PREFACE

In compliance to kVS(HQ), New Delhi letter no.28350(SM)/2012-13KVSPR/ dated 01.08.2012 the responsibility of preparation of Study/ Support Material class X mathematics has been entrusted to this Region.

KVS Patna Region acknowledges the sincere efforts of Sh. Sudhakar Singh, Principal, KV No.2 Gaya, Ms. Archita Gupta, PGT(Maths), Dr. A.K. Tiwari, TGT(Maths), Sh. S. Ram, TGT(Maths) and Sh. P. N. Chaki, PGT(Comp.Sc).

I am confident that the study/support material class X mathematics will directly help the students to understand the concept well and meet quality expectation.

Wish you all the best.

(.....)

Dy. Commissioner
STUDY MATERIAL – MATHEMATICS

FOR

CLASS – X

(2012-2013)

PATRON
DY. COMMISSIONER

COORDINATOR
SH. SUDHAKAR SINGH
PRINCIPAL, KV NO.2 GAYA

RESOURCE PERSONS
MS. ARCHITA GUPTA, PGT(MATHS), KV NO.2 GAYA
DR. A.K.TIWARI, TGT(MATHS), KV NO.2 GAYA
SH. S.RAM, TGT(MATHS), KV NO.2 GAYA
SH. P.N.CHAKI, PGT(COMP.SC), KV NO.2 GAYA
How to use this study material?

Dear Children,

This study material contains gist of the topics/units along with the assignments for self assessment. Here are some tips to use this study material while revision during pre-boards and finally in board examination.

- Go through the syllabus given in the beginning. Identify the units carrying more weightage.
- Suggestive blue print and design of question paper is a guideline for you to have clear picture about the form of the question paper.
- Revise each of the topics/units and attempt the questions given for self assessment.
- After attempting the self assessment part, consult the question bank where questions carrying one, two, three, four marks are given. Revise them.
- After revision of all the units, solve the sample paper, and do self assessment with the value points.
- Must study the marking scheme/solution for CBSE previous year paper which will enable you to know the coverage of content under different questions.
- Underline or highlight key ideas to have birds eye view of all the units at the time of examination.
- Write down your own notes and make summaries with the help of this study material.
- Turn the theoretical information into outline mind maps.
- Make a separate revision note book for diagrams and numericals as well.
- Discuss your ‘DOUBTS’ with your teacher/other students.
- Use part 2 for FA-2 and FA-4

Important:

- Slow learners may revise the knowledge part first.
- Bright students may emphasize the application part of the question paper.
<table>
<thead>
<tr>
<th>SLNO</th>
<th>TOPIC</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>PART -1</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Real Numbers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Polynomials</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A pair of linear equations in two variables</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Introduction to Trigonometry</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Statistics</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Model Question paper SA-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>SA- 2</strong></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Quadratic Equation</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Arithmetic Progression</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Coordinate Geometry</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Some Applications of Trigonometry</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Circle</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Construction</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Area Related to Circle</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Surface Area and Volume</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Probability</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Model Question paper SA-2</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>PART – 2</strong></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Activities (Term I)</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Activities (Term II)</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Projects</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Quiz/oral Test</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Puzzles</td>
<td></td>
</tr>
</tbody>
</table>
As per CCE guidelines, the syllabus of Mathematics for class X has been divided term wise.

The units specified for each term shall be assessed through both formative and summative assessment.

In each term, there shall be two formative assessments each carrying 10% weightage.

The summative assessment in I term will carry 30% weightage and the summative assessment in the II term will carry 30% weightage.

Listed laboratory activities and projects will necessarily be assessed through formative assessments.

### SUMMATIVE ASSESSMENT -1

<table>
<thead>
<tr>
<th>FIRST TERM (SA I)</th>
<th>MARKS: 90</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNITS</strong></td>
<td>MARKS</td>
</tr>
<tr>
<td>I NUMBER SYSTEM</td>
<td>11</td>
</tr>
<tr>
<td>Real Numbers</td>
<td></td>
</tr>
<tr>
<td>II ALGEBRA</td>
<td>23</td>
</tr>
<tr>
<td>Polynomials, pair of linear equations in two variables.</td>
<td></td>
</tr>
<tr>
<td>III GEOMETRY</td>
<td>17</td>
</tr>
<tr>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td>V TRIGONOMETRY</td>
<td>22</td>
</tr>
<tr>
<td>Introduction to trigonometry, trigonometric identity.</td>
<td></td>
</tr>
<tr>
<td>VII STATISTICS</td>
<td>17</td>
</tr>
<tr>
<td>TOTAL</td>
<td>90</td>
</tr>
</tbody>
</table>

### SUMMATIVE ASSESSMENT -2

<table>
<thead>
<tr>
<th>SECOND TERM (SA II)</th>
<th>MARKS: 90</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNITS</strong></td>
<td>MARKS</td>
</tr>
<tr>
<td>II ALGEBRA(contd)</td>
<td>23</td>
</tr>
<tr>
<td>Quadratic equations, arithmetic progressions</td>
<td></td>
</tr>
<tr>
<td>III GEOMETRY(contd)</td>
<td>17</td>
</tr>
<tr>
<td>Circles, constructions</td>
<td></td>
</tr>
<tr>
<td>IV MENSURATION</td>
<td>23</td>
</tr>
<tr>
<td>Areas related to Circles, Surface Area &amp; Volumes</td>
<td></td>
</tr>
<tr>
<td>V TRIGONOMETRY(Contd)</td>
<td>08</td>
</tr>
<tr>
<td>Heights and Distances.</td>
<td></td>
</tr>
<tr>
<td>VI COORDINATE GEOMETRY</td>
<td>11</td>
</tr>
<tr>
<td>VII PROBABILITY</td>
<td>08</td>
</tr>
<tr>
<td>TOTAL</td>
<td>90</td>
</tr>
</tbody>
</table>
**SYMBOLS USED**
*:-Important Questions, **:- Very important Questions, ***:- Very very important Questions

<table>
<thead>
<tr>
<th>S.No</th>
<th>TOPIC</th>
<th>CONCEPTS</th>
<th>DEGREE OF IMPORTANCE</th>
<th>References(NCERT BOOK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Real Number</td>
<td>Euclid’s division Lemma &amp; Algorithm</td>
<td>***</td>
<td>Example -1,2,3,4 Ex:1.1 Q:1,2,4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fundamental Theorem of Arithmetic</td>
<td>***</td>
<td>Example -5,7,8 Ex:1.2 Q:4,5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Revisiting Irrational Numbers</td>
<td>***</td>
<td>Example -9,10,11 Ex: 1.3 Q:1.2 Th:1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Revisiting Rational Number and their decimal Expansion</td>
<td>**</td>
<td>Ex -1.4 Q:1</td>
</tr>
<tr>
<td>02</td>
<td>Polynomials</td>
<td>Meaning of the zero of Polynomial</td>
<td>*</td>
<td>Ex -2.1 Q:1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relationship between zeroes and coefficients of a polynomial</td>
<td>**</td>
<td>Ex -2.3 Ex-2.2 Q:1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Forming a quadratic polynomial</td>
<td>**</td>
<td>Ex -2.2 Q:2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Division algorithm for a polynomial</td>
<td>*</td>
<td>Ex -2.3 Q:1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Finding the zeroes of a polynomial</td>
<td>***</td>
<td>Example: 9 Ex -2.3 Q:1,2,3,4,5 Ex-2.4,3,4,5</td>
</tr>
<tr>
<td>03</td>
<td>Pair of Linear Equations in two variables</td>
<td>Graphical algebraic representation</td>
<td>*</td>
<td>Example:2,3 Ex -3.4 Q:1,3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Consistency of pair of liner equations</td>
<td>**</td>
<td>Ex -3.2 Q:2,4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Graphical method of solution</td>
<td>***</td>
<td>Example: 4,5 Ex -3.2 Q:7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Algebraic methods of solution</td>
<td>**</td>
<td>Ex -3.3 Q:1,3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. Substitution method</td>
<td></td>
<td>Example-13 Ex:3.4 Q:1,2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Elimination method</td>
<td></td>
<td>Example-15,16 Ex:3.5 Q:1,2,4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c. Cross multiplication method</td>
<td></td>
<td>Example-19 Ex:3.6 Q.:1(ii),(viii),2 (ii),(iii)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d. Equation reducible to pair of liner equation in two variables</td>
<td></td>
<td>Theo:6.1 Example:1,2,3 Ex:6.2 Q:2,4,6,9,10</td>
</tr>
<tr>
<td>04</td>
<td>TRIANGLES</td>
<td>1) Similarity of Triangles</td>
<td>***</td>
<td>Theo:6.1 Example:1,2,3 Ex:6.2 Q:2,4,6,9,10</td>
</tr>
<tr>
<td>05</td>
<td>Introduction to Trigonometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>--------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 2) | Criteria for Similarity of Triangles | ** | Example: 6, 7
Ex: 6.3
Q: 4, 5, 6, 10, 13, 16 |
| 3) | Area of Similar Triangles | *** | Example: 9 The: 6.6
Ex: 6.4 Q: 3, 5, 6, 7 |
| 4) | Pythagoras Theorem | *** | Theo: 6.8 & 6.9
Example: 10, 12, 14,
Ex: 6.5
Q: 4, 5, 6, 7, 13, 14, 15, 16 |

<table>
<thead>
<tr>
<th>06</th>
<th>STATISTICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONCEPT 1</td>
<td>Mean of grouped data</td>
</tr>
</tbody>
</table>
| 1. | Direct Method | *** | Example: 2
Ex: 14.1 Q: 1&3 |
| 2. | Assumed Mean Method | * | Ex: 14.1 Q: 6 |
| CONCEPT 2 | Mode of grouped data | *** | Example: 5
Ex: 14.2 Q: 1, 5 |
| CONCEPT 3 | Median of grouped data | *** | Example: 7, 8
Ex: 14.3 Q: 1, 3, 5 |
| CONCEPT 4 | Graphical representation of c.f.(ogive) | ** | Example: 9
Ex: 14.4 Q: 1, 2, 3 |
1. **Euclid’s Division lemma**: Given positive integers a and b there exist unique integers q and r satisfying
   \[ a = bq + r, \] where \( 0 \leq r < b \), where a, b, q and r are respectively called as dividend, divisor, quotient and remainder.

2. **Euclid’s division Algorithm**: To obtain the HCF of two positive integers say c and d, with c>0, follow the steps below:
   - **Step I**: Apply Euclid’s division lemma, to c and d, so we find whole numbers, q and r such that \( c = dq + r \), \( 0 \leq r < d \).
   - **Step II**: If r=0, d is the HCF of c and d. If \( r \neq 0 \), apply the division lemma to d and r.
   - **Step III**: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

3. **The Fundamental theorem of Arithmetic**: Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

   Ex.: \( 24 = 2 \times 2 \times 2 \times 3 = 3 \times 2 \times 2 \times 2 \)

   **Theorem**: LET \( x \) be a rational number whose decimal expansion terminates. Then \( x \) can be expressed in the form
   \[ \frac{p}{q} \] where p and q are co-prime and the prime factorisation of q is the form of \( 2^n \times 5^m \), where \( n, m \) are non negative integers.

   Ex. \( \frac{7}{10} = \frac{7}{2 \times 5} = 0.7 \)

   **Theorem**: LET \( x = \frac{p}{q} \) be a rational number such that the prime factorisation of q is not of the form \( 2^n \times 5^m \), where \( n, m \) are non negative integers. Then \( x \) has a decimal expansion which is non terminating repeating (recurring).

   Ex. \( \frac{7}{6} = \frac{7}{2 \times 3} = 1.1666 \ldots \)

   **Theorem**: For any two positive integers a and b,

   \[ \text{HCF (a,b) \times LCM (a,b)} = a \times b \]

   Ex.: 4 & 6; HCF (4,6) = 2, LCM (4,6) = 12; HCF x LCM = 2 \times 12 = 24

   Ans. : \( a \times b = 24 \)

   **(Level-1)**

1. If \( \frac{p}{q} \) is a rational number (\( q \neq 0 \)). What is the condition on q so that the decimal representation of \( \frac{p}{q} \) is terminating?

   Ans. q is of the form \( 2^n \times 5^m \) where \( n, m \) are non negative integers.

2. Write a rational number between \( \sqrt{2} \) and \( \sqrt{3} \).

   Ans. 1.5

3. The decimal expansion of the rational number \( \frac{43}{2^4 \times 5^3} \) will terminate after how many of decimals?

   Ans. After 4 places of decimal.

4. Find the (HCF \( \times \) LCM) for the numbers 100 and 190.
5. State whether the number \((\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})\) is rational or irrational justify.
   Ans. Rational

6. Write one rational and one irrational number lying between 0.25 and 0.32.
   Ans. One rational no. = 0.26, one irrational no. = 0.27010010001.........

7. Express 107 in the form of \(4q + 3\) for some positive integer.
   Ans. 4 \(\times\) 26 + 3

8. Write whether the rational number \(\frac{51}{1380}\) will have a terminating decimal expansion or a non terminating repeating decimal expansion.
   Ans. Terminating.

**(level - 2)**

1. Use Euclid’s division algorithm to find the HCF of 1288 and 575.
   Ans. 23.

2. Check whether \(5 \times 3 \times 11 + 11\) and \(5 \times 7 + 7 \times 3 + 3\) are composite number and justify.
   Ans. Composite number.

3. Check whether \(6^n\) can end with the digit 0, where \(n\) is any natural number.
   Ans. No, \(6^n\) can not end with the digit 0.

4. Given that LCM (26, 169) = 338, write HCF (26, 169 ).]
   Ans. 13

5. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.
   Ans. HCF = 6
   LCM = 360

**level - 3**

1. Show that \(\sqrt{3}\) is an irrational number.
2. Show that \(5 + 3\sqrt{2}\) is an irrational number.
3. Show that square of an odd positive integer is of the form \(8m + 1\), for some integer \(m\).
4. Find the LCM & HCF of 26 and 91 and verify that \(LCM \times HCF = product\ of\ the\ two\ numbers\).
   Ans. LCM=182, HCF=13

**PROBLEMS FOR SELF EVALUATION/HOTS**

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime factors.
3. Show that the square of an odd positive integers is of the form \(8m + 1\) for some whole number \(m\).
4. Find the LCM and HCF of 17, 23 and 29.
5. Prove that $\sqrt{2}$ is not a rational number.

6. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5, 7 and 11 respectively.

7. Show that there is no positive integer $n$ for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

8. Using prime factorization method, find the HCF and LCM of 72, 126 and 168. Also show that $HCF \times LCM \neq \text{product of the three numbers}$.

2. Polynomials

( Key Points )

Polynomial:

An expression of the form $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ where $a_n \neq 0$ is called a polynomial in variable $x$ of degree $n$. where; $a_0, a_1, \ldots, a_n$ are real numbers and each power of $x$ is a non-negative integer.

Ex.: $2x^2 - 5x + 1$ is a polynomial of degree 2.

Note: $\sqrt{x} + 3$ is not a polynomial.

- A polynomial $p(x) = ax + b$ of degree 1 is called a linear polynomial. Ex. $5x - 3$, $2x$ etc
- A polynomial $p(x) = ax^2 + bx + c$ of degree 2 is called a quadratic polynomial. Ex. $2x^2 + x - 1, 1 - 5x + x^2$ etc.
- A polynomial $p(x) = ax^3 + bx^2 + cx + d$ of degree 3 is called a cubic polynomial.

Ex. $\sqrt{3}x^3 - x + \sqrt{5}$, $x^3 - 1$ etc.

Zeroes of a polynomial: A real number $k$ is called a zero of polynomial $p(x)$ if $p(x) = 0$. The graph of $y = p(x)$ intersects the X-axis.

- A linear polynomial has only one zero.
- A Quadratic polynomial has two zeroes.
- A Cubic polynomial has three zeroes.

For a quadratic polynomial: If $\alpha$, $\beta$ are zeroes of $P(x) = ax^2 + bx + c$ then:

1. Sum of zeroes $= \alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{coefficient of } x^2}$
2. Product of zeroes $= \alpha \cdot \beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

A quadratic polynomial whose zeroes are $\alpha$ and $\beta$, is given by:

$p(x) = x^2 - (\alpha + \beta) x + \alpha \beta = x^2 - (\text{sum of zeroes}) x + \text{product of zeroes}$.

If $\alpha, \beta$ and $\gamma$ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:

* $\alpha + \beta + \gamma = -\frac{b}{a}$
* $a \beta + b \gamma + c \alpha = \frac{c}{a}$
* $a \beta \gamma = -\frac{d}{a}$

Division algorithm for polynomials: If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that:
\[ p(x) = q(x)X \ g(x) + r(x), \text{ where } r(x) = 0 \text{ or degree of } r(x) < \text{ degree of } g(x). \]

( Level - 1 )

1. In a graph of \( y = p(x) \), find the number of zeroes of \( p(x) \).
   \[
   \text{Ans. 3.}
   \]

2. If \( \alpha, \beta \) are the zeroes of \( f(x) = x^2 + x + 1 \), then find \( \frac{1}{\alpha} + \frac{1}{\beta} \).
   \[
   \text{Ans. (-1)}
   \]

3. Find a quadratic polynomial whose zeroes are \( -\frac{2}{\sqrt{3}} \) and \( \frac{\sqrt{3}}{2} \).
   \[
   \text{Ans. } x^2 - \left( -\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2} \right) x + \left(-\frac{1}{2}\right)
   \]

4. If \( p(x) = \frac{1}{2}x^2 - 5x + \frac{3}{2} \) then find its sum and product of zeroes.
   \[
   \text{Ans. Sum=15, Product = } \frac{9}{2}
   \]

5. If the sum of zeroes of a given polynomial \( f(x) = x^3 - 3kx^2 - x + 30 \) is 6. Find the value of \( k \).
   \[
   \text{Ans. } \alpha + \beta + \gamma = \frac{-b}{a} = \frac{3k}{1} = 6
   \]
   \[
   \therefore k = 2
   \]

6. Find the zero of polynomial \( 3x + 4 \).
   \[
   \text{Ans. } \frac{-4}{3}
   \]

7. Write the degree of zero polynomial.
   \[
   \text{Ans. Not defined.}
   \]

( Level - 2 )

1. Form a cubic polynomial with zeroes 3, 2 and \(-1\).
   \[
   \text{Hints/Ans. } p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma
   \]

2. Find the zeroes of the quadratic polynomial \( 6x^2 - 3 - 7x \) and verify the relationship between the zeroes and the coefficients.
   \[
   \text{Ans. Zeroes are } 3/2 \& -1/3.
   \]

3. For what value of \( k \), \((-4)\) is a zero of polynomial \( x^2 - x - (2k + 2) \)?
   \[
   \text{Ans. } k=9
   \]

4. Give an example of polynomials \( p(x), g(x), q(x) \) and \( r(x) \) which satisfy division algorithm and \( \text{deg. } p(x) = \text{deg. } g(x) \).
   \[
   \text{Ans. } 3x^2 + 2x + 1, \ x^2, \ 3, \ 2x + 1
   \]

5. Find the zeroes of \( 4u^2 + 8u \).
   \[
   \text{Ans. } 0, \ -2
   \]

6. Find a quadratic polynomial, whose the sum and product of its zeroes are \( \frac{1}{4}, -1 \).
   \[
   \text{Ans. } x^2 - \frac{1}{4}x - 1
   \]

( Level - 3 )

1. Find the zeroes of polynomial \( x^3 - 2x^2 - x + 2 \)
   \[
   \text{Ans. } -1, \ 1, \ 2
   \]

2. If the zeroes of the polynomial \( x^3 - 3x^2 + x + 1 \) are \( \alpha - \beta, \ \alpha, \ \alpha + \beta \). Find \( \alpha \) and \( \beta \)
   \[
   \text{Ans. } \alpha = 1, \ \beta = \pm \sqrt{2}
   \]

3. Divide \( f(x) = 6x^3 + 11x^2 - 39x - 65 \) by \( g(x) = x^2 - 1 + x \)
   \[
   \text{Ans. Quotient=6x + 5; Remainder = } -38x - 60
   \]

4. Check whether the polynomial \( t^2 - 3 \) is a factor of polynomial \( 2t^4 + 3t^3 - 2t^2 - 9t - 12 \)
   \[
   \text{by applying the division algorithm.}
   \]
   \[
   \text{Ans. Remainder=0, Quotient=2t^2 + 3t + 4, Given Polynomial is a factor.}
   \]

( Level - 4 )
1. Obtain all zeroes of \( f(x) = x^3 + 13x^2 + 32x + 20 \)
   Ans. \(-1, -2, -10\)

2. Obtain all other zeroes of \( 3x^4 + 6x^3 - 2x^2 - 10x - 5 \), if two of its zeroes are \( \sqrt{\frac{5}{3}} \) and \( -\sqrt{\frac{5}{3}} \)
   Ans. \(-1 & -1\)

3. On dividing \( x^3 - 3x^2 + x + 2 \) by a polynomial \( g(x) \), the quotient and remainder were \( x - 2 \) and \(-2x + 4 \) respectively, find \( g(x) \).
   Ans. \( x^2 - x + 1 \)

(PROBLEMS FOR SELF-EVALUATION)

1. Check whether \( g(x) = 3x - 2 \) is a factor of \( p(x) = 3x^3 + x^2 - 20x + 12 \).

2. Find quotient and remainder applying the division algorithm on dividing \( p(x) = x^3 - 6x^2 + 2x - 4 \) by \( g(x) = x - 1 \).

