Comparison of Analytical Methods for Calculating
Fastener Elongation with Test Results – Part 1

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Objective: Nearly any analytical analysis of bolted joints requires either direct calculation of fastener elongation or calculation of the components that determine elongation. Even in instances when tension can be measured directly during testing, calculating elongation can be desirable when direct measurement is impractical. For example, when clamp load at conditions not tested is desired, calculation of elongation or its components is required to extrapolate the test data.

A primary challenge of calculating elongation is translating the stress profile in the head and engaged threads into equations that can be solved. As opposed to the portion of the shank that lies within the grip length, these transition areas can contain complex stress profiles. This is further complicated by the fact that it is the head and nut member that contain most of the differences that differentiate the various forms of threaded fasteners available. As a result, there are several published approaches to elongation calculation that contain subtle differences for solving this problem.

In this article, we use two popular calculation methods to compare these estimates to actual test measurements of elongation. In light of the challenge of estimating the contribution of the head and nut areas, a test matrix was created to test various head/nut combinations (even some that might not represent practical application). Since the transition areas become more influential as the grip length gets shorter, tests were conducted at two different grip-length-to-diameter ratios. In this first phase of testing, fully threaded cap screws were used so that the shank transitions tested could be preserved. Each test would be a constant cross section and the effect of the various head and nut transitions tested could be preserved. Each test was run at two loads as a check of elongation measurement by reviewing the ratio of high/low values.

Calculation Methods

The two estimation methods used as a comparison to tested values of elongation are those from Bickford and VDI 2230. In their simplest form of relating force to elongation, both methods are identical:

\[ y = \frac{FL}{EA} \]

where: \( y \) = elongation, \( F \) = force/tension, \( L \) = length, \( E \) = modulus of elasticity and \( A \) = area

But this relationship is only valid for samples with a constant cross-section, so the relationship is expanded to handle the fact that portions of a fastener have different cross sections.

\[ y = \frac{F}{A_N} \]

where: \( \delta_0 \) = bolt resilience = \( \delta_0 + \delta_1 + \delta_n = \frac{L_n/E_A}{A_n} + \frac{L_l/E_A}{A_l} + \frac{L_n/E_A}{A_n} \)

Bolt resilience (the inverse of stiffness) consists of three terms: head resilience, thread resilience and nut resilience. It is in the length and area variables of these terms where the difference in the two estimates lie.

First, the VDI method expands the nut resilience into two terms which replaces \( \delta_n \). These terms separate the contribution of the nut and the engaged threads.

\[ \delta_{ne} = \frac{L_n}{EA_n} + \frac{L_l}{EA_l} \]

The remaining differences are summarized in the table shown below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Bickford</th>
<th>VDI 2230</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_3 )</td>
<td>0.5( h_n )</td>
<td>0.4d</td>
</tr>
<tr>
<td>( a_n )</td>
<td>( a_l )</td>
<td>( a_n )</td>
</tr>
<tr>
<td>( l_e^2 )</td>
<td>( l_e^1 )</td>
<td>( l_e )</td>
</tr>
<tr>
<td>( a_e )</td>
<td>( a_e )</td>
<td>( a_e )</td>
</tr>
<tr>
<td>( L_n, L_e )</td>
<td>0.5( h_n )</td>
<td>0.4d, 0.5d</td>
</tr>
<tr>
<td>( A_n, A_e )</td>
<td>( a_n )</td>
<td>( a_n )</td>
</tr>
</tbody>
</table>

NOTES:
1. Use \( a_n \) when shank is not fully threaded.
2. This test uses fully threaded bolts. In cases where there is an unthreaded portion of the shank \( h_n \) is broken into a threaded and unthreaded term.

Test Method

Figure 1 summarizes the test matrix chosen for the test. English fasteners were chosen because all configurations of fully threaded metric bolts and jamb nuts were not readily available in a common property class. Figure 2 provided better visualization of the test variants. Load was applied with a universal test machine (UTM). Elongation was measured with a pair of lever-style gage heads contacting the ends of the fastener. The gage heads were supported from a common point on the UTM’s fixed platen. The gages were output to an amplifier set to read at 0.00005" resolution.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Fastener</th>
<th>L/D</th>
<th>Nut 1</th>
<th>Nut 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hex Cap Screw - Long</td>
<td>5.78</td>
<td>Tapped Block</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>Hex Cap Screw - Long</td>
<td>6.80</td>
<td>Hex</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>Hex Cap Screw - Long</td>
<td>6.31</td>
<td>Jamb</td>
<td>N/A</td>
</tr>
<tr>
<td>4</td>
<td>Stud</td>
<td>4.59</td>
<td>Hex</td>
<td>Tapped Block</td>
</tr>
<tr>
<td>5</td>
<td>Stud</td>
<td>5.81</td>
<td>Hex</td>
<td>Hex</td>
</tr>
<tr>
<td>6</td>
<td>Hex Cap Screw - Short</td>
<td>1.20</td>
<td>Tapped Block</td>
<td>N/A</td>
</tr>
<tr>
<td>7</td>
<td>Hex Cap Screw - Short</td>
<td>1.70</td>
<td>Hex</td>
<td>N/A</td>
</tr>
<tr>
<td>8</td>
<td>Hex Cap Screw - Short</td>
<td>2.55</td>
<td>Jamb</td>
<td>N/A</td>
</tr>
</tbody>
</table>

NOTES:
1. All fasteners SAE J429 Gr 8 except tapped block which is AISI 1018
2. All fasteners 3/8-16 UNC fully threaded
3. Tapped block 0.75" deep
4. All tests run with sample size of 6
5. A settling run was performed before the measurement run
6. Load was applied at a rate of 16,000 lb/min
7. L/D represents the ratio of grip length to nominal fastener dia.
8. Applied loads: 7,440 lb and 3,720 lb
9. Modulus of Elasticity: 29.3x10^6 lb/in^2
Test Results
Test results are tabulated in Figure 3A and Figure 3B, and graphed in Figure 4A and Figure 4B. The test set-up appeared to be reasonably robust as the ratio between elongation at high and low loads for the six tests in which they were both run was 1.99, 2.06, 2.03, 2.03, 1.96 and 2.07.

The average deviation from test values for the Bickford and VDI 2230 calculations was 7.6% and 9.4%, respectively. Having calculations independent of thread engagement seems to have affected the correlation of VDI estimates with tests utilizing the tapped block. It is not immediately apparent why the calculations didn’t correlate very well with tests conducted with a stud.

Next Steps
Our plan for the next phase of this study is to gain better fidelity of the relative contributions of the shank and the nut and head by taking extensometer readings on the shank including those that are partially unthreaded. We are considering other fastener configurations that might widen the scope of comparison as well as compiling other analytical variants to test against.

If readers have any suggestions or comments, contact the author at darcher@archetypejoint.com.

Company Profile:
Archetype Joint, LLC offers joint design, test and validation services for OEMs and suppliers of assembly equipment and fasteners. Located in Orion, MI, USA, the firm has a complete independent test lab accredited to ISO 17025 by A2LA. Archetype Joint uses the diversity of its professional experience and its customer base to provide clients interpretation and recommendations and not just test data. www.archetypejoint.com

About the Author:
David Archer is President of Archetype Joint, which he founded after over 20 years experience in design, manufacturing and consulting, focusing on enhancing product value through reducing design/process complexity. The realization that joint design was the most widely ignored barrier to achieving product cost, performance and quality goals was the genesis for Archetype Joint. Archer has a Bachelor of Science Degree in Mechanical Engineering/Applied Mechanics and a Master of Science degree in Manufacturing Engineering from the University of Rhode Island.