{o Using the quadratic formula.} \]

• Solve simultaneous equations with two unknowns algebraically and graphically, where the one equation is linear and the other equation is quadratic.

• Solve quadratic inequalities in one variable and represent the solution in the following ways:
  
  \[ \text{o In set builder notation} \]
  
  \[ \text{o Interval notation} \]
  
  \[ \text{o On the number line.} \]
excluded, e.g. \( \frac{x}{2} \leq \frac{2}{x} \)

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<thead>
<tr>
<th>ASSESSMENT TASKS OR ACTIVITIES</th>
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<tbody>
<tr>
<td>Practical Exercises (solve simultaneous equations algebraically and then graphically)</td>
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<td>Assignments</td>
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<tr>
<th>SUBJECT OUTCOMES</th>
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<tr>
<td><strong>2.4 Use Mathematical models to investigate linear programming problems.</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
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</thead>
</table>
| Linear programming problems are solved by optimizing a function in two variables, subject to one or more given linear constraints, by numerical search along the boundary of the feasible region. Method:  
  - Sketch the given functions/constraints.  
  - Determine and shade the feasible region.  
  - Complete a boundary search to find the vertices of the feasible region.  
  - Optimise the maximum or minimum from the given objective function. | Solve linear programming problems by optimizing a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region. Note: Explicit constraints will be given in all examples Method:  
  - Sketch the given functions/constraints.  
  - Determine and shade the feasible region.  
  - Complete a boundary search to find the vertices of the feasible region.  
  - Optimise the maximum or minimum from the given objective function. |

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<tr>
<th>ASSESSMENT TASKS OR ACTIVITIES</th>
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<tr>
<td>Practical Exercises</td>
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<td>Assignments</td>
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<tr>
<td>Test</td>
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<td>Examination</td>
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<thead>
<tr>
<th>SUBJECT OUTCOME</th>
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<tbody>
<tr>
<td><strong>2.5 Investigate and use instantaneous rate of change</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
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</thead>
</table>
| Limits of functions are determined intuitively.  
A distinction is made between the value of a function at a particular point and the limit of that function at that point.  
Derivatives are established by using first principles. | Determine limits of functions intuitively.  
Distinguish between the value of a function at a particular point and the limit of that function at that point.  
Establish the derivatives of the following functions from first principles: |
The derivative of functions in the form:

\[ f(x) = a x^n \quad \text{or} \quad y = ax^n \]

where \( f'(x) = n a x^{n-1} \) or \( \frac{dy}{dx} = n a x^{n-1} \)

are found.

Derivatives are found using the constant, sum/difference rule after simplifying the expression.

Range: The following rules of differentiation are used:

- If \( y = f(x) = a \) and \( a \) is a constant function, then
  \[ \frac{dy}{dx} = f'(x) = 0 \]
- If \( y = k f(x) \) then
  \[ \frac{d}{dx} [k f(x)] = k \frac{d}{dx} [f(x)] \]
- If \( y = f(x) \pm g(x) \) then
  \[ \frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} [f(x)] \pm \frac{d}{dx} [g(x)] \]

Maxima and minima problems about real life situations are solved from given equations excluding calculations using second order derivatives.

Find the derivative of functions in the form:

\[ f(x) = ax^n \quad \text{or} \quad y = ax^n \]

where \( f'(x) = n ax^{n-1} \) or \( \frac{dy}{dx} = n ax^{n-1} \)

Examples to include are

\[ 4x^2 ; \frac{3}{x^3} ; \frac{2}{\sqrt{x^2}} ; \frac{5}{3x^2} \]

(All examples within this range)

Use the constant, sum and/or difference rule by first simplifying the expression.

- If \( y = f(x) = a \) and \( a \) is a constant function, then
  \[ \frac{dy}{dx} = f'(x) = 0 \]
- If \( y = k f(x) \) then
  \[ \frac{d}{dx} [k f(x)] = k \frac{d}{dx} [f(x)] \]
- If \( y = f(x) \pm g(x) \) then
  \[ \frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} [f(x)] \pm \frac{d}{dx} [g(x)] \]

Solve maxima and minima problems about real life situations from given equations.