3. Find zeros of the polynomial \( 2x^2 - 8x + 6 \)

4. Find the quadratic polynomial whose sum and product of its zeros are \( \frac{2}{3}, \frac{-1}{3} \) respectively.

5. Find the zeroes of polynomial \( x^3 - 2x^2 - x + 2 \)

6. If one of the zeroes of the polynomial \( 2x^2 + px + 4 = 0 \) is 2, find the other root, also find the value of \( p \).

7. If \( \alpha \) and \( \beta \) are the zeroes of the polynomial \( kx^2 + 4x + 4 \) show that \( \alpha^2 + \beta^2 = 24 \), find the value of \( k \).

8. If \( \alpha \) and \( \beta \) are the zeroes of the equation \( 6x^2 + x - 2 = 0 \), find \( \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \)

---xxx---
3. **Pair of linear equations in two variables**  
*(Key Points)*

- An equation of the form $ax + by + c = 0$, where $a$, $b$, $c$ are real nos ($a \neq 0$, $b \neq 0$) is called a linear equation in two variables $x$ and $y$.

  Ex: (i) $x - 5y + 2 = 0$
  (ii) $\frac{3}{2}x - y = 1$

- The general form for a pair of linear equations in two variables $x$ and $y$ is

  $a_1x + b_1y + c_1 = 0$
  $a_2x + b_2y + c_2 = 0$

  where $a_1$, $b_1$, $c_1$, $a_2$, $b_2$, $c_2$ are all real nos and $a_1 \neq 0$, $b_1 \neq 0$, $a_2 \neq 0$, $b_2 \neq 0$.

  Examples: $x + 3y - 6 = 0$
  $2x - 3y - 12 = 0$

- Graphical representation of a pair of linear equations in two variables:

  $a_1x + b_1y + c_1 = 0$
  $a_2x + b_2y + c_2 = 0$

  (i) will represent intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

  i.e. unique solution. And this type of equations are called consistent pair of linear equations.

  Ex: $x - 2y = 0$
  $3x + 4y - 20 = 0$

  (ii) will represent overlapping or coincident lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

  i.e. Infinitely many solutions, consistent or dependent pair of linear equations

  Ex: $2x + 3y - 9 = 0$
  $4x + 6y - 18 = 0$

  (iii) will represent parallel lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

  i.e. no solution and called inconsistent pair of linear equations.

  Ex: $x + 2y - 4 = 0$
  $2x + 4y - 12 = 0$

- **Algebraic methods of solving a pair of linear equations:**

  (i) **Substitution method**
  (ii) **Elimination Method**
  (iii) **Cross multiplication method**

  *(Level - 1)*

1. Find the value of ‘$a$’ so that the point $(3,9)$ lies on the line represented by $2x - 3y = 5$

   Ans: $a = \frac{1}{3}$

2. Find the value of $k$ so that the lines $2x - 3y = 9$ and $kx - 9y = 18$ will be parallel.

   Ans: $k = 6$

3. Find the value of $k$ for which $x + 2y = 5$, $3x + ky + 15 = 0$ is inconsistent
4. Check whether given pair of lines is consistent or not $5x - 1 = 2y$, $y = \frac{-1}{2} + \frac{5}{2} \cdot x$

   Ans: consistent

5. Determine the value of ‘$a$’ if the system of linear equations $3x + 2y - 4 = 0$ and $9x - y - 3 = 0$ will represent intersecting lines.

   Ans: $a \neq \frac{-3}{2}$

6. Write any one equation of the line which is parallel to $\sqrt{2}x - \sqrt{3}y = 5$

   Ans: $5\sqrt{2}x - 5\sqrt{3}y = 5\sqrt{5}$

7. Find the point of intersection of line $-3x + 7y = 3$ with x-axis

   Ans: $(-1, 0)$

8. For what value of $k$ the following pair has infinite number of solutions.

   $(k-3)x + 3y = k$
   $k(x+y)=12$

   Ans: $k=6$

9. Write condition so that $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ have unique solution.

   Ans: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   (Level - 2)

1. 5 pencils and 7 pens together cost Rs. 50 whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

   Ans: Cost of one pencil = Rs. 3
   Cost of one pen = Rs. 5

2. Solve the equations:

   $3x - y = 3$
   $7x + 2y = 20$

   Ans: $x=2$, $y=3$

3. Find the fraction which becomes to $2/3$ when the numerator is increased by 2 and equal to $4/7$

   when the denominator is increased by 4

   Ans: $28/45$

4. Solve the equation:

   $px + qy = p - q$
   $qx - py = p + q$

   Ans: $x = 1$, $y = -1$

   (Level - 3)
1. Solve the equation using the method of substitution:

\[3x - 5y = -1\]
\[x - y = -1\]

\[\text{Ans. } x = -2, \ y = -1\]

2. Solve the equations:

\[\frac{1}{2x} - \frac{1}{y} = -1\]
\[\frac{1}{x} + \frac{1}{2y} = 8 \quad \text{Where, } x \neq 0, y \neq 0\]

\[\text{Ans. } x = \frac{1}{6}, \ y = \frac{1}{4}\]

3. Solve the equations by using the method of cross multiplication:

\[x + y = 7\]
\[5x + 12y = 7\]

\[\text{Ans. } x = 11, \ y = -4\]

4. A man has only 20 paisa coins and 25 paisa coins in his purse, if he has 50 coins in all totaling Rs. 11.25, how many coins of each kind does he have.

\[\text{Ans. 25 coins of each kind}\]

5. For what value of \( k \), will the system of equations

\[x + 2y = 5\]
\[3x + ky - 15 = 0\]

has a unique solution.

\[\text{Ans. } k \neq 6\]

(level - 4)

1. Draw the graphs of the equations

\[4x - y = 4\]
\[4x + y = 12\]

Determine the vertices of the triangle formed by the lines representing these equations and the x-axis. Shade the triangular region so formed

\[\text{Ans: } (2,4)(1,0)(3,0)\]

2. Solve Graphically

\[x - y = -1\] and
\[3x + 2y = 12\]

Calculate the area bounded by these lines and the x-axis,

\[\text{Ans: } x = 2, \ y = 3 \ \text{ and area } = 7.5 \ \text{unit}^2\]

3. Solve:

\[\frac{10}{x+y} + \frac{2}{x-y} = 4\]
4. Ritu can row downstream 20 km in 2 hr, and upstream 4 km in 2 hr. Find her speed of rowing in still water and the speed of the current.  
   \[ \text{Ans: Speed of rowing in still water} = 6 \text{ km/hr} \]  
   \[ \text{Speed of the current} = 4 \text{ km/hr} \]  

5. In a \( \triangle ABC \), \( \angle C = 3 \), \( \angle B = 2(\angle A + \angle B) \) find these angles.  
   \[ \text{Ans:} \quad \angle A = 20^0, \quad \angle B = 40^0, \quad \angle C = 120^0. \]  

6. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by 1 man alone and that by 1 boy alone to finish the work.  
   \[ \text{Ans: One man can finish work in 140 days} \]  
   \[ \text{One boy can finish work in 280 days} \]  

7. Find the value of \( K \) for which the system of linear equations  
   \[ 2x + 5y = 3, \quad (k + 1)x + 2(k + 2)y = 2K \]  
   will have infinite number of solutions.  
   \[ \text{Ans:} \quad K = 3 \]  

1. Solve for \( x \) and \( y \):  
   \[ x + y = a + b \]  
   \[ ax - by = a^2 - b^2 \]  

2. For what value of \( k \) will the equation \( x + 5y - 7 = 0 \) and \( 4x + 20y + k = 0 \) represent coincident lines?  

3. Solve graphically:  
   \[ 3x + y + 1 = 0 \]  
   \[ 2x - 3y + 8 = 0 \]  

4. The sum of digits of a two-digit number is 9. If 27 is subtracted from the number, the digits are reversed. Find the number.  

5. Draw the graph of \( x + 2y - 7 = 0 \) and \( 2x - y - 4 = 0 \). Shade the area bounded by these lines and the \( y \)-axis.
6. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of the students in the class.

7. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by the car it takes him 4 hours, but if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

8. Given linear equation \(2x + 3y - 8 = 0\), write another linear equation such that the geometrical representation of the pair so formed is (i) intersecting lines, (ii) Parallel Lines.
TRIANGLES

KEY POINTS

1. **Similar Triangles**:- Two triangles are said to be similar, if (a) their corresponding angles are equal and (b) their corresponding sides are in proportion (or are in the same ration).

2. Basic proportionality Theorem [ or Thales theorem ].

3. Converse of Basic proportionality Theorem.

   (a) AA or AAA similarity criterion.
   (b) SAS similarity criterion.
   (c) SSS similarity criterion.

5. Areas of similar triangles.

6. Pythagoras theorem.

7. Converse of Pythagoras theorem.

( Level -1)

1. If in two triangles, corresponding angles are equal, then the two triangles are…………… Ans. Equiangular then similar

2. $\Delta$ABC is a right angled at B. BD is perpendicular upon AC. If AD=a, CD=b, then $AB^2=$ Ans. $a(a+b)$

3. The area of two similar triangles are $32cm^2$ and $48cm^2$.If the square of a side of the first $\Delta$ is $24cm^2$,then the square of the corresponding side of 2nd triangle will be Ans. $36cm^2$

4. $\Delta$ABC is a triangle with DE || BC. If AD=2cm, BD=4cm then find the value DE:BC Ans. 1:3

5. In $\Delta$ABC,DE || BC, if AD=4x-3,DB=3x-1,AE=8x-7and BC=5x-3,then find the values of x are:
   Ans. $1, \frac{1}{2}$

6. The perimeters of two similar triangles are 40cm and 50 cm respectively, find the ratio of the area of the first triangle to the area of the 2nd triangle:
   Ans. 16:25

7. A man goes 150m due east and then 200m due north. How far is he from the starting point?
   Ans. 250 m

8. A ladder reaches a window which is 12m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 9m high. If the length of the ladder is 15m, find the width of the street.
   Ans. 21m
9. BO and CO are respectively the bisector of ∠B and ∠C of ∆ABC. AO produced meets BC at P, then find AB/AC

Ans. \( \frac{BP}{PC} \)

10. In ▲ABC, the bisectors of ∠B intersects the side AC at D. A line parallel to side AC intersects line segments AB, DB and CB at points P, R, Q respectively. Then, Find AB × CQ

Ans. BC × AP

11. If ∆ABC is an equilateral triangle such that AD ⊥ BC, then AD^2 = ............

Ans. 3CD^2

12. If ∆ABC and ∆DEF are similar triangles such that ∠A = 47°, and ∠E = 83°, then find ∠C

Ans. 50°

13. Two isosceles triangles have equal angles and their areas are in the ratio 16:25, then find the ratio of their corresponding heights

Ans. 4:5

14. Two poles of heights 6m and 11m stand vertically upright on a plane ground. If the distance between their feet is 12m, then find the distance between their tops.

Ans. 13m

15. The lengths of the diagonals of a rhombus are 16cm and 12cm. Then, find the length of the side of the rhombus.

Ans. 10cm

(Level - 2)

1. In given fig. BD ⊥ AC and CE ⊥ AB then prove that
   (a) △AEC ~ △ADB
   (b) CA/AB = CE/DB

![Diagram](image)

2. In the given figure fig. \( \frac{PS}{SQ} = \frac{PT}{TR} \) and ∠PST = ∠PQR. Prove that ∆PQR is an isosceles triangle.

![Diagram](image)
3. In given fig AD \perp BC and \angle B < 90^0, prove that \( AC^2 = AB^2 + BC^2 - 2BC \times BD \)

![Diagram 1]

4. In given fig. \( \triangle ABC \) is right angled at C and DE \perp AB. Prove that \( \triangle ABC \sim \triangle ADE \) and hence find length of AE and DE.

\[ \text{Ans.} \frac{15}{17}, \frac{36}{17} \]

5. In a \( \triangle ABC \) if DE \| AC and DF \| AE, prove that \( \frac{EF}{BF} = \frac{EC}{BE} \)

6. In given fig. \( \triangle ABC \) if \( \frac{BD}{DA} = \frac{DA}{DC} \) prove that \( \triangle ABC \) is a right angled triangle.

![Diagram 2]

7. Two \( \triangle \)s ABC and DEF are similar. If \( \text{ar}(\triangle DEF) = 243 \text{cm}^2 \), \( \text{ar}(\triangle ABC) = 108 \text{cm}^2 \) and BC = 6 cm, find EF.

\[ \text{Ans.} 9 \text{ cm} \]

8. What is the value of K in given figure if DE \| BC.

\[ \text{Ans.} K = 4, -1 \]

9. A pole of length 10 m casts a shadow 2 m long on the ground. At the same time a tower casts a shadow of length 60 m on the ground then find the height of the tower.

\[ \text{Ans.} 300 \text{ m} \]

**Level - 3**

1. In given figure, AB \| DC and \( \frac{AO}{OC} = \frac{BO}{OD} \) then find the value of x, if.

\[ OA = 2x + 7, OB = 4x, OD = 4x - 4 \text{ and } OC = 2x + 4 \]

\[ \text{Ans.} 7 \]
2. PQR is a right angled triangle with $\angle P=90^0$. If PM $\perp$ QR, then show that $PM^2 = QM \times MR$.

3. In given fig. $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1=\angle 2$. Show that $\Delta PQS \sim \Delta TQR$.

4. Find the length of altitude of an equilateral triangle of side 2cm.
   Ans. $\sqrt{3}$ cm

5. In a trapezium ABCD, O is the point of intersection of AC and BD, AB $\parallel$ CD and AB=2CD. If the area of $\Delta AOB=84$ cm$^2$ then find area of $\Delta COD$.
   Ans. 21 cm$^2$

6. In given fig. $\frac{PS}{PT} = 3$. If area of $\Delta PQR$ is 32 cm$^2$, then find the area of the quad. STQR
   Ans. 14 cm$^2$

7. M is the mid-point of the side CD of a ||gm ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL=2BL.

8. Prove that the ratio of the area of two similar $\Delta$s is equal to the square of the ratio of their corresponding medians.

9. D and E are points on the sides CA and CB respectively of $\Delta ABC$, right angled at C. Prove that $AE^2+BD^2=AB^2+DE^2$.

10. ABC and DBC are two $\Delta$s on the same base BC and on the same side of BC with $\angle A=\angle D=90^0$. If CA and BD meet each other at E, show that $AE \times EC=BE \times ED$. 

**Level - 4**
1. Prove that in a right angled triangle the square of hypotenuse is equal to the sum of the squares of the other two sides.

2. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided into the same ratio.

3. Δ ABC is right angled at B and D is midpoint of side BC. Prove that \( AC^2 = 4AD^2 - 3AB^2 \)

4. Prove that the ratio of the areas of two similar triangles is equal to the ratio of square of their corresponding sides.

5. In a Δ, if the square of one side is equal to the sum of the squares of the other two sides, prove that the angle opposite to the first side is a right angle.

6. In an equilateral Δ ABC, D is a point on the side BC, such that BD = \( \frac{1}{3}BC \). Prove that \( 9AD^2 = 7AB^2 \)

7. P and Q are the mid points of side CA and CB respectively of Δ ABC right angled at C. Prove that \( 4(AQ^2 + BP^2) = 5AB^2 \).

8. CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC~ΔPQR, prove that
   (i) ΔAMC~ΔPNR
   (ii) CM/RN = AB/PQ
   (iii) ΔCMB~ΔRNQ

**SELF EVALUATION**

1. The diagonal BD of a ||gm ABCD intersects the line segment AE at the point F, where E is any point on the side BC. Prove that \( DF \times EF = FB \times FA \).

2. In fig DB ⊥ BC, DE ⊥ AB and AC ⊥ BC. Prove that \( BE/DE = AC/BC \).

3. In given fig. PA, QB, RC are each perpendicular to AC. Prove that \( \frac{1}{x} + \frac{1}{z} = \frac{1}{y} \).
4. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

5. ABC is a right triangle with $\angle A = 90^\circ$, A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm. find the radius of the incircle. Ans. 4 cm

6. ABC is a right triangle, right angled at C. If $p$ is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that
   (i) $cp = ab$ (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

7. In a trapezium ABCD, AB || DC and DC=2AB.EF || AB, where E and F lie on the side BC and AD respectively such that BE/EC=4/3. Diagonal DB intersects EF at G. Prove that EF=11AB.

