# Contents

**Introduction** ................................................................. 3  
  Purpose of This Document ............................................... 4  
  Features of This Document ................................................ 4  
  The Tasks ............................................................................. 5  
  The Rubrics ........................................................................... 5  
  Development of the Tasks ..................................................... 7  
  Assessment and Selection of the Samples ............................... 8  
  Use of the Student Samples .................................................. 8  
    Teachers and Administrators .............................................. 8  
    Parents ............................................................................... 9  
    Students ............................................................................ 10  

**Number Sense and Numeration** ........................................... 11  
  What's the Question? ............................................................ 12  
    The Task ........................................................................... 12  
    Expectations ....................................................................... 12  
    Prior Knowledge and Skills ................................................ 13  
    Task Rubric ......................................................................... 14  
    Student Samples ................................................................... 15  
    Teacher Package ................................................................... 49  

**Measurement / Geometry and Spatial Sense** .......................... 55  
  Polygons on the Geoboard ...................................................... 56  
    The Task ........................................................................... 56  
    Expectations ....................................................................... 56  
    Prior Knowledge and Skills ................................................ 57  
    Task Rubric ......................................................................... 58  
    Student Samples ................................................................... 59  
    Teacher Package ................................................................... 99  

**Patterning and Algebra** ...................................................... 105  
  Investigating Patterns With Tiles .......................................... 106  
    The Task ........................................................................... 106  
    Expectations ....................................................................... 106  
    Prior Knowledge and Skills ................................................ 107  
    Task Rubric ......................................................................... 108  
    Student Samples ................................................................... 110  
    Teacher Package ................................................................... 143  

This publication is available on the Ministry of Education’s website at  
http://www.edu.gov.on.ca.
In 1997, the Ministry of Education and Training published a new mathematics curriculum policy document for Ontario elementary students entitled *The Ontario Curriculum, Grades 1–8: Mathematics, 1997*. The new curriculum is more specific than previous curricula with respect to both the knowledge and the skills that students are expected to develop and demonstrate in each grade. The document contains the curriculum expectations for each grade and an achievement chart that describes four levels of student achievement to be used in assessing and evaluating student work.

The present document is part of a set of eight documents – one for each grade – that contain samples (“exemplars”) of student work in mathematics at each of the four levels of achievement described in the achievement chart. The exemplar documents are intended to provide assistance to teachers in their assessment of student achievement of the curriculum expectations. The samples represent work produced at the end of the school year in each grade.

Ontario school boards were invited by the Ministry of Education to participate in the development of the exemplars. Teams of teachers and administrators from across the province were involved in developing the assessment materials. They designed the performance tasks and scoring scales (“rubrics”) on the basis of selected Ontario curriculum expectations, field-tested them in classrooms, suggested changes, administered the final tasks, marked the student work, and selected the exemplars used in this document. During each stage of the process, external validation teams and Ministry of Education staff reviewed the tasks and rubrics to ensure that they reflected the expectations in the curriculum policy documents and that they were appropriate for all students. External validation teams and ministry staff also reviewed the samples of student work.

The selection of student samples that appears in this document reflects the professional judgement of teachers who participated in the project. No students, teachers, or schools have been identified.

The procedures followed during the development and implementation of this project will serve as a model for boards, schools, and teachers in designing assessment tasks within the context of regular classroom work, developing rubrics, assessing the achievement of their own students, and planning for the improvement of students’ learning.
The samples in this document will provide parents\(^1\) with examples of student work to help them monitor their children's progress. They also provide a basis for communication with teachers.

Use of the exemplar materials will be supported initially through provincial in-service training.

**Purpose of This Document**

This document was developed to:

- show the characteristics of student work at each of the four levels of achievement for Grade 6;
- promote greater consistency in the assessment of student work across the province;
- provide an approach to improving student learning by demonstrating the use of clear criteria applied to student work in response to clearly defined assessment tasks;
- show the connections between what students are expected to learn (the curriculum expectations) and how their work can be assessed using the levels of achievement described in the curriculum policy document for the subject.

Teachers, parents, and students should examine the student samples in this document and consider them along with the information in the Teacher's Notes and Comments/Next Steps sections. They are encouraged to examine the samples in order to develop an understanding of the characteristics of work at each level of achievement and the ways in which the levels of achievement reflect progression in the quality of knowledge and skills demonstrated by the student.

The samples in this document represent examples of student achievement obtained using only one method of assessment, called performance assessment. Teachers will also make use of a variety of other assessment methods and strategies in evaluating student achievement over a school year.

**Features of This Document**

This document contains the following:

- a description of each of three performance tasks (each task focuses on a particular strand or combination of strands), as well as a listing of the curriculum expectations related to the task
- a task-specific assessment chart (“rubric”) for each task
- two samples of student work for each of the four levels of achievement for each task
- Teacher's Notes, which provide some details on the level of achievement for each sample

---

\(^1\) In this document, *parent(s)* refers to parent(s) and guardian(s).
• Comments/Next Steps, which offer suggestions for improving achievement
• the Teacher Package that was used by teachers in administering each task

It should be noted that each sample for a specific level of achievement represents the characteristics of work at that level of achievement.

The Tasks
The performance tasks were based directly on curriculum expectations selected from The Ontario Curriculum, Grades 1–8: Mathematics, 1997. The tasks encompassed the four categories of knowledge and skills (i.e., problem solving; understanding of concepts; application of mathematical procedures; communication of required knowledge related to concepts, procedures, and problem solving), requiring students to integrate their knowledge and skills in meaningful learning experiences. The tasks gave students an opportunity to demonstrate how well they could use their knowledge and skills in a specific context.

Teachers were required to explain the scoring criteria and descriptions of the levels of achievement (i.e., the information in the task rubric) to the students before they began the assignment.

The Rubrics
In this document, the term rubric refers to a scoring scale that consists of a set of achievement criteria and descriptions of the levels of achievement for a particular task. The scale is used to assess students’ work; this assessment is intended to help students improve their performance level. The rubric identifies key criteria by which students’ work is to be assessed, and it provides descriptions that indicate the degree to which the key criteria have been met. The teacher uses the descriptions of the different levels of achievement given in the rubric to assess student achievement on a particular task.

The rubric for a specific performance task is intended to provide teachers and students with an overview of the expected product with regard to the knowledge and skills being assessed as a whole.

The achievement chart in the curriculum policy document for mathematics provides a standard province-wide tool for teachers to use in assessing and evaluating their students’ achievement over a period of time. While the chart is broad in scope and general in nature, it provides a reference point for all assessment practice and a framework within which to assess and evaluate student achievement. The descriptions associated with each level of achievement serve as a guide for gathering and tracking assessment information, enabling teachers to make consistent judgements about the quality of student work while providing clear and specific feedback to students and parents.

For the purposes of the exemplar project, a single rubric was developed for each performance task. This task-specific rubric was developed in relation to the achievement chart in the curriculum policy document.
The differences between the achievement chart and the task-specific rubric may be summarized as follows:

- The achievement chart contains broad descriptions of achievement. Teachers use it to assess student achievement over time, making a summative evaluation that is based on the total body of evidence gathered through using a variety of assessment methods and strategies.
- The rubric contains criteria and descriptions of achievement that relate to a specific task. The rubric uses some terms that are similar to those in the achievement chart but focuses on aspects of the specific task. Teachers use the rubric to assess student achievement on a single task.

The rubric contains the following components:

- an identification (by number) of the expectations on which student achievement in the task was assessed
- the four categories of knowledge and skills
- the relevant criteria for evaluating performance of the task
- descriptions of student performance at the four levels of achievement (level 3 on the achievement chart is considered to be the provincial standard)

As stated earlier, the focus of performance assessment using a rubric is to improve students' learning. In order to improve their work, students need to be provided with useful feedback. Students find that feedback on the strengths of their achievement and on areas in need of improvement is more helpful when the specific category of knowledge or skills is identified and specific suggestions are provided than when they receive only an overall mark or general comments. Student achievement should be considered in relation to the criteria for assessment stated in the rubric for each category, and feedback should be provided for each category. Through the use of a rubric, students' strengths and weaknesses are identified and this information can then be used as a basis for planning the next steps for learning. In this document, the Teacher's Notes indicate the reasons for assessing a student's performance at a specific level of achievement, and the Comments/Next Steps give suggestions for improvement.

In the exemplar project, a single rubric encompassing the four categories of knowledge and skills was used to provide an effective means of assessing the particular level of student performance in each performance task, to allow for consistent scoring of student performance, and to provide information to students on how to improve their work. However, in the classroom, teachers may find it helpful to make use of additional rubrics if they need to assess student achievement on a specific task in greater detail for one or more of the four categories. For example, it may be desirable in evaluating a written report on an investigation to use separate rubrics for assessing understanding of concepts, problem-solving skills, ability to apply mathematical procedures, and communication skills.
The rubrics for the tasks in the exemplar project are similar to the scales used by the Education Quality and Accountability Office (EQAO) for the Grade 3, Grade 6, and Grade 9 provincial assessments in that both the rubrics and the EQAO scales are based on the Ontario curriculum expectations and the achievement charts. The rubrics differ from the EQAO scales in that they were developed to be used only in the context of classroom instruction to assess achievement in a particular assignment.

Although rubrics were used effectively in this exemplar project to assess responses related to the performance tasks, they are only one way of assessing student achievement. Other means of assessing achievement include observational checklists, tests, marking schemes, or portfolios. Teachers may make use of rubrics to assess students’ achievement on, for example, essays, reports, exhibitions, debates, conferences, interviews, oral presentations, recitals, two- and three-dimensional representations, journals or logs, and research projects.

**Development of the Tasks**

The performance tasks for the exemplar project were developed by teams of educators in the following way:

- The teams selected a cluster of curriculum expectations that focused on the knowledge and skills that are considered to be of central importance in the subject area. Teams were encouraged to select a manageable number of expectations. The particular selection of expectations ensured that all students would have the opportunity to demonstrate their knowledge and skills in each category of the achievement chart in the curriculum policy document for the subject.

- The teams drafted three tasks for each grade that would encompass all of the selected expectations and that could be used to assess the work of all students.

- The teams established clear, appropriate, and concrete criteria for assessment, and wrote the descriptions for each level of achievement in the task-specific rubric, using the achievement chart for the subject as a guide.

- The teams prepared detailed instructions for both teachers and students participating in the assessment project.

- The tasks were field-tested in classrooms across the province by teachers who had volunteered to participate in the field test. Student work was scored by teams of educators. In addition, classroom teachers, students, and board contacts provided feedback on the task itself and on the instructions that accompanied the task. Suggestions for improvement were taken into consideration in the revision of the tasks, and the feedback helped to finalize the tasks, which were then administered in the spring of 2001.

In developing the tasks, the teams ensured that the resources needed for completing the tasks – that is, all the worksheets and support materials – were available.

Prior to both the field tests and the final administration of the tasks, a team of validators – including research specialists, gender and equity specialists, and subject experts – reviewed the instructions in the teacher and student packages, making further suggestions for improvement.
Assessment and Selection of the Samples

After the final administration of the tasks, student work was scored at the district school board level by teachers of the subject who had been provided with training in the scoring. These teachers evaluated and discussed the student work until they were able to reach a consensus regarding the level to be assigned for achievement in each category. This evaluation was done to ensure that the student work being selected clearly illustrated that level of performance. All of the student samples were then forwarded to the ministry. A team of teachers from across the province, who had been trained by the ministry to assess achievement on the tasks, rescored the student samples. They chose samples of work that demonstrated the same level of achievement in all four categories and then, through consensus, selected the samples that best represented the characteristics of work at each level of achievement. The rubrics were the primary tools used to evaluate student work at both the school board level and the provincial level.

The following points should be noted:

• Two samples of student work are included for each of the four achievement levels. The use of two samples is intended to show that the characteristics of an achievement level can be exemplified in different ways.

• Although the samples of student work in this document were selected to show a level of achievement that was largely consistent in the four categories (i.e., problem solving; understanding of concepts; application of mathematical procedures; communication of required knowledge), teachers using rubrics to assess student work will notice that students’ achievement frequently varies across the categories (e.g., a student may be achieving at level 3 in understanding of concepts but at level 4 in communication of required knowledge).

• Although the student samples show responses to most questions, students achieving at level 1 and level 2 will often omit answers or will provide incomplete responses or incomplete demonstrations.

• Students’ effort was not evaluated. Effort is evaluated separately by teachers as part of the “learning skills” component of the Provincial Report Card.

• The document does not provide any student samples that were assessed using the rubrics and judged to be below level 1. Teachers are expected to work with students whose achievement is below level 1, as well as with their parents, to help the students improve their performance.

Use of the Student Samples

Teachers and Administrators

The samples of student work included in the exemplar documents will help teachers and administrators by:

• providing student samples and criteria for assessment that will enable them to help students improve their achievement;

• providing a basis for conversations among teachers, parents, and students about the criteria used for assessment and evaluation of student achievement;
• facilitating communication with parents regarding the curriculum expectations and levels of achievement for each subject;
• promoting fair and consistent assessment within and across grade levels.

Teachers may choose to:
• use the teaching/learning activities outlined in the performance tasks;
• use the performance tasks and rubrics in the document in designing comparable performance tasks;
• use the samples of student work at each level as reference points when assessing student work;
• use the rubrics to clarify what is expected of the students and to discuss the criteria and standards for high-quality performance;
• review the samples of work with students and discuss how the performances reflect the levels of achievement;
• adapt the language of the rubrics to make it more “student friendly”;
• develop other assessment rubrics with colleagues and students;
• help students describe their own strengths and weaknesses and plan their next steps for learning;
• share student work with colleagues for consensus marking;
• partner with another school to design tasks and rubrics, and to select samples for other performance tasks.

Administrators may choose to:
• encourage and facilitate teacher collaboration regarding standards and assessment;
• provide training to ensure that teachers understand the role of the exemplars in assessment, evaluation, and reporting;
• establish an external reference point for schools in planning student programs and for school improvement;
• facilitate sessions for parents and school councils using this document as a basis for discussion of curriculum expectations, levels of achievement, and standards.

Parents
The performance tasks in this document exemplify a range of meaningful and relevant learning activities related to the curriculum expectations. In addition, this document invites the involvement and support of parents as they work with their children to improve their achievement. Parents may use the samples of student work and the rubrics as:
• resources to help them understand the levels of achievement;
• models to help monitor their children’s progress from level to level;
• a basis for communication with teachers about their children’s achievement;
• a source of information to help their children monitor achievement and improve their performance;
• models to illustrate the application of the levels of achievement.
**Students**

Students are asked to participate in performance assessments in all curriculum areas. When students are given clear expectations for learning, clear criteria for assessment, and immediate and helpful feedback, their performance improves. Students’ performance improves as they are encouraged to take responsibility for their own achievement and to reflect on their own progress and “next steps”.

It is anticipated that the contents of this document will help students in the following ways:

- Students will be introduced to a model of one type of task that will be used to assess their learning, and will discover how rubrics can be used to improve their product or performance on an assessment task.
- The performance tasks and the exemplars will help clarify the curriculum expectations for learning.
- The rubrics and the information given in the Teacher’s Notes section will help clarify the assessment criteria.
- The information given under Comments/Next Steps will support the improvement of achievement by focusing attention on two or three suggestions for improvement.
- With an increased awareness of the performance tasks and rubrics, students will be more likely to communicate effectively about their achievement with their teachers and parents, and to ask relevant questions about their own progress.
- Students can use the criteria and the range of student samples to help them see the differences in the levels of achievement. By analysing and discussing these differences, students will gain an understanding of ways in which they can assess their own responses and performances in related assignments and identify the qualities needed to improve their achievement.
Number Sense and Numeration
What’s the Question?

The Task
This task required students to:
• use base-ten blocks to investigate many tasks involving the multiplication of decimals;
• add decimals in finding all the rectangles with a given perimeter, and multiply decimals in finding the area of the rectangles discovered.

Students worked with base-ten blocks to find as many instances as possible of a decimal number and a whole number that, when multiplied, give a specified product. They showed their work and explained how they found their solutions. Then students discovered whether any products could be found in only one way or in only two ways (and justified their thinking), and whether any product could be found in four ways (and explained the process they used). Finally, students drew as many rectangles as possible with a perimeter of 12.8 cm, one dimension being a whole number and the other dimension being no less than 1 cm, and found the areas of the rectangles drawn.

Expectations
This task gave students the opportunity to demonstrate their achievement of all or part of each of the following selected overall and specific expectations from the strand Number Sense and Numeration. Note that the codes that follow the expectations are from the Ministry of Education’s Curriculum Unit Planner (CD-ROM).

Students will:
1. select and perform computation techniques appropriate to specific problems involving unlike denominations in fractions and the multiplication and division of decimals, and determine whether the results are reasonable (6m7);
2. solve and explain multi-step problems using the multiplication and division of decimals and percents (6m8);
3. justify and verify the method chosen for calculations with whole numbers, fractions, decimals, and percents (6m9);
4. represent the place value of whole numbers and decimals from 0.001 to 1 000 000 using concrete materials, drawings, and symbols (6m14);
5. explain processes and solutions with fractions and decimals using mathematical language (6m19);
6. multiply and divide numbers using concrete materials, drawings, and symbols (6m31);
7. multiply and divide decimal numbers to thousandths by a one-digit whole number (6m37).
Prior Knowledge and Skills

To complete this task, students were expected to have some knowledge or skills relating to the following:

- using base-ten materials to add, subtract, multiply, and divide
- representing and exploring the relationships between fractions and decimals
- applying the formulas for perimeter and area

For information on the process used to prepare students for the task and on the materials, resources, and equipment required, see the Teacher Package reproduced on pages 49–54 of this document.
Task Rubric – What’s the Question?