**ASSESSMENT TASKS OR ACTIVITIES**

- Practical Exercises (work with minima and maxima problems)
- Assignments
- Test
- Examinations
### Topic 3: Space, Shape and Measurement.
(Minimum of 26 hours face to face teaching which excludes time for revision, test series and internal and external examination)

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
</tr>
</thead>
</table>
| 3.1 Calculate the surface area and volume of two and three dimensional shapes | - Formulae are used to calculate surface area and volume of geometrical objects
  - *Range: right pyramids (with square, equilateral triangle or regular hexagonal bases), right cones, spheres*
  - The surface area and volume of a combination of the above mentioned geometrical objects are calculated. | - Calculate the surface area and volume of the following geometrical objects:
  - right pyramids (with square, equilateral triangle or regular hexagonal bases)
  - right cones
  - spheres
  - Calculate the surface area and volume of a combination of the above mentioned geometrical objects. |

#### ASSESSMENT TASKS OR ACTIVITIES
- Practical tasks in groups with actual objects
- Assignments
- Test
- Examination

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
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</tr>
</thead>
</table>
| 3.2 Use the Cartesian co-ordinate system to derive and apply equations. | - The Cartesian co-ordinate system is used to derive the equation of a line through two given points.
  - The Cartesian co-ordinate system is used to derive the equation of a line parallel or perpendicular to another line.
  - The Cartesian co-ordinate system is used to find the angle of inclination and apply it to find the equation of a line. | - Use the Cartesian co-ordinate system to derive the equation of a line through two given points.
  - Use the Cartesian co-ordinate system to derive the equation of a line parallel or perpendicular to another line.
  - Use the Cartesian co-ordinate system to derive and use the angle of inclination of a line. |

#### ASSESSMENT TASKS OR ACTIVITIES
- Assignments
- Test
- Examination

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3 Solve problems by constructing and interpreting trigonometric models.</td>
<td></td>
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</tr>
</tbody>
</table>

#### ASSESSMENT TASKS OR ACTIVITIES
- |
• The values of the trigonometric functions for special angles are derived

Range: 30°, 45° and 60° in all four quadrants

• The reduction formulae and special angles are used to solve trigonometric expressions and prove equations in all four quadrants (without the use of a calculator) for the following functions.

\[
\sin (90^\circ \pm \theta); \cos (90^\circ \pm \theta)
\]

\[
\sin (180^\circ \pm \theta); \cos (180^\circ \pm \theta); \tan (180^\circ \pm \theta)
\]

\[
\sin (360^\circ - \theta); \cos (360^\circ - \theta); \tan (360^\circ - \theta)
\]

• Trigonometric identities are used to simplify expressions and prove equations.

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

• Trigonometric equations are solved for the three trigonometric functions in all four quadrants with calculator.

Range: [0°; 360°]

Note - positive angles only.

\[
\sin (90^\circ \pm \theta); \cos (90^\circ \pm \theta)
\]

\[
\sin (180^\circ \pm \theta); \cos (180^\circ \pm \theta); \tan (180^\circ \pm \theta)
\]

\[
\sin (360^\circ - \theta); \cos (360^\circ - \theta); \tan (360^\circ - \theta)
\]

• The sine, cosine and area rules are applied.

• Two dimensional problems are solved using the sine, cosine and area rules by interpreting given geometric and trigonometric models.

• Derive and use the values of the trigonometric functions (in surd form where applicable) of 30°, 45° and 60°.

• Use the reduction formulae and special angles to solve trigonometric expressions and prove equations in all four quadrants (without the use of a calculator) for the following functions.

\[
\sin (90^\circ \pm \theta); \cos (90^\circ \pm \theta)
\]

\[
\sin (180^\circ \pm \theta); \cos (180^\circ \pm \theta); \tan (180^\circ \pm \theta)
\]

\[
\sin (360^\circ - \theta); \cos (360^\circ - \theta); \tan (360^\circ - \theta)
\]

• Use the following trigonometric identities to simplify expressions and prove equations.

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

• Solve trigonometric equations (with the use of a calculator) involving reduction formulae using special triangles for the three trigonometric functions in all four quadrants.

\[
\sin (90^\circ \pm \theta); \cos (90^\circ \pm \theta)
\]

\[
\sin (180^\circ \pm \theta); \cos (180^\circ \pm \theta); \tan (180^\circ \pm \theta)
\]

\[
\sin (360^\circ - \theta); \cos (360^\circ - \theta); \tan (360^\circ - \theta)
\]

• Apply the sine, cosine and area rules.