8. Sides AB, AC and median AD of a triangle ABC are respectively proportional to sides PQ, PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$. 
INTRODUCTION TO TRIGONOMETRY

IMPORTANT CONCEPTS

TAKE A LOOK:

1. Trigonometric ratios of an acute angle of a right angled triangle.

\[
\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC}
\]

\[
\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC}
\]

\[
\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB}
\]

\[
cot \theta = \frac{1}{\tan \theta} = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC}
\]

\[
\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta} = \frac{AC}{AB}
\]

\[
\csc \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle \theta} = \frac{AC}{BC}
\]

2. Relationship between different trigonometric ratios

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
cot \theta = \frac{\cos \theta}{\sin \theta}
\]

\[
\tan \theta = \frac{1}{\cot \theta}
\]

\[
\cos \theta = \frac{1}{\sec \theta}
\]

\[
\sin \theta = \frac{1}{\csc \theta}
\]

3. Trigonometric Identities.

(i) \(\sin^2 \theta + \cos^2 \theta = 1\)

(ii) \(1 + \tan^2 \theta = \sec^2 \theta\)

(iii) \(1 + \cot^2 \theta = \csc^2 \theta\)

4. Trigonometric Ratios of some specific angles.

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \theta)</td>
<td>0</td>
<td>½</td>
<td>1/√2</td>
<td>√3/2</td>
<td>1</td>
</tr>
<tr>
<td>(\cos \theta)</td>
<td>1</td>
<td>√3/2</td>
<td>1/√2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>(\tan \theta)</td>
<td>0</td>
<td>1/√3</td>
<td>1</td>
<td>√3</td>
<td>Not defined</td>
</tr>
<tr>
<td>(\cot \theta)</td>
<td>Not defined</td>
<td>√3</td>
<td>1</td>
<td>1/√3</td>
<td>0</td>
</tr>
<tr>
<td>(\sec \theta)</td>
<td>1</td>
<td>2/√3</td>
<td>√2</td>
<td>2</td>
<td>Not defined</td>
</tr>
<tr>
<td>(\csc \theta)</td>
<td>Not defined</td>
<td>2</td>
<td>√2</td>
<td>2/√3</td>
<td>1</td>
</tr>
</tbody>
</table>
5. Trigonometric ratios of complementary angles.
   (i) \( \sin (90^\circ - \theta) = \cos \theta \)
   (ii) \( \cos (90^\circ - \theta) = \sin \theta \)
   (iii) \( \tan (90^\circ - \theta) = \cot \theta \)
   (iv) \( \cot (90^\circ - \theta) = \tan \theta \)
   (v) \( \sec (90^\circ - \theta) = \cosec \theta \)
   (vi) \( \cosec (90^\circ - \theta) = \sec \theta \)

(Level – 1)

1. If \( \theta \) and \( 3\theta - 30^\circ \) are acute angles such that \( \sin \theta = \cos (3\theta - 30^\circ) \), then find the value of \( \tan \theta \).
   Ans. \( \frac{1}{\sqrt{3}} \)

2. Find the value of \( \frac{\cos 30^\circ + \sin 60^\circ}{\cos 60^\circ + \sin 30^\circ} \)
   Ans. \( \sqrt{3} \)

3. Find the value of \( (\sin \theta + \cos \theta)^2 + (\cos \theta - \sin \theta)^2 \)
   Ans. \( 2 \)

4. If \( \tan \theta = \frac{3}{4} \) then find the value of \( \cos^2 \theta - \sin^2 \theta \)
   Ans. \( \frac{7}{25} \)

5. If \( \sec \theta + \tan \theta = p \), then find the value of \( \sec \theta - \tan \theta \)
   Ans. \( \frac{1}{p} \)

6. Change \( \sec^4 \theta - \sec^2 \theta \) in terms of \( \tan \theta \).
   Ans. \( \tan^4 \theta + \tan^2 \theta \)

7. If \( \cot \theta = \frac{1}{\sqrt{3}} \) then find the value of \( \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} \)
   Ans. \( \frac{3}{5} \)

8. If \( \cot \theta + \frac{1}{\cot \theta} = 2 \) then find the value of \( \cot^2 \theta + \frac{1}{\cot^2 \theta} \)
   Ans. \( 2 \)

9. If \( \sin \theta = \frac{a}{b} \), then find the value of \( \sec \theta + \tan \theta \)
   Ans. \( \frac{b + a}{\sqrt{b^2 - a^2}} \)

10. If \( \cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ \), then find the value of \( x \)
    Ans. \( 30^\circ \)

11. If \( 0^\circ \leq x \leq 90^\circ \) and \( 2\sin^2 x = 1/2 \), then find the value of \( x \)
    Ans. \( 30^\circ \)

12. Find the value of \( \cosec^2 30^\circ - \sin^2 45^\circ \cdot \sec^2 60^\circ \)
    Ans. \(-2\)

13. Simplify \( (\sec \theta + \tan \theta)(1 - \sin \theta) \)
    Ans. \( \cos \theta \)
1. If secα=5/4 then evaluate tanα/(1+tan²α).  
   Ans: 12/25

2. If A+B =90°, then prove that \( \sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B}} - \frac{\sin^2 B}{\cos^2 B} = \tan A \)

3. Prove that cosA/(1-sinA)+cosA/(1+sinA)=2secA.

4. Prove that \( \frac{\sec A - 1}{\sec A + 1} + \frac{\sec A + 1}{\sec A - 1} = 2\csc A \)

5. Prove that (sinθ+cosecθ)² + (cosθ+secθ)² = 7+tan²θ+cot²θ.

6. Evaluate \( \frac{11 \sin 70^\circ}{4 \cos 53^\circ \csc 37^\circ} \) \( \frac{\tan 15^\circ \tan 35^\circ \tan 55^\circ \tan 75^\circ}{7 \cos 20^\circ} \)  
   Ans: 1

7. Prove that \( \sqrt{\frac{\csc A - 1}{\csc A + 1}} + \frac{\csc A + 1}{\csc A - 1} = 2\sec A \)

8. In a right angle triangle ABC, right angled at B, if tanA=1, then verify that 2sinA cosA = 1.

9. If tan (A-B)=\sqrt{3}, and sinA =1, then find A and B.  
   Ans: 90° & 30°

10. If θ is an acute angle and sinθ=cosθ, find the value of 3tan²θ + 2sin²θ - 1.  
    Ans: 3

11. If \( \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \) and \( \frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1 \), prove that \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \).

---

**Level - 3**

1. Evaluate the following: \(- \sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ)\).  
   Ans: 2

2. If \( \frac{\cos A}{\cos \beta} = m \) and \( \frac{\cos A}{\sin \beta} = n \), show that \( (m^2+n^2) \cos^2 \beta = n^2 \).

3. Prove that tan²θ + cot²θ + 2 = cosec²θ sec²θ.

4. Prove that \( (\tan A - \tan B)^2 + (1+\tan A \tan B)^2 = \sec^2 A \sec^2 B \).

5. If (cosθ-sinθ) = \sqrt{2} sinθ, then show that cosθ + sinθ = \sqrt{2} cos θ.

6. Prove that \( (\sin θ + \sec θ)^2 + (\cos θ + \csc θ)^2 = (1+\sec θ \cosec θ)^2 \).
7. Prove that $\sin\theta/(1-\cos\theta) + \tan\theta/(1+\cos\theta) = \sec\theta\csc\theta + \cot\theta$.

8. Prove that $(\sin\theta - \csc\theta) (\cos\theta - \sec\theta) = \frac{1}{\tan\theta + \cot\theta}$.

9. If $\cot\theta = \frac{15}{8}$, evaluate $(2 + 2\sin\theta) (1 - \sin\theta)/(1+\cos\theta) (2 - 2\sin\theta)$.

**Level - 4**

1. Prove that $(\sec\theta + \tan\theta - 1)/(\tan\theta - \sec\theta + 1) = \cos\theta/(1 - \sin\theta)$.

2. If $x = r \sin Acos C$, $y = r \sin Asin C$, $z = r \cos A$, Prove that $r^2 = x^2 + y^2 + z^2$.

3. Prove that $\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}$.

4. If $x = \sin\theta$, $y = \tan\theta$, prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

5. Prove that: $\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta$

6. Evaluate $\frac{\sin^2\theta + \sin^2(90^\circ - \theta)}{3(\sec^2 61^\circ - \cot^2 29^\circ)} - \frac{3\cot^2 30^\circ \sin^2 54^\circ \sec^2 36^\circ}{2(\cosec^2 65^\circ - \tan^2 25^\circ)}$.  
   Ans. $\frac{25}{6}$

7. Prove that $\frac{1 + \cos A + \sin A}{1 + \cos A - \sin A} = \frac{1 + \sin A}{\cos A}$.

8. Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

9. Prove that $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$.

10. If $\cot\theta = \frac{7}{8}$, evaluate (i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$.  
   Ans. 1, $-\frac{15}{113}$

**Self Evaluation**

1. If $a \cos\theta + b \sin\theta = c$, then prove that $a \sin\theta - b \cos\theta = \pm \sqrt{a^2 + b^2 - c^2}$.

2. If $A, B, C$ are interior angles of triangle $ABC$, show that $\cosec^2\left(\frac{B+C}{2}\right) - \tan^2 \frac{A}{2} = 1$.

3. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$. 
4. If $\tan A = n\tan B$, $\sin A = m\sin B$, prove that $\cos^2 A = (m^2 - 1)/(n^2 - 1)$.

5. Evaluate $[\sec \theta \csc(90^\circ - \theta) - \tan \theta \cot(90^\circ \theta) + \sin^2 55^\circ \sin^2 35^\circ] / (\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ)$. Ans: $\frac{2}{\sqrt{3}}$

6. If $\sec \theta + \tan \theta = p$, prove that $\sin \theta = (p^2 - 1)/(p^2 + 1)$. 
The three measures of central tendency are:

i. Mean
ii. Median
iii. Mode

- Mean of grouped frequency distribution can be calculated by the following methods.

(i) **Direct Method**
\[
\text{Mean} = \bar{X} = \frac{\sum_{i=1}^{n} f_{i}X_{i}}{\sum_{i=1}^{n} f_{i}}
\]
Where \(X_{i}\) is the class mark of the \(i^{th}\) class interval and \(f_{i}\) frequency of that class

(ii) **Assumed Mean method or Shortcut method**
\[
\text{Mean} = \bar{X} = a + \frac{\sum_{i=1}^{n} f_{i}d_{i}}{\sum_{i=1}^{n} f_{i}} \times h
\]
Where \(a = \text{assumed mean}\)
And \(d_{i} = X_{i} - a\)

(iii) **Step deviation method**
\[
\text{Mean} = \bar{X} = a + \frac{\sum_{i=1}^{n} f_{i}u_{i}}{\sum_{i=1}^{n} f_{i}} \times h
\]
Where \(a = \text{assumed mean}\)
\(h = \text{class size}\)
And \(u_{i} = (X_{i} - a)/h\)

- Median of a grouped frequency distribution can be calculated by
\[
\text{Median} = l + \left(\frac{n/2 - cf}{f}\right) \times h
\]
Where
\(l = \text{lower limit of median class}\)
\(n = \text{number of observations}\)
\(cf = \text{cumulative frequency of class preceding the median class}\)
\(f = \text{frequency of median class}\)
\(h = \text{class size of the median class}\).

- Mode of grouped data can be calculated by the following formula.
\[
\text{Mode} = l + \left(\frac{f_{1} - f_{0}}{2f_{1} - f_{0} - f_{2}}\right) \times h
\]
Where
\(l = \text{lower limit of modal class}\)
\(h = \text{size of class interval}\)
\(f_{1} = \text{Frequency of the modal class}\)
\(f_{0} = \text{frequency of class preceding the modal class}\)
\(f_{2} = \text{frequency of class succeeding the modal class}\)

- Empirical relationship between the three measures of central tendency.
\[3 \times \text{Median} = \text{Mode} + 2 \times \text{Mean}\]
Or, \(\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}\)

- Ogive
Ogive is the graphical representation of the cumulative frequency distribution. It is of two types:
(i) Less than type ogive.
(ii) More than type ogive
Median by graphical method
The x-coordinates of the point of intersection of ‘less than ogive’ and ‘more than ogive’ gives the median.

LEVEL – 1

<table>
<thead>
<tr>
<th>Slno</th>
<th>Question</th>
<th>Ans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What is the mean of 1st ten prime numbers?</td>
<td>12.9</td>
</tr>
<tr>
<td>2</td>
<td>What measure of central tendency is represented by the abscissa of the point where less than ogive and more than ogive intersect?</td>
<td>Median</td>
</tr>
<tr>
<td>3</td>
<td>If the mode of a data is 45 and mean is 27, then median is ____________.</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>Find the mode of the following</td>
<td></td>
</tr>
</tbody>
</table>
|      | \[
<table>
<thead>
<tr>
<th>X_i</th>
<th>f_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>5</td>
</tr>
<tr>
<td>38</td>
<td>9</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>42</td>
<td>7</td>
</tr>
<tr>
<td>44</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Mode =40</td>
</tr>
<tr>
<td>5</td>
<td>Write the median class of the following distribution.</td>
</tr>
<tr>
<td></td>
<td>Class</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
</tr>
</tbody>
</table>

LEVEL – 2

<table>
<thead>
<tr>
<th>Slno</th>
<th>Question</th>
<th>Ans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Calculate the mean of the following distribution</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>Class interval</td>
<td>50-60</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>Find the mode of the following frequency distribution</td>
<td>33.33</td>
</tr>
<tr>
<td></td>
<td>Marks</td>
<td>10-20</td>
</tr>
<tr>
<td></td>
<td>No. of students</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>Find the median of the following distribution</td>
<td>28.5</td>
</tr>
<tr>
<td></td>
<td>Class interval</td>
<td>0-10</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>A class teacher has the following absentee record of 40 students of a class for the whole term.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. of days</td>
<td>0-6</td>
</tr>
<tr>
<td></td>
<td>No. of students</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Write the above distribution as less than type cumulative frequency distribution.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Answer:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. of days</td>
<td>Less Than 6</td>
</tr>
<tr>
<td></td>
<td>No. of students</td>
<td>11</td>
</tr>
</tbody>
</table>
### LEVEL – 3

<table>
<thead>
<tr>
<th>Slno</th>
<th>Question</th>
<th>Ans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If the mean distribution is 25. Then find p.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Class</strong></td>
<td>0-10</td>
</tr>
<tr>
<td></td>
<td><strong>Frequency</strong></td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Find the mean of the following frequency distribution using step deviation method</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td><strong>Class</strong></td>
<td>0-10</td>
</tr>
<tr>
<td></td>
<td><strong>Frequency</strong></td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Find the value of p if the median of the following frequency distribution is 50.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Class</strong></td>
<td>20-30</td>
</tr>
<tr>
<td></td>
<td><strong>Frequency</strong></td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>Find the median of the following data.</td>
<td>76.36</td>
</tr>
<tr>
<td></td>
<td><strong>Marks</strong></td>
<td>Less than 10</td>
</tr>
<tr>
<td></td>
<td><strong>Frequency</strong></td>
<td>0</td>
</tr>
</tbody>
</table>

### LEVEL – 4

<table>
<thead>
<tr>
<th>Slno</th>
<th>Question</th>
<th>Ans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The mean of the following frequency distribution is 57.6 and the sum of the observations is 50. Find the missing frequencies $f_1$ and $f_2$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Class</strong></td>
<td>0-20</td>
</tr>
<tr>
<td></td>
<td><strong>Frequency</strong></td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>The following distribution give the daily income of 65 workers of a factory</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Daily income (in Rs)</strong></td>
<td>100-120</td>
</tr>
<tr>
<td></td>
<td><strong>No. of workers</strong></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Convert the above to a more than type cumulative frequency distribution and draw its ogive.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Draw a less than type and more than type ogives for the following distribution on the same graph. Also find the median from the graph.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Marks</strong></td>
<td>30-39</td>
</tr>
<tr>
<td></td>
<td><strong>No. of students</strong></td>
<td>14</td>
</tr>
</tbody>
</table>
SELF – EVALUATION

1. What is the value of the median of the data using the graph in figure of less than ogive and more than ogive?

2. If mean =60 and median =50, then find mode using empirical relationship.

3. Find the value of p, if the mean of the following distribution is 18.

   | Variate ($x_i$) | 13 | 15 | 17 | 19 | 20+p | 23 |
   | Frequency ($f_i$) | 8  | 2  | 3  | 4  | 5p   | 6  |

4. Find the mean, mode and median for the following data.

   | Classes     | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
   | frequency   | 5    | 8     | 15    | 20    | 14    | 8     | 5     |

5. The median of the following data is 52.5. find the value of x and y, if the total frequency is 100.

   | Class Interval | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
   | frequency     | 2    | 5     | X     | 12    | 17    | 20    | Y     | 9     | 7     | 4      |

6. Draw ‘less than ogive’ and ‘more than ogive’ for the following distribution and hence find its median.

   | Classes     | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 |
   | frequency   | 10    | 8     | 12    | 24    | 6     | 25    | 15    |

7. Find the mean marks for the following data.

   | Marks     | Below 10 | Below 20 | Below 30 | Below 40 | Below 50 | Below 60 | Below 70 | Below 80 | Below 90 | Below 100 |
   | No. of students | 5        | 9        | 17        | 29        | 45        | 60        | 70        | 78        | 83        | 85        |

8. The following table shows age distribution of persons in a particular region. Calculate the median age.

   | Age in     | Below 10 | Below 20 | Below 30 | Below 40 | Below 50 | Below 60 | Below 70 | Below 80 | Below 90 | Below 100 |
   |           |          |          |          |          |          |          |          |          |          |            |
9. If the median of the following data is 32.5. Find the value of $x$ and $y$.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-10</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>$x$</td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>$y$</td>
<td>3</td>
<td>2</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>years</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of persons</td>
<td>200</td>
<td>500</td>
<td>900</td>
<td>1200</td>
<td>1400</td>
<td>1500</td>
<td>1550</td>
<td>1560</td>
</tr>
</tbody>
</table>
SAMPLE PAPER FOR SA-1
CLASS – X
MATHEMATICS

Time : 3 hours
Maximum Marks : 90

General Instructions:
1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 8 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 10 questions of 4 marks each.
3. Question numbers 1 to 8 in Section A are multiple choice questions where you are to select on correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.
6. An additional 15 minutes time has been allotted to read this question paper only.

SECTION A

[1] If the system of liner equations x - ky =2 and 3x + 2y =-5 has a unique solution, then the value of k is: -
   [a] k = \frac{2}{3}  [b] k ≠ \frac{2}{3}  [c] k = \frac{3}{2}  [d] k ≠ \frac{3}{2}

[2] If tan\( \theta = \frac{3}{4} \) then the value of \( \frac{1-cos\theta}{1+cos\theta} \) is:-
   [a]– \frac{1}{9}  [b] \frac{2}{9}  [c] 1  [d] \frac{1}{9}

[3] If sin3x = cos(x-26°) and 3x is an actual angle, then the value of x is :-

[4] If x=3² x 5² , y=2²x3³ , then HCF [x, y] is :-

[5] If a positive integer p is divided by 3, then the remainder can be:-
   [a] 1 or 3  [b] 1, 2 or 3  [c] 0, 1 or 2  [d] 2 or 3

[6] If the given figure, the value of tanP – cotR is:-
   [a] 1  [b] 0  [c] -1  [d] 2

[7] Construction of a cumulative frequency table is useful in determining the :-

[8] In the given figure, if \( \angle A = \angle D = 90° \), AD=6cm, CD = 8cm and BC =26cm then ar(\( \Delta ABC \)) is :-
   [a] 240cm²  [b] 48cm²  [c] 120²  [d] 260cm²
SECTION – B

[9] Find the value of \( p \) and \( q \) in the given figure, if \( ABCD \) is a rectangle

\[
\begin{align*}
3p + q & \quad \text{at point A} \\
13 & \quad \text{at point B}
\end{align*}
\]

[10] If \( \alpha \) and \( \frac{1}{\alpha} \) are the zeroes of the polynomial \( p(x) = 4x^2 - 2x + k - 4 \), then find the value of \( k \)

Or

Divide the polynomial \( p(x) = 5 - 3x + 3x^2 - x^3 \) by \( g(x) = x^2 - x + 1 \) and find the quotient and remainder

[11] Without actually performing the long division, state whether \( \frac{39}{343} \) will have a terminating or non-terminating, repeating decimal expansion

[12] Find the value of \( k \), if

\[
\frac{\cos 35^\circ}{\sin 55^\circ} + \frac{2 \sin \theta}{\cos (90^\circ - \theta)} = \frac{k}{2}
\]

[13] \( ABC \) is right angle triangle with \( \angle ABC = 90^\circ \), \( BD \perp AC \), \( DM \perp BC \), and \( DN \perp AB \). Prove that \( DM^2 = DN \times BC \).

\[
\begin{align*}
&D \quad (BD) \\
&m \quad (DN) \\
&A \quad (AB) \\
&C \quad (BC) \\
&D \quad (BD)
\end{align*}
\]

[14] The following table gives production yield per hectare of wheat of 100 farms of village:-

<table>
<thead>
<tr>
<th>Production (in kg/hec)</th>
<th>25-35</th>
<th>35-45</th>
<th>45-55</th>
<th>55-65</th>
<th>65-75</th>
<th>75-85</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of farms</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>26</td>
<td>35</td>
<td>19</td>
</tr>
</tbody>
</table>

Write the above distribution to a more than type distribution.