<table>
<thead>
<tr>
<th>Expectations*</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem solving</strong></td>
<td><strong>The student:</strong></td>
<td><strong>The student:</strong></td>
<td><strong>The student:</strong></td>
<td><strong>The student:</strong></td>
</tr>
<tr>
<td>1, 2</td>
<td>– selects and applies a problem-solving strategy that leads to an incomplete or inaccurate solution</td>
<td>– selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution</td>
<td>– selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution</td>
<td>– selects and applies an appropriate problem-solving strategy that leads to a thorough and accurate solution</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Understanding of concepts</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 4</td>
<td>– demonstrates a limited understanding of processes and solutions involving place value and decimals by providing limited explanations and illustrations</td>
<td>– demonstrates some understanding of processes and solutions involving place value and decimals by providing partial explanations and illustrations</td>
<td>– demonstrates a general understanding of processes and solutions involving place value and decimals by providing appropriate and complete explanations and illustrations</td>
<td>– demonstrates a thorough understanding of processes and solutions involving place value and decimals by providing appropriate and detailed explanations and illustrations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Application of mathematical procedures</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 6, 7</td>
<td>– uses computations and mathematical procedures that include many errors and/or omissions</td>
<td>– uses computations and mathematical procedures that include some errors and/or omissions</td>
<td>– uses computations and mathematical procedures that include few errors and/or omissions</td>
<td>– uses computations and mathematical procedures that include few, if any, minor errors and/or omissions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Communication of required knowledge</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
<th><strong>The student:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 5</td>
<td>– uses mathematical language and notation with limited clarity to explain and justify responses to reverse-processing problems involving decimals</td>
<td>– uses mathematical language and notation with some clarity to explain and justify responses to reverse-processing problems involving decimals</td>
<td>– uses mathematical language and notation clearly to explain and justify responses to reverse-processing problems involving decimals</td>
<td>– uses mathematical language and notation clearly and precisely to explain and justify responses to reverse-processing problems involving decimals</td>
</tr>
</tbody>
</table>

*The expectations that correspond to the numbers given in this chart are listed on page 12.

**Note:** This rubric does not include criteria for assessing student performance that falls below level 1.
Exemplar Task

Jasmine and Laciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product. [Note: Do not put more than one digit inside a ‘box’]

- 16.4
- 16.4
- 16.4

2. Explain what you did to find your solutions.

I picked these solutions because first I added them in my head then I simply multiplied them:

- x 2
- x 4

3. Here is another problem Laciano and Jasmine found. Use this information to find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.

I picked these numbers because they are the only numbers I could think of that work for the product.
4. Do you think some products can only be found in one way? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
1.0 & 5.3 & 2.3 \\
x & 1 & x & 3 \\
\hline
1.0 & 5.3 & 16.9
\end{array}
\]

5. Is it possible to find a product that can only be found in two ways? Investigate showing all your work in the space below.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
8.0 & 4.0 \\
x & 3 & x & 6 \\
\hline
24.0 & 24.0
\end{array}
\]

I figured out that 24 would work for my understanding and then figured out what goes into 24.

\[
\begin{align*}
24 \div 3 &= 8 \\
24 \div 6 &= 4
\end{align*}
\]
6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in four different ways using the template shown below.

   a) Use the template below to investigate the question.
      [Note: Do not put more than one digit inside a ‘box’]

      \[
      \begin{array}{cccc}
        1 & 2 & 3 & 4 \\
        5 & 6 & 7 & 8 \\
      \end{array}
      \]

   b) Explain how you found the answer to this question.

   I got these answers by multiplying the bottom number to the top then all four by crossing out 2 of the decimals and put in the right.

7. a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
    Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

    \[
    \begin{array}{c}
      4 \text{ cm} \\
      2.4 \text{ cm} = 12.8 \text{ cm} \quad 2.4 \text{ cm} \\
      4 \text{ cm} \\
    \end{array}
    \]

    b) Find the areas of the rectangles you have drawn.
    What are some of the things you notice?

    \[
    \begin{array}{c}
      2.4 \div 4 = 0.6 \\
      1.2 \div 3 = 0.4 \\
      0.6 \div 2 = 0.3 \\
    \end{array}
    \]

    I noticed that the lower you go in the decimal of the perimeter the lower you
Teacher’s Notes

Problem Solving
- The student selects and applies a problem-solving strategy that leads to an incomplete or inaccurate solution (e.g., in questions 2, 3, and 4, attempts to use diagrams to aid in problem solving; in question 4, uses diagrams that do not support the solutions provided).

Understanding of Concepts
- The student demonstrates a limited understanding of processes and solutions involving place value and decimals by providing limited explanations and illustrations (e.g., in questions 2 and 3, provides a few appropriate explanations to support his or her solutions: “I picked these solutions because first I added them in my head then I simply multiplied them”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include many errors and/or omissions (e.g., in question 4, shows three ways a product can be generated, but does not provide justifications; in question 7a, draws two of the rectangles accurately, but omits several other possible ones).

Communication of Required Knowledge
- The student uses mathematical language and notation with limited clarity to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 3, “I picked these numbers because these were the only numbers I could think of and I think they work for this product”; in question 6b, focuses on the multiplication process, not the problem-solving process: “Now I got all four by crossing out 2 of the decimals and put in the right answer”).

Comments/Next Steps
- The student should use base-ten blocks for solving problems involving decimals.
- The student needs to develop strategies required for reverse processing in order to solve problems.
- The student should develop his or her use of mathematical language for explaining and justifying his or her responses.
What's the Question? Level 1, Sample 2

A

Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a 'box']

\[
\begin{array}{c}
\times 2 \\
\hline
16.4
\end{array}
\]

2. Explain what you did to find your solutions.

\[
\begin{array}{c}
\sqrt{8.2} \\
+8.2 \\
\hline
16.4
\end{array}
\]

B

3. Here is another problem Luciano and Jasmine found. Use this information to find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.

\[
\begin{array}{c}
7.5 \\
\times 1 \\
\hline
7.5
\end{array}
\]

\[
\begin{array}{c}
7.5 \\
\times 1 \\
\hline
7.5
\end{array}
\]

\[
\begin{array}{c}
7.5 \\
\times 3 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
7.5
\end{array}
\]
4. Do you think some products can only be found in **one way**? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
  1 & 1 & 2 & 3 & 5 & 5 \\
  \times 1 & \times 7 & \times 5 \\
\end{array}
\]

\[
\begin{array}{c}
  1 & 1 & 1 & 6 & 1 & 8 & 5 & 5 \\
\end{array}
\]

One reason why these products can only be made up in one way because the numbers the two digits are all the same uneven numbers and can only be divided by 1 or themselves and the numbers multiplied by are uneven.

5. Is it possible to find a product that can only be found in **two ways**?
Investigate showing all your work in the space below.
(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
  6 & 2 & 3 & 1 \\
  \times 3 & \times 6 \\
\end{array}
\]

\[
\begin{array}{c}
  1 & 8 & 6 & 1 & 8 & 6 \\
\end{array}
\]

\[
\begin{array}{c}
  6.2 \\
  \times 3 \\
\end{array}
\]

\[
\begin{array}{c}
  18.6 \\
\end{array}
\]

\[
\begin{array}{c}
  18.6 \div 3 = 6.2 \\
\end{array}
\]

\[
\begin{array}{c}
  3.1 \\
\end{array}
\]

\[
\begin{array}{c}
  3.1 \\
\end{array}
\]

\[
\begin{array}{c}
  3.1 \\
\end{array}
\]

\[
\begin{array}{c}
  3.1 \\
\end{array}
\]

\[
\begin{array}{c}
  3.1 \\
\end{array}
\]

\[
\begin{array}{c}
  3.1 \\
\end{array}
\]

\[
\begin{array}{c}
  18.6 \div 6 = 3.1 \\
\end{array}
\]
6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

   a) Use the template below to investigate the question.
   [Note: Do not put more than one digit inside a ‘box’]

   
   \[
   \begin{array}{cccc}
   7.0 & 2.0 & 3.5 & 6.5 \\
   \times & 2 & 7 & 4 & 2 \\
   \hline
   14.0 & 14.0 & 14.0 & 13.0 \\
   \end{array}
   \]

   b) Explain how you found the answer to this question.

   \[
   \begin{array}{c}
   7.0 \\
   +7.0 \\
   \hline
   14.0
   \end{array}
   \]

   I guessed and checked everything.

   \[
   \begin{array}{c}
   2.0 \\
   +2.0 \\
   \hline
   4.0
   \end{array}
   \]

   \[
   \begin{array}{c}
   3.5 \\
   +3.5 \\
   \hline
   7.0
   \end{array}
   \]

   \[
   \begin{array}{c}
   0.5 \\
   +0.5 \\
   \hline
   1.0
   \end{array}
   \]

7. a) Draw as many rectangles as you can with a perimeter of 12.8 cm. Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

   \[
   \begin{array}{c}
   2.0 \\
   10.4 \text{ cm} \\
   \hline
   \end{array}
   \]

   \[
   \begin{array}{c}
   12.8 \text{ cm} \\
   1.4 \text{ cm}
   \end{array}
   \]

   b) Find the areas of the rectangles you have drawn. What are some of the things you notice?

   \[
   \begin{array}{c}
   A = \text{L} \times \text{W} \quad A = \text{L} \times \text{W} \\
   A = 10 \text{ cm} \times 1.4 \text{ cm} \quad A = 10.4 \text{ cm} \times 1 \text{ cm} \\
   A = 19.00 \text{ cm}^2 \quad A = 10.40 \text{ cm}^2
   \end{array}
   \]
Teacher’s Notes

Problem Solving
- The student selects and applies a problem-solving strategy that leads to an incomplete or inaccurate solution (e.g., in question 6b, applies a portion of a guess-and-check strategy: “I guessed and checked everything”, and includes an incorrect solution).

Understanding of Concepts
- The student demonstrates a limited understanding of processes and solutions involving place value and decimals by providing limited explanations and illustrations (e.g., in question 3, attempts to use an illustration and notation, makes only limited use of base-ten diagrams).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include many errors and/or omissions (e.g., in question 7, attempts to draw a few of the required rectangles, but the measurements do not result in the required perimeter).

Communication of Required Knowledge
- The student uses mathematical language and notation with limited clarity to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, “I found my solutions by guessing in my head and then writing them down and trying them out”).

Comments/Next Steps
- The student should use base-ten blocks for solving problems involving decimals.
- The student should develop strategies required for reverse processing in order to solve problems.
- The student should explore the relationship between the factors (dimensions) and the area (product) of a rectangle, as well as the concept of perimeter.
- The student should develop the use of mathematical language for explaining and justifying his or her responses.
- The student should refer to word charts and a dictionary for correct spellings.
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a ‘box’]

   \[
   \begin{array}{c}
   8 \times 2 = 16.4 \\
   4 \times 4 = 16.4 \\
   1 \times 1 = 16.4 \\
   \end{array}
   \]

2. Explain what you did to find your solutions.

   \[
   \begin{array}{c}
   \begin{array}{c}
   8.2 \\
   \end{array}
   \end{array}
   \]
3. Here is another problem Luciano and Jasmine found. Use this information to find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.

\[
\begin{array}{ccc}
7.5 & 3.7 & 1.6 \\
x & 1 & x 2  \\
7.5 & 7.5 & 7.5 \\
\end{array}
\]

\(a) \quad 7.5 \times 1 = 7.5\)

\(b) \quad 3.7 \times 2 = 7.4\)

\(c) \quad 1.6 \times 5 = 8\)

4. Do you think some products can only be found in one way? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
0.1 & 0.7 & 0.9 \\
x & 1 & x \ 1  \\
0.1 & 0.7 & 0.9 \\
\end{array}
\]

\(A) \quad x 0.1 = 0.1\)

\(B) \quad \underline{0.7} \times 1 = 0.7\)

\(C) \quad \underline{0.9} \times 1 = 0.9\)

These numbers can only be found in one way because they are all prime numbers and they are uneven.
5. Is it possible to find a product that can only be found in two ways? Investigate showing all your work in the space below. (You don’t have to use all of the boxes in the product)

\[ \begin{array}{ccc}
1 & . & 1 \\
\times & 4 & x \\
\hline
4 & . & 4 \\
\end{array} \]

\[ \begin{array}{ccc}
3 & . & 5 \\
\times & 2 & \hline
7 & . & 0 \\
\end{array} \]

I know this because 4.4 can be found by 1.1 \times 4 and 2.2 \times 2. Those are the only ways.

6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in four different ways using the template shown below.

\[ \begin{array}{cccc}
1 & . & 0 & 0.5 \\
\times & 4 & x & 8 \\
\hline
4 & . & 0 & 4 & 0 \\
\end{array} \]

\[ \begin{array}{cccc}
2 & . & 0 \\
\times & 2 & \hline
4 & . & 0 \\
\end{array} \]

\[ \begin{array}{cccc}
4 & . & 0 \\
\times & 1 & \hline
4 & . & 0 \\
\end{array} \]

a) Use the template below to investigate the question. [Note: Do not put more than one digit inside a ‘box’]

b) Explain how you found the answer to this question.

how I figured this out is I timed some numbers to get 4.0. If you take away the decimal, it would be 40 so things that go into 40 is what I put (10, 5, 20, 40).
7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
Draw the rectangle such that one of the dimensions is always a whole number
and the other dimension is not shorter than 1 unit.

b) Find the areas of the rectangles you have drawn.
What are some of the things you notice?

A) \( P = 3 \times 3 + 3.4 + 3.4 = 12.8 \)  
I noticed that my perimeters are the same.

B) \( P = 4.4 + 4.4 + 2 + 2 = 12.8 \)

**Teacher’s Notes**

**Problem Solving**
- The student selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution (e.g., in question 5, uses illustrations to attempt to solve the problem, arriving at inaccurate solutions).

**Understanding of Concepts**
- The student demonstrates some understanding of processes and solutions involving place value and decimals by providing partial explanations and illustrations (e.g., in question 5, states, “I know this because 4.4 can be found by 1.1 \times 4 and 2.2 \times 2. Those are the only ways”, and uses an illustration to support the statement).

**Application of Mathematical Procedures**
- The student uses computations and mathematical procedures that include some errors and/or omissions (e.g., in question 3, does not multiply correctly; in question 7, does not attempt to find the area).

**Communication of Required Knowledge**
- The student uses mathematical language and notation with some clarity to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 1, “I put my products into 3 groups”; in question 3, “I put my products into 3 groups and then drew the pictures of the base ten blocks”; in question 4, “These numbers can only be found in one way because they are all prime numbers and they are uneven”).

**Comments/Next Steps**
- The student should expand his or her use of mathematical language to explain and justify his or her responses.
- The student should use base-ten blocks to find products with a given number of factors.
- The student needs to further develop his or her understanding of perimeter and area as well as explore the relationship between the dimensions (factors) and the area (product) of a rectangle.
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a ‘box’]

2. Explain what you did to find your solutions.

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
4. Do you think some products can only be found in one way? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
2 & \cdot & 5 \\
\times & 2 & 1 & \cdot & 5 \\
\times & 2 & 3 & \cdot & 5 \\
\hline
0 & 5 & 0 & 3 & 0 & 7 & .
\end{array}
\]

Yes, because it got a little. I doubled a half.

5. Is it possible to find a product that can only be found in two ways? Investigate showing all your work in the space below.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
1 & \cdot & 5 \\
\times & 4 & 1 & \cdot & 5 \\
\times & 6 & & & \\
\hline
6 & 0 & 9 & 0 &
\end{array}
\]

6 can be found in 2 ways. 9 can be found 2 ways.
6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in four different ways using the template shown below.

a) Use the template below to investigate the question.
[Note: Do not put more than one digit inside a ‘box’]

\[
\begin{array}{ccc}
7 & . & 5 \\
\times & 3 & \phantom{0}
\end{array}
\quad \begin{array}{ccc}
6 & . & 0 \\
\times & 5 & \phantom{0}
\end{array}
\quad \begin{array}{ccc}
5 & . & 0 \\
\times & 6 & \phantom{0}
\end{array}
\]

b) Explain how you found the answer to this question.

I tried different number sentences to get the answer and I found 3 ways for the answer.

7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm. Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

b) Find the areas of the rectangles you have drawn. What are some of the things you notice?

\[
\begin{array}{l}
A = L \times W \\
A = 5\text{cm} \times 1.4\text{cm} \\
A = 2.4\text{cm} \times 4\text{cm} \\
A = 3.4\text{cm} \times 3\text{cm} \\
A = 7\text{cm}^2 \\
A = 9.6\text{cm}^2 \\
A = 10.2\text{cm}^2
\end{array}
\]
Teacher’s Notes

Problem Solving
– The student selects and applies an appropriate problem-solving strategy that leads to a partially complete and/or partially accurate solution (e.g., in question 6a, uses a guess-and-check strategy, trying different number sentences to get the solution).