• Solve problems in two dimensions by using the sine, cosine and area rules by interpreting given geometric and trigonometric models.

### ASSESSMENT TASKS OR ACTIVITIES

- Practical Exercises
- Assignments
- Test
- Examination
### Topic 4: Data Handling

(Minimum of 20 hours face to face teaching which excludes time for revision, test series and internal and external examination)

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
</tr>
</thead>
</table>
| 4.1 Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical ungrouped data | • The five number summary is calculated to construct the box and whisker diagram.  
  o Calculating the maximum, minimum and quartiles  
  o Determining the fences  
  o Constructing the box and whisker diagram  
  o Indicating any outliers | • Work out the five number summary by:  
  o Calculating the maximum, minimum and quartiles  
  o Determining the fences  
  o Constructing the box and whisker diagram  
  o Indicating any outliers  
  • The meaning of the representation of the box and whisker diagram with its outliers is interpreted.  
  • Interpret the meaning of the representation of the box and whisker diagram with its outliers. |

#### ASSESSMENT TASKS OR ACTIVITIES

- Research Project
- Practical Exercises
- Assignments
- Test
- Examination

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<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
<th>ASSESSMENT STANDARD</th>
<th>LEARNING OUTCOME</th>
</tr>
</thead>
</table>
| 4.2 Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical grouped data | • A frequency distribution table is constructed by grouping data into classes.  
  *Note: The class width for each class will be given.*  
  • The Ogive curve is plotted by using the cumulative frequency.  
  • Quartile values are estimated from the Ogive curve.  
  • Histograms are constructed from grouped data.  
  • The mean, median and modal values of grouped data are calculated using the following formulae: | • Construct a frequency distribution table by grouping data into classes.  
  • Calculate the Cumulative frequency and plot the Ogive curve.  
  • Use the Ogive curve to estimate quartile values.  
  • Construct histograms using tabulated grouped data.  
  • Calculate the mean \( \bar{x} \), median \( \text{Me} \) and modal \( \text{Mo} \) values of grouped data using the formulae: |
\[ \bar{x} = \frac{\sum f_i x_i}{n} \]

\[ Me = l + \left( \frac{n^2 F}{f} \right) \times c \]

\[ Mo = l + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times c \]

**ASSESSMENT TASKS OR ACTIVITIES**

- Research Project
- Practical Exercises
- Assignments
- Test
- Examination

### Topic 5: Financial Mathematics

(Minimum of 10 hours face to face teaching which excludes time for revision, test series and internal and external examination)

<table>
<thead>
<tr>
<th>SUBJECT OUTCOME</th>
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</thead>
<tbody>
<tr>
<td><strong>5.1 Plan and describe how to manage finances of social clubs.</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>ASSESSMENT STANDARD</strong></th>
<th><strong>LEARNING OUTCOME</strong></th>
</tr>
</thead>
</table>
| • Describe financial concepts related to social clubs, methods of financing and financial control.  
  *Range: normal operating expenses telephone, stationery, water & electricity, consumables plus club specific income and expenses: membership fees, donations, sponsorships, affiliation fees, license fees, honorarium, function fees, cutlery and crockery, catering and other expenses to host specific events e.g. prize giving functions, year end functions, any other club events* | • Describe financial concepts related to social clubs, methods of financing and financial control.  
  *Range: normal operating expenses telephone, stationery, water & electricity, consumables plus club specific income and expenses: membership fees, donations, sponsorships, affiliation fees, license fees, honorarium, function fees, cutlery and crockery, catering and other expenses to host specific events e.g. prize giving functions, year end functions, any other club events* |
| • A yearly budget for a social club is drawn up from given information.  
  *Note: Detailed examples and templates are available on the internet.* | • Use given information to draw up a yearly budget for a social club.  
  *Note: Detailed examples and templates are available on the internet.* |
| • Actual income and expenditure is recorded for a year from given information. | • Use given information to record actual income and expenditure for a year. |
| • The actual income and expenditure recorded is | • Compare actual income and expenses to the projected budget figures. |
• Variances are identified and possible explanations are provided.
• Possible corrective actions to control future finances of the club are provided.