SECTION - C

[15] Prove that \( \frac{\sqrt{7}}{4} \) is irrational.

Or

Prove that \( (16-5\sqrt{7}) \) is irrational.

[16] If one diagonal of a trapezium divides the other diagonal in the ratio 1:2. Prove that one of the parallel sides is double the other?

[17] Prove that:

\[
\frac{\sin \theta}{\cot \theta + \csc \theta} = 2 + \frac{\sin \theta}{\cot \theta - \csc \theta}
\]

[18] The sum of the numerator and denominator, of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction become \( \frac{3}{4} \). Find the fraction.

Or

Seven times a two digit number is equal to 4 times the number obtained by reversing the order of its digits. If the difference of digit is 3, find the number.

[19] If one zero of the polynomial \( p(x) = 3x^2 - 8x + 2k + 1 \) is seven times of other, then find the zeroes and the value of \( k \).

[20] If \( \sin \theta + \sin^2 \theta + \sin^3 \theta = 1 \), prove that \( \cos^5 \theta - 4 \cos^4 \theta + 8 \cos^2 \theta = 4 \).
[21] Find the mean of the following data, using step-deviation method:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-100</th>
<th>100-120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>7</td>
<td>8</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

Or

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>17</td>
<td>28</td>
<td>32</td>
<td>p</td>
<td>19</td>
</tr>
</tbody>
</table>

If the mean of the above data is 50, then find the value of p?

[22] Prove that \( \tan \theta \cdot \cot \theta = \frac{2 \sin^2 \theta - 1}{\sin \theta \cos \theta} \)

[23] In \( \triangle ABC \), if AD is the median, then show that \( AB^2 + AC^2 = 2[AD^2 + BD^2] \).

[24] Find the median of the following data:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>12</td>
<td>30</td>
<td>34</td>
<td>65</td>
<td>46</td>
<td>25</td>
<td>18</td>
</tr>
</tbody>
</table>

SECTION-D

[25] Prove that, if a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points the other two sides are divided into the same ratio.

Or

Prove that in a right triangle the square of the hypotenuse is equal to the sum of the square of the other two sides?

[26] If \( x = a \sin \theta \), \( y = b \tan \theta \), prove that \( \frac{a^2}{x^2} + \frac{b^2}{y^2} = 1 \).

[27] On dividing \( 3x^3 + 4x^2 + 5x - 13 \) by a polynomial \( g(x) \), the quotient and remainder are \( 3x + 10 \) and \( 16x + 43 \) respectively, Find the polynomial \( g(x) \).

[28] The fraction becomes \( \frac{9}{11} \) if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes \( \frac{5}{6} \). Find the fraction.

[29] The following table shows the ages of the patients admitted in a hospital during a year:

<table>
<thead>
<tr>
<th>AGE(In years)</th>
<th>5-15</th>
<th>15-25</th>
<th>25-35</th>
<th>35-45</th>
<th>45-55</th>
<th>55-65</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of patients</td>
<td>6</td>
<td>11</td>
<td>21</td>
<td>23</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the mode and mean of the data given above.

[30] The perpendicular from A on the side BC of \( \triangle ABC \) intersects BC at D such that DB=3CD Prove that \( 2AB^2 = 2AC^2 + BC^2 \).

[31] Draw the graph of following eqn:-

\[ 2x + 3y = 12 \text{ and } x - y = 1 \]

Shade the region between the two lines and x – axis. Also, determine the vertices of the triangle so formed.
[32] Prove that: \( \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin^2 \theta}{\sin \theta - \cos \theta} = \sin \theta + \cos \theta \)

Or

Evaluate: \( \frac{\sin^2 \theta + \sin^2(90^\circ - \theta)}{3(\sec^261^\circ - \cot^229^\circ)} \cdot \frac{3 \cot^230^\circ \sin^254^\circ \sec^236^\circ}{2(\csc^265^\circ - \tan^225^\circ)} \).

[33] In a sports meet, the number of players in Football, Hockey and Athletics are 48, 60, 132, respectively. Find the minimum number of rooms required, if in each room the same number of players are to be seated and all of them being in the same sports?

[34] The following distribution gives the daily income of 65 workers of a factory:

<table>
<thead>
<tr>
<th>Daily income (in Rs)</th>
<th>100-120</th>
<th>120-140</th>
<th>140-160</th>
<th>160-180</th>
<th>180-200</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of worker</td>
<td>14</td>
<td>16</td>
<td>10</td>
<td>16</td>
<td>9</td>
</tr>
</tbody>
</table>

Convert the distribution above to a more than type cumulative frequency distribution and draw its ogive.
MARKING SCHEME

SUMMATIVE ASSESSMENT – 1

<table>
<thead>
<tr>
<th>Sl.no</th>
<th>marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (b)</td>
<td>1</td>
</tr>
<tr>
<td>2. (d)</td>
<td>1</td>
</tr>
<tr>
<td>3. (a)</td>
<td>1</td>
</tr>
<tr>
<td>4. (a)</td>
<td>1</td>
</tr>
<tr>
<td>5. (c)</td>
<td>1</td>
</tr>
<tr>
<td>6. (b)</td>
<td>1</td>
</tr>
<tr>
<td>7. (b)</td>
<td>1</td>
</tr>
<tr>
<td>8. (c)</td>
<td>1</td>
</tr>
</tbody>
</table>

SECTION – B

9. Since opposite sides of the rectangle are equal
So, \( P + 3q = 13, \quad 3p + q = 7 \)
Solving \( p = 1, \quad q = 4 \)

10. Since \( \alpha \) and \( \frac{1}{\alpha} \) are the zeros of the polynomial
\( P(x) = 4x^2 - 2x + k - 4 \)
So, \( \alpha \times \frac{1}{\alpha} = \frac{k - 4}{4} \)
\( \Rightarrow 1 = \frac{k - 4}{4} \)
\( \Rightarrow k = 8 \)
or
\[ x^2 - x + 1 - x^3 + 3x^2 - 3x + 5(x + 2) \]
\[ - x^3 + x^2 - x \]
\[ + \]
\[ 2x^2 - 2x + 5 \]
\[ 2x^2 - 2x + 2 \]
\[ \square \]
\[ 3 \]
\[ \square \]
\[ \square \]

So, quotient = \(-x + 2\), remainder = 3

11. Here, \( \frac{39}{343} = \frac{3 \times 13}{7 \times 7 \times 7} \)
Since denominator contains prime factor 7 other than 2 or 5
So, \( \frac{39}{343} \) will have a non-terminating repeating decimal expansion.

12. we have \( \frac{\cos 35^0}{\sin 55^0} + \frac{2 \sin \theta}{\cos (90^0 - \theta)} = \frac{k}{2} \)
\[ \cos 35^0 + \frac{2 \sin \theta}{\sin \theta} = \frac{k}{2} \]
\[ 1 + 2 \frac{k}{2} \]
\[ K = 6 \]

13. \( \because DN \perp AB \) and \( \angle B = 90^0, DM \perp BC \)
So, \( DN \parallel BC \) and \( DM \parallel AB \)
so, \( DNB \) is a \( || \) grmn.
\( \Rightarrow DN = BM \)

In \( \triangle BDM \) and \( \triangle DCM, \)
\( \angle 1 = \angle 3, \quad \angle 2 = \angle 4, \)
by AA-Similarity \( \triangle BDM \sim \triangle DCM \)
\( \Rightarrow \frac{DM}{BM} = \frac{CM}{DM} \)
\( \Rightarrow DM^2 = CM \times BM \)
\( \Rightarrow DM^2 = CM \times DN \quad \text{Proved.} \)

14. \( 1 + 1 \)
More than type | Commutative frequency
---|---
more than 25 | 100
more than 35 | 96
more than 45 | 90
more than 55 | 80
more than 65 | 54
more than 75 | 19

**SECTION-C**

15. Let us assume that, on contrary $\frac{\sqrt{7}}{4}$ is rational
\[
\therefore \frac{\sqrt{7}}{4} = \frac{a}{b}, \text{where } a, b \text{ are integers with } b \neq 0
\]
\[
\Rightarrow \sqrt{7} = \frac{4a}{7b}
\]
\[
\therefore \frac{4a}{7b} \text{ is a rational number. So, } \sqrt{7} \text{ is rational.}
\]
but this contradicts the fact that $\sqrt{7}$ is irrational.
so our assumption is wrong.
\[
\therefore \sqrt{7} \text{ is irrational.}
\]

**OR**

on contrary, Let $16-5\sqrt{7}$ is rational
so, $16-5\sqrt{7}=\frac{a}{b}$, where $a, b$ are integers with $b \neq 0$
\[
\Rightarrow \sqrt{7} = \frac{16b-a}{5b} = \text{integer} \div \text{integer}
\]
\[
\Rightarrow \sqrt{7} = \text{rational}
\]
But, this contradicts the fact that $\sqrt{7}$ is irrational.
so, $16-5\sqrt{7}$ is irrational.

16. Given that
to prove
\[
\frac{CP}{AP} = \frac{1}{2}
\]
In $\triangle ABP$ and $\triangle CDP$,
\[
\angle ABP = \angle CDP \quad \text{ (alt. } \angle \text{s)}
\]
\[
\angle BAP = \angle DCP \quad \text{ (alt. } \angle \text{s)}
\]
\[
\therefore \triangle ABP \sim \triangle CDP \quad \text{(BY AA-Similarity)}
\]
\[
\Rightarrow \frac{AB}{DC} = \frac{AP}{CP} = \frac{2}{1}
\]
\[
\Rightarrow AB = 2DC \quad \text{ Proved.}
\]

17. Given:
\[
\frac{\sin \theta}{\cot \theta + \cosec \theta} = 2 + \frac{\sin \theta}{\cot \theta - \cosec \theta}
\]
\[
\Rightarrow \frac{\sin \theta}{\cot \theta + \cosec \theta} - \frac{\sin \theta}{\cot \theta - \cosec \theta} = 2
\]
L.H.S.\(=\sin \theta \left[ \frac{-2\cosec \theta}{\cot \theta + \cosec \theta} \right]
\]
\[
=\sin \theta \left[ \frac{-\sin \theta}{\cot \theta + \cosec \theta} \right]
\]
\[
=\frac{-2\sin \theta}{\cot \theta + \cosec \theta} = 2 \quad \text{Proved.}
\]

18. Let the numerator be $x$ and denominator be $y$, then fraction $= \frac{x}{y}$

As per question,
\[
x + y = 8
\]
\[
x + 3 = 3
\]
\[
y + 3 = 4
\]
solving, $x = 3$, $y = 5$
\[
\text{fraction} = \frac{x}{y} = \frac{3}{5}
\]

**OR**
Let the unit digit be \( x \) and tens digit be \( y \)
then \( \text{no.} = 10y + x \)

As per question
\[
7(10y + x) = 4(10x + y)
\]
and \( x - y = 3 \)
solving \( x = 6, \ y = 3 \)
so No. = 36

19. Let zeroes of the given polynomial \( p(x) \) be \( \alpha \) and \( \beta \)
then, as per question
\[
\beta = 7\alpha
\]
sum of zeroes \( \alpha + \beta = \frac{b}{a} \)
\[
\Rightarrow \alpha + 7\alpha = \frac{8}{3}
\]
\[
\Rightarrow \alpha = \frac{1}{3}
\]
also, Product of zeroes \( \alpha \beta = \frac{c}{a} \)
\[
\Rightarrow 7\alpha \times \alpha = \frac{2k + 1}{3}
\]
solving, \( k = \frac{2}{3} \)
zeros are \( \frac{1}{3}, \frac{7}{3} \)

20. We have,
\[
\sin\theta + \sin^2\theta + \sin^3\theta = 1
\]
\[
\sin\theta(1+\sin^2\theta)=1-\sin\theta
\]
\[
\Rightarrow \sin^2\theta(1+\sin\theta)=\cos^2\theta \text{ squaring both sides}
\]
\[
\Rightarrow (1-\cos\theta)(1+1-\cos\theta)^2= \cos^3\theta
\]
solving,
\[
\cos^6\theta-4\cos^4\theta+8\cos^2\theta=4
\]

21.

<table>
<thead>
<tr>
<th>Class - Interval</th>
<th>Mid value ((x_i))</th>
<th>frequency ((f_i))</th>
<th>(u_i = \frac{x_i - d}{h})</th>
<th>(f_i \times u_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>10</td>
<td>7</td>
<td>-2</td>
<td>-14</td>
</tr>
<tr>
<td>20-40</td>
<td>30</td>
<td>8</td>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>40-60</td>
<td>A = 50</td>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60-80</td>
<td>70</td>
<td>10</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>80-100</td>
<td>90</td>
<td>8</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>100-120</td>
<td>110</td>
<td>5</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>(\sum f_i = 50)</td>
<td></td>
<td>(\sum f_i \times u_i = 19)</td>
<td></td>
</tr>
</tbody>
</table>

Here, \( A = 50, \ h = 20, \ \sum f_i = 50, \ \sum f_i \times u_i = 19 \)

\[
\text{Mean}(\bar{x}) = A + \frac{\sum f_i \times u_i}{\sum f_i} \times h
\]
\[
\text{Mean}(\bar{x}) = 50 + \frac{19}{50} \times 20
\]
\[
= 57.6
\]

OR

<table>
<thead>
<tr>
<th>Class</th>
<th>frequency ((f_i))</th>
<th>class – mark ((x_i))</th>
<th>(f_i \times x_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>17</td>
<td>10</td>
<td>170</td>
</tr>
<tr>
<td>20-40</td>
<td>28</td>
<td>30</td>
<td>840</td>
</tr>
<tr>
<td>40-60</td>
<td>32</td>
<td>50</td>
<td>1600</td>
</tr>
<tr>
<td>60-80</td>
<td>(p)</td>
<td>70</td>
<td>(70p)</td>
</tr>
<tr>
<td>80-100</td>
<td>19</td>
<td>90</td>
<td>1710</td>
</tr>
<tr>
<td></td>
<td>(\sum f_i = 96 + p)</td>
<td>(\sum f_i \times x_i = 4320 + 70p)</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Mean}(\bar{x}) = \frac{\sum f_i \times x_i}{\sum f_i}
\]
Given \( x = 50 \)
\[ \Rightarrow 50 = \frac{4320 + 70p}{96 + p} \]

Solving \( p = 24 \)

22. \( L.H.S. = \tan \theta - \cot \theta \)
\[ = \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \]
\[ = \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} \]
\[ = \frac{\sin^2 \theta - 1 + \sin^2 \theta}{\sin \theta \cos \theta} \]
\[ = \frac{1}{\sin \theta \cos \theta} \]

23. \( \text{Construction: - Draw } AE \perp BC \)

In \( \triangle ADE \), \( AD^2 = AE^2 + ED^2 \)
\[ \Rightarrow AE^2 = AD^2 - DE^2 \quad (I) \]

In \( \triangle ABE, AB^2 = AE^2 + BE^2 \)
\( \angle B = 90^\circ, \text{ Pythagoras theorem} \)
\[ \Rightarrow AB^2 = AD^2 + BD^2 + 2BD.DE \quad \text{(II)} \]

In \( \triangle ACE, AC^2 = AE^2 + EC^2 \)
\[ \Rightarrow AC^2 = AD^2 - DE^2 + (CD + DE)^2 \]
\[ \Rightarrow AC^2 = AD^2 + BD^2 + 2BD.DE \quad \text{(As } BD = CD) \quad \text{(III)} \]

Adding (II) and (III)
\[ \Rightarrow AB^2 + AC^2 = 2[AD^2 + BD^2] \quad \text{Proved} \]

24. \[ \begin{array}{ccc}
\text{Class – Interval} & \text{Frequency \( f \)} & \text{Cumulative frequency} \\
10-20 & 12 & 12 \\
20-30 & 30 & 42 \\
30-40 & 34 & 76 \\
40-50 & 65 & 141 \\
50-60 & 46 & 187 \\
60-70 & 25 & 212 \\
70-80 & 18 & 230 \\
N = \sum f_i = 230 \\
\end{array} \]
\[ \frac{N}{2} = \frac{230}{2} = 115 \]

Median class is 40-50
\[ l = 40, \quad cf = 76, \quad f = 65, \quad h = 10 \]
\[ \therefore \text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h \]
\[ = 46 \]

25. \( \text{Correct given that, to prove, construction, fig.} \)
\( \text{Correct proof} \)
\( \text{OR} \)

Correct fig., given that, to prove, construction
\( \text{Correct proof} \)

26. \( \text{Given, } x = \sin \theta, \quad y = \tan \theta \)
\[ \Rightarrow \frac{a}{x} = \frac{1}{\sin \theta} \quad \text{and} \quad \frac{b}{y} = \frac{1}{\tan \theta} \]
\[ \Rightarrow \frac{a}{x} = \csc \theta \quad \text{and} \quad \frac{b}{y} = \cot \theta \]
\[ \text{LHS} = \frac{a^2}{x} - \frac{b^2}{y} = \csc^2 \theta - \cot^2 \theta = 1 = \text{RHS} \quad \text{Proved.} \]

27. \( \text{Here, } \text{Dividend } p(x) = 3x^3 + 4x^2 + 5x - 13 \)
\( \text{Quotient } q(x) = 3x + 10 \)
\( \text{Remainder } r(x) = 16x - 43 \)
By division algorithm

\[ p(x) = g(x)q(x) + r(x) \]
\[ g(x) = \frac{p(x) - r(x)}{q(x)} \]
\[ = \frac{(3x^3+4x^2+5x-13)-(16x-43)}{3x+10} \]
\[ = \frac{3x^3+4x^2-11x+30}{3x+10} \]

Correct division
\[ \therefore g(x) = x^2 - 2x + 3 \]

Let the fraction be \( \frac{x}{y} \)

As per question
\[ \frac{x+2}{y+2} = \frac{9}{11} \]
\[ \Rightarrow 11x - 9y + 4 = 0 \]

\[ \frac{x+3}{y+3} = \frac{5}{6} \]
\[ \Rightarrow 6x - 5y + 3 = 0 \]
Solving above, \( x = 7, y = 9 \)
\[ \therefore \text{Fraction} = \frac{7}{9} \]

29. 