Understanding of Concepts
– The student demonstrates some understanding of processes and solutions involving place value and decimals by providing partial explanations and illustrations (e.g., in question 5, provides illustrations to support two ways in which the products can be found, but provides no explanation of the second way).

Application of Mathematical Procedures
– The student uses computations and mathematical procedures that include some errors and/or omissions (e.g., in question 4, assumes that a whole answer is needed and proceeds to make an incorrect argument; in question 7, draws three rectangles but misses two possible solutions).

Communication of Required Knowledge
– The student uses mathematical language and notation with some clarity to explain and justify responses to reverse-processing problems involving decimals (e.g., relies heavily on illustrations to explain answers, providing only brief written statements to support them).

Comments/Next Steps
– The student should expand his or her use of mathematical language to explain and justify his or her responses.
– The student needs to further develop the use of base-ten blocks for solving problems related to products.
– The student should explore the relationship between the dimensions (factors) and the area (product) of rectangles, as well as the concept of perimeter.
What’s the Question?  Level 3, Sample 1

Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a ‘box’]

   \[
   \begin{array}{c}
   8 \times 2 \\
   4 \times 1 \\
   16.4
   \end{array}
   \]

2. Explain what you did to find your solutions.

   I found my solutions by knowing that 8.2 was multiplied by 2 you will get 16.4, and when 4.1 was multiplied by 4 you will get 16.4 also.

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.

   \[
   \begin{array}{c}
   7.5 \\
   1.5 \\
   2.5
   \end{array}
   \]

   \[
   \begin{array}{c}
   7 \times 1 \\
   7 \times 5 \\
   7 \times 5
   \end{array}
   \]

   \[
   \begin{array}{c}
   7.5 \\
   7.5 \\
   7.5
   \end{array}
   \]

   \[
   \begin{array}{c}
   1.5 \\
   3.
   \end{array}
   \]

   I know this because I divided 7.5 by 3 also, and got 2.5.
4. Do you think some products can only be found in one way? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

\[
\begin{align*}
3.1 \times 1 &= 3.1 \\
1.3 \times 1 &= 1.3 \\
2.1 \times 1 &= 2.1
\end{align*}
\]

I think that 3.1 can only be found in one way because it can only be divided by 1. I think that 13 can only be found in one way because it is an odd number.

5. Is it possible to find a product that can only be found in two ways? Investigate showing all your work in the space below.

(You don’t have to use all of the boxes in the product)

\[
\begin{align*}
8.0 \times 2 &= 16.0 \\
4.0 \times 4 &= 16.0
\end{align*}
\]

This can only be found in 2 ways, because 16.0 can only be divided by 2 numbers: 8 and 4.
6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in four different ways using the template shown below.

   a) Use the template below to investigate the question.
   [Note: Do not put more than one digit inside a ‘box’]

   \[
   \begin{array}{cccc}
   3 & 0 & \times & 3 \\
   4 & 5 & \times & 2 \\
   9 & 1 & \times & 1 \\
   1 & 5 & \times & 6 \\
   \hline
   9.0 & 9.0 & 9.0 & 9.0
   \end{array}
   \]

   b) Explain how you found the answer to this question.

   I found the answer to this question by knowing that \(3 \times 3 = 9\) and \(4.5 \times 2 = 9\) and of course \(9 \times 1 = 9\), \(6 \times 1\).

   I knew the last one by knowing if \(1.5 \times 5 = 7.5\) than \(1.5 \times 6 = 9\).

7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm.
   Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

    \[
    \begin{array}{cccc}
    a) & 1.4 \times 0.4 & \text{cm} \\
    b) & 4 \times 1.4 & \text{cm} \\
    c) & 3.4 \times 3.4 & \text{cm} \\
    d) & 2 \times 4.4 & \text{cm} \\
    e) & 5 \times 1 & \text{cm} \\
    \end{array}
    \]

   b) Find the areas of the rectangles you have drawn. What are some of the things you notice?

   \[
   \begin{array}{llll}
   a) & A = 1 \times w = 5 \times 1.4 \\
   & = 1.4 \\
   & = 1.4 \\
   & A = 9.6 \\
   \end{array}
   \]

   \[
   \begin{array}{llll}
   b) & A = 1 \times w = \frac{4 \times 2.4}{x} \\
   & = 2.4 \\
   & = 2.4 \\
   & A = 9.6 \\
   \end{array}
   \]

   \[
   \begin{array}{llll}
   c) & A = 1 \times w = \frac{3 \times 3.4}{x} \\
   & = 3.4 \\
   & = 3.4 \\
   & A = 10.2 \\
   \end{array}
   \]

   \[
   \begin{array}{llll}
   d) & A = 1 \times w = \frac{2 \times 4.4}{x} \\
   & = 4.4 \\
   & = 4.4 \\
   & A = 8.8 \\
   \end{array}
   \]

   \[
   \begin{array}{llll}
   e) & A = 1 \times w = \frac{1 \times 5.4}{x} \\
   & = 5.4 \\
   & = 5.4 \\
   & A = 5.4 \\
   \end{array}
   \]
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., in question 4, uses his or her knowledge of number properties to arrive at solutions; in question 7, uses a pattern to draw all of the possible rectangles).

Understanding of Concepts
- The student demonstrates a general understanding of processes and solutions involving place value and decimals by providing appropriate and complete explanations and illustrations (e.g., in question 3, “I know this because I divided 7.5 by 5 and got 1.5” and provides appropriate base-ten block illustrations).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include few errors and/or omissions (e.g., in question 4, two of the three solutions provided are accurate; in question 6, there are no computational errors; in question 5, 16.0 is an incorrect solution because it has more than two sets of factors).

Communication of Required Knowledge
- The student uses mathematical language and notation clearly to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, uses base-ten blocks, numbers, and a written explanation; in question 3, “I know this because I divided 7.5 by 5 and got 1.5”; in question 6, states a variety of combinations, including “I knew the last one by knowing if 1.5 x 5 = 7.5 than 1.5 x 6 = 9”).

Comments/Next Steps
- The student should further explore the use of base-ten blocks for solving a variety of problems involving decimals.
- The student should explore the relationship between the dimensions (factors) and the area (product) of rectangles.
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product. [Note: Do not put more than one digit inside a ‘box’]

\[
\begin{array}{ccc}
4 \times 1 & = 4 \\
8 \times 2 & = 16 \\
1 \times 6 & = 6 \\
\end{array}
\]

\[
\begin{array}{ccc}
16 \times 4 & = 64 \\
16 \times 2 & = 32 \\
1 \times 6 & = 6 \\
\end{array}
\]

2. Explain what you did to find your solutions.

3. Here is another problem Luciano and Jasmine found. Use this information to find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.

\[
\begin{array}{ccc}
7 \times 5 & = 35 \\
1 \times 5 & = 5 \\
2 \times 5 & = 10 \\
\end{array}
\]

\[
\begin{array}{ccc}
7 & 5 \\
7 & 5 \\
7 & 5 \\
\end{array}
\]

I found out my answers by figuring out what numbers multiply to 7 or 7. If it ended in 7, I would find a decimal that would add to it and make it 5 or a number that would make the answer 7.
4. Do you think some products can only be found in one way? Give reasons to justify your thinking.
   (You don’t have to use all of the boxes in the product)

   \[
   \begin{array}{cc}
   1 & 0 \\
   \times & 1 \\
   \hline
   1 & 0 \\
   \end{array}
   \quad
   \begin{array}{cc}
   1 & 1 \\
   \times & 1 \\
   \hline
   1 & 1 \\
   \end{array}
   \quad
   \begin{array}{cc}
   2 & 0 \\
   \times & 2 \\
   \hline
   2 & 0 \\
   \end{array}
   \]

   I think some products can be found only one way because you can only times it by 1. That would be prime numbers you would have to use though. The examples I have shown can only be found 1 way.

5. Is it possible to find a product that can only be found in two ways? Investigate showing all your work in the space below.
   (You don’t have to use all of the boxes in the product)

   \[
   \begin{array}{cc}
   1 & 2 \\
   \times & 1 \\
   \hline
   \end{array}
   \quad
   \begin{array}{cc}
   0 & 3 \\
   \times & 4 \\
   \hline
   \end{array}
   \]

   Work:
   \[
   \begin{array}{c}
   2 \frac{4}{1} \\
   \times 1 \\
   \hline
   2 \frac{4}{1} \\
   \end{array}
   \quad
   \begin{array}{c}
   1 \frac{2}{3} \\
   \times \frac{1}{2} \\
   \hline
   1 \frac{1}{2} \\
   \end{array}
   \quad
   \begin{array}{c}
   0 \frac{3}{4} \\
   \times \frac{3}{4} \\
   \hline
   1 \frac{1}{2} \\
   \end{array}
   \quad
   \begin{array}{c}
   0 \frac{4}{4} \\
   \times \frac{3}{4} \\
   \hline
   0 \frac{3}{4} \\
   \end{array}
   \]
6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

   a) Use the template below to investigate the question.
   [Note: Do not put more than one digit inside a “box”]

   \[
   \begin{array}{cccc}
   6 & 4 & 4 & 2 \\
   \times & 1 & \times 2 & \times 4 & \times 3 \\
   \hline
   6 & 4 & 8 & 4 & 8 & 4 & 8 & 4
   \end{array}
   \]

   b) Explain how you found the answer to this question.

   I found the answer to this question by trial and error. I tried different combinations to see what worked. I started with 6.2 and it didn’t work so I tried 6.4. That didn’t work so I tried 6.9 and it worked!

7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm. Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

   b) Find the areas of the rectangles you have drawn. What are some of the things you notice?

   \[
   \begin{align*}
   4 \times 2 &= 8 \text{ cm}^2 \\
   5.2 \times 2 &= 10.4 \text{ cm}^2 \\
   2.4 \times 4.1 &= 9.84 \text{ cm}^2 \\
   5 \times 3 &= 15 \text{ cm}^2 \\
   4.2 \times 2 &= 8.4 \text{ cm}^2 \\
   \end{align*}
   \]
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy that leads to a generally complete and accurate solution (e.g., in question 1, uses base-ten blocks and a knowledge of multiplication and division to arrive at three sets of factors for the product, one of which is inaccurate).

Understanding of Concepts
- The student demonstrates a general understanding of processes and solutions involving place value and decimals by providing appropriate and complete explanations and illustrations (e.g., in question 3, uses illustrations of base-ten blocks to support his or her conclusion: “I found out my answers by figuring out what numbers multiply to 7 or are very close to 7”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include few errors and/or omissions (e.g., in question 4, two of the three solutions are accurate, with a minor computational error in the third: “2.0 x 2 = 2.0”).

Communication of Required Knowledge
- The student uses mathematical language and notation clearly to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, uses base-ten blocks, numbers, and a written statement: “I thought of numbers that multiplied into 16.4 then I tried different combinations”; in question 4, mentions prime numbers and their significance to the solution).

Comments/Next Steps
- The student needs to justify his or her responses by providing more than one example or by supplying multiple reasons for his or her responses.
- The student should further explore the use of base-ten blocks to find products with a given number of factors.
- The student should explore the relationship between the dimensions (factors) and the area (product) of a rectangle.
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a ‘box’]

\[
\begin{array}{ccc}
4 \cdot 1 & 16 \cdot 4 & 8 \cdot 2 \\
\times 4 & \times 1 & \times a \\
164 & 164 & 164
\end{array}
\]

2. Explain what you did to find your solutions.

This is 164 in base-ten blocks. I grouped them into three groups. For instance:
- 1 group of 164
- 2 groups of 82
- 4 groups of 41

Since \(2 \times 8 = 16\)

\[16/4 \text{ factors of 16} \]
\[1 \times 16 = 16 \quad 4 \times 4 = 16 \]
\[2 \times 8 = 16 \quad 4 \times 4 = 16 \]

Since \(4 \times 4 = 16\)

\[1 \times 0.4 = 0.4 \quad 2 \times 0.2 = 0.4 \]

3. Here is another problem Luciano and Jasmine found.
   Use this information to find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.

\[
\begin{array}{ccc}
1 \cdot 5 & 7 \cdot 5 & 2 \cdot 5 \\
\times 5 & \times 1 & \times 3 \\
75 & 75 & 75
\end{array}
\]

I grouped these blocks into three groups which is:
- a) 2 groups of 1.5
- b) 1 group of 7.5
- c) 5 groups of 1.5

If you take away the decimal instead of 7.5, you will have 75. 75 can be made multiplying.

\[1 \times 75 \quad 5 \times 15 \]

I know that there are 8 hundred base blocks and 5 ten base blocks in a.)
4. Do you think some products can only be found in one way? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

The first number is prime if you take away the decimal number.
You can only group these one way.
There is no product that can only be found in one way if you are allowed to go into decimals, because if you add another 0 at the end of the decimal number, you can divide the number by 10.

a) \[
\begin{array}{c}
0.72 \\
10 \\
\hline
7.2 \\
70 \\
\hline
30 \\
30 \\
\hline
0 \\
\end{array}
\]

b) \[
\begin{array}{c}
0.11 \\
10 \\
\hline
1.1 \\
10 \\
\hline
0.1 \\
\end{array}
\]
5. Is it possible to find a product that can only be found in two ways?
Investigate showing all your work in the space below.
(You don’t have to use all of the boxes in the product)

\[
\begin{array}{c}
7 \cdot 5 \\
\times 2 \\
\hline
15 \\
\end{array} 
\quad 
\begin{array}{c}
2 \cdot 5 \\
\times 6 \\
\hline
15 \\
\end{array}
\]

It is impossible to find a product which can only be found in 2 ways because every multiple can be divided by 2 so you must add another 2 to the other multiple for example
\[
\frac{25}{5} = 2 - 3.75 \\
\times 2 = 4 \\
\frac{150}{5} = 30 \\
\]

This procedure always works no matter whether you are using prime numbers or composite numbers it works.

6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in four different ways using the template shown below.

a) Use the template below to investigate the question.
[Note: Do not put more than one digit inside a “box”]

\[
\begin{array}{c}
3 \cdot 0 \\
\times 5 \\
\hline
15 \\
\end{array} 
\quad 
\begin{array}{c}
2 \cdot 5 \\
\times 6 \\
\hline
15 \\
\end{array} 
\quad 
\begin{array}{c}
3 \cdot 0 \\
\times 3 \\
\hline
15 \\
\end{array} 
\quad 
\begin{array}{c}
7 \cdot 5 \\
\times 2 \\
\hline
15 \\
\end{array}
\]

b) Explain how you found the answer to this question.

Well I used the process of counting money I removed all the decimals first. Then I thought of using bills.
7.a) Draw as many rectangles as you can with a perimeter of 12.8 cm. Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

- Rectangle A
  \[
  \text{Length} = 5.4 \text{ cm}, \quad \text{Width} = 1.6 \text{ cm}
  \]
  \[
  5.4 \times 1.6 = 8.64 \text{ cm}^2
  \]
- Rectangle B
  \[
  \text{Length} = 5.0 \text{ cm}, \quad \text{Width} = 1.4 \text{ cm}
  \]
  \[
  5.0 \times 1.4 = 7.0 \text{ cm}^2
  \]
- Rectangle C
  \[
  \text{Length} = 4.4 \text{ cm}, \quad \text{Width} = 2.0 \text{ cm}
  \]
  \[
  4.4 \times 2.0 = 8.8 \text{ cm}^2
  \]

b) Find the areas of the rectangles you have drawn. What are some of the things you notice?
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy that leads to a thorough and accurate solution (e.g., in question 6, uses the concept of money to find four sets of factors – “I removed all the decimals first. Then I thought of using bills” – supported by clearly labelled illustrations).

Understanding of Concepts
- The student demonstrates a thorough understanding of processes and solutions involving place value and decimals by providing appropriate and detailed explanations and illustrations (e.g., in questions 2 and 3, uses broken lines, colour coding, and circling of base-ten blocks to demonstrate his or her understanding of grouping).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include few, if any, minor errors and/or omissions (e.g., in questions 1, 4, 5, and 6, provides accurate answers; in question 7, provides all of the possible rectangles).

Communication of Required Knowledge
- The student uses mathematical language and notation clearly and precisely to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, explains the process used to find three sets of factors for a given product: “Since 2 x 8 = 16 & 2 x 0.2 = 0.4 . . . 2 can be a multiple of 16.4”).

Comments/Next Steps
- The student has a very good understanding of prime numbers, factors, and multiples. Additional rich investigations could be assigned to this student.
Exemplar Task

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers which, when multiplied, give the products shown.
   Use the blank space to show your work and to show any other combinations that would give you the same product.
   [Note: Do not put more than one digit inside a “box”]

2. Explain what you did to find your solutions.
   (a) I divided 16.4 by a number I knew would work. I divide a number such as 2 into 16.4 when I get my answer 8.2 I place it in the first row of the question and because I divided it by 2 I have to use the 2 to multiply 8.2 with equals 16.4.

3. Here is another problem Luciano and Jasmine found. Use this information to find the numbers which, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.
4. Do you think some products can only be found in one way? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
2 & 9 & 8 & 3 \\
\times & 1 & x & 5 \\
\hline
2 & 9 & 4 & 1 & 3 \\
2 & 9 & 1 \\
\end{array}
\]

This is a different approach this time. I used 8 x 5. 9 x 1 is 29, if you multiply 29 by 1. Other numbers have remainders. I found this out by dividing each number (149) into 29. I found out that 3 goes into 29 evenly.