• Identify variances and provide possible corrective actions to be taken to control future finances of the club.

### ASSESSMENT TASKS OR ACTIVITIES
- Practical Exercises
- Assignments
- Test
- Examination

### SUBJECT OUTCOME

#### 5.2 Use simple and compound interest to explain and define a variety of situations.

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<thead>
<tr>
<th>ASSESSMENT STANDARD</th>
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<tbody>
<tr>
<td>Time lines are constructed and used to solve problems relating to finance.</td>
<td>Construct and make use of time lines to solve problems relating to finance.</td>
</tr>
<tr>
<td>The simple growth formula ( A = P(1 + i n) ) is used and manipulated to solve real life problems.</td>
<td>Use the simple growth formula ( A = P(1 + i n) ) to solve real life problems.</td>
</tr>
<tr>
<td>The compound growth formulae ( A = P(1 + i)^n ) or ( A_t = A_0 \left(1 + \frac{r}{100 \times m}\right)^{t \times m} ) is used and manipulated to solve problems subject to compounding being done:</td>
<td>Use the compound growth formulae ( A = P(1 + i)^n ) or ( A_t = A_0 \left(1 + \frac{r}{100 \times m}\right)^{t \times m} ) to solve problems subject to the following compounding:</td>
</tr>
<tr>
<td>o annually,</td>
<td>o annually,</td>
</tr>
<tr>
<td>o semi – annually</td>
<td>o semi – annually</td>
</tr>
<tr>
<td>o quarterly</td>
<td>o quarterly</td>
</tr>
<tr>
<td>o monthly</td>
<td>o monthly</td>
</tr>
<tr>
<td>o daily</td>
<td>o daily</td>
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</tbody>
</table>

**Range:** unknown values to calculate will only include \( A; P; i \)

### ASSESSMENT TASKS OR ACTIVITIES
- Projects
- Practical Exercises
- Assignments
- Test
- Examination
4 EXTERNAL ASSESSMENT IN MATHEMATICS - LEVEL 3

Apart from the internal assessment a national examination is also conducted annually in October or November each year by means of two three hour examination papers set externally. The national examination is subjected to external moderation by Umalusi or an appropriate Education and Training Quality Assurance (ETQA) body, appointed by the Umalusi Council in terms of Section 28(2) of the General and Further Education and Training Quality Assurance Act, 2001 (Act No. 58 of 2001).

The level 3 papers will be structured as follows:

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<th>LEVEL 3</th>
<th>KNOWLEDGE</th>
<th>COMPREHENSION &amp; APPLICATION</th>
<th>ANALYSIS</th>
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<tr>
<td></td>
<td>20%</td>
<td>60%</td>
<td>20%</td>
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</table>

The following mark distribution between paper 1 and paper 2 is proposed for setting national examination papers

**Paper 1 (3 hours)**

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<tr>
<th>TOPICS</th>
<th>MARKS</th>
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<tr>
<td>1. Complex Numbers</td>
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<tr>
<td>2. Functions and Algebra</td>
<td></td>
</tr>
<tr>
<td>2.1 Functions</td>
<td>20</td>
</tr>
<tr>
<td>2.2 Algebra</td>
<td></td>
</tr>
<tr>
<td>2.2.1 Manipulate and simplify algebraic expressions, solve algebraic equations and inequalities</td>
<td>20</td>
</tr>
<tr>
<td>2.2.2 Linear Programming</td>
<td>10</td>
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<td>2.2.3 Differential Calculus</td>
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<td><strong>TOTAL</strong></td>
<td><strong>100</strong></td>
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</table>

**Paper 2 (3 hours)**

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<tbody>
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<td>3. Space, Shape and Measurement</td>
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<tr>
<td>3.1 Geometry</td>
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</tr>
<tr>
<td>3.2 Trigonometry</td>
<td>25</td>
</tr>
<tr>
<td>4. Data Handling</td>
<td>30</td>
</tr>
<tr>
<td>5. Financial Mathematics</td>
<td>20</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>100</strong></td>
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</tbody>
</table>