<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>No. Of Patients ( (f_i) )</th>
<th>Class Mark ( (x_i) )</th>
<th>( f_i x_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-15</td>
<td>6</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>15-25</td>
<td>11</td>
<td>20</td>
<td>220</td>
</tr>
<tr>
<td>25-35</td>
<td>21</td>
<td>30</td>
<td>630</td>
</tr>
<tr>
<td>35-45</td>
<td>23</td>
<td>40</td>
<td>920</td>
</tr>
<tr>
<td>45-55</td>
<td>14</td>
<td>50</td>
<td>700</td>
</tr>
<tr>
<td>55-65</td>
<td>5</td>
<td>60</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>( \sum f_i = 80 )</td>
<td>( \sum f_i x_i = 2830 )</td>
<td></td>
</tr>
</tbody>
</table>

The modal class is 35-45
Here \( l = 35, f_1 = 23, f_2 = 14, f_0 = 21, h = 10 \)
Mode = \( l + \frac{\left( \frac{f_2-f_0}{2f_1-f_0-f_2} \right) \times h}{2} \)
\[ = 36.8 \]
Mean(\( \bar{x} \)) = \( \frac{\sum f_i x_i}{\sum f_i} \)
\[ = 35.68 \]

30. Given that, figure, to prove.

\[ \therefore DB = 3 \text{ CD} \]
\[ \therefore DB = \frac{3}{4} BC \]
and \( CD = \frac{1}{4} BC \)
In right \( \triangle ADB \)
\[ AB^2 = AD^2 + DB^2 \]
In right \( \triangle ACB \)
\[ AC^2 = AD^2 + CD^2 \]
\[ \therefore AB^2 - AC^2 = DB^2 - CD^2 \]
\[ = \left( \frac{3}{4} BC \right)^2 - \left( \frac{1}{4} BC \right)^2 = \frac{9}{16} BC^2 - \frac{1}{16} BC^2 = \frac{1}{2} BC^2 \]
\[ \Rightarrow AB^2 - AC^2 = \frac{1}{2} BC^2 \]
\[ \Rightarrow 2AB^2 = 2AC^2 + BC^2 \]
Proved.
31. Given Equations are
\[ \begin{align*}
2x + 3y &= 12 \quad \text{(i)} \\
x - y &= 1 \quad \text{(ii)}
\end{align*} \]

From (i) \[ y = \frac{12 - 2x}{3} \]

From (ii) \[ y = x - 1 \]

Graph for above equations

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Vertices of triangle are \((1,0), (6,0), (3,2)\)

32. \[ \text{LHS} = \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin ^2 \theta}{\sin \theta - \cos \theta} \]

\[ = \frac{\cos ^2 \theta - \sin \theta}{\cos \theta - \sin \theta} \]

\[ = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \]

OR

\[ \frac{\sin ^2 \theta + \sin ^2 (90^\circ - \theta)}{3(\sec ^2 61^\circ - \tan ^2 29^\circ)} = \frac{3 \cot ^2 62^\circ \sin ^2 54^\circ \sec ^2 36^\circ}{2(\csc ^2 65^\circ - \tan ^2 25^\circ)} \]

\[ = \frac{3^2 \times 2^2 \times 2 \times 2}{3^2 \times 2 \times 6} \]

\[ = 2 \]

\[ = \frac{3 	imes 1}{3 	imes 2} \]

\[ = \frac{1}{2} \]

33. Prime factorization of 48 = \(2^4 \times 3\)
Prime factorization of 60 = \(2^2 \times 3 \times 5\)
Prime factorization of 132 = \(2^2 \times 3 \times 11\)

HCF of (48, 60, 132) = 2 \times 2 \times 3 = 12

\[ \therefore \text{In each room 12 players of same sports can be accommodated.} \]

\[ \therefore \text{number of rooms required} = \frac{\text{Total number of players}}{\text{Number of players in a room}} \]

\[ \Rightarrow \text{number of rooms required} = \frac{48 + 60 + 132}{12} = 20 \]

34. correct table
correct graph
<table>
<thead>
<tr>
<th>SYMBOLS USED</th>
<th><strong>Important Questions</strong>,</th>
<th><strong>Very important questions</strong>,</th>
<th><strong>Very, Very Important questions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>**</td>
<td>***</td>
<td></td>
</tr>
</tbody>
</table>

### 01 Quadratic Equation
- **Standard form of quadratic equation**
  - NCERT Text book Q.1.2, Ex 4.1
- **Solution of quadratic equation by factorization**
  - Example 3,4,5, Q.1, 5 Ex. 4.2
- **Solution of quadratic equation by completing the square**
  - Example 8,9 Q.1 Ex. 4.3
- **Solution of quadratic equation by quadratic formula**
  - Example. 10,11,13,14,15 , Q.2,3(ii) Ex.4.3
- **Nature of roots**
  - Example 16 Q.1.2, Ex. 4.4

### 02 Arithmetic progression
- **General form of an A.P.**
  - Exp-1,2, Ex. 5.1 Q.s2(a), 3(a),4(v)
- **nth term of an A.P.**
  - Exp. 3,7,8 Ex. 5.2 Q.4,7,11,16,17,18
- **Sum of first n terms of an A.P.**
  - Exp.11,13,15 Ex. 5.3, Q.No.1(i, ii) Q3(i,iii) Q.7,10,12,11,6, Ex5.4, Q-1

### 03 Coordinate geometry
- **Distance formula**
  - Exercise 7.1, Q.No 1,2,3,4,7,8
- **Section formula**
  - Mid point formula
  - Example No. 6,7,9 Exercise 7.2, Q.No. 1,2,4,5 Example 10. Ex.7.2, 6,8,9, Q.No.7
- **Area of Triangle**
  - Ex.12,14 Ex 7.3 QNo-12,4 Ex.7.4, Qno-2

### 04 Some application of Trigonometry
- **Heights and distances**
  - Example-2,3,4 Ex 9.1 Q 2,5,10,12,13,14,15,16

### 05 Circles
- **Tangents to a circle**
  - Q3(Ex10.1) Q 1,Q6,Q7(Ex 10.2),4
- **Number of tangents from a point to a circle**
  - Theorem 10.1,10.2 Eg 2.1 Q8,9,,10,12,13 (Ex 10.2)

### 06 Constructions
- **Division of line segment in the given ratio**
  - Const 11.1 Ex 11.1 Qno 1
- **Construction of triangle similar to given triangle as per given scale**
  - Ex 11.1 Qno-2,4,5,7
- **Construction of tangents to a circle**
  - Ex 11.2 Qno 1,4

### 07 Area related to circles
- **Circumference of a circle**
  - Example 1 Exercise 12.1 Q.No 1,2,4
- **Area of a circle**
  - Example 5,3
<table>
<thead>
<tr>
<th>Topic</th>
<th>Type</th>
<th>Example(s)</th>
<th>Exercise(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of an arc of a circle</td>
<td>*</td>
<td></td>
<td>Exercise 12.2 Q No 5</td>
</tr>
<tr>
<td>Area of sector of a circle</td>
<td>**</td>
<td></td>
<td>Example 2, Exercise 12.2 QNo 1.2</td>
</tr>
<tr>
<td>Area of segment of a circle</td>
<td>**</td>
<td></td>
<td>Exercise 12.2 Qno 4,7,9,3</td>
</tr>
<tr>
<td>Combination of figures</td>
<td>***</td>
<td></td>
<td>Ex 12.3 Example 4.5 1,4,6,7,9,12,15</td>
</tr>
<tr>
<td>Surface area and volumes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface area of a combination of solids</td>
<td>**</td>
<td></td>
<td>Example 1,2,3, Exercise 13.1 Q1,3,6,7,8</td>
</tr>
<tr>
<td>Volume of combination of a solid</td>
<td>**</td>
<td></td>
<td>Example 6, Exercise 13.2 Q 1,2,5,6</td>
</tr>
<tr>
<td>Conversion of solids from one shape to another</td>
<td>***</td>
<td></td>
<td>Example 8 &amp; 10, Exercise 13.3 Q 1,2,6,4,5</td>
</tr>
<tr>
<td>Frustum of a cone</td>
<td>***</td>
<td></td>
<td>Example 12&amp; 14, Exercise 13.4 Q 1,3,4,5 Ex-13.5, Q. 5</td>
</tr>
<tr>
<td>Probability</td>
<td>Events</td>
<td>*</td>
<td>Ex 15.1 Q4,8,9</td>
</tr>
<tr>
<td>Probability lies between 0 and 1</td>
<td>**</td>
<td></td>
<td>Exp- 1,2,4,6,13</td>
</tr>
<tr>
<td>Performing experiment</td>
<td>***</td>
<td></td>
<td>Ex 15 1,13,15,18,24</td>
</tr>
</tbody>
</table>
QUADRATIC EQUATIONS

KEY POINTS

1. The general form of a quadratic equation is $ax^2+bx+c=0$, $a \neq 0$. $a$, $b$ and $c$ are real numbers.
2. A real number $x$ is said to be a root of the quadratic equation $ax^2+bx+c=0$ where $a \neq 0$ if $ax^2+bx+c=0$. The zeroes of the quadratic equation polynomial $ax^2+bx+c=0$ and the roots of the corresponding quadratic equation $ax^2+bx+c=0$ are the same.
3. Discriminant: The expression $b^2-4ac$ is called discriminant of the equation $ax^2+bx+c=0$ and is usually denoted by $D$. Thus discriminant $D=b^2-4ac$.
4. Every quadratic equation has two roots which may be real, coincident or imaginary.
5. If $\alpha$ and $\beta$ are the roots of the equation $ax^2+bx+c=0$ then

\[
\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{And} \quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}
\]

6. Sum of the roots, $\alpha + \beta = \frac{-b}{a}$ and product of the roots, $\alpha\beta = \frac{c}{a}$
7. Forming quadratic equation, when the roots $\alpha$ and $\beta$ are given.

\[
x^2-(\alpha + \beta)x + \alpha \cdot \beta = 0
\]
8. Nature of roots of $ax^2+bx+c=0$
   i. If $D>0$, then roots are real and unequal.
   ii. $D=0$, then the equation has equal and real roots.
   iii. $D<0$, then the equation has no real roots

LEVEL-I

1. IF $\frac{1}{2}$ is a root of the equation $x^2+kx-5/4=0$, then the value of $K$ is
   (a) $2$ \hspace{2cm} (b) $-2$ \hspace{2cm} (c) $\frac{1}{4}$ \hspace{2cm} (d) $\frac{1}{2}$ \hspace{2cm} [Ans(d)]
2. IF $D>0$, then roots of a quadratic equation $ax^2+bx+c=0$ are
   (a) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ \hspace{2cm} (b) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ \hspace{2cm} (c) $\frac{-b-\sqrt{b^2-4ac}}{2a}$ \hspace{2cm} (d) None of these \hspace{2cm} [Ans(a)]
3. Discriminant of $x^2+5x+5=0$ is
   (a) $\frac{5}{2}$ \hspace{2cm} (b) $-5$ \hspace{2cm} (c) $5$ \hspace{2cm} (d) $-4$ \hspace{2cm} [Ans(c)]
4. The sum of roots of a quadratic equation $x^2+4x-320=0$ is \hspace{2cm} [Ans(a)]
   (a) $-4$ \hspace{2cm} (b) $4$ \hspace{2cm} (c) $1/4$ \hspace{2cm} (d) $1/2$
5. The product of roots of a quadratic equation $2x^2+7x-4=0$ is \hspace{2cm} [Ans(d)]
   (a) $2/7$ \hspace{2cm} (b) $-2/7$ \hspace{2cm} (c) $-4/7$ \hspace{2cm} (d) $-2$
6. Values of $K$ for which the equation $9x^2+2kx-1=0$ has real roots are:
   (a) $k \geq \pm 3$ \hspace{2cm} (b) $k \geq 3$ or $K \leq -3$ \hspace{2cm} (c) $K \geq -3$ \hspace{2cm} (d) $k \leq \pm 3$ \hspace{2cm} [Ans(b)]

LEVEL-II

1. For what value of $k$, $x=a$ is a solution of equation $x^2-(a+b)x+k=0$?
2. Represent the situation in the form of quadratic equation:
Rohan’s mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan’s present age.
Ans. \( x^2 + 32x - 273 = 0 \) where \( x \) (in years) is Rohan’s present age

3. Find the roots of \( x^2 - 3x - 10 = 0 \)
Ans. \(-2, 5\)

4. Find two consecutive positive integers, sum of whose squares is 365.
Ans. 13, 14

5. Find the roots of Quadratic equation \( 4x^2 + 4\sqrt{3}x + 3 = 0 \) by using the quadratic formula.
Ans. \(-\sqrt{3}/2, -\sqrt{3}/2\)

6. Find the discriminant of the Quadratic equation \( 2x^2 - 4x + 3 = 0 \) and hence find the nature of its roots.
Ans. \( D = -8 < 0 \) its no real roots.

LEVEL - 3

1. If \( x = 2 \) and \( x = 3 \) are roots of the equation \( 3x^2 - 2kx + 2m = 0 \) find the value of \( k \) and \( m \).
Ans. \( K = \frac{15}{2}, m = 9 \)

2. Solve the equation:
\( \frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}, x \neq 0, x \neq -1 \)
Ans. \( x = \frac{3}{2} \) or \( x = -\frac{5}{2} \)

3. Solve the equation \( 2x^2 - 5x + 3 = 0 \) by the method of completing square.
Ans. \( x = \frac{3}{2} \) or \( x = 1 \)

4. Using quadratic formula, solve the equation: \( p^2x^2 + (p^2 - q^2)x - q^2 = 0 \).
Ans. \( x = -1, \text{or} \, x = \frac{q^2}{p^2} \)

5. The sum of two numbers is 15, if the sum of their reciprocals is \( \frac{3}{10} \), find the numbers.
Ans. 10 and 5

LEVEL - 4

1. In a class test, the sum of Shefali’s marks in maths and English is 30. Had she got 2 marks more in maths and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.
Ans. Marks in maths = 12, marks in English = 18 or, marks in maths = 13, marks in English = 17

2. Two water taps together can fill a tank in \( 9 \frac{3}{8} \) hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
Ans. 15 hours, 25 hours.

3. Find the roots of equation \( \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{13} \), \( x \neq -4, 7 \)
Ans. 1, 2

4. Solve the following equation for ‘x’ \( 9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0 \)
5. If the roots of the equation \((a-b)x^2+(b-c)x+(c-a)=0\) are equal, prove that \(2a=b+c\).

**Self Evaluation**

1. Find the value of \(p\) so that the equation \(3x^2-5x+2p=0\) has equal roots. Also find the roots.

2. The sum of two numbers is 15. If the sum of their reciprocals is \(\frac{3}{10}\), find the two numbers.

3. Find \(a\) and \(b\) such that \(x+1\) and \(x+2\) are factors of the polynomials \(x^3+ax^2-bx+10\).

4. Find the quadratic equation whose roots are \(2+\sqrt{3}\) and \(2-\sqrt{3}\).

5. A person on tour has Rs. 360 for his daily expenses. If he exceeds his tour program by four days, he must cut down his daily expenses by Rs 3 per day. Find the number of days of his tour program me.

6. Divide 29 into two parts so that the sum of squares of the parts is 425.

7. Solve for \(x\): \(9x^2-6ax+(a^2-b^2)=0\)

8. If the equation \((1+m^2)x^2+2mcx+c^2-a^2=0\) has equal roots, show that \(c^2=a^2(1+m^2)\).
**ARITHMETIC PROGRESSION**

**(Key Points)**

- Arithmetic progression (A.P.): An A.P. is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- This fixed number is called the common difference of the A.P.
- If $a$ is first term and $d$ is common difference of an A.P., then the A.P is $a, a+d, a+2d, 2+3d$ ....
- The $n^{th}$ term of an A.P is denoted by $a_n$ and $a_n = a+(n-1)d$, where $a =$ first term and $d =$ common difference.
- $n^{th}$ term from the end = $l - (n-1)d$, where $l =$ last term.
- Three terms $a-d, a, a+d$ are in A.P with common difference $d$.
- Four terms $a-3d, a-d, a+d ,a+3d$ are in A.P with common diff. $2d$.
- The sum of first $n$ natural numbers is $\frac{n(n+1)}{2}$.
- The sum of $n$ terms of an A.P with first term $a$ and common difference $d$ is denoted by $s_n = \frac{n}{2}\{2a+(n-1)d\}$ also, $s_n = \frac{n}{2}(a+l)$ where, $l =$ last term.
- $a_n = s_n - s_{n-1}$. Where $a_n = n^{th}$ term of an A.P
- $D = a_n-a_{n-1}$. Where $d =$ common difference of an A.P.

**[LEVEL - 1]**

1. Find $n^{th}$ term of $– 15 , -18 , -21 , ...........
   
   Ans. $-3 (n+4)$

2. Find the common diff. of A.P $1 , -2 , -5 , -8 ,........$
   
   Ans. $-3$

3. Find the A.P whose first term is 4 and common difference is $–3$
   
   Ans. a.p = $4 , 1 -2, -5, -8.............$

4. Find $5^{th}$ term from end of the AP : $17 , 14,11........................-40$.
   
   Ans. $-28$

5. If $2p, p+10, 3p+2$ are in AP then find $p$.
   
   Ans. $p= 6$

6. If arithmetic mean between $3a$ and $2a-7$ is $a+4$, then find $a$.
   
   Ans. $a= 5$

7. Find sum of all odd numbers between 0 & 50.
   
   Ans. $625$

8. If $a = 5, d = 3$ and $a_n = 50$, then find $n$.
   
   Ans. $n =16$

9. For what value of $n$ are the $n^{th}$ term of two AP, $63, 65, 67, ........$ and $3, 10, 17, ........$equal?
   
   Ans. $n = 13$.

10. If sum of $n$ terms of an AP is $2n^2+5n$, then find its $n^{th}$ term.
    
    Ans. $4n+3$.

**[LEVEL - 2]**
1. Find $n^{th}$ term of an AP is $7-4n$. find its common difference.

   Ans. -4.

2. Which term of an AP $5,2,-1,...$ will be -22?

   Ans. $10^{th}$ term.

3. Write the next term of an AP $\sqrt{3}, \sqrt{18}, \sqrt{32},...$

   Ans. $5\sqrt{2}$.

4. Determine $27^{th}$ term of an AP whose $9^{th}$ term is -10 and common difference is $1\frac{1}{4}$

   Ans. $927 = \frac{25}{2}$.

5. Find the sum of series $103=+101+99+....+49$.

   Ans. 2128.

6. Which term of the AP $3,15,27,39,...$ will be 132 more than its $54^{th}$ term?

   Ans. $65^{th}$ term.

7. How many three digit numbers are divisible by 7?

   Ans. 128.