5. Is it possible to find a product that can only be found in two ways? Investigate showing all your work in the space below.

(You don’t have to use all of the boxes in the product)

\[
\begin{array}{ccc}
4 & 6 \\
\times & 2 & 3 \\
\hline
9 & 2 \\
\end{array}
\]

4 x 7 = 28
2 x 14 = 28
14 x 2 = 28
28 x 1 = 28

Yes, 2 and 4 are the only two numbers that can equally go into 28.

I got my answer by choosing a number and seeing if 1 or 2 could go into it. To my surprise, I found out that 2 and 2 are the only numbers.
6. Jasmine and Luciano’s classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

   a) Use the template below to investigate the question.  
   [Note: Do not put more than one digit inside a ‘box’]

   \[
   \begin{array}{cccc}
   3 & 3 & 4 & 6 \\
   \times & 4 & \times & \times \\
   \hline
   13.8 & 13.8 & 13.8 & 13.8 \\
   \end{array}
   \]

   b) Explain how you found the answer to this question.

   I found my answer by multiplying different numbers by different numbers like \(11 \times 3 = 33\) then I multiplied other numbers together to see if it would equal the same.

7. a) Draw as many rectangles as you can with a perimeter of 12.8 cm. Draw the rectangle such that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

   b) Find the areas of the rectangles you have drawn. What are some of the things you notice?

   - 1.4 cm \( \times \) 1.4 cm
   - 3.4 cm \( \times \) 3.4 cm
   - 4.1 cm \( \times \) 4.1 cm
   - 5.4 cm \( \times \) 5.4 cm

   I notice that the first number(s) of each area has a pattern. One number gets bigger 1.4, 2.1, 3.4, 4.1, 5.4 and the other gets smaller 1.4, 2.1, 3.4, 4.1, 5.4.
**Teacher’s Notes**

**Problem Solving**
- The student selects and applies an appropriate problem-solving strategy that leads to a thorough and accurate solution (e.g., in question 4, uses a systematic problem-solving approach: “I found this out by dividing every number (1 to 9) into 2.9 ...”).

**Understanding of Concepts**
- The student demonstrates a thorough understanding of processes and solutions involving place value and decimals by providing appropriate and detailed explanations and illustrations (e.g., in question 3, shows multiple ways of solving the problem to find three sets of factors for 7.5; uses diagrams to support written statements).

**Application of Mathematical Procedures**
- The student uses computations and mathematical procedures that include few, if any, minor errors and/or omissions (e.g., in questions 1, 3, 4, and 6, provides accurate responses; in question 7, draws five rectangles, including the one with the largest area).

**Communication of Required Knowledge**
- The student uses mathematical language and notation clearly and precisely to explain and justify responses to reverse-processing problems involving decimals (e.g., in question 2, uses a clear statement and supporting diagrams; in question 5, “I got my answer by choosing a number (9.2) and seeing if 1 to 10 could go into it to my results I found out that 4 and 2 are the only numbers”).

**Comments/Next Steps**
- The student should expand his or her responses to include all the processes and procedures used (e.g., regrouping).
- The student could further investigate the use of base-ten blocks for solving problems involving the multiplication and division of decimals.
Title: What's the Question?

Time requirements: 160–200 minutes (total)
• 40–50 minutes for the pre-task
• three periods of 40–50 minutes each for the exemplar task

Description of the Task

This task requires students to:
• use base-ten blocks to investigate many tasks involving the multiplication of decimals;
• add decimals in finding all the rectangles with a given perimeter, and multiply decimals in finding the area of the rectangles discovered.

Students will work with base-ten blocks to find as many instances as possible of a decimal number and a whole number that, when multiplied, give a specified product. They will show their work and explain how they found their solutions. Then students will discover whether any products can be found in only one way or in only two ways (and justify their thinking), and whether any product can be found in four ways (and explain the process they used). Finally, students will draw as many rectangles as possible with a perimeter of 12.8 cm, one dimension being a whole number and the other dimension being no less than 1 cm, and will find the areas of the rectangles drawn.

Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from the Ministry of Education's Curriculum Unit Planner (CD-ROM).

Students will:
1. select and perform computation techniques appropriate to specific problems involving unlike denominations in fractions and the multiplication and division of decimals, and determine whether the results are reasonable (6m7);
2. solve and explain multi-step problems using the multiplication and division of decimals and percents (6m8);
3. justify and verify the method chosen for calculations with whole numbers, fractions, decimals, and percents (6m9);
4. represent the place value of whole numbers and decimals from 0.001 to 1 000 000 using concrete materials, drawings, and symbols (6m14);
5. explain processes and solutions with fractions and decimals using mathematical language (6m19);
6. multiply and divide numbers using concrete materials, drawings, and symbols (6m31);
7. multiply and divide decimal numbers to thousandths by a one-digit whole number (6m37).

Teacher Instructions

Prior Knowledge and Skills Required

To complete this task, students should have some knowledge or skills related to the following:
• using base-ten materials to add, subtract, multiply, and divide
• representing and exploring the relationships between fractions and decimals
• applying the formulas for perimeter and area

The Rubric*

The rubric provided with this exemplar task is to be used to assess students' work. The rubric is based on the achievement chart given on page 9 of The Ontario Curriculum, Grades 1–8: Mathematics, 1997.

Before asking students to do the task outlined in this package, review with them the concept of a rubric. Rephrase the rubric so that students can understand the different levels of achievement.

Accommodations

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar task.

Materials and Resources Required

Before attempting the tasks, students should be provided with the following materials:
• a copy of the Student Package (see Appendix 1) for each student
• base-ten blocks

*The rubric is reproduced on page 14 of this document.
Task Instructions

Introductory Activities
The pre-tasks are designed to review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task.

Multiplication of Decimals Using Base-ten Blocks: Notes for the Teacher
The following examples model how to use base-ten blocks to deepen students’ understanding of decimals and to show what happens when students perform multiplication with decimals. In the exemplar task, students will be using base-ten blocks to perform the reverse of the multiplication process, that is, they will find the numbers, which, when multiplied, result in a given decimal product.

Please note: The figures of base-ten blocks that appear in this package have not been drawn to scale, owing to spatial constraints. Figures and models that are presented to students should, however, be proportional.

The example that follows illustrates question (a) of the pre-task.

The flat base-ten block has a value of 1.

A long base-ten block has a value of \( \frac{1}{10} \) or 0.1.

A small base-ten block has a value of \( \frac{1}{100} \) or 0.01.

Example
We can illustrate the following operation with base-ten blocks.

4.8 x 2
This can be shown with the base-ten blocks as two groups of 4.8:

By combining the two groups – two groups of 4.8 – we get:
We now proceed by exchanging 10 long (0.1) blocks for one flat (1.0), just as we would do if there were no decimals involved (exchanging ten ones for one ten or ten tens for one hundred or ten hundreds for one thousand).

The answer would be 9.6, as shown below:
Appendix 1: Student Worksheets

Jasmine and Luciano discovered some ancient mathematical problems where only the products could be seen. They wondered if it was possible to use the clues they had been given to figure out the original questions. For each of the following questions use base-ten blocks.

1. Find the numbers that, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.

[Note: Do not put more than one digit inside a box.]

\[
\begin{array}{ccc}
\square & \times & \square \\
\times & \square & \square \\
\hline
1 & 6 & 4 \\
\end{array}
\]

2. Explain what you did to find your solutions.

3. Here is another problem Luciano and Jasmine found. Use this information to find the numbers that, when multiplied, give the products shown. Use the blank space to show your work and to show any other combinations that would give you the same product.

\[
\begin{array}{ccc}
\square & \times & \square \\
\times & \square & \square \\
\hline
7 & 5 \\
\end{array}
\]

7

8
4. Do you think some products can only be found in **one way**? Give reasons to justify your thinking.

(You don’t have to use all of the boxes in the product.)

5. Is it possible to find a product that can only be found in **two ways**? Investigate showing all your work in the space below.

(You don’t have to use all of the boxes in the product.)
6. Jasmine and Luciano's classmates began to wonder if they could find a product that could be written in **four different ways** using the template shown below.

   a) Use the template below to investigate the question.
   [Note: Do not put more than one digit inside a box.]

   b) Explain how you found the answer to this question.

---

7. a) Draw as many rectangles as you can with a perimeter of 12.8 cm. Draw the rectangles in such a way that one of the dimensions is always a whole number and the other dimension is not shorter than 1 unit.

   b) Find the areas of the rectangles you have drawn. What are some of the things you notice?
Measurement / Geometry and Spatial Sense
Polygons on the Geoboard

The Task
This task required students to:

- draw non-congruent rectangles of a given perimeter;
- investigate the relationship between the area and perimeter of rectangles;
- draw a graph comparing the lengths and widths of rectangles with a given perimeter;
- investigate the area relationships of rectangles, triangles, and parallelograms.

Students used geoboards and geopaper to make as many different rectangles as possible with a given perimeter. They looked for a pattern in lengths and widths, and used the pattern to make predictions. Next, they drew a graph comparing the lengths and widths of rectangles with a given perimeter, and described the patterns shown. Finally, students justified their answers to questions about the relationship between the area and dimensions of rectangles, and demonstrated how to construct a rectangle, an isosceles triangle, and a right-angled triangle having the same area.

Expectations
This task gave students the opportunity to demonstrate their achievement of all or part of each of the following selected overall and specific expectations from two strands – Measurement, and Geometry and Spatial Sense. Note that the codes that follow the expectations are from the Ministry of Education’s Curriculum Unit Planner (CD-ROM).

Measurement
Students will:
1. solve problems related to the calculation and comparison of the perimeter and the area of regular polygons (6m44);
2. relate dimensions of rectangles and area to factors and products (in a rectangle 2 cm by 3 cm the side lengths are factors and the area, 6 cm², is the product of the factors) (6m56);
3. understand the relationship between the area of a parallelogram and the area of a rectangle, between the area of a triangle and the area of a rectangle, and between the area of a triangle and the area of a parallelogram (6m57);
4. understand the relationship between area and lengths of sides and between perimeter and lengths of sides for squares, rectangles, triangles, and parallelograms (6m59);
5. sketch a rectangle, square, triangle, or parallelogram given its area and/or perimeter (6m60).

Geometry and Spatial Sense
Students will:
6. identify, describe, compare, and classify geometric figures (6m64);
7. use mathematical language effectively to describe geometric concepts, reasoning, and investigations, and coordinate systems (6m69);
8. use mathematical language to describe geometric ideas (6m82);
9. explain, make conjectures about, and articulate hypotheses about geometric properties and relationships (6m85).
Prior Knowledge and Skills

To complete this task, students were expected to have some knowledge or skills relating to the following:

- identifying, describing, comparing, and classifying geometric figures
- calculating the area and perimeter of rectangles and the area of triangles and parallelograms
- finding factors of a number
- the properties of right-angled and isosceles triangles

For information on the process used to prepare students for the task and on the materials, resources, and equipment required, see the Teacher Package reproduced on pages 99–104 of this document.
## Task Rubric – Polygons on the Geoboard

<table>
<thead>
<tr>
<th>Expectations*</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem solving</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>selects and applies a problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at an incomplete or inaccurate solution</td>
<td>selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a partially complete and/or partially accurate solution</td>
<td>selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a complete and accurate solution</td>
<td>selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a thorough and accurate solution</td>
</tr>
<tr>
<td><strong>Understanding of concepts</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3, 4, 6, 9</td>
<td>identifies and describes a few of the measurement relationships and geometric properties by providing limited explanations and illustrations</td>
<td>identifies and describes some of the measurement relationships and geometric properties by providing partially complete explanations and illustrations</td>
<td>identifies and describes many of the measurement relationships and geometric properties by providing complete explanations and illustrations</td>
<td>identifies and describes most of the measurement relationships and geometric properties by providing thorough explanations and illustrations</td>
</tr>
<tr>
<td><strong>Application of mathematical procedures</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2, 5</td>
<td>constructs a few of the possible rectangles on the geoboard</td>
<td>constructs some of the possible rectangles on the geoboard</td>
<td>constructs most of the possible rectangles on the geoboard</td>
<td>constructs all or almost all of the possible rectangles on the geoboard</td>
</tr>
<tr>
<td><strong>Communication of required knowledge</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7, 8, 9</td>
<td>uses mathematical language with limited clarity to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles</td>
<td>uses mathematical language with some clarity to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles</td>
<td>uses mathematical language clearly to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles</td>
<td>uses mathematical language clearly and precisely to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles</td>
</tr>
</tbody>
</table>

*The expectations that correspond to the numbers given in this chart are listed on page 56.

Note: This rubric does not include criteria for assessing student performance that falls below level 1.
Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

Part One

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.
   a) 16 units?

   ![Diagram of rectangles with various dimensions]

b) 18 units?

   ![Diagram of a rectangle with dimensions 8 cm x 8 cm]
c) 20 units?

2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

lots have one x something
2 of them have 2 x something
the bigger ones have a bigger area

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

I probably could do lots because I would have a very large geoboard.
c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

10 \times 3 would be the greatest area.

Part Two
1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle. Justify your answer.

Yes, you would because you would have length down the side width across the top.
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

\[ \text{[Diagram of a rectangle and two triangles]} \]

c) Finally, Maria stated that:

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas.

I think that Maria is correct because I showed it on page 10.

\[ \text{[Diagram of two isosceles triangles]} \]
Teacher’s Notes

Problem Solving
- The student selects and applies a problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at an incomplete or inaccurate solution (e.g., in Part One, question 1c, draws three combinations, but does not use a systematic approach; in Part One, question 2c, constructs only two of the possible rectangles and calculates their areas to decide which rectangle has the largest area).

Understanding of Concepts
- The student identifies and describes a few of the measurement relationships and the geometric properties by providing limited explanations and illustrations (e.g., in Part Two, question 1a, attempts to explain the relationship between the dimensions and the area of a rectangle: “Yes you would because you would have length down the side width across the top”; in Part Two, question 1b, attempts to draw triangles with the same area).

Application of Mathematical Procedures
- The student constructs a few of the possible rectangles on the geoboard (e.g., in Part One, omits many of the possible rectangles in questions 1a, 1b, 1c, and 2c).

Communication of Required Knowledge
- The student uses mathematical language with limited clarity to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., solutions are short, with little supporting evidence; in Part One, question 2a, “lots have one x somthing, 2 of them have 2 x somthing, the bigger ones have a bigger area”; in Part One, question 2c, “10 x 3 would be the greats area”).

Comments/Next Steps
- The student needs to explore different strategies for constructing rectangles of different perimeters.
- The student needs to use effective problem-solving strategies such as making a systematic list and making a chart to assist with a systematic approach to constructing all the possible rectangles with specific perimeters.
- The student needs to develop the use of mathematical language when explaining geometric concepts and relationships.
- The student needs to investigate the relationship between the dimensions and the area of rectangles.
- The student should refer to word charts or a dictionary for correct spellings.
Polygons on the Geoboard  Level 1, Sample 2

Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

Part One

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.
   a) 16 units?

   \[ W = 3 \quad L = 5 \]
   \[ W = 1 \quad L = 7 \]
   \[ W = 2 \quad L = 6 \]
   \[ W = 5 \quad L = 5 \]
   \[ W = 4 \quad L = 5 \]

b) 18 units?

   \[ W = 5 \quad L = 4 \]
   \[ W = 2 \quad L = 7 \]
   \[ W = 4 \quad L = 6 \]
c) 20 units?

\[
\begin{align*}
W &= 11 \quad L = 9 \\
W &= 2L = 8 \\
W &= 3L = 7
\end{align*}
\]

2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

I noticed that if two lengths are doubled because they would equal the amount needed it wouldn't work because the width must be conted for. For example: \(6 \times 2 = 12\) pretend this is the length it won't work because there is the width too odd.

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

I can only think of 1 because I think 42 is a hard number to try because I can't experiment with something: graph paper, geoboard, graph paper.
c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

40 is the greatest one because that is the one that is can find.

\[
\begin{array}{c}
3 \\
10
\end{array}
\begin{array}{c}
5 \\
8
\end{array}
\]

Part Two

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle? Justify your answer.

Yes, you do because the area is almost the dimensions. The dimensions are the area, so yes.
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas. Yes.

c) Finally, Maria stated that:

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas.
Teacher’s Notes

Problem Solving
- The student selects and applies a problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at an incomplete or inaccurate solution (e.g., in Part One, questions 1a, 1b, and 1c, lists lengths and widths to find possible combinations, but makes errors in 1a and 1b and an omission in 1c; does not apply the same approach in question 2b: “I can only think of 1 because I think 42 is a hard number to try...”).

Understanding of Concepts
- The student identifies and describes a few of the measurement relationships and the geometric properties by providing limited explanations and illustrations (e.g., in Part two, question 1b, attempts to draw triangles with the same area; in Part two, question 1c, attempts to use an illustration to show how a triangle and a rectangle can have the same area).

Application of Mathematical Procedures
- The student constructs a few of the possible rectangles on the geoboard (e.g., in Part One, questions 1a, 1b, 1c, and 2c, lists a few possible rectangles).

