SECTION – II

Q. 4. (A) Select and write the correct answer from the given alternatives in each of the following:  (6)[12]

i. If \( y = 1 - \cos \theta, x = 1 - \sin \theta \), then \( \frac{dy}{dx} \) at \( \theta = \frac{\pi}{4} \) is
   (A) \(-1\)  
   (B) 1  
   (C) \(\frac{1}{2}\)  
   (D) \(\frac{1}{\sqrt{2}}\)

ii. The integrating factor of linear differential equation
    \( \frac{dy}{dx} + y \sec x = \tan x \) is
    (A) \(\sec x - \tan x\)  
    (B) \(\sec x \cdot \tan x\)  
    (C) \(\sec x + \tan x\)  
    (D) \(\sec x \cdot \cot x\)

iii. The equation of tangent to the curve \( y = 3x^2 - x + 1 \) at the point \((1, 3)\) is
    (A) \(y = 5x + 2\)  
    (B) \(y = 5x - 2\)  
    (C) \(y = \frac{1}{5}x + 2\)  
    (D) \(y = \frac{1}{5}x - 2\)

(B) Attempt any THREE of the following:  (6)

i. Examine the continuity of the function
   \( f(x) = \sin x - \cos x \), for \( x \neq 0 \)
   \( = -1 \), for \( x = 0 \)
   at the point \( x = 0 \).

ii. Verify Rolle’s theorem for the function
    \( f(x) = x^2 - 5x + 9 \) on \([1, 4]\)

iii. Evaluate: \( \int \sec^4 x \cdot \tan x \, dx \)

iv. The probability mass function (p.m.f.) of \( X \) is given below:

<table>
<thead>
<tr>
<th>( X = x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>(\frac{1}{5})</td>
<td>(\frac{2}{5})</td>
<td>(\frac{2}{5})</td>
</tr>
</tbody>
</table>

Find \( E(X^2) \)

v. Given that \( X \sim B (n = 10, p) \). If \( E(X) = 8 \), find the value of \( p \).
Q.5. (A) Attempt any TWO of the following: (6)[14]

i. If \( y = f(u) \) is a differentiable function of \( u \) and \( u = g(x) \) is a differentiable function of \( x \), then prove that \( y = f[g(x)] \) is a differentiable function of \( x \) and 
\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.
\]

ii. Obtain the differential equation by eliminating the arbitrary constants \( A, B \) from the equation:
\( y = A \cos (\log x) + B \sin (\log x) \)

iii. Evaluate:
\[
\int \frac{x^2}{(x^2 + 2)(2x^2 + 1)} \, dx
\]

(B) Attempt any TWO of the following: (8)

i. An open box is to be made out of a piece of a square cardboard of sides 18 cms by cutting off equal squares from the corners and turning up the sides. Find the maximum volume of the box.

ii. Prove that:
\[
\int_0^{2a} f(x) \, dx = \int_0^a f(x) \, dx + \int_0^{a} f(2a - x) \, dx
\]

iii. If the function \( f(x) \) is continuous in the interval \([-2, 2]\), find the values of \( a \) and \( b \), where
\begin{align*}
  f(x) &= \frac{\sin ax}{x} - 2, & \text{for } -2 < x < 0 \\
  &= 2x + 1, & \text{for } 0 \leq x \leq 1 \\
  &= 2b \sqrt{x^2 + 3} - 1, & \text{for } 1 < x < 2
\end{align*}

Q.6. (A) Attempt any TWO of the following: (6)[14]

i. Solve the differential equation:
\[
\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}
\]

ii. A fair coin is tossed 8 times. Find the probability that it shows heads at least once.

iii. If \( x^p y^q = (x + y)^{p+q} \), then prove that 
\[
\frac{dy}{dx} = \frac{y}{x}.
\]

(B) Attempt any TWO of the following: (8)

i. Find the area of the sector of a circle bounded by the circle \( x^2 + y^2 = 16 \) and the line \( y = x \) in the first quadrant.

ii. Prove that:
\[
\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C
\]

iii. A random variable \( X \) has the following probability distribution:

<table>
<thead>
<tr>
<th>( X = x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P[X = x] )</td>
<td>( k )</td>
<td>( 3k )</td>
<td>( 5k )</td>
<td>( 7k )</td>
<td>( 9k )</td>
<td>( 11k )</td>
<td>( 13k )</td>
</tr>
</tbody>
</table>

(a) Find \( k \)
(b) Find \( P(0 < X < 4) \)
(c) Obtain cumulative distribution function (c.d.f.) of \( X \).