8. Given $a = 2$, $d = 8$, $s_n = 90$, find $n$ and $a_n$.

   Ans. $N = 5$ & $a_n = 34$

(LEVEL- 3)

1. Which term of the sequence -1, 3, 7, 11 ............. Is 95?

   Ans. $25^{th}$ term

2. How many terms are there in the sequence 3, 6, 9, 12, ......111?

   Ans. 37 terms

3. The first term of an AP is -7 and the common difference 5, find its $18^{th}$ term and the general term.

   Ans. $a_{18} = 78n$ & $a_n = 5n - 12$

4. How many numbers of two digits are divisible by 3?

   Ans. 30

5. If the $n^{th}$ term of an AP is $(2n+1)$, find the sum of first $n$ terms of the AP

   Ans. $S_n = \frac{n(n+2)}{2}$

6. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

   Ans. 156375.

Problems for self evaluation.

1. Show that the sequence defined by $t_n=4n+7$ is an AP.

2. Find the number of terms for given AP : $7,13,19,25,...,205$.

3. The $7^{th}$ term of an AP is 32 and it $13^{th}$ term is 62. Find AP.

4. Find the sum of all two digit odd positive nos.

5. Find the value of ‘x’ for AP. $1+6+11+16+....+x=148$.

6. Find the $10^{th}$ term from the end of the AP $8,10,12,...,126$.

7. The sum of three numbers of AP is 3 and their product is -35. Find the numbers.

8. A man repays a loan of Rs3250 by paying Rs20 in the first month and then increase the payment by Rs15 every month. How long will it take him to clear the loan?
9. The ratio of the sums of m and n terms of an AP is \( m^2 : n^2 \). Show that the ratio of the mth and nth terms is \((2m-1) : (2n-1)\).

10. In an AP, the sum of first n terms is \( \frac{3n^2}{2} + \frac{5n}{2} \), Find it 25th term.
CO-ORDINATE GEOMETRY

IMPORTANT CONCEPTS
TAKE A LOOK

1. Distance Formula:
The distance between two points A(x₁,y₁) and B(x₂,y₂) is given by the formula.
\[ AB = \sqrt{(x₂-x₁)^2 + (y₂-y₁)^2} \]

COROLLARY: The distance of the point P(x,y) from the origin O(0,0) is given by
\[ OP = \sqrt{x^2 + y^2} \]

2. Section Formula:
The co-ordinates of the point P(x,y) which divides the line segment joining A(x₁,y₁) and B(x₂,y₂) internally in the ratio m:n are given by
\[ \frac{mx₂ + nx₁}{m+n}, \frac{my₂ + ny₁}{m+n} \]

3. Midpoint Formula:
If R is the mid-point, then \( m₁ = m₂ \) and the coordinates of R are
\[ \left( \frac{x₁ + x₂}{2}, \frac{y₁ + y₂}{2} \right) \]

4. Co-ordinates of the centroid of triangle:
The co-ordinates of the centroid of a triangle whose vertices are P(x₁,y₁), Q(x₂,y₂) and R(x₃,y₃) are
\[ \left( \frac{x₁ + x₂ + x₃}{3}, \frac{y₁ + y₂ + y₃}{3} \right) \]

5. Area of a Triangle:
The area of the triangle formed by the points P(x₁,y₁) Q(x₂,y₂) and R(x₃,y₃) is the numerical value of the expression.
\[ \text{ar (∆PQR)} = \frac{1}{2} \left( x₁(y₂-y₃) + x₂(y₃-y₁) + x₃(y₁-y₂) \right) \]

LEVEL- 1

1. If the coordinates of the points P and Q are (4,-3) and (-1,7). Then find the abscissa of a point R on the line segment PQ such that \( \frac{PR}{PQ} = \frac{3}{5} \)
   Ans. 1

2. If P (\( \frac{a}{3},4 \)) is the midpoint of the line segment joining the points Q (-6, 5 ) and R (-2 , 3) , then find the value of a .
   Ans. -12

3. A line intersects y-axis and x-axis at the points P and Q respectively . If ( 2 , -5) is the midpoint of PQ , then find the coordinates of P and Q respectively .
   Ans. (0,-10) and (4,0)

4. If the distance between the points (4,p)&(1,0) is 5,then find the value of p
   Ans. ±4

5. If the point A(1,2), B(0,0) and C(a,b)are collinear, then find the relation between a and b.
6. Find the coordinate of the point on x-axis which is equidistant from (2,-5) and (-2,9).

7. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, -3) and B is (1, 4).

8. Find the centroid of triangle whose vertices are (3, -7), (-8, 6) and (5, 10).

LEVEL-2

1. Point P (5, -3) is one of the two points of trisection of the line segment joining the points A (7, -2) and B (1, -5) near to A. Find the coordinates of the other point of trisection.

2. Show that the point P (-4, 2) lies on the line segment joining the points A (-4, 6) and B (-4, -6).

3. If A (-2, 4), B (0, 0), C (4, 2) are the vertices of a ΔABC, then find the length of median through the vertex A.

4. Find the value of x for which the distance between the points P (4, -5) and Q(12, x) is 10 units.

5. If the points A (4,3) and B (x,5) are on the circle with centre O(2,3) then find the value of x.

6. What is the distance between the point A (c, 0) and B (0, -c)?

7. For what value of p, are the points (-3, 9), (2, p) and (4, -5) collinear?

LEVEL-3

1. Show that the points (3, 2), (0, 5), (-3,2) and (0, -1) are the vertices of a square.

2. Point P divides the line segment joining the points A(2,1) and B(5,-8) such that AP:AB=1:3. If P lies on the line 2x-y+k=0, then find the value of k.

3. Points P, Q, R, and S in that order are dividing a line segment joining A (2, 6) and B (7, -4) in five equal parts. Find the coordinates of point P and R?

4. Find a relation between x and y if the points (2, 1), (x, y) and (7, 5) are collinear.

5. If A (-4, -2), B (-3, -5), C (3, -2) and D (2, 3) are the vertices of a quadrilateral, then find the area of the quadrilateral.

6. Find the values of x for which the distance between the points P(2, -3) and Q(x, 5) is 10 units.

7. Find the point on y-axis which is equidistant from the points (5, -2) and (-3, 2).

LEVEL-4

1. A (6, 1), B (8, 2), C (9, 4) are the three vertices of a parallelogram ABCD. If E is the midpoint of DC, then find the area of ΔADE.
2. In each of following, find the value of ‘k’ for which the points are collinear.
   (a) (7, -2), (5, 1), (3, k)    (b) (8, 1), (k, -4), (2, -5)
   Ans. (a) $k = \frac{4}{3}$   (b) $k = -3$

3. Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
   Ans. 1:4

4. Find the coordinates of the points which divides the line segment joining the points (-2, 0) and (0, 8) in four equal parts.
   Ans. \((\frac{-3}{2}, 2), (-1, 4), (-\frac{1}{2}, 6)\)

5. Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2) and (2, 3)
   Ans. 28 sq. units

6. Find the area of the rhombus, if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.
   Ans. 24 sq. units

**HOTS /SELF EVALUATION**

1. Two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices.
   [Ans. (1, 0) and (1, 4)]

2. Find the centre of a circle passing through the points (6, -6), (3, 7) and (3, 3).
   [Ans. 3, -2]

3. If the distance between the points (3, 0) and (0, y) is 5 units and y is positive, then what is the value of y?
   [Ans. 4]

4. If the points \((x, y), (-5, -2)\) and \((3, -5)\) are collinear, then prove that \(3x + 8y + 31 = 0\).

5. Find the ratio in which the Y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of division.
   Ans. 5:1; (0, -13/3)

6. Find k so that the point P(-4, 6) lies on the line segment joining A (k, 0) and B (3, -8). Also find the ratio in which P divides AB.
   [Ans. 3:7 externally; k = -1]

7. By distance formula, show that the points (1, -1), (5, 2) and (9, 5) are collinear.
APPLICATIONS OF TRIGONOMETRY
(HEIGHT AND DISTANCES)

KEY POINTS

Line of sight
Line segment joining the object to the eye of the observer is called the line of sight.

Angle of elevation
When an observer sees an object situated in upward direction, the angle formed by line of sight with horizontal line is called angle of elevation.

Angle of depression
When an observer sees an object situated in downward direction the angle formed by line of sight with horizontal line is called angle of depression.

LEVEL - 1

1. A ploe 6cm high casts a shadow $2\sqrt{3}$m long on the ground, then find the sun’s elevation?
   Ans. $60^\circ$

2. If $\sqrt{3} \tan \theta = 1$, then find the value of $\sin^2 \theta - \cos^2 \theta$
   Ans. $-1/2$

3. An observer 1.5m tall is 20.5 metres away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.
   Ans. $45^\circ$

4. A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle $60^\circ$ with the wall, find the height of the wall
   Ans. $15/2$ m

5. In a rectangle ABCD, AB =20cm $\angle BAC=60^\circ$ then find the length of the side AD.
   Ans. $20\sqrt{3}$cm

6. Find the angle of elevation of the sun’s altitude when the height of the shadow of a vertical pole is equal to its height.
   Ans. $45^\circ$
7. From a point 20m away from the foot of a tower, the angle of elevation of top of the tower is 30°, find the height of the tower.
   \[ \text{Ans.} \frac{20}{\sqrt{3}} \text{ m} \]

8. In the adjacent figure, what are the angles of depression of the top and bottom of a pole from the top of a tower h m high:
   \[ \text{Ans.} 45^0, 60^0 \]

**LEVEL - 2**

1. In \( \triangle ABC \), \( \angle B = 45^0 \), \( \angle C = 45^0 \), \( AB = 5 \text{ cm} \) then find the length of the other two sides.
   \[ \text{Ans.} 5 \text{ cm, } 5\sqrt{2} \text{ cm} \]

2. From a point 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°, find the height of the tower.
   \[ \text{Ans.} \frac{20\sqrt{3}}{3} \text{ m} \]

3. A ladder 50 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.
   \[ \text{Ans.} 25 \text{ m} \]

4. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.
   \[ \text{Ans.} 10 \text{ m} \]

5. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.
   \[ \text{Ans.} 8\sqrt{3} \text{ m} \]

**LEVEL - 3**

1. The shadow of a tower standing on a level plane is found to be 50 m longer when sun’s elevation is 30° then when it is 60°. Find the height of the tower.
   \[ \text{Ans.} 25\sqrt{3} \text{ m} \]

2. The angle of depression of the top and bottom of a tower as seen from the top of a 100 m high cliff are 30° and 60° respectively. Find the height of the tower. \[ \text{[Ans.} 66.67 \text{ m}] \]

3. From a window (9 m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 60° respectively. Find the height of the opposite house and width of the street.
4. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30° and 45°. Find the height of the hill.

Ans. 1.37 km

5. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30°. Find the heights of pole and the distance of the point from the poles.

[Ans; h=34.64m; 20m, 60m].

6. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720 km/hr, find the constant height at which the jet is flying.

[Ans; 1500m]

7. A window in a building is at a height of 10m above the ground. The angle of depression of a point P on the ground from the window is 30°. The angle of elevation of the top of the building from the point P is 60°. Find the height of the building.

[Ans; 30m]

8. A boy, whose eye level is 1.3m from the ground, spots a ballon moving with the wind in a horizontal line at same height from the ground. The angle of elevation of the ballon from the eyes of the boy at any instant is 60°. After 2 seconds, the angle of elevation reduces to 30°. If the speed of the wind at that moment is $29\sqrt{3}$ m/s, then find the height of the ballon from the ground.

[Ans; 88.3m]

9. A man on the deck on a ship 14m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

[Ans; h=56m, distance 24.25m]

10. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six minutes later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower.

[Ans. 3 minutes]

SELF EVALUATION/HOTS

1. An aeroplane when flying at a height of 3125m from the ground passes vertically below another
plane at an instant when the angle of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at that instant.

[Ans; 6250m]

2. From the top of a building 60m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find [i] horizontal distance between the building and the lamp post [ii] height of the lamp post.

[Ans. 34.64m h=40m]

3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h m. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β, respectively. Prove that the height of the tower is

\[
\frac{htan\alpha}{tan\beta - tan\alpha}
\]

4. The angle of elevation of a cloud from a point 60m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.

[Ans 120m]
Circle

KEY POINTS

Tangent to a circle:

A tangent to a circle is a line that intersects the circle at only one point.

- There is only one tangent at a point on a circle.
- There are exactly two tangents to a circle through a point lying outside the circle.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- The length of tangents drawn from an external point to a circle are equal.

(1 Mark Questions)

1. If radii of the two concentric circles are 15cm and 17cm, then find the length of each chord of one circle which is tangent to one other.
   Ans. 16cm

2. If two tangents making an angle of 120° with each other, are drawn to a circle of radius 6cm, then find the angle between the two radii, which are drawn to the tangents.
   Ans. 60°

3. In the adjoining figure, Δ ABC is circumscribing a circle, then find the length of BC.
   Ans. 9cm

4. PQ is a chord of a circle and R is point on the minor arc. If PT is a tangent at point P such that ∠QPT = 60° then find <PRQ.
   Ans. 120°

5. If a tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q such that OQ = 12 cm then find the length of PQ.
   Ans. \( \sqrt{119} \) cm

6. From a point P, two tangents PA and PB are drawn to a circle C(O,r). If OP =2r, then what is the type of Δ APB.
   Ans. Equilateral triangle

7. If the angle between two radii of a circle is 130°, then find the angle between the tangents at the end of the radii.
   Ans. 50°

8. ABCD is a quadrilateral. A circle centred at O is inscribed in the quadrilateral. If AB = 7cm, BC = 4cm, CD = 5cm then find DA.
   Ans. 8 cm

9. In a Δ ABC, AB = 8cm, ∠ABC = 90°. Then find the radius of the circle inscribed in the triangle.
   Ans. 2cm

(Two Marks Questions)
1. Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that OAPB is a cyclic quadrilateral.

2. If PA and PB are two tangents drawn to a circle with centre O, from an external point P such that PA=5cm and \( \angle APB = 60^\circ \), then find the length of the chord AB.

   Ans. 5cm

3. CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R and intersects PC and QC at A and B respectively. If CP = 11cm and BR = 4cm, then find the length of BC.

   Ans. 7cm

4. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

6. In adjacent figure; AB & CD are common tangents to two circles of unequal radii. Prove that AB=CD.

   [Diagram of two circles with common tangents AB and CD]

(Three Marks Questions)

1. If quadrilateral ABCD is drawn to circumscribe a circle then prove that AB+CD=AD+BC.

2. Prove that the angle between the two tangents to a circle drawn from an external point, is supplementary to the angle subtended by the line segment joining the points of contact to the centre.

3. AB is a chord of length 9.6cm of a circle with centre O and radius 6cm. If the tangents at A and B intersect at point P then find the length PA.

   Ans. 8cm

4. The incircle of a \( \triangle ABC \) touches the sides BC, CA & AB at D, E and F respectively. If AB=AC, prove that BD=CD.

5. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre of the circle.

6. PQ and PR are two tangents drawn to a circle with centre O from an external point P. Prove that \( \angle QPR=2\angle OQR \).
(Four Marks Questions)

1. Prove that the length of tangents drawn from an external point to a circle are equal. Hence, find BC, if a circle is inscribed in a \(\triangle ABC\) touching AB, BC & CA at P, Q & R respectively, having AB=10cm, AR=7cm & RC=5cm.

   Ans. 8cm

2. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: If O is the centre of two concentric circles, AB is a chord of the larger circle touching the smaller circle at C, then prove that AC=BC.

3. A circle touches the side BC of a \(\triangle ABC\) at a point P and touches AB and AC when produced, at Q & R respectively. Show that AQ=1/2 (perimeter of \(\triangle ABC\)).

4. From an external point P, a tangent PT and a line segment PAB is drawn to circle with centre O, ON is perpendicular to the chord AB. Prove that \(PA \cdot PB = PN^2 - AN^2\).

5. If AB is a chord of a circle with centre O, AOC is diameter and AT is the tangent at the point A, then prove that \(\angle BAT = \angle ACB\).

6. The tangent at a point C of a circle and diameter AB when extended intersect at P. If \(\angle PCA = 110^0\), find \(\angle CBA\).

   Ans. 70^0

[Self Evaluation/HOTS Questions]

1. If PA and PB are tangents from an external point P to the circle with centre O, the find \(\angle AOP + \angle OPA\).

   Ans. 90^0

2. ABC is an isosceles triangle with AB=AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.

3. AB is diameter of a circle with centre O. If PA is tangent from an external point P to the circle with \(\angle POB = 115^0\) then find \(\angle OPA\).

   Ans. 25^0

4. PQ and PR are tangents from an external point P to a circle with centre O. If \(\angle RPQ = 120^0\), Prove that OP=2PQ.

5. If the common tangents AB and CD to two circles \(C(O,r)\) and \(C'(O'r')\) intersect at E, then prove that AB=CD.

6. If a, b, c are the sides of a right triangle where c is the hypotenuse, then prove that radius r of the circle touches the sides of the triangle is given by \(r = (a+b-c)/2\).
CONSTRUCTION

KEY POINTS

1. Division of line segment in the given ratio.

2. Construction of triangles:-
   a. When three sides are given.
   b. When two sides and included angle given.
   c. When two angles and one side given.
   d. Construction of right angled triangle.

3. Construction of triangle similar to given similar to given triangle as per given scale.

4. Construction of triangles to a circle.

LEVEL - I

1. Divide a line segment in given ratio.

2. Draw a line segment AB=8cm and divide it in the ratio 4:3.

3. Divide a line segment of 7cm internally in the ratio 2:3.

4. Draw a circle of radius 4 cm. Take a point P on it. Draw tangent to the given circle at P.

5. Construct an isosceles triangle whose base 7.5 cm and altitude is 4.2 cm.

LEVEL –II

1. Construct a triangle of sides 4cm , 5cm and 6cm and then triangle similar to it whose side are 2/3 of corresponding sides of the first triangle.

2. Construct a triangle similar to a given ΔABC such that each of its sides is 2/3\textsuperscript{rd} of the corresponding sides of ΔABC. It is given that AB=4cm BC=5cm and AC=6cm also write the steps of construction.

3. Draw a right triangle ABC in which ∠B=90° AB=5cm, BC=4cm then construct another triangle ABC whose sides are 5/3 times the corresponding sides of ΔABC.

4. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle of 60°.

5. Draw a circle of radius 5cm from a point 8cm away from its centre construct the pair of tangents to the circle and measure their length.

6. Construct a triangle PQR in which QR=6cm ∠Q=60° and ∠R=45°. Construct another triangle similar to ΔPQR such that its sides are 5/6 of the corresponding sides of ΔPQR.
AREAS RELATED TO TWO CIRCLES

KEY POINTS

1. Circle: The set of points which are at a constant distance of \( r \) units from a fixed point \( o \) is called a circle with centre \( o \).

2. Circumference: The perimeter of a circle is called its circumference.

3. Secant: A line which intersects a circle at two points is called secant of the circle.

4. Arc: A continuous piece of circle is called and arc of the circle.

5. Central angle: An angle subtended by an arc at the center of a circle is called its central angle.

6. Semi Circle: A diameter divides a circle into two equal arc. Each of these two arcs is called a semi circle.

7. Segment: A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.

8. Sector of a circle: The region enclosed by and an arc of a circle and its two bounding radii is called a sector of the circle.