Communication of Required Knowledge
- The student uses mathematical language with limited clarity to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part Two, question 1a, when describing the relationship between area and dimensions, states, “Yes, you do because the area is almost the dimensions. The dimensions are the area so yes”).

Comments/Next Steps
- The student should use a variety of strategies, such as making a systematic list and making a chart, for constructing and checking the accuracy of rectangles with different perimeters.
- The student needs to use mathematical language when explaining geometric concepts and relationships.
- The student should use strategies such as making drawings to help solve problems involving geometric concepts and relationships.
- The student should explore the relationship between the areas of rectangles and the areas of different types of triangles.
- The student should refer to word charts or a dictionary for correct spellings.
Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

**Part One**

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.

   a) 16 units?

   ![Rectangle Diagram 1]

   b) 18 units?

   ![Rectangle Diagram 2]
c) 20 units?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1u</td>
<td>9u</td>
<td>20q</td>
</tr>
<tr>
<td>2u</td>
<td>8u</td>
<td>20q</td>
</tr>
<tr>
<td>4u</td>
<td>6u</td>
<td>20q</td>
</tr>
<tr>
<td>5u</td>
<td>5u</td>
<td>25q</td>
</tr>
</tbody>
</table>

S

S

2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

\[ A = 24 \]
\[ 4 \times 6 = 24 \]

length \times width = area

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

\[ 6, 14, 18, 20, 15 \]
c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

\[ \begin{align*}
1 \times 12 &= 12u^2 \\
2 \times 11 &= 22u^2 \\
3 \times 8 &= 20u^2 \\
4 \times 7 &= 28u^2 \\
5 \times 6 &= 30u^2 \\
6 \times 4 &= 24u^2 \\
\end{align*} \]

Part Two

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle. Justify your answer.

Yes, if the area was 16, the dimensions are 2x8 or 4x4.
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

---

c) Finally, Maria stated that:

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas. Yes because all you have to do is double the size and then choose one of the two to make an isosceles or right angle triangle.
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a partially complete and/or partially accurate solution (e.g., in Part One, questions 1a, 1b, and 1c, attempts to apply a systematic method, using charts and diagrams, to find the possible combinations, but makes omissions in 1b and 1c; in Part One, question 2b, uses drawings to explore some rectangles that have a perimeter of 42 units, but misses many of the possible combinations).

Understanding of Concepts
- The student identifies and describes some of the measurement relationships and the geometric properties by providing partially complete explanations and illustrations (e.g., in Part Two, questions 1b and 1c, draws some appropriate shapes to support a brief statement).

Application of Mathematical Procedures
- The student constructs some of the possible rectangles on the geoboard (e.g., in Part One, there are omissions in questions 1b, 1c, and 2b).

Communication of Required Knowledge
- The student uses mathematical language with some clarity to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part Two, question 1c, produces a partial explanation that is supported by a diagram).

Comments/Next Steps
- The student should explore all possibilities for constructing rectangles of different perimeters.
- The student needs to incorporate mathematical language when explaining geometric concepts and relationships.
- The student should explore the relationship between the dimensions and the areas of rectangles, right-angled triangles, and isosceles triangles.
- The student should use geoboards to explore geometric concepts and relationships.
- The student should refer to word charts or a dictionary for correct spellings.
Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

**Part One**

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.

a) 16 units?

b) 18 units?
2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

all of the dimensions of the width grow by one centimetre and all the length get one less

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

you could make 8 because the rectangles all add up to a perimeter of 42 units.

22 by 5   22 by 10
28 by 7   20 by 11
26 by 8   18 by 12
24 by 9   16 by 13
c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

The rectangle that is 6 by 7 units because if you times 6 by 7 it comes out to more than 8 by 4.

\[ 8 \times 5 = 40 \]

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle. Justify your answer.

Yes you would know the length and width because to find out the area you need to times the length and width together.

\[ A = 4 \times 2 = 8 \text{ units}^2 \]
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

I think you can because you could just stretch one side of the rectangle.

Maria is right because she could stretch one side of the point on the rectangle and it would have the same area and it would be a right angle triangle or an isosceles triangle.

c) Finally, Maria stated that:

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas.
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a partially complete and/or partially accurate solution (e.g., in Part One, questions 1a, 1b, and 1c, draws possible combinations but misses at least one in each question; in Part One, question 2b, does not apply the same systematic approach as in previous questions).

Understanding of Concepts
- The student identifies and describes some of the measurement relationships and the geometric properties by providing partially complete explanations and illustrations (e.g., in Part One, question 2c, constructs two of the possible rectangles and concludes that “the rectangle that is 6 by 7 units [has the greatest area] because if you times 6 by 7 it comes out to more than 8 by 4”).

Application of Mathematical Procedures
- The student constructs some of the possible rectangles on the geoboard (e.g., in Part One, in questions 1a, 1b, and 1c, draws three of the possible rectangles in each case; in question 2b, one of the two dimensions given for each of the possible rectangles is twice what it should be – “32 by 5” should be “16 by 5”, and so on).

Communication of Required Knowledge
- The student uses mathematical language with some clarity to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part One, question 2a, “all of the dimensions of the width grow by one centimetre and all the length get one less”; in Part Two, question 1c, “Maria is right because she could stretch one side of the point on the rectang and it would have the same area and it would be a right angle-triangle or a isosceles triangle”).

Comments/Next Steps
- The student should explore all possibilities for constructing rectangles of different perimeters using a manipulative such as a geoboard.
- The student needs to incorporate mathematical language when explaining geometric concepts and relationships.
- The student should explore the relationship between the dimensions and the areas of rectangles.
Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

Part One

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.
   a) 16 units?

   Rule: increase 1 more cm (centimeter) for length, and decrease the cm (centimeters) by 1 for width.
   Pattern: length = #5,6,7; add 1
to width = #3,2,1; subtract 1
2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

rule - When you increase the length, the width decreases.

pattern - length = add 1
width = subtract 1

you increase 1 cm for length and you decrease the cm, by 1 cm, for width.

No matter what unit amount there is.

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

a) $20 + 20 = 40 + 1 = 41 + 1 = 42$

b) $19 + 19 = 38 + 2 = 40 + 2 = 42$

c) $18 + 18 = 36 + 3 = 39 + 3 = 42$

d) $17 + 17 = 34 + 4 = 38 + 4 = 42$

e) $16 + 16 = 32 + 5 = 37 + 5 = 42$

f) $15 + 15 = 30 + 6 = 36 + 6 = 42$

g) $14 + 14 = 28 + 7 = 35 + 7 = 42$

h) $13 + 13 = 26 + 8 = 34 + 8 = 42$

i) $12 + 12 = 24 + 9 = 33 + 9 = 42$

Answer: 11 units
c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

Perimeter:
- a) $10 + 10 + 3 + 3 = 26$ units
- b) $9 + 9 + 4 + 4 = 26$ units
- c) $8 + 8 + 5 + 5 = 26$ units
- d) $7 + 7 + 6 + 6 = 26$ units

Area:
- a) $10 \times 3 = 30$
- b) $9 \times 4 = 36$
- c) $8 \times 5 = 40$
- d) $7 \times 6 = 42$

"d" would be the greatest because when you multiply it comes out the greatest.

Part Two
1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle? Justify your answer.

No, because there is more than one set of dimensions for most areas.

ex. $6 \times 5 = 30$

$15 \times 2 = 30$
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

---

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas:

ex.

- rectangles have right angles, so no matter what there will be a right angles. And since you put a line in the middle of the rectangle to get 2 triangles, the line is exactly the same as one other one so you can get an isosceles triangle.
Teacher’s Notes

Problem Solving
– The student selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a complete and accurate solution (e.g., in Part One, questions 1a, 1b, and 1c, applies a systematic approach, using diagrams and calculations, to find all the possible rectangles).

Understanding of Concepts
– The student identifies and describes many of the measurement relationships and the geometric properties by providing complete explanations and illustrations (e.g., in Part One, question 2c, provides diagrams and shows calculations of the area and perimeter of four of the six possible rectangles, and provides an explanation regarding the one with the greatest area).

Application of Mathematical Procedures
– The student constructs most of the possible rectangles on the geoboard (e.g., in Part One, questions 1a, 1b, and 1c, constructs all possible rectangles; in question 2c, omits two rectangles because they are not necessary to find the answer).

Communication of Required Knowledge
– The student uses mathematical language clearly to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part One, question 2a, “When you increase the length, the width decreases, pattern – length = add 1 width = subtract 1”; in Part Two, question 1c, identifies that right-angled triangles are always present in rectangles and that all rectangles can be divided into two equal triangles).

Comments/Next Steps
– The student should expand his or her use of mathematical terms when explaining geometric concepts and relationships.
– The student needs to provide more detail and supporting evidence to explain his or her mathematical reasoning.
– The student should use mathematical notation that reflects the formula for finding the perimeter of a rectangle (e.g., in question 2b, use $2(20 + 1)$ rather than “$20 + 20 = 40 + 1 = 41 + 1 = 42$”, which is misleading and inaccurate).
Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

**Part One**

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.

   a) 16 units?

   b) 18 units?

You can make 3 (non-regular) rectangles.

You can make 4 (non-congruent) rectangles.
2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

I notice that every time you go to do a new rectangle one dimension increases while the other dimension decreases. I also noticed that for different numbers that the more things you can multiply to get that number that will be more rectangles than if you can only multiply by a few numbers to get that number.

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

You could make 10 (non-congruent) rectangles, I know because I wrote all of them out over there. To find the first combination I divide 42 in half and then with that number - found 20 + 20 + 14 equalled 42 and from then on I just had one dimension increase and one the dimension decrease.
E

a) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

\[ \frac{13}{2} + \frac{13}{2} \]

\[ 10 + 6 + 10 + 6 = 36 \]

\[ 8 + 8 + 5 + 5 = 26 \]

1 + 1 + 6 + 6 would have the greatest area.

I know because I wrote out all the perimeters that equal 26, then multiplied them all. I also know because 1 + 1 + 6 + 6 has the biggest numbers that when put together would equal the most.

F

Part Two

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle. Justify your answer.

No, you do not know the dimensions. I know this because say the area was 20cm². There is more than one way to get 20 by multiplying. So you could say it was one way, but it could really be another.
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

\[
\begin{align*}
\text{Triangle} & \quad 10 \times 3 = 30 \div 2 = 15 \\
\text{Rectangle} & \quad 5 \times 3 = 15 \\
\end{align*}
\]

Yes it is possible to make an isosceles triangle and a right-angled triangle for every rectangle you can make on the geoboard. I know this is true because all you have to do is make the height on the triangle twice as big as the length on the rectangle and keep the base equal to the width of the rectangle. And the when you multiply the base and the height together it should be equal as much as when you multiply the length and the width on the rectangle. Then when you divide the number you got for the triangle in half it will equal the area of the rectangle.

c) Finally, Maria stated that:

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas.
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a complete and accurate solution (e.g., in Part One, question 2b, finds a pattern and then works it out numerically to find all ten solutions).

Understanding of Concepts
- The student identifies and describes many of the measurement relationships and the geometric properties by providing complete explanations and illustrations (e.g., in Part Two, question 1a, identifies that multiple factors exist for 20, so knowing the area does not automatically mean the dimensions are also known; in Part Two, question 1b, uses a partial illustration; in Part Two, question 1c, explains the method for making one of the two types of triangle).

Application of Mathematical Procedures
- The student constructs most of the possible rectangles on the geoboard (e.g., in Part One, there is one omission in each of questions 1a and 1c).

Communication of Required Knowledge
- The student uses mathematical language clearly to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part Two, question 1c, supports the following statement with mathematical notation: “Yes it is possible to make an isosceles triangle and a right angled triangle for every rectangle you can make on the geoboard. I know this is true because all’s you have to do is make the height on the triangle twice a big as the length on the rectangle and keep the base equal to the width of the rectangle”).

Comments/Next Steps
- The student should explore all possibilities for constructing rectangles of different perimeters.
- The student needs to expand his or her use of mathematical language when explaining geometric concepts and relationships.
- The student should use more illustrations when explaining geometric concepts and relationships.
- The student needs to use geoboards to explore geometric concepts and relationships.
Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

Part One

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters?
   Show all of your work.
   
   a) 16 units?

   I noticed a pattern when I was finding perimeter of the shapes. That pattern was, to find the perimeter you always add length + length and width + width. I know I have all the possible answers because I started at 1 and 4. If you follow my pattern and add 1 + 1 and 7 + 7 (length + length and width + width) it equals 16 units. If you subtract 1 from the length and add one to the width, this would be 2 and 6. This still has 16 units for perimeter. If you follow this pattern, you know you have all possible answers. I also noticed that the areas of the rectangles have a different area. There are 4 non-congruent rectangles you can make with the perimeter of 16 units.
b) 18 units?

To better explain the pattern I found while calculating the perimeter of 16 units, I will give you an example: 4 units $\rightarrow$ 3 units

5 units $\rightarrow$ 6 units

P = 18 units

Again none of the areas are the same. I know I have all the possible answers because I followed the pattern above. There is 4 non-congruent rectangles you can make with the perimeter of 18 units.

\[
\begin{array}{c|c}
L & W \\
8 & 1 \\
4 & 2 \\
6 & 4 \\
5 & 3 \\
\end{array}
\]

c) 20 units?

I used the same pattern. All the areas are different. I know I have all possible answers because I followed my pattern.

There are 5 different rectangles you can make with the perimeter of 20 units.
2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

All the rectangles I have just drawn have different dimensions. No shape is the same, unless it is a congruent shape. Length x Width = Area
2(L+W) = Perimeter

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

<table>
<thead>
<tr>
<th>L</th>
<th>W</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>6</td>
<td>42</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>

You would be able to make 10 non-congruent rectangles. I know I have all possible answers is because I made a chart and followed the pattern I discovered.

c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

The one with length of 6 and width of 7, the area of this shape is 42 units. It would have the greatest area because when you use the pattern above, 6+9, W=4+1

L=69, W=4+1

They both have the same perimeter, but their areas are different. But, there is also a pattern in the area. If you start finding the area at the bottom of the chart, that will be the smallest area. The top shape will have the greatest area because the top one is the closest to a square.
Part Two

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle? Justify your answer.

You know a few possible dimensions. Ex. If the area was 12 units², then you know that one possible set of dimensions is 3 by 4. You could also do 6 by 2 or 12 by 1. There, you already have 3 possibilities! So, the answer is no.

b) For every rectangle you can make, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

Yes, it is possible to make an isosceles and a right-angled triangle with the same area as every rectangle you can make!
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a thorough and accurate solution (e.g., in Part One, questions 1a, 1b, and 1c, consistently applies the systematic approach of adding to one dimension and subtracting from the other to arrive at a list of possible combinations; in question 2b, uses the same approach, correctly, once again).

Understanding of Concepts
- The student identifies and describes most of the measurement relationships and the geometric properties by providing thorough explanations and illustrations (e.g., in Part Two, questions 1b and 1c, uses labelled illustrations, mathematical notation, and descriptions to explain the relationship between the area of a rectangle, a right-angled triangle, and an isosceles triangle).

Application of Mathematical Procedures
- The student constructs all or almost all of the possible rectangles on the geoboard (e.g., in Part One, questions 1a, 1b, and 1c, identifies and/or draws all possible rectangles; in question 2b, lists all of the ten possible combinations in a table).

Communication of Required Knowledge
- The student uses mathematical language clearly and precisely to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part Two, question 1a, states, “If the area was 12 units\(^2\), then you know that one possible set of dimensions is 3 by 4. You could also do 6 by 2 or 12 by 1. There, you already have 3 possibilities! So, the answer is no!”; in Part Two, question 1c, provides a concise explanation and uses appropriate drawings and mathematical notations to show the solution).

Comments/Next Steps
- The student should expand his or her use of precise mathematical language to explain geometric concepts and relationships.
- The student should include more diagrams to clarify his or her answers.
Polygons on the Geoboard  Level 4, Sample 2

Exploring Rectangles with Geoboards

You may use your geoboard or geopaper to help you for all of the questions below.

Part One

1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with the following perimeters? Show all of your work.

   a) 16 units?
      You can make 4 non-congruent rectangles.
      
      \[ 1 \times 7 \]
      \[ 2 \times 6 \]
      \[ 3 \times 5 \]
      \[ 4 \times 4 \]

   A rule is?
   Divide the expected perimeter into two groups. Then the number right below \[ \frac{1}{2} \times \text{number to 1 then } 1 \text{ less than } \text{ that with a and so on till you start repeating.} \]

   \[ N = \frac{2}{3} \text{ the number from } x \text{ and so on.}

   \text{Non-number}
   \[ x = \text{a numbers less than } N \]

   \[ 1 \times 7 \] \[ A=7 \]
   \[ 2 \times 6 \] \[ A=12 \]
   \[ 3 \times 5 \] \[ A=15 \]
   \[ 4 \times 4 \] \[ A=16 \]

b) 18 units?
   You can make 4 non-congruent rectangles with a perimeter of 18 units.
   
   \[ 1 \times 8 \]
   \[ 2 \times 7 \]
   \[ 3 \times 6 \]
   \[ 4 \times 5 \]

   A rule is?
   This number
   \[ \frac{w+1}{w} \frac{1}{w} \text{ etc.} \]
   \[ N = \text{number} \frac{w}{w+1} \text{ divide } N \text{ by } a, \text{ subtract 1,} \]
   \[ \frac{w}{w+1} \text{ with } x = n \text{ number put it with } a, \text{ then subtract } 1 \text{ by 1, put it with } a, \]
   \[ \text{then subtract } \frac{w}{w+1} \text{ with } 1, \text{ put it with } 3, \]
   \[ \text{and so on till you start repeating.} \]
C)

You can make 5 non-congruent rectangles with a perimeter of 30 units.

\[ \begin{align*}
1 \times 9 & \quad \text{A rule is?} \\
2 \times 8 & \quad \text{this number}
3 \times 7 & \quad N = -\frac{1}{\sqrt{3}}, x = \sqrt{3}, y = \sqrt{3} \frac{1}{\sqrt{3}}
4 \times 6 & \\
5 \times 5 & \\
\end{align*} \]

N: number
n: with
x: number divided by 6.

When you have a number; by \( \frac{1}{\sqrt{3}} \), subtract number by \( \frac{1}{\sqrt{3}} \), subtract 3 from number, and so on till you start repeating yourself.

\( \begin{align*}
2 \times 6 & \\
3 \times 5 & \\
4 \times 4 & \\
5 \times 3 & \\
6 \times 2 & \\
\end{align*} \)

D)

2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

- Length \times Width = Area
- The closer the rectangles shape gets to a square, the larger the area gets.

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

You could make 10 non-congruent rectangles if you had a geoboard with a perimeter of 42 units.

\( \begin{align*}
21 \times 1 & \\
19 \times 2 & \\
17 \times 3 & \\
15 \times 4 & \\
13 \times 5 & \\
9 \times 6 & \\
7 \times 7 & \\
5 \times 8 & \\
3 \times 9 & \\
\end{align*} \)

I divided 42 by 6 and subtracted 1 from 42. 42 \div 6 = 7, 7 - 1 = 6

Then multiplied 20 \times 1.

Then: 19 \times 2, 17 \times 3, 15 \times 4, 13 \times 5, etc.
Till I started repeating myself.
c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?

\[ 26 \div 2 = 13 \div 2 = 12 \]

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Area</th>
<th>If out of all the rectangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 12 x 1</td>
<td>12u²</td>
<td>rectangle f (area: 48 units²).</td>
</tr>
<tr>
<td>b) 11 x 2</td>
<td>22u²</td>
<td>Its area is 44 units².</td>
</tr>
<tr>
<td>c) 10 x 3</td>
<td>30u²</td>
<td>It would have the greatest area because (7 \times 6 = 42).</td>
</tr>
<tr>
<td>d) 9 x 4</td>
<td>36u²</td>
<td>The area of a square because (9 \times 4 = 36).</td>
</tr>
<tr>
<td>e) 8 x 5</td>
<td>40u²</td>
<td>If a rectangle and a square (40 units²) both had the same perimeter as the square always has the greater area, because of its dimensions.</td>
</tr>
<tr>
<td>f) 7 x 6</td>
<td>42u²</td>
<td></td>
</tr>
</tbody>
</table>

Part Two

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle. Justify your answer.

The formula for area \(A = \text{length} \times \text{width}\) are the dimensions of that quadrilateral. This only works for rectangles and squares because if you do it with a triangle you have to go \((1/2 \times \text{base} \times \text{height})\) and you could find some of the dimensions but not all of them.
b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.

![Diagram showing a right angle and measurements]

Yes, you could make a right-angled isosceles triangle if you do so.

![Diagram showing a triangle with specific measurements]

c) Finally, Maria stated that:

For every different sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas.

![Diagram showing calculations and reasoning]

For the area of a rectangle, double its dimension divided by 2, and you get the area of a triangle with the same area.
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy related to the calculation and comparison of the perimeters of rectangles and the areas of rectangles and triangles, arriving at a thorough and accurate solution (e.g., in Part One, question 2b, calculates the first two dimensions, then finds the other possible combinations with the aid of a table).

Understanding of Concepts
- The student identifies and describes most of the measurement relationships and the geometric properties by providing thorough explanations and illustrations (e.g., in Part One, question 2c, uses an organized chart to show all six possible rectangles with a perimeter of 26 units and identifies that they all have different areas).

Application of Mathematical Procedures
- The student constructs all or almost all of the possible rectangles on the geoboard (e.g., in Part One, questions 1a, 1b, 1c, 2b, and 2c, shows all possible combinations).

Communication of Required Knowledge
- The student uses mathematical language clearly and precisely to describe geometric concepts and relationships involving the dimensions, areas, and perimeters of rectangles and triangles (e.g., in Part One, question 1a, explains the relationship between the length and width of a rectangle and how these dimensions affect the area of the rectangle; in Part Two, question 1c, “Find the area of a rectangle double its dimension divide by 2 and you got the area of a triangle with the same area as [rectangle]”).

Comments/Next Steps
- The student should use multiple examples to reinforce his or her statements and explanations.
- The student needs to expand his or her use of illustrations when completing investigations involving relationships between the areas of rectangles and triangles.
Title: Polygons on the Geoboard

Time requirements: 160–200 minutes (total)
- 40–50 minutes for the pre-task
- three periods of 40–50 minutes each for the exemplar task

Description of the Task

This task requires students to:
- draw non-congruent rectangles of a given perimeter;
- investigate the relationship between the area and perimeter of rectangles;
- draw a graph comparing the lengths and widths of rectangles with a given perimeter;
- investigate the area relationships of rectangles, triangles, and parallelograms.

Students will use geoboards and geopaper to make as many different rectangles as possible with a given perimeter. They will look for a pattern in lengths and widths, and will use the pattern to make predictions. Next, they will draw a graph comparing the lengths and widths of rectangles with a given perimeter, and will describe the patterns shown. Finally, students will justify their answers to questions about the relationship between the area and dimensions of rectangles, and will demonstrate how to construct a rectangle, an isosceles triangle, and a right-angled triangle having the same area.

Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from the Ministry of Education’s Curriculum Unit Planner (CD-ROM).

Measurement

Students will:
1. solve problems related to the calculation and comparison of the perimeter and the area of regular polygons (6m44);
2. relate dimensions of rectangles and area to factors and products (in a rectangle 2 cm by 3 cm the side lengths are factors and the area, 6 cm², is the product of the factors) (6m56);
3. understand the relationship between the area of a parallelogram and the area of a rectangle, between the area of a triangle and the area of a rectangle, and between the area of a triangle and the area of a parallelogram (6m57);
4. understand the relationship between area and lengths of sides and between perimeter and lengths of sides for squares, rectangles, triangles, and parallelograms (6m59);
5. sketch a rectangle, square, triangle, or parallelogram given its area and/or perimeter (6m60).

Geometry and Spatial Sense

Students will:
6. identify, describe, compare, and classify geometric figures (6m64);
7. use mathematical language effectively to describe geometric concepts, reasoning, and investigations, and coordinate systems (6m69);
8. use mathematical language to describe geometric ideas (6m82);
9. explain, make conjectures about, and articulate hypotheses about geometric properties and relationships (6m85).

Teacher Instructions

Prior Knowledge and Skills Required

To complete this task, students should have some knowledge or skills related to the following:
- identifying, describing, comparing, and classifying geometric figures
- calculating the area and perimeter of rectangles and the area of triangles and parallelograms
- finding factors of a number
- the properties of right-angled and isosceles triangles

The Rubric*

The rubric provided with this exemplar task is to be used to assess students’ work. The rubric is based on the achievement chart given on page 9 of The Ontario Curriculum, Grades 1–8: Mathematics, 1997.

Before asking students to do the task outlined in this package, review with them the concept of a rubric. Rephrase the rubric so that students can understand the different levels of achievement.

Accommodations

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar task.

*The rubric is reproduced on page 58 of this document.
Materials and Resources Required
Before attempting the task, students should be provided with the following materials:
• a copy of the Student Package (see Appendix 1) for each student
• rulers
• erasers
• geoboards (1 for each student)
• geopaper
• rubber bands

Task Instructions
Introductory Activities
The pre-task is designed to review and reinforce the skills and concepts that students will be using in the exemplar task, and to model strategies useful in completing the task.

Pre-task: Making Quadrilaterals
Students work individually at their desks for this whole class activity.

Begin by posing the following question: “How many different quadrilaterals that have a perimeter of 16 units can be made on a geoboard?”

Give students time to investigate possible solutions and to record their solutions on geopaper.

You may use the following prompts for class discussion:
– “What do you notice about the area of your shapes?”
– “How did you know that you were finished?”
– “How do you know when you have found all the possible quadrilaterals?”
– “What patterns did you notice?”

Exemplar Task
1. Distribute a copy of the Student Package to each student.
2. Tell students that they are to work independently to complete the assigned task.
3. Make sure that each student has a large geoboard and geopaper to use for investigating and drawing the required figures.
4. Remind students about the rubric and make sure that each student has a copy of it.
5. The problem that the students will solve independently is provided in the worksheets in Appendix 1.

Appendix 1: Student Worksheets
Exploring Rectangles With Geoboards
You may use your geoboard and geopaper to help you for all of the questions below.

Part One
1. How many different (non-congruent) rectangles can you make on your geoboard or draw on your geopaper with each of the following perimeters? Show all of your work.
   a) 16 units?
b) 18 units?

c) 20 units?
2. a) What do you notice about the dimensions (length, width) and areas of the rectangles you have just drawn?

b) If you had a very large geoboard, how many non-congruent rectangles with a perimeter of 42 units would you be able to make? Give reasons to support your answer.

c) Of all the rectangles you can construct with a perimeter of 26 units, which would have the greatest area? Why?
Part Two

1. a) If you know the area of a rectangle, do you also know the dimensions (length and width) of the rectangle? Justify your answer.

b) For every rectangle you can make on the geoboard, would it be possible to make an isosceles and a right-angled triangle with the same area? Use diagrams to show your ideas.
c) Finally, Maria stated that:

For every different-sized rectangle you can build on the geoboard, you can also make an isosceles triangle with the same area and a right-angled triangle with the same area.

Give reasons to show why Maria is correct. Use diagrams to show your ideas.
Investigating Patterns With Tiles

The Task
This task required students to:
• investigate patterning problems using colour tiles and pattern blocks;
• explain or describe what they observed about continuations of the tile and pattern block patterns that they make.

Students used different-coloured tiles to create a linear pattern, recorded the pattern, and explained how to find the eightieth tile in the pattern. Next, they used colour tiles to create a pattern in which the fiftieth tile was red, described the process they used, and explained how to find the eightieth tile in the pattern. Then students used tiles and pattern blocks to make additional patterns, described what they observed about the patterns, and answered questions posed about continuations of the patterns. Finally, they each used pattern blocks to create a pattern, posed a question about the pattern, and showed how they would answer the question.

Expectations
This task gave students the opportunity to demonstrate their achievement of all or part of each of the following selected overall and specific expectations from the strand Patterning and Algebra. Note that the codes that follow the expectations are from the Ministry of Education’s Curriculum Unit Planner (CD-ROM).

Students will:
1. recognize and discuss the mathematical relationships between and among patterns (6m90);
2. identify, extend, and create patterns in a variety of contexts (6m91);
3. analyse and discuss patterning rules (6m92);
4. apply patterning strategies to problem-solving situations (6m94);
5. recognize relationships and use them to summarize and generalize patterns (e.g., in the number pattern 1, 2, 4, 8, 16, …, recognize and report that each term is double the term before it) (6m95);
6. use a calculator and computer applications to explore patterns (6m99);
7. pose and solve problems by recognizing a pattern (6m100);
8. discuss and defend the choice of a pattern rule (6m102).
Prior Knowledge and Skills

To complete this task, students were expected to have some knowledge or skills relating to the following:

• the properties of geometric figures
• pattern-rule development and articulation

For information on the process used to prepare students for the task and on the materials, resources, and equipment required, see the Teacher Package reproduced on pages 143–147 of this document.
The Ontario Curriculum – Exemplars, Grade 6: Mathematics

Task Rubric – Investigating Patterns With Tiles

<table>
<thead>
<tr>
<th>Expectations*</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem solving</strong></td>
<td>The student:</td>
<td>selects and applies a problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at an incomplete or inaccurate solution</td>
<td>selects and applies a problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a partially complete and/or partially accurate solution</td>
<td>selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a generally complete and accurate solution</td>
</tr>
<tr>
<td>4, 7</td>
<td>selects and applies a problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at an incomplete or inaccurate solution</td>
<td>selects and applies a problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a partially complete and/or partially accurate solution</td>
<td>selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a generally complete and accurate solution</td>
<td>selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a thorough and accurate solution</td>
</tr>
<tr>
<td><strong>Understanding of concepts</strong></td>
<td>The student:</td>
<td>demonstrates a limited understanding of patterns when identifying and creating patterns using the colour tiles</td>
<td>demonstrates some understanding of patterns when identifying and creating patterns using the colour tiles</td>
<td>demonstrates a general understanding of patterns when identifying and creating patterns using the colour tiles</td>
</tr>
<tr>
<td>1, 2, 5</td>
<td>demonstrates a limited understanding of how to explain and generalize a pattern</td>
<td>demonstrates some understanding of how to explain and generalize a pattern</td>
<td>demonstrates a general understanding of how to explain and generalize a pattern</td>
<td>demonstrates a thorough understanding of how to explain and generalize a pattern</td>
</tr>
<tr>
<td>Expectations*</td>
<td>Level 1</td>
<td>Level 2</td>
<td>Level 3</td>
<td>Level 4</td>
</tr>
<tr>
<td>--------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td><strong>Application of mathematical procedures</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The student:</td>
<td>uses computations and mathematical procedures that include many errors and/or omissions</td>
<td>uses computations and mathematical procedures that include some errors and/or omissions</td>
<td>uses computations and mathematical procedures that include few errors and/or omissions</td>
<td>uses computations and mathematical procedures that include few, if any, minor errors and/or omissions</td>
</tr>
<tr>
<td>2, 4, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Communication of required knowledge</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The student:</td>
<td>uses mathematical language and notation with limited clarity to analyse and describe patterning rules</td>
<td>uses mathematical language and notation with some clarity to analyse and describe patterning rules</td>
<td>uses mathematical language and notation clearly to analyse and describe patterning rules and extensions of patterns</td>
<td>uses mathematical language and notation clearly and precisely to analyse and describe patterning rules and extensions of patterns</td>
</tr>
<tr>
<td>1, 3, 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The expectations that correspond to the numbers given in this chart are listed on page 106.

Note: This rubric does not include criteria for assessing student performance that falls below level 1.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern. 
   Record your pattern below.

   
   Gybrybrybrybrybrybrybryb6b

   b) Explain how you could find the colour of the 80th tile.

   I would find it by seeing which colour is number 78. Then after 8 of them it would be 80.

   c) How would a calculator help you to find the colour of the 80th tile?
      You could times the tenth colour by 8 and you would get 80.

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.

   a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

   
   GbbyGyb6byfrbrbr

   b) Describe how you made sure that your pattern included a red tile for the 50th tile.

   The red one has to be number 10 then after five of them have passed it will be number 50.
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

I would find it by getting the 10th colour and after 8 of them have gone by I would know it was number 80.

d) Explain how a calculator would help you to find the colour of the 80th tile.

I would times the tenth colour by 8.

3. For this task you will be joining squares side by side as shown below to make rectangles.

Stage 1  Stage 2  Stage 3

a) Describe all the patterns you observe as you add squares as shown above.

| The first one goes up by 3 the second one goes up by 4 the third goes up by 6 and so on. |
|---|---|---|
| before | after |
| 1 | 4 |
| 2 | 8 |
| 3 | 10 |

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

You just add 2 onto the aster.

<table>
<thead>
<tr>
<th>before</th>
<th>after</th>
<th>best aster</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12</td>
<td>16 32</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>17 34</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>18 36</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>19 38</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>20 40</td>
</tr>
</tbody>
</table>
4. Let us now repeat the same task, this time with equilateral triangles.
   Select triangles from your pattern blocks.
   
   a) Place them side by side as shown below.

   Stage 1  Stage 2  Stage 3

   b) Describe all the patterns you notice as triangles are added as shown above.
   
   \[\text{Each time another triangle is added, the perimeter goes up by one.}\]

   c) Describe how you would find the perimeter of the shape in Stage 30. Since it goes up by one, if 29 triangles were put together it would equal 30.

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

   \[\text{\begin{tikzpicture}
   \node (a) at (0,0) [draw] {}; \node (b) at (1,0) [draw] {}; \node (c) at (2,0) [draw] {};
   \end{tikzpicture}}\]

   b) Pose a question related to your pattern.
   \[\text{How many blocks would you have to use to get 20?}\]

   c) Show how you would answer the question you posed.
   \[\text{You would have to put 18 blocks on to get 20.}\]
Teacher’s Notes

Problem Solving
- The student selects and applies a problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at an incomplete or inaccurate solution (e.g., draws diagrams of the coloured tiles; in question 1b, “I would find it by seeing which colour is number is 10, then after 8 of them it would be 80”; in question 2a, repeats only part of the pattern trying to arrive at the solution).
- The student selects and applies a problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at an incomplete or inaccurate solution (e.g., in question 3a, draws a T-chart and identifies the pattern to determine perimeter; in question 3b, applies the same strategy with limited accuracy).