9. Quadrant: One fourth of a circle disc is called a quadrant. The central angle of a quadrant is \( 90^\circ \).
a. Length of an arc AB = \( \frac{\theta}{360} \cdot 2\pi r \)

b. Area of major segment = Area of a circle – Area of minor segment

c. Distance moved by a wheel in
   1 rotation=circumference of the wheel

d. Number of rotation in 1 minute
   
   \[ \text{Number of rotation} = \frac{\text{Distance moved in 1 minute}}{\text{circumference}} \]
LEVEL-I

1. If the perimeter of a circle is equal to that of square, then the ratio of their areas is
   i. 22/7  
   ii. 14/11  
   iii. 7/22  
   iv. 11/14  [Ans-ii]

2. The area of the square that can be inscribed in a circle of 8 cm is
   i. 256 cm$^2$  
   ii. 128 cm$^2$  
   iii. $64\sqrt{2}$ cm$^2$  
   iv. 64 cm$^2$  [Ans-ii]

3. Area of a sector to circle of radius 36 cm is $54\pi$ cm$^2$. Find the length arc of the corresponding arc of the circle is
   i. 6 $\pi$ cm  
   ii. 3 $\pi$ cm  
   iii. 5 $\pi$ cm  
   iv. 8 $\pi$ cm  [Ans –ii]

4. A wheel has diameter 84 cm. The number of complete revolution it will take to cover 792 m is.
   i. 100  
   ii. 150  
   iii. 200  
   iv. 300  [Ans-iv]

5. The length of an arc of a circle with radius 12 cm is $10\pi$ cm. The central angle of this arc is .
   i. 120$^0$  
   ii. 60$^0$  
   iii. 75$^0$  
   iv. 150$^0$  [Ans-iv]

6. The area of a quadrant of a circle whose circumference is 22 cm is
   i. $7/2$ cm$^2$  
   ii. 7 cm$^2$  
   iii. 3 cm$^2$  
   iv. 9.625 cm$^2$  [Ans-iv]

LEVEL-II

1. In figure ‘o’ is the centre of a circle. The area of sector OAPB is 5/18 of the area of the circle find x.  
   [Ans 100]

2. If the diameter of a semicircular protractor is 14 cm, then find its perimeter .  [Ans-36 cm]
3. The radius of two circles are 3 cm and 4 cm. Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

   [Ans: 5 cm]

4. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

   [Ans: 154/3 cm²]

5. The radii of two circles are 3 cm and 4 cm. Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

   [Ans: 5 cm]

---

**LEVEL-III**

1. Find the area of the shaded region in the figure if AC=24 cm, BC=10 cm and O is the center of the circle (use \( \pi = 3.14 \))

   [Ans: 145.33 cm²]

2. The inner circumference of a circular track is 440 m. The track is 14 m wide. Find the diameter of the outer circle of the track. [Take \( \pi = \frac{22}{7} \)]

   [Ans: 168]

3. Find the area of the shaded region.

   [Ans: 4.71 cm²]

4. A copper wire when bent in the form of a square encloses an area of 121 cm². If the same wire is bent into the form of a circle, find the area of the circle (Use \( \pi = \frac{22}{7} \))

   [Ans: 154 cm²]

5. A wire is looped in the form of a circle of radius 28 cm. It is rebent into a square form. Determine the side of the square (use \( \pi = \frac{22}{7} \))

   [Ans: 44 cm]
LEVEL-IV

1. In fig, find the area of the shaded region [use \( \pi = 3.44 \)]

2. In fig find the shape of the top of a table in restaurant is that of a sector a circle with centre O and \( \angle \text{bod}=90^\circ \). If OB=OD=60cm find
   i. The area of the top of the table [Ans 8478 cm\(^2\)]
   ii. The perimeter of the table top (Take \( \pi = 3.44 \)) [Ans 402.60 cm]

3. An arc subtends an angle of 90\(^0\) at the centre of the circle of radius 14 cm. Write the area of minor sector thus form in terms of \( \pi \).
   [Ans \( 49\pi \) cm\(^2\)]

4. The length of a minor arc is 2/9 of the circumference of the circle. Write the measure of the angle subtended by the arc at the center of the circle.
   [Ans \( 80^\circ \)]

5. The area of an equilateral triangle is \( 49\sqrt{3} \) cm\(^2\). Taking each angular point as center, circle are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles.
   [Take \( \sqrt{3}=1.73 \)]
   [Ans 777cm\(^2\)]

SELF EVALUATION

1. Two circles touch externally the sum of the areas is 130 \( \pi \) cm\(^2\) and distance between there center is 14 cm. Find the radius of circle.

2. Two circle touch internally. The sum of their areas is 116 \( \pi \) cm\(^2\) and the distance between there centers is 6 cm. Find the radius of circles.

3. A pendulum swings through an angle of 30\(^0\) and describes and arc 8.8 cm in length. Find length of pendulum.

4. What is the measure of the central angle of a circle?

5. The perimeter and area of a square are numerically equal. Find the area of the square.
### SURFACE AREAS AND VOLUMES

#### IMPORTANT FORMULA

#### TAKE A LOOK

<table>
<thead>
<tr>
<th>SNo</th>
<th>NAME</th>
<th>FIGURE</th>
<th>LATERAL CURVED SURFACE AREA</th>
<th>TOTAL SURFACE AREA</th>
<th>VOLUME</th>
<th>NOMENCLATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cuboid</td>
<td><img src="image" alt="Cuboid Image" /></td>
<td>$2(l+b) x h$</td>
<td>$2(lb + bxh + hx l)$</td>
<td>$lx b x h$</td>
<td>$l$=length, $b$=breadth, $h$=height</td>
</tr>
<tr>
<td>2</td>
<td>Cube</td>
<td><img src="image" alt="Cube Image" /></td>
<td>$4l^2$</td>
<td>$6l^2$</td>
<td>$l^3$</td>
<td>$l$=edge of cube</td>
</tr>
<tr>
<td>3</td>
<td>Right Circular Cylinder</td>
<td><img src="image" alt="Right Circular Cylinder Image" /></td>
<td>$2\pi rh$</td>
<td>$2\pi r(r+h)$</td>
<td>$\pi r^3h$</td>
<td>$r$=radius of base, $h$=height</td>
</tr>
<tr>
<td>4</td>
<td>Right Circular Cone</td>
<td><img src="image" alt="Right Circular Cone Image" /></td>
<td>$\pi rl$</td>
<td>$\pi (l+r)$</td>
<td>$\frac{1}{3}\pi r^2h$</td>
<td>$r$=radius of base, $h$=height, $l$=slant height = $\sqrt{r^2 - h^2}$</td>
</tr>
<tr>
<td>5</td>
<td>Sphere</td>
<td><img src="image" alt="Sphere Image" /></td>
<td>$4\pi r^2$</td>
<td>$4\pi r^2$</td>
<td>$\frac{4}{3}\pi r^3$</td>
<td>$r$=radius of the sphere</td>
</tr>
<tr>
<td>6</td>
<td>Hemisphere</td>
<td><img src="image" alt="Hemisphere Image" /></td>
<td>$2\pi r^2$</td>
<td>$3\pi r^2$</td>
<td>$\frac{2}{3}\pi r^3$</td>
<td>$r$=radius of hemisphere</td>
</tr>
<tr>
<td>7</td>
<td>Spherical shell</td>
<td><img src="image" alt="Spherical Shell Image" /></td>
<td>$2\pi(R^2 + r^2)$</td>
<td>$3\pi(R^2 - \pi r^2)$</td>
<td>$\frac{4}{3}\pi(R^3 - r^3)$</td>
<td>$R$=External radius, $r$=internal radius</td>
</tr>
<tr>
<td>8</td>
<td>Frustum of a cone</td>
<td><img src="image" alt="Frustum of a Cone Image" /></td>
<td>$\pi(R+r)$ where $l^2=h^2+(R-r)^2$</td>
<td>$\pi[R^2 + r^2 + l(R+r)]$</td>
<td>$\frac{\pi h}{3}[R^2 + r^2 + Rr]$</td>
<td>$R$ and $r$ = radii of the base, $h$=height, $l$=slant height.</td>
</tr>
</tbody>
</table>

9. Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$
10. Diagonal of Cube = $\sqrt{3l}$
(LEVEL - 1)

[1] The height of a cone is 60 cm. A small cone is cut off at the top by a plane parallel to the base and its volume is \( \frac{1}{64} \) th the volume of original cone. Find the height from the base at which the section is made?

ANS: 45 cm

[2] Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm?

ANS: 19.4 cm³.

[3] A cubical ice cream brick of edge 22 cm is to be distributed among some children by filling ice cream cones of radius 2 cm and height 7 cm up to its brim. How many children will get ice cream cones?

ANS: 363.

[4] Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.9 cm is?

ANS: 30.8 cm³.

[5] The slant height of a frustum of a cone is 4 cm and the perimeter of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum [use \( \pi = \frac{22}{7} \)].

ANS: 48 cm².

[6] A plumbline is a combination of which geometric shapes?

ANS: A cone with hemisphere.

(LEVEL - 2)

[1] The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm. Write the height of the frustum.

ANS: 3 cm

[2] A cylinder, a cone and a hemisphere are of same base and of same height. Find the ratio of their volumes?

ANS: [3:1:2].

[3] A cone of radius 4 cm is divided into two parts by drawing a plane through the midpoint of its axis and parallel to its base, compare the volume of the two parts.

ANS: 1:7

[4] How many spherical lead shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm X 11 cm X 12 cm?

ANS: 84

[5] Three metallic solid cubes whose edges are 3 cm, 4 cm, and 5 cm are melted and converted into a single cube. Find the edge of the cube so formed?

ANS: 6 cm.

(LEVEL - 3)

[1] How many shots each having diameter 4.2 cm can be made from a cuboidal lead solid of dimensions 66 cm X 42 cm X 21 cm?

ANS: 1500

[2] Find the number of metallic circular disk with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm?

ANS: 450
[3] From a solid cube of side 7cm, a conical cavity of height 7cm and radius 3cm is hollowed out. Find the volume of remaining solid?

ANS: 277 cm$^3$.

[4] A cubical block of side 7cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the surface area of the solid?

ANS: 7 cm, 332.5 cm$^2$.

[5] A heap of rice is in the form of a cone of diameter 9m and height 3.5m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

ANS: 74.25 m$^3$, 80.61 m$^2$.

[6] A square field and an equilateral triangle park have equal perimeter. If the cost of ploughing the field at the rate of Rs 5/m$^2$ is Rs 720. Find the cost of maintaining the park at the rate of Rs 10/m$^2$?

ANS: Rs 1108.48

**LEVEL - 4**

[1] A well of diameter 3cm and 14m deep is dug. The earth, taken out of it, has been evenly spread all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of embankment?

ANS: $\frac{9}{8}$ m.

[2] 21 glass spheres each of radius 2cm are packed in a cuboidal box of internal dimensions 16cmx8cmx8cm and then the box is filled with water. Find the volume of water filled in the box?

ANS: 320 cm$^3$.

[3] The slant height of the frustum of a cone is 4cm and the circumferences of its circular ends are 18cm and 6cm. Find curved surface area and total surface area of the frustum.

ANS: 48 cm$^2$, 76.63 cm$^2$.

[4] A farmer connects a pipe of internal diameter 25cm from a canal into a cylindrical tank in his field, which is 12m in diameter and 2.5m deep. If water flows through the pipe at the rate of 3.6 km/hr, in how much time will the tank be filled? Also find the cost of water, if the canal department charges at the rate of Rs 0.07/m$^3$?

ANS: 96 min, Rs 19.80

[5] A spherical glass vessel has a cylindrical neck 7cm long and 4cm in diameter. The diameter of the spherical part is 21cm. Find the quantity of water it can hold.

ANS: 4939 cm$^3$.

[6] The surface area of a solid metallic sphere is 616 cm$^2$. It is melted and recast into a cone of height 28cm. Find the diameter of the base of the cone so formed.

ANS: 14 cm.

**SELF EVALUATION/HOTS QUESTIONS**

[1] A spherical copper shell, of external diameter 18cm, is melted and recast into a solid cone of base radius 14cm and height 4 cm. Find the inner diameter of the shell.

ANS: 16 cm.
[2] A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm³. The radii of the top and bottom circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of metal sheet used in making it [take $\pi = 3.14$]?

ANS: $l = 14 \text{ cm}$, $\text{AREA} = 2160.32 \text{ cm}^2$.

[3] The volume of a solid metallic sphere is 616 cm³. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed?

ANS: 21 cm.

[4] From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid [take $\pi = 3.14$]?

ANS: $603.19 \text{ cm}^3$, $603.19 \text{ cm}^2$.

[5] A cylindrical vessel, with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the volume of:

(i) water displaced out of the cylindrical vessel.

(ii) water left in the cylindrical vessel.

ANS: (i) $77 \text{ cm}^3$, (ii) $748 \text{ cm}^3$.

[6] A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 20 cm, and radius of the base is 3.5 cm, find the total surface area of the article.

ANS: $544 \text{ cm}^2$.

[7] A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41 \frac{19}{21} \text{ m}^3$ of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building?

ANS: 4 m.

[8] A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find the external surface area.

ANS: $74.38 \text{ cm}^2$. 
1. **Probability**: The theoretical probability of an event $E$, written as $P(E)$ is defined as.

$$P(E) = \frac{\text{Number of outcomes Favorable to } E}{\text{Number of all possible outcomes of the experiment}}$$

Where we assume that the outcomes of the experiment are equally likely.

2. The probability of a sure event (or certain event) is 1.

3. The probability of an impossible event is 0.

4. The probability of an Event $E$ is number $P (E)$ such that $0 \leq P(E) \leq 1$.

5. Elementary events: An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

6. For any event $E$, $P(E) + P(\bar{E}) = 1$, where $\bar{E}$ stands for not $E$, $E$ and $\bar{E}$ are called complementary event.

7. Performing experiments: -
   a. Tossing a coin.
   b. Throwing a die.
   c. Drawing a card from deck of 52 cards.

8. **Sample space**: The set of all possible outcomes in an experiment is called sample space.

---

**LEVEL-1**

1. The probability of getting bad egg in a lot of 400 is 0.035. Then find the no. of bad eggs in the lot. [Ans. 14]

2. Write the probability of a sure event. [Ans. 1]

3. What is the probability of an impossible event. [Ans. 0]

4. When a dice is thrown, then find the probability of getting an odd number less than 3. [Ans. $\frac{1}{6}$]

5. A girl calculates that the probability of her winning the third prize in a lottery is 0.08. If 6000 tickets are sold, how many ticket has she brought. [Ans. 480]

6. What is probability that a non-leap year selected at random will contain 53 Sundays. [Ans. $\frac{1}{7}$]

7. A bag contains 40 balls out of which some are red, some are blue and remaining are black. If the probability of drawing a red ball is $\frac{11}{20}$ and that of black ball is $\frac{1}{5}$, then what is the no. of black ball. [Ans. 10]

8. Two coins are tossed simultaneously. Find the probability of getting exactly one head. [Ans. $\frac{1}{2}$]

9. A card is drawn from a well suffled deck of 52 cards. Find the probability of getting an ace. [Ans. $\frac{1}{13}$]

10. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize. [Ans. $\frac{2}{7}$]

---

**LEVEL-2**

1. Find the probability that a no. selected at random from the number 3, 4, 5, 6,...........25 is prime. [Ans. $\frac{8}{23}$]

2. A bag contains 5 red, 4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (a) of red colour (b) not of green colour. [Ans. $\frac{5}{12, \frac{3}{4}}$]
3. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing
(a) A face card (b) a card which is neither a king nor a red card

(Ans. \( \frac{3}{13}, \frac{6}{13} \))

4. A dice is thrown once. What is the probability of getting a number greater than 4?

(Ans. \( \frac{1}{3} \))

5. Two dice are thrown at the same time. Find the probability that the sum of two numbers appearing on the top of the dice is more than 9.

(Ans. \( \frac{1}{6} \))

6. Two dice are thrown at the same time. Find the probability of getting different numbers on both dice.

(Ans. \( \frac{5}{6} \))

7. A coin is tossed two times. Find the probability of getting almost one head.

(Ans. \( \frac{3}{4} \))

8. Cards with numbers 2 to 101 are placed in a box. A card selected at random from the box. Find the probability that the card which is selected has a number which is a perfect square.

(Ans. \( \frac{9}{100} \))

9. Find the probability of getting the letter M in the word “MATHEMATICS”.

(Ans. \( \frac{2}{11} \))

**LEVEL-3**

1. Cards bearing numbers 3, 5, ..........., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing
(a) a prime number less than 15  
(b) a number divisible by 3 and 5.

(Ans. \( \frac{5}{17}, \frac{2}{17} \))

2. Two dice are thrown at the same time. Find the probability of getting
(a) same no. on the both side  
(b) different no. on both sides.

(Ans. \( \frac{1}{6}, \frac{5}{6} \))

3. A child game has 8 triangles of which three are blue and rest are red and ten squares of which six are blue and rest are red. One piece is lost at random. Find the probability of that is
(a) A square  
(b) A triangle of red colour.

(Ans. \( \frac{5}{9}, \frac{5}{18} \))

4. Two dice are thrown simultaneously. What is the probability that:
(a) 5 will not come up either of them?  
(b) 5 will come up on at least one?  
(c) 5 will come at both dice?

(Ans. \( \frac{25}{36}, \frac{11}{36}, \frac{1}{36} \))

5. The king, queen and jack of clubs are removed from a deck of 52 playing cards and remaining cards are suffled. A card is drawn from the remaining cards. Find the probability of getting a card of
(a) heart  
(b) queen  
(c) clubs.

(Ans. \( \frac{13}{49}, \frac{3}{49}, \frac{10}{49} \))

6. A game consist of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., 3 heads or three tails and looses otherwise. Calculate the probability that hanif will lose the game.

(Ans. \( \frac{3}{4} \))

7. Cards bearing numbers 1, 3, 5, ..........., 37 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing
(a) a prime number less than 15

(Ans. \( \frac{5}{19} \))

(b) a number divisible by 3 and 5.

(Ans. \( \frac{2}{19} \))
8. A dice has its six faces marked 0,1,1,1,6,6. Two such dice are thrown together and total score is recorded. (a) how many different scores are possible? (b) what is the probability of getting a total of seven?

[Ans. (a) 5 scores (0, 1, 2, 6, 7, 12) (b) \(\frac{1}{3}\)]

**Self Evaluation/Hots**

1. Three unbiased coins are tossed together. Find the probability of getting
   
   (i) all heads  
   Ans. \(\frac{1}{8}\)
   
   (ii) two heads  
   Ans. \(\frac{3}{8}\)
   
   (iii) one heads  
   Ans. \(\frac{3}{8}\)
   
   (iv) at least two heads  
   Ans. \(\frac{1}{2}\)

2. Two dice are thrown simultaneously. Find the probability of getting an even number as the sum.

   Ans. \(\frac{1}{2}\)

3. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is:

   (i) An even number  
   Ans. \(\frac{1}{2}\)
   
   (ii) A number less than 14  
   Ans. \(\frac{3}{25}\)
   
   (iii) A number is perfect square  
   Ans. \(\frac{9}{100}\)
   
   (iv) A prime number less than 20  
   Ans. \(\frac{2}{25}\)

4. Out of the families having three children, a family is chosen random. Find the probability that the family has

   (i) Exactly one girl  
   Ans. \(\frac{3}{8}\)
   
   (ii) At least one girl  
   Ans. \(\frac{7}{8}\)
   
   (iii) At most one girl  
   Ans. \(\frac{1}{2}\)

5. Five card the ten, jack, queen, king, and ace of diamonds are well shuffled with their face downward. One card is picked up at random

   (i) What is the probability that the card is the queen?  
   Ans. \(\frac{1}{5}\)
   
   (ii) If the queen is drawn and put aside what is the probability that the second card picked up is (a) an ace (b) a queen  
   Ans. \(\frac{1}{4}\)
MODEL PAPER (SA-II) 2013
CLASS – X
SUB : MATHS

TIME ALLOWED : 3 HRS
M.M. = 90

General Instructions:
(i) All questions are compulsory.
(ii) The question paper consists of 34 questions divided into four sections A, B, C and D.
(iii) Section A contains 8 questions of 1 marks each, which are MCQ. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 10 questions of 4 marks each.
(iv) There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, three question of 3 marks and two questions of 4 marks.
(v) Use of calculator is not permitted.