Understanding of Concepts
- The student demonstrates a limited understanding of patterns when identifying and creating patterns using the colour tiles (e.g., in questions 1a and 2a, uses all required tiles but alters the patterns initiated part-way through the pattern).
- The student demonstrates a limited understanding of how to explain and generalize a pattern (e.g., in question 3a, “The first one goes up by 3 the second one goes up by 4 the third goes up by 5 and so on”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include many errors and/or omissions (e.g., all patterning activities are attempted, but many incorrect solutions are presented).

Communication of Required Knowledge
- The student uses mathematical language and notation with limited clarity to analyse and describe patterning rules (e.g., provides short statements that are lacking in detail and supporting evidence, such as in question 3b, “you just add 2 onto the after”).

Comments/Next Steps
- The student should use mathematical notation and language to explain processes and patterns.
- The student should try to extend patterns using colour tiles and other concrete materials.
- The student should use a calculator to complete calculations, increase his or her accuracy, and explore the use of repeated addition for extending patterns.
- The student should try to make more complex repeating patterns with colour tiles.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern. Record your pattern below.

GBB BRRRYYG BBB BRRRYYGB …

b) Explain how you could find the colour of the 80th tile.

You could continue the pattern or use a calculator.

c) How would a calculator help you to find the colour of the 80th tile? You could keep adding 8 until you got to 80.

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.

a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

YYGGGRRRBBB

b) Describe how you made sure that your pattern included a red tile for the 50th tile.

I did this:

YYGGGRRRYYGGGRRRYYGGGRRR
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

I know it would be red because of what I did for part b.

d) Explain how a calculator would help you to find the colour of the 80th tile.

You would just have to keep adding 10.
Or you could just look at the 10th colour and you would see it is red.

3. For this task you will be joining squares side by side as shown below to make rectangles.

Stage 1  Stage 2  Stage 3

a) Describe all the patterns you observe as you add squares as shown above.

For stage one there is one square for step 2 there are 2 squares etc. When you add a square the rectangle becomes longer.

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

The perimeter of stage 20 would be 80 units.
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

![Triangle Illustrations]

Stage 1  Stage 2  Stage 3

b) Describe all the patterns you notice as triangles are added as shown above.

Stage one has 1 shape. Stage 2 has 2 shapes. Each time you add to it the shape will become either a rhombus or trapezoid.

c) Describe how you would find the perimeter of the shape in Stage 30.

The perimeter of the stage 30 would be $30^2$ units.

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

![Pattern Illustration]

b) Pose a question related to your pattern.

How could you find out what the 30th shape would look like? Would it be like this:

or like this:

or like this:

c) Show how you would answer the question you posed.

Have them continue the pattern to the 30th one or find their own way.
Teacher’s Notes

Problem Solving
- The student selects and applies a problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at an incomplete or inaccurate solution (e.g., in question 1a, draws a pattern that indicates a break before repeating the pattern, but makes an error; in question 2a, develops a simple pattern by grouping the tiles of each colour).
- The student selects and applies a problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at an incomplete or inaccurate solution (e.g., in questions 3b and 4c, recognizes some patterns, but uses an inappropriate strategy to find the perimeter of the 20th and 30th stages).

Understanding of Concepts
- The student demonstrates a limited understanding of patterns when identifying and creating patterns using the colour tiles (e.g., in question 2a, creates a pattern; however, in question 2b, omits part of the pattern when identifying the 50th tile).
- The student demonstrates a limited understanding of how to explain and generalize a pattern (e.g., in question 3a, “For stage one there is one square, for step 2 there are 2 squares ect. When you add a square the rectangle becomes longer”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include many errors and/or omissions (e.g., in question 2a and 2b, creates a pattern that will not result in the 50th tile being red).

Communication of Required Knowledge
- The student uses mathematical language and notation with limited clarity to analyse and describe patterning rules (e.g., in questions 3b and 4c, uses exponents incorrectly; in question 4b, “Stage one has one Shape Stage 2 has 2 Shapes ect. Each time you add to it the shape will become either a rhombus or trapezoid”).

Comments/Next Steps
- The student should develop his or her use of mathematical language and notation to explain processes and patterns clearly.
- The student should explore the extension of patterns using colour tiles and other concrete materials to relate them to the first term.
- The student should try to make more complex repeating patterns using colour tiles.
- The student needs to use a calculator to complete calculations, increase his or her accuracy, and explore the use of repeated addition for extending patterns.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern. Record your pattern below.

c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

To find the colour of the 80th tile you have to divide 15 from 80 so you get 75 so subtract it from 80 after you do this you get 5 left you go through your pattern you can figure out your colour.

```
    1 2 3 4 5
    6 7 8 9 0
    1 2 3 4 5
```
your colour is red it's the 5th one.

d) Explain how a calculator would help you to find the colour of the 80th tile.

You could divide 15 from 80 on the calculator and get 75 so in all the steps you go through on the question can be easily used on a calculator.

3. For this task you will be joining squares side by side as shown below to make rectangles.

```
    
    
    
```

Stage 1  Stage 2  Stage 3

a) Describe all the patterns you observe as you add squares as shown above.

Each stage you go on to you add 1 on and it first starts out as a square and as you move on it gets to be a long rectangle it make sense because example stage

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

The perimeter is 42...... because if you follow the rule you will get the answer really quick.
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

![Stage 1 Stage 2 Stage 3]

b) Describe all the patterns you notice as triangles are added as shown above.

Every time you go to a different stage there are new triangles there are. Example $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_4$
you make different shapes all the time.

c) Describe how you would find the perimeter of the shape in Stage 30, you could draw a picture or a chart.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
</tr>
<tr>
<td>11</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>31</td>
</tr>
<tr>
<td>13</td>
<td>32</td>
</tr>
<tr>
<td>14</td>
<td>33</td>
</tr>
<tr>
<td>15</td>
<td>34</td>
</tr>
<tr>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>17</td>
<td>36</td>
</tr>
</tbody>
</table>

I got this answer by stopping at 10 and 8 by 3 and that's 30 than + 12 = 32 and got 36.

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

b) Pose a question related to your pattern.

Explain how you can find how many demands there are in stage 20.

c) Show how you would answer the question you posed. You answer it like this:

every time you go to a new stage like $s_1$ you add 1 on to it so when you get to 20 you should have 20 blocks on stage 20.
Teacher’s Notes

Problem Solving
– The student selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a partially complete and/or partially accurate solution (e.g., in question 1a, only uses 9 of 26 coloured tiles in the pattern).
– The student selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a partially complete and/or partially accurate solution (e.g., in question 3b, draws the square tiles to find the perimeter with 20 tiles, but in question 4c, uses an inappropriate strategy to find the perimeter of 30 shapes, resulting in an inaccurate answer).

Understanding of Concepts
– The student demonstrates some understanding of patterns when identifying and creating patterns using the colour tiles (e.g., in question 1a, “If you put the pattern with 26 blocks you would be 1 short because there are only 3 grey [green] blocks”; in question 2a, provides a more accurate pattern by drawing 75 individual tiles).
– The student demonstrates some understanding of how to explain and generalize a pattern (e.g., in question 3a, “Each stage you go on to you add 1 on”; in question 4b, “Every time you go to a different stage thats how many triangles there are”).

Application of Mathematical Procedures
– The student uses computations and mathematical procedures that include some errors and/or omissions (e.g., in question 4c, identifies a pattern for the increasing perimeters and attempts to extend triangle patterns, but errors are evident in the reasoning: “I got this answer by stopping at 10 and x by 3 and thats 30 then 1 x 12 by 3 and got 36”).

Communication of Required Knowledge
– The student uses mathematical language and notation with some clarity to analyse and discuss patterning rules (e.g., attempts to use pictures, numbers, and words in his or her explanations; in question 4b, describes what he or she sees, but provides descriptions that are not precise: “you make different shapes all the time”).

Comments/Next Steps
– The student needs to investigate a number of different ways to describe patterns.
– The student should expand his or her use of mathematical language and notation to describe extensions of patterns.
– The student should try to make more complex patterns with a given number of tiles.
– The student needs to use a calculator to complete calculations, increase his or her accuracy, and explore the use of repeated addition for extending patterns.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern. Record your pattern below.

   [pattern illustration]

   b) Explain how you could find the colour of the 80th tile.

      You can find the answer if you keep on going with the pattern. The colour is blue.

   c) How would a calculator help you to find the colour of the 80th tile?

      You just times it by 3, 3, 3.

---

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.

   a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

   [pattern illustration]

   b) Describe how you made sure that your pattern included a red tile for the 50th tile.

      Because you can times it by 15 times 3 = 45 add 5 = 50 then it's red then you get your answer.
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

If the colour is red at the 50th tile you just have to add 30 and its 80th tile and its red, red.

d) Explain how a calculator would help you to find the colour of the 80th tile.

Well I would times 15 by 5 = 75 + 5 = 80 then its red.

3. For this task you will be joining squares side by side as shown below to make rectangles.

```
  Stage 1  Stage 2  Stage 3
  _______  _______  _______
```

a) Describe all the patterns you observe as you add squares as shown above.

I just add 2 on each time then I got the answer.

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

I got a scrap piece of paper and put stage and perimeter and 1/5 20 for stage and 9 put 4 units at the start to the finish and I got 42 units.
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

![Triangle stages](image)

Stage 1  Stage 2  Stage 3

b) Describe all the patterns you notice as triangles are added as shown above.

I see a pyramid, triangles together.

c) Describe how you would find the perimeter of the shape in Stage 30.

I got a scrap piece of paper and put stage and its 20 for stage and put 3 units at the start to the finish and I got 61 units.
4c) Stage | perimeter

1 | 3 units
2 | 5 units
3 | 7 units
4 | 9 units
5 | 11 units
6 | 13 units
7 | 15 units
8 | 17 units
9 | 19 units
10 | 21 units
11 | 23 units
12 | 25 units
13 | 27 units
14 | 29 units
15 | 31 units
16 | 33 units
17 | 35 units
18 | 37 units
19 | 39 units
20 | 41 units
21 | 43 units
22 | 45 units
23 | 47 units
24 | 49 units
25 | 51 units
26 | 53 units
27 | 55 units
28 | 57 units
29 | 59 units
30 | 61 units

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

b) Pose a question related to your pattern.

What would stage 12 be for the perimeter?

C) Show how you would answer the question you posed.

I would answer it like this: stage 12 and perimeter 48.
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a partially complete and/or partially accurate solution (e.g., in question 1a, draws the coloured tiles, but the repeating pattern is not accurate with the inclusion of 2 red tiles in the 25th and 26th places; in question 2b, appropriately uses groups of 15 to extend the pattern to find the 50th tile).
- The student selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a partially complete and/or partially accurate solution (e.g., in question 3b, uses a table accurately to find the perimeter of the rectangle made by joining 20 squares, but in questions 4c and 5c, applies the same strategy inaccurately).

Understanding of Concepts
- The student demonstrates some understanding of patterns when identifying and creating a pattern using colour tiles (e.g., in question 1a, creates a repeating pattern of 8 colour tiles but adds an extra 2 red tiles at the end to use up the 26 tiles).
- The student demonstrates some understanding of how to explain and generalize a pattern (e.g., explains the attempted pattern from question 1a as 3, 2, 2, 1 as evident in question 1c, but does not explain the two extra red tiles at the end of the linear pattern).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include some errors and/or omissions (e.g., in question 4c, uses “a scrap piece of paper and put stage and perimeter and its 20 for Stage and i put 3 units at the start to the finish and i got 61 units”, resulting in an incorrect solution).

Communication of Required Knowledge
- The student uses mathematical language and notation with some clarity to analyse and describe patterning rules (e.g., in question 5a, “I just add 2 on each time” [referring to perimeter units], and in question 3b, describes the chart made to answer the question: “put stage and perimeter and its 20 for stage and i put 4 units at the start to the finish and i got 42 units”).

Comments/Next Steps
- The student should use appropriate and clear mathematical language to describe the processes required to extend patterns.
- The student should develop effective strategies for extending patterns.
- The student should use a calculator to complete calculations, increase his or her accuracy, and explore the use of repeated addition for extending patterns.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern.
   Record your pattern below.

   \[ \text{G G S Y Y Y Y Y R R R R R R R B B B B B} \]
   \[ \text{B - BLUE} \quad \text{G - GREEN} \]
   \[ \text{R - RED} \quad \text{Y - YELLOW} \]

   b) Explain how you could find the colour of the 80th tile.
   I know that there are 36 numbers, so I multiplied 36 by 3. The answer was 72. So the 78th tile was the colour was blue. The next three numbers were all green. I only needed two more. The colour of the 80th tile was green.

   c) How would a calculator help you to find the colour of the 80th tile?
   A calculator could help me by getting the number 72, and then adding two would help me get the answer.

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.

   a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

   \[ \text{G G G G G R R R R R R Y Y Y B B B B B B} \]
   \[ \text{G - GREEN} \]
   \[ \text{R - RED} \]
   \[ \text{Y - YELLOW} \]
   \[ \text{B - BLUE} \]

   b) Describe how you made sure that your pattern included a red tile for the 50th tile.
   I know that 15 x 3 = 45, so I knew that the fifth number after that had to be red. So I made my pattern so that the fifth number after 45, and I made red the 50th number in the pattern.
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.
   I am already at 50, so I just continue the pattern and the 80th tile is the colour is red.

   d) Explain how a calculator would help you to find the colour of the 80th tile.
   A calculator would help me find the number 75 and then I would only have to add five.

D

3. For this task you will be joining squares side by side as shown below to make rectangles.

   Stage 1
   Stage 2
   Stage 3

   a) Describe all the patterns you observe as you add squares as shown above.
   as you move up to a higher stage the rectangle gets longer and longer.

   b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.
   The perimeter of the rectangle in stage 20 would be 40 units. I found out the answer by figuring out the width and the height of the rectangle. The width of the rectangle is 30. 20 + 20 equals 40. The height of the rectangle is 40.
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

```
\[ \begin{array}{ccc}
\text{Stage 1} & \text{Stage 2} & \text{Stage 3} \\
\end{array} \]
```

b) Describe all the patterns you notice as triangles are added as shown above.

In Stage 1, the shape was a triangle, in Stage 2, the triangle turned into a rhombus, and in Stage 3, the rhombus turned into a trapezoid.

c) Describe how you would find the perimeter of the shape in Stage 30.

I know that in Stage 30 the answer to the perimeter is 35 because in Stage 30 the perimeter is 35.

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

```
\[ \begin{array}{ccc}
\text{Stage 1} & \text{Stage 2} & \text{Stage 3} \\
\end{array} \]
```

b) Pose a question related to your pattern.

In Stage 4 what would the pattern look like.

c) Show how you would answer the question you posed.

```
\[ \begin{array}{ccc}
\end{array} \]
```

This is what Stage 4 would look like.
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a generally complete and accurate solution (e.g., in questions 1a and 2a, attempts to solve the problems through the use of diagrams; in question 2b, uses number patterns to find the answer: “I know that $15 \times 3 = 45$, so I knew that the fifth number after that had to be red”).
- The student selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a generally complete and accurate solution (e.g., in question 3b, uses numbers and logical reasoning to find the perimeter in stage 20).

Understanding of Concepts
- The student demonstrates a general understanding of patterns when identifying and creating patterns using the colour tiles (e.g., in question 1b, extends the pattern to determine the colour of the 80th tile).
- The student demonstrates a general understanding of how to explain and generalize a pattern (e.g., in question 2b, “I know that $15 \times 3 = 45$, so I knew that the fifth number after that had to be red”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include few errors and/or omissions (e.g., creates all patterns accurately, although the patterns are simple with tiles of the same colour grouped together throughout the pattern).

Communication of Required Knowledge
- The student uses mathematical language and notation clearly to analyse and describe patterning rules and extensions of patterns (e.g., in question 1b, “I know that there are 26 numbers, so I multiplied $26 \times 3$. The answer was 78 … I only needed two more”; uses clear, appropriately labelled diagrams).

Comments/Next Steps
- The student should expand his or her use of mathematical language and notation to analyse and discuss patterning rules and extensions of patterns.
- The student should try to make more complex patterns using colour tiles and other concrete materials.
- The student should investigate a number of ways of describing patterns.
- The student should explore the use of the repeat function on a calculator for extending patterns.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern.
   Record your pattern below.

\[
\begin{align*}
BBB & BB RR RR RR \\ Y & YY YY YG G
\end{align*}
\]

b) Explain how you could find the colour of the 80th tile.
   You could find the colour of the 80th tile by multiplying 36 x a number. You find 
   that number by multiplying 6 x 1-2-3-4 or until the answer is close to 80, so 
   in this case it is 6 x 3 which is 78 
   so you have to add 2 which means 
   you 'move a tiles in, the 2nd tile in' 
   c) How would a calculator help you to find the colour of the 80th tile? It would help you multiply quicker.

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.
   a) Construct a linear pattern with all of the tiles so that the 50th tile is red.
   
   \[
   \begin{align*}
   B & B B R R R R Y Y G G G B Y Y
   \end{align*}
   \]

b) Describe how you made sure that your pattern included a red tile for the 50th tile.
   I multiplied 15 x 3 which is 45 but I needed fifty so I added 45 + 5 = 50. 
   Then I had to move 5 tiles down because I added 5 and it was RED. So I know RED was the 50th tile
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

You find the colour of the 80th square by looking at the 50th tile which is red so the 80th tile is going to be red. I knew this because there are 15 tiles so you add 15 + 15 = 30 then 30 + 50 = 80 so it’ll be the same colour.

d) Explain how a calculator would help you to find the colour of the 80th tile.

It would help you add quicker with high numbers.

3. For this task you will be joining squares side by side as shown below to make rectangles.

<table>
<thead>
<tr>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Describe all the patterns you observe as you add squares as shown above.

It gets bigger and bigger every time you add a square.

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

$$\text{perimeter} = 42 \text{ cm}$$
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

Stage 1  Stage 2  Stage 3

b) Describe all the patterns you notice as triangles are added as shown above.

I noticed that in stage 1 it's a triangle in stage 2 it's a parallelogram in stage 3 it's a trapezoid in stage 4 it would be a \( \square \) parallelogram.

c) Describe how you would find the perimeter of the shape in Stage 30.

\[
\text{perimeter} = 3a \text{ cm} \\
\frac{\text{area}}{15} = \frac{15}{3a} \text{ cm} \\
\]

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

b) Pose a question related to your pattern.

What would the area be in stage 15.

c) Show how you would answer the question you posed.

\[
\begin{array}{cccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\end{array}
\]

Area = 15 hexagonal units
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a generally complete and accurate solution (e.g., attempts to solve problems through the use of diagrams; in question 1b, uses trial and error).
- The student selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a generally complete and accurate solution (e.g., in question 4c, draws the shape and creates his or her own notation).

Understanding of Concepts
- The student demonstrates a general understanding of patterns when identifying and creating patterns using colour tiles (e.g., in question 1b, extends the pattern to determine the colour of the 80th tile).
- The student demonstrates a general understanding of how to explain and generalize a pattern (e.g., in question 1b, “... 26 x 3 which is 78 so you have to add 2...”; in question 4b, “in Stage 1 it’s a triangle in stage 2 it’s a parallelogram in Stage 3 it’s a trapezoid in Stage 4 it would be a parallelogram”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include few errors and/or omissions (e.g., the pattern in question 1a is simple, with tiles of the same colour grouped together – 9 blue, 8 red, and so on – but in question 2a, the pattern is more complex).

Communication of Required Knowledge
- The student uses mathematical language and notation clearly to analyse and discuss patterning rules and extensions of patterns (e.g., in question 2b, “I multiplied 15 x 3 which is 45 but I needed fifty so I added 45 + 5 = 50. Then I had to move 5 tiles down ...”; uses clear, appropriately labelled diagrams throughout the task).

Comments/Next Steps
- The student should expand his or her use of mathematical language and appropriate notation to explain processes and patterns.
- The student should examine and clearly communicate patterns involving perimeter.
- The student should explore the use of the repeat function on a calculator for extending patterns.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

a) Use all of the pieces to make a linear (straight-line) pattern. Record your pattern below.

My pattern is:

b) Explain how you could find the colour of the 80th tile.

I could multiply 26 by 3 to get 78. Then I just counted from the beginning of the linear 2 tiles and the 80th tile is blue.

c) How would a calculator help you to find the colour of the 80th tile?

I could use the calculator to multiply 26 by 3.

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.

a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

My pattern is:

gbbyrgbyrrgbbyr

b) Describe how you made sure that your pattern included a red tile for the 50th tile.

I made sure by first counting the number of tiles. With the number tiles, 15 times I multiplied 15 by 3 to get 45. Next I just counted from the beginning of the linear 5 tiles.
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

I would find it the same way I multiplied the number of tiles 15 by 5 to get 75. Then I counted from the beginning of the linear 5 tiles and it ends up being the same tile as the 50th tile.

d) Explain how a calculator would help you to find the colour of the 80th tile.

I could use a calculator to multiply 15 by 5 and add 5.

\[
\frac{15 \times 5}{75} + 5 = 80
\]
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

Stage 1  Stage 2  Stage 3

b) Describe all the patterns you notice as triangles are added as shown above.

<table>
<thead>
<tr>
<th>Stage #</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1x1 + 2 = 3</td>
</tr>
<tr>
<td>2</td>
<td>2x1 + 2 = 4</td>
</tr>
<tr>
<td>3</td>
<td>3x1 + 2 = 5</td>
</tr>
<tr>
<td>4</td>
<td>4x1 + 2 = 6</td>
</tr>
<tr>
<td>5</td>
<td>5x1 + 2 = 7</td>
</tr>
</tbody>
</table>

Stage #1 x 1 + 2 = perimeter

| Stage 30 - 30 x 1 + 2 = 32 |

b) Pose a question related to your pattern.

Explain what the pattern is and tell how you would find the area and perimeter for stage 9.5.

(c) Show how you would answer the question you posed.

<table>
<thead>
<tr>
<th>Stage #</th>
<th>P = Stage # x 2 + 2</th>
<th>A = LW</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 x 2 + 2 = 6 units</td>
<td>1 x 2 = 2 units²</td>
</tr>
<tr>
<td>3</td>
<td>3 x 2 + 2 = 8 units</td>
<td>1 x 3 = 3 units²</td>
</tr>
<tr>
<td>4</td>
<td>4 x 2 + 2 = 10 units</td>
<td>1 x 4 = 4 units²</td>
</tr>
<tr>
<td>5</td>
<td>5 x 2 + 2 = 12 units</td>
<td>1 x 5 = 5 units²</td>
</tr>
<tr>
<td>95</td>
<td>95 x 2 + 2 = 196 units</td>
<td>1 x 95 = 95 units²</td>
</tr>
</tbody>
</table>
Teacher’s Notes

Problem Solving
- The student selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a thorough and accurate solution (e.g., in question 2b, organizes the pattern in groups of 15 to find that the 50th tile is red).
- The student selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a thorough and accurate solution (e.g., in question 3b, develops an equation to find the perimeter: “For the perimeter of the rectangles one pattern is multiply the stage number by 2 and add 2”).

Understanding of Concepts
- The student demonstrates a thorough understanding of patterns when identifying and creating patterns using the colour tiles (e.g., in question 1b, extends the pattern to determine the colour of the 80th tile; in question 3a, uses an equation).
- The student demonstrates a thorough understanding of how to explain and generalize a pattern (e.g., in question 4b, “stage # x 1 + 2 = perimeter”).

Application of Mathematical Procedures
- The student uses computations and mathematical procedures that include few, if any, minor errors and/or omissions (e.g., in questions 3b and 4c, uses equations to solve the problems).

Communication of Required Knowledge
- The student uses mathematical language and notation clearly and precisely to analyse and describe patterning rules and extensions of patterns (e.g., in question 2c, “… I multiplied the number of tiles 15 by 5 to get 75. Then I counted from the beginning of the linear 5 tiles and it ends up being the same tile as the 50th tile”; throughout the task, provides diagrams that are clearly labelled and accurate).

Comments/Next Steps
- The student should explore the use of the repeat function on a calculator for extending patterns.
- The student should continue to develop his or her use of mathematical language to describe patterns.
Investigating Patterns with Tiles

For question 1 you will need 8 red tiles, 9 blue tiles, 3 green tiles and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern.
   Record your pattern below.
   \[ \text{Red} = \text{Red} + \text{Red} \]
   \[ \text{Blue} = \text{Blue} + \text{Blue} \]
   \[ \text{Green} = \text{Green} + \text{Green} \]
   \[ \text{Yellow} = \text{Yellow} + \text{Yellow} \]

b) Explain how you could find the colour of the 80th tile.

2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles and 3 yellow tiles.

   a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

   b) Describe how you made sure that your pattern included a red tile for the 50th tile.
c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

The pattern is actually a simple repeating sequence. The pattern repeats every 5 tiles, and the colour of the 80th tile can be found by dividing 80 by 5 and determining the remainder of the division. The remainder will tell you which colour is the 80th tile. For example, if 80 divided by 5 equals 16 with a remainder of 0, the 80th tile will be red (the 0th tile in the pattern).

d) Explain how a calculator would help you to find the colour of the 80th tile.

A calculator would be useful for determining the number of complete cycles of the pattern within 80 tiles. To find the remainder, you can use the modulo function on a calculator. For example, if you divide 80 by 5, the calculator will show a quotient of 16 and a remainder of 0, indicating that the 80th tile is red.

3. For this task you will be joining squares side by side as shown below to make rectangles.

Stage 1

Stage 2

Stage 3

a) Describe all the patterns you observe as you add squares as shown above.

Stage 1: 1 square
Stage 2: 3 squares
Stage 3: 5 squares

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

Stage 20: When you observe the above pattern and find out some information, you find out that 40 squares is the shape number from Stage 1, so it equals 40 squares. To find the shape number from Stage 20, you multiply 1 by 20. So, Stage 20 will be 40 squares. Multiply 40 squares by 4 to get 160 squares. So, the perimeter of the rectangle is 480 squares.
4. Let us now repeat the same task, this time with equilateral triangles.

Select triangles from your pattern blocks.

a) Place them side by side as shown below.

Stage 1  Stage 2  Stage 3

b) Describe all the patterns you notice as triangles are added as shown above.

One pattern is that on every odd stage a new triangle is added. Also, the number of triangles on an even stage is twice the number of triangles on the preceding odd stage.

In the pattern above, the number of triangles on Stage 2 is 2, and the number on Stage 3 is 4, which is twice the number on Stage 2.

5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

b) Pose a question related to your pattern.

What patterns do you see in the above growth pattern? How can you use these patterns to find how many squares and how many triangles would be in Stage 15?

c) Show how you would answer the question you posed.
Teacher’s Notes

Problem Solving
– The student selects and applies an appropriate problem-solving strategy when using colour tiles and pattern blocks to explore and investigate patterns, arriving at a thorough and accurate solution (e.g., in question 1a, develops a repeating pattern of 9 tiles and shows it in three ways, omitting the 27th tile, since only 26 tiles are required; does not restrict the pattern to or organize it around 26 tiles).
– The student selects and applies an appropriate problem-solving strategy to recognize patterns in the lengths, widths, and perimeters of linear pattern shapes formed by joining squares and by joining triangles, arriving at a thorough and accurate solution (e.g., in question 3a, uses a table to organize data).

Understanding of Concepts
– The student demonstrates a thorough understanding of patterns when identifying and creating patterns using the colour tiles (e.g., in question 5a, accurately develops a complex pattern).
– The student demonstrates a thorough understanding of how to explain and generalize a pattern (e.g., in question 4b, provides a detailed solution, including small illustrations, to support the statement made).

Application of Mathematical Procedures
– The student uses computations and mathematical procedures that include few, if any, minor errors and/or omissions (e.g., in question 1, represents patterns in multiple ways; in question 2, creates a repeating pattern that is 10 tiles long and uses this information to find the 50th tile accurately; in question 3b, correctly applies the formula for finding the perimeter of a rectangle to find the solution).

Communication of Required Knowledge
– The student uses mathematical language and notation clearly and precisely to analyse and describe patterning rules and extensions of patterns (e.g., in question 5c, identifies patterns and fully explains them using words, charts, and numbers; provides diagrams that are accurate and clearly labelled).

Comments/Next Steps
– The student should explore the use of the repeat function on a calculator for extending patterns.
– The student needs to continue using precise mathematical language to explain his or her thinking.
# Teacher Package

## Mathematics Exemplar Task

**Grade 6 – Patterning and Algebra**

**Title:** Investigating Patterns With Tiles

**Time requirements:**
- 160–200 minutes (total)
- 40–50 minutes for the pre-task
- three periods of 40–50 minutes for the exemplar task

### Description of the Task

This task requires students to:

- investigate patterning problems using colour tiles and pattern blocks;
- explain or describe what they observe about continuations of the tile and pattern block patterns that they make.

Students will use different-coloured tiles to create a linear pattern, record the pattern, and explain how to find the eightieth tile in the pattern. Next, they will use colour tiles to create a pattern in which the fiftieth tile is red, describe the process they used, and explain how to find the eightieth tile in the pattern. Then students will use tiles and pattern blocks to make additional patterns, describe what they observe about the patterns, and answer questions posed about continuations of the patterns. Finally, they will each use pattern blocks to create a pattern, pose a question about the pattern, and show how they would answer the question.

### Expectations Addressed in the Exemplar Task

Note that the codes that follow the expectations are from the Ministry of Education’s *Curriculum Unit Planner* (CD-ROM).

<table>
<thead>
<tr>
<th>Students will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. recognize and discuss the mathematical relationships between and among patterns (6m90);</td>
</tr>
<tr>
<td>2. identify, extend, and create patterns in a variety of contexts (6m91);</td>
</tr>
<tr>
<td>3. analyse and discuss patterning rules (6m92);</td>
</tr>
<tr>
<td>4. apply patterning strategies to problem-solving situations (6m94);</td>
</tr>
</tbody>
</table>

### Teacher Instructions

**Prior Knowledge and Skills Required**

To complete this task, students should have some knowledge or skills related to the following:

- the properties of geometric figures
- pattern-rule development and articulation

**The Rubric**

The rubric provided with this exemplar task is to be used to assess students’ work. The rubric is based on the achievement chart given on page 9 of *The Ontario Curriculum, Grades 1–8: Mathematics, 1997*.

Before asking students to do the task outlined in this package, review with them the concept of a rubric. Rephrase the rubric so that students can understand the different levels of achievement.

**Accommodations**

Accommodations that are normally provided in the regular classroom for students with special needs should be provided in the administration of the exemplar task.

**Materials and Resources Required**

Before attempting the task, students should be provided with the following materials:

- a copy of the Student Package (see Appendix 1) for each student
- colour tiles
- pattern blocks (squares and triangles)
- a calculator for each student

---

5. recognize relationships and use them to summarize and generalize patterns (e.g., in the number pattern 1, 2, 4, 8, 16, ..., recognize and report that each term is double the term before it) (6m95);

6. use a calculator and computer applications to explore patterns (6m99);

7. pose and solve problems by recognizing a pattern (6m100);

8. discuss and defend the choice of a pattern rule (6m102).

---

*The rubric is reproduced on pages 108–109 of this document.*
Task Instructions

Introductory Activities

The pre-task is designed to review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task.

Pre-task: Creating a Pattern

Students may work individually, in pairs, or in small groups.

1. Ask students to:
   - select seven red tiles, eight blue tiles, and nine green tiles;
   - arrange the tiles to create a pattern;
   - record the pattern on a piece of paper;
   - describe the pattern.

2. Follow up in any of the following ways:
   - by asking students to describe their patterns
   - by having the class brainstorm a list of words that might be used to describe a pattern
   - by choosing one of the student patterns and having the class work together to write a description of it

3. Then ask students to:
   - create a different pattern with the same pieces;
   - record the pattern on a piece of paper;
   - describe the pattern.

4. Pose these questions:
   - “How many different patterns can you make with the pieces?”
   - “How do you know when you have made all the possible patterns?”

5. Record the answers on a sheet of paper.

6. Have students select two of the patterns that they have made and state why they are patterns, how they are different from one another, and how they are similar.

Exemplar Task

1. Distribute a copy of the Student Package to each student.

2. Tell students that they will be working independently to complete the task.

3. Make sure that each student has a set of tiles, a set of pattern blocks, and a calculator.

4. The problem that the students will solve independently is provided in the worksheets in Appendix 1.

Appendix 1: Student Worksheets

Investigating Patterns With Tiles

For question 1, you will need 8 red tiles, 9 blue tiles, 3 green tiles, and 6 yellow tiles.

1. a) Use all of the pieces to make a linear (straight-line) pattern.
   Record your pattern below.

   b) Explain how you could find the colour of the 80th tile.

   c) How would a calculator help you find the colour of the 80th tile?
2. For this task you will need 4 red tiles, 5 blue tiles, 3 green tiles, and 3 yellow tiles.

   a) Construct a linear pattern with all of the tiles so that the 50th tile is red.

   b) Describe how you made sure that your pattern included a red tile for the 50th tile.

6. c) For the pattern you have just made, explain how you would find the colour of the 80th tile.

   d) Explain how a calculator would help you to find the colour of the 80th tile.
3. For this task you will be joining squares side by side as shown below to make rectangles.

Stage 1  Stage 2  Stage 3

a) Describe all the patterns you observe as you add squares as shown above.

b) Use the information you have discovered from the previous task to find the perimeter of the rectangle in Stage 20. Show how you solved the problem.

4. Let us repeat the same task, this time with equilateral triangles. Select triangles from your pattern blocks.

a) Place them side by side as shown below.

Stage 1  Stage 2  Stage 3

b) Describe all the patterns you notice as triangles are added as shown above.

c) Describe how you would find the perimeter of the shape in Stage 30.
5. a) Use your pattern blocks to construct a pattern similar to those in questions 3 and 4. Record your pattern below.

b) Pose a question related to your pattern.

c) Show how you would answer the question you posed.
The Ministry of Education wishes to acknowledge the contribution of the many individuals, groups, and organizations that participated in the development and refinement of this resource document.