SECTION – A

1. The roots of a quadratic equation $px^2 + 6x + 1 = 0$ have real roots then value of $p$ is
   (A) $p \geq 9$  (B) $p < 9$  (C) $p \leq 9$  (D) None of these

2. The number of terms in the AP 7,13, 19, .........., 205 are
   (A) 35  (B) 36  (C) 38  (D) 34

3. For what value of $k$, 10, $k-2$ are in A.P.
   (A) $k=4$  (B) $k=3$  (C) $k=2$  (D) $k=1$

4. In the figure given, PA= 4 cm, AB= 9 cm, then value of PT is
   (A) 9 cm  (B) 4 cm  (C) 6 cm  (D) None of these

5. The height of a tower is $\sqrt{3}$ times of its shadow. The angle of elevation of the source of height is
   (A) $30^0$  (B) $60^0$  (C) $45^0$  (D) None of these

6. The probability of selecting a queen of hearts is
   (A) $\frac{1}{4}$  (B) $\frac{1}{52}$  (C) $\frac{1}{13}$  (D) $\frac{12}{13}$

7. If the points P(1,2), Q(0,0) and R(a,b) are collinear, then
   (A) $a=b$  (B) $a=2b$  (C) $2a=b$  (D) $a=-b$

8. A cone, a hemisphere and a cylinder stand on equal bases and have the same height then their volumes are in the ratio of
   (A) 3:1:2  (B) 1:2:3  (C) 2:1:3  (D) 3:2:1

SECTION – B

9. Find the value of $k$, so that the quadratic equation $kx(x-2) + 6 = 0$ has two equal roots.

10. In the figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB= 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.
11. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

12. Draw a line segment AB of length 7 cm. Using ruler and compasses, find a point P on AB such that \( \frac{AP}{AB} = \frac{3}{5} \).

13. Two cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.
   OR
   A sphere of radius 8 cm is melted and recast into a right circular cone of height 32 cm. Find the radius of the base of the cone.

14. Calculate the area of the shaded region shown in the figure.

15. Find the roots of the quadratics equation \( 3x^2 - 4\sqrt{3}x + 4 = 0 \).

16. The sum of three numbers of AP is 3 and their product is -35. Find the numbers.
   OR
   Which term of the AP 3, 10, 17, ........ will be 84 more than its 13th term?

17. In the given figure, AOC is a diameter of the circle. If AB = 7 cm, BC = 6 cm and CD = 2 cm. Find the perimeter of the cyclic quadrilateral ABCD.

18. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 60°.
   OR
   Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are \( \frac{3}{5} \) times the corresponding sides of the given triangle.

19. The shadow of a tower standing on a level ground is found to be 40 m longer when the sun’s altitude is 30° than when it is 60°. Find the height of the tower.

20. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.
   OR
   What is the probability that a leap year, selected at random will contain 53 Sundays?
21. Find the ratio in which the segment joining the points (-3,10) and (6,-8) is divided by (-1,6)

22. Find the area of the quadrilateral whose vertices taken in order are (-4,-2); (-3,-5); (3,-2);(2,3)

23. The circumference of a circle is 88 cm. Find the area of the sector, whose angle at the centre is 45°.

24. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

SECTION – D

25. Solve for x.
\[
\frac{1}{a + b + x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, \quad a + b \neq 0
\]
OR
A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

26. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

27. Which term of the sequence 20, 19\(\frac{1}{4}\), 18\(\frac{1}{2}\), 17\(\frac{3}{4}\).... is the first negative term?

28. A circle is touching the side BC of \(\triangle ABC\) at P and touching AB and AC produced at Q and R respectively. Prove that \(AQ = \frac{1}{2} (\text{Perimeter of } \triangle ABC)\)
OR
If all the side of a parallelogram touch a circle, show that the parallelogram is a rhombus.

29. From the top of a building 60m. high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

30. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the well shuffled. One card is selected from the remaining cards. Find the probability of getting (i) a king (ii) a heart (iii) a club (iv) the 10 of hearts.

31. Find the value of ‘k’ for the points (7,-2);(5,1);(3,k); are collinear

32. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately, how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

33. Water is flowing at the rate of 5 km/hr through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Determine the time in which the level of the water in the tank will rise by 7 cm.

34. A toy is in the form of a cone mounted on hemisphere of diameter 7 cm. The total height of the toy is 14.5 m. Find the volume and the total surface area of the toy.
# MARKING SCHEME

## CLASS-X (MATHS)

### EXPECTED ANSWERS/VALUE POINTS

## MARKING SCHEME FOR SA-2

### SECTION-A

<table>
<thead>
<tr>
<th>Q. No.</th>
<th>Solution</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(C)</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>(D)</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>(A)</td>
<td>1</td>
</tr>
<tr>
<td>4.</td>
<td>(C)</td>
<td>1</td>
</tr>
<tr>
<td>5.</td>
<td>(B)</td>
<td>1</td>
</tr>
<tr>
<td>6.</td>
<td>(B)</td>
<td>1</td>
</tr>
<tr>
<td>7.</td>
<td>(C)</td>
<td>1</td>
</tr>
<tr>
<td>8.</td>
<td>(B)</td>
<td>1</td>
</tr>
</tbody>
</table>

### SECTION-B

9. Since, we know that for equal roots
   \[ D=0 \]
   \[ b^2-4ac=0 \]
   \[ (-2k)^2-4 \times k \times 6=0 \]
   \[ 4k^2-24k=0 \]
   \[ 4k(k-6)=0 \]
   \[ k=0, \text{ or } k=6 \]
   \[ k=0, 6 \text{ Ans.} \]

10. Here the circle touches the all sides of the Quadrilateral
    So, \[ AB+CD=AD+BC \]
    \[ 6+8=AD+9 \]
    \[ AD=14-9=5 \text{ cm Ans.} \]

11. Required Fig., Given and to prove
    Proof:

12. Drawing \[ \overline{AB}=7 \text{ cm} \]
    Correct division by any method
    Correct location of point i.e; \[ AP/AB=3/5 \]

13. \[ \because \text{ vol. of the cube}= \text{side}^3 \]
    \[ 64 = \text{side}^3 \]
    \[ \therefore \text{ side of the cube}=\sqrt[3]{64}=4 \text{ cm} \]
    Now S.A of the resultant cuboid=2(\text{lxb+lb+xh})
    \[ =2(8 \times 4+4 \times 4+4 \times 8) \]
    \[ =2(32+16+32) \]
    \[ =2(80) \]
    \[ =160 \text{ cm}^2 \text{ Ans.} \]
    Or

By question
\[ \text{Vol.of the cone}=\frac{\theta}{360} \times \pi \text{R}^3 \]
\[ \text{Or, } \frac{1}{3} \pi r^2 h = \frac{4}{3} \pi R^3 \]
\[ \therefore r^2 \times 32 = 4 \times 8 \times 8 \times 8 \]
\[ r = 8 \text{ cm} \]
so, the radius of the base of the cone=8 cm Ans.

14. Ar. of the shaded portion = \[ \frac{\theta}{360} \times \pi (R^2-r^2) \]
    \[ =(60/360) \times (22/7) (7^2-4^2) \]
15. \( x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \) and putting the correct value
\[ x = \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3} \]
\[ = \frac{4\sqrt{3} \pm 0}{6} \]
\[ = \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}} \]
Ans

16. Let the three nos. of the AP are \( \alpha-\beta, \alpha, \alpha+\beta \)
By queston,
\[ \alpha-\beta+\alpha+\alpha+\beta = 3 \]
or,
\[ 3\alpha = 3 \]
\[ \therefore \alpha = 1 \]
And \( (\alpha-\beta) \times \alpha \times (\alpha+\beta) = -35 \)
or,
\[ \alpha(\alpha^2-\beta^2) = -35 \]
Putting the value of \( \alpha = 1 \) then
\[ 1(1-\beta^2) = -35 \]
or,
\[ -\beta^2 = -36 \]
or,
\[ \beta = \pm 6 \]
Hence the no. are 7, 1, -5, or, -5, 1, 7 respectively. Ans.

Or
Here \( t_{13} = a + 12d \)
\[ = 3 + 12(7) \]
\[ = 87 \]
Let \( t_n = t_{13} + 84 \)
or,
\[ a + (n-1)d = 87 + 84 \]
or,
\[ 3 + (n-1)7 = 171 \]
or,
\[ (n-1) = 168/7 = 24 \]
or,
\[ n = 25 \]
\[ \therefore \text{the required term} = 25^{th} \]
Ans.

17. Since, AOC is a diameter of the circle.
\[ \therefore \angle ABC = 90^0 \]
So, in right triangle ABC
\[ AC^2 = 7^2 + 6^2 \]
\[ = 85 \]
Similarly, \( \angle ADC = 90^0 \)
So, in right triangle ADC
\[ AD^2 = AC^2 - CD^2 \]
\[ = 85 - 4 \]
\[ = 81 \]
\[ \therefore AD = 9 \text{ cm} \]
So, the perimeter of the cyclic Quad. ABCD = (7+6+2+9) cm
\[ = 24 \text{ cm} \]
Ans.

18. Constructing 120° at the centre with radii
Drawing tangents at the end of radii
Angle 60° between both tangents at the intersection point

Or
For drawing correct triangle
For correct construction steps for making similar triangle
Required triangle whose sides are 3/5 times the corresponding sides
19. For correct figure.

In triangle ABC, \( \tan 60^\circ = \frac{AB}{BC} \)

or \( \sqrt{3} = \frac{h}{BC} \)

\( \sqrt{3} = \frac{h}{x} \)

\( \therefore h = \sqrt{3}x \)

Now in triangle ABD

\( \tan 30^\circ = \frac{h}{40+x} \)

or \( \frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{40+x} \)

or, \( x = 20 \)

\( \therefore h = 20\sqrt{3} \text{m} \) Ans.

20. Here, no. Of red balls = 5

let no. Of blue balls = \( x \)

\( \therefore \) Total no. of balls = (5 + \( x \))

By question,

\( P(B) = 2P(R) \)

or, \( \frac{x}{5+x} = 2 \left( \frac{x}{5+x} \right) \)

or, \( x = 10 \)

so, No. Of blue balls = 10 Ans.

In a leap year = 366 days = 52 weeks and 2 days

The remaining two days can be

(i) SUN, MON

(ii) MON, TUE

(iii) TUE, WED

(iv) WED, THU

(v) THU, FRI

(vi) FRI, SAT

(vii) SAT, SUN

There are total seven possibilities i.e. \( n(s) = 7 \)

and \( n(E) = 2 \) i.e. SUN, MON & SAT, SUN

\( \therefore P(E) = \frac{n(E)}{n(s)} = \frac{2}{7} \) Ans.

21. By question,

\[ -1 = \frac{6k-3}{k+1} \]

\( k = \frac{2}{7} \)

Hence required ration is 2:7
22. 
\[ \text{Ar. Of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \]
\[ = \frac{1}{2}[-4(-5 + 2) + (-3)(-2 + 2) + (-4)(-2 - 3)] \]
\[ = \frac{21}{2} \text{ unit}^2 \]

\[ \text{Ar. Of } \triangle CDA = \frac{1}{2}[3(3 + 2) + 2(-2 + 2) + (-4)(-2 - 3)] \]
\[ = \frac{35}{2} \text{ unit}^2 \]

so, \[ \text{Ar. of qua. } ABCD = \frac{21}{2} + \frac{35}{2} = 28 \text{ unit}^2 \] Ans.

23. Since, the Circumference of the circle = 88 cm
or, \[ 2\pi r = 88 \]
or, \[ 88 \times \frac{7}{28} = 14 \text{ cm} \]

So, \[ \text{Ar. of the required sector} = \frac{\theta}{360} \times \pi r^2 \]
\[ = \frac{45}{360} \times \frac{22}{7} \times 14 \times 14 \]
\[ = 77 \text{ cm}^2 \]

24. Vol. Of Glass (Shaped frustum of a cone) = \[ \frac{1}{3}\pi(R^2 + r^2 + Rr)h \]
\[ = \frac{1}{3} \times \frac{22}{7} \left( 2^2 + 1^2 + 2 \times 1 \right) \times 14 \]
\[ = \frac{1}{3} \times \frac{22}{7} \times 7 \times 14 \]
\[ = 102.67 \text{ cm}^3 \]

25. \[ \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b} \]
\[ \frac{x}{x-a-b-x} = \frac{1}{ab} \]
or, \[ x(a+b) = ab \]
\[ \frac{1}{x-a} = \frac{1}{a+b} \]
or, \[ x(a+b) + ab = 0 \]
or, \[ x^2 + ax + bx + ab = 0 \]
or, \[ (x+a)(x+a) = 0 \]
\[ x = -a \text{ or } x = -b \] Ans.

Or

Let the usual speed of the plane be \( x \) km/hr.

Then, By question,
\[ \frac{1500}{x} - \frac{1500}{x+250} = 1 \]
\[ \frac{1500}{x+250} - \frac{2}{x} = 0 \]
or, \[ x^2 + 250x - 750000 = 0 \]
or, \[ (x+1000)(x-750) = 0 \]

Or, \( x = -1000 \) (rejected) or, \( x = 750 \)

Hence, the usual speed of the plane is 750 km/hr. Ans.

26. Required nos. are 252,255, 258, .......999

Here, \( a + (n-1)d = 999 \)
or, 252 + (n-1)3 = 999
\[ \therefore n = 250 \]

So, Required sum = \( S_n = \frac{n}{2} [a + l] = \frac{250}{2} (252 + 999) = 156375. \) Ans.

27. Let the \( n^{th} \) term of the given AP be the first negative term.

Then \( a_n < 0 \)
or, \( a + (n-1)d < 0 \)
or, 20 + (n-1) \[ \frac{3}{4} \] < 0
or, 83 -3n <0
28. Required fig.

Thus, 28th term of the given sequence is the first negative term. Ans.

29. For correct fig.

Let AB = Building, CD = Tower

In, ΔDEB,

\[ \tan 30^\circ = \frac{BE}{DE} \]
or, \( \frac{1}{\sqrt{3}} = \frac{60-h}{x} \)

\[ \therefore x = (60 - h)\sqrt{3} \quad \text{(i)} \]

In \( \triangle CAB \),

\[ \tan 60^\circ = \frac{AB}{CA} \]

or,

\[ \sqrt{3} = \frac{60}{x} \]

\[ \therefore x = \frac{60}{\sqrt{3}} \quad \text{--------- (ii)} \]

By (i) & (ii)

\[ (60-h)\sqrt{3} = \frac{60}{\sqrt{3}} \]

\[ h = 40 \text{ m} \]

Thus, the height of the tower is 40 m. Ans.

30. Here, \( n(s) = 49 \)

(i) \[ P(E_1) = \frac{n(E_1)}{n(s)} = \frac{3}{49} \]

(ii) \[ P(E_2) = \frac{n(E_2)}{n(s)} = \frac{13}{49} \]

(iii) \[ P(E_3) = \frac{n(E_3)}{n(s)} = \frac{10}{49} \]

(iv) \[ P(E_4) = \frac{n(E_4)}{n(s)} = \frac{1}{49} \]

31. Three points are collinear if \( \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0 \)

or, \( \frac{1}{2} [7(1 - k) + 5(k + 2) + 3(-2 - 1)] = 0 \)

or, \(-2k + 8 = 0 \)

or, \( k = 4 \)

\[ \therefore k = 4 \quad \text{Ans.} \]

32. For correct Fig.

Vol. of 1 Gulab Jamun = Vol. of cylindrical part + 2(Vol. of hemispherical part)

\[ = \pi r^2 h + 2 \left( \frac{2}{3} \pi r^3 \right) \]

\[ = \pi r^2 + \left( \frac{4}{3} \pi r^3 \right) \]

\[ = \frac{22}{7} \times 1.4 \times 1.4 \times [2.2 + \frac{4}{3} \times 1.4] \]

\[ = 25.05 \text{ cm}^3 \]

So, vol. of 45 gulab jamuns = \( 45 \times 25.05 = 1127.28 \text{ cm}^3 \)

Hence, Vol. of sugar syrup = \( 30/100 \times 1127.28 = 338.18 \text{ cm}^3 = 338 \text{ cm}^3 \) (approx.)

33. Let the level of the water in the tank will rise by 7 cm in \( x \) hrs

So, vol of the water flowing through the cylindrical pipe in \( x \) hrs = \( \pi r^2 h \)

\[ = \frac{22}{7} \times \left( \frac{7}{100} \right)^2 \times 5000 \times x \text{ m}^3 \]

\[ = 77 x \text{ m}^3 \]

Also, Vol of water that falls into the tank in \( x \) hrs = \( 50 \times 44 \times \frac{7}{100} \text{ m}^3 = 154 \text{ m}^3 \)

By ques \( 77 x = 154 \)

\[ x = 2 \]

So, the level of the water in the tank will rise by 7 cm in 2 hours
Radius of hemisphere = $7/2 = 3.5$ cm
Height of cone = $(14.5 - 3.5)$
= 11 cm
Slant height of cone = $\sqrt{r^2 + h^2}$
= $\sqrt{(3.5)^2 + (11)^2}$
= 11.55 cm

Now, volume of toy = Vol of hemisphere + Vol of cone
= $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$
= $\frac{1}{3}\pi r^2 (2r + h)$
= $2\times \frac{\pi}{7} \times \frac{7}{2} \times \frac{7}{2} (2 \times \frac{7}{2} + 11) \text{ cm}^3$
= 231 cm$^3$

And, TSA of the toy = SA of hemisphere + SA of cone
= $2\pi r^2 + \pi rl$
= $\pi r (2r + l)$
= $2\times \frac{\pi}{7} \times \frac{7}{2} (2 \times \frac{7}{2} + 11.55)$
= 204.05 cm$^2$
ACTIVITES (TERM-I)
(Any Eight)

Activity1: To find the HCF of two Numbers Experimentally Based on Euclid Division Lemma

Activity2: To Draw the Graph of a Quadratic Polynomial and observe:
   i. The shape of the curve when the coefficient of $x^2$ is positive
   ii. The shape of the curve when the coefficient of $x^2$ is negative
   iii. Its number of zero

Activity3: To obtain the zero of a linear Polynomial Geometrically

Activity4: To obtain the condition for consistency of system of linear Equations in two variables

Activity5: To Draw a System of Similar Squares, Using two intersecting Strips with nails

Activity6: To Draw a System of similar Triangles Using Y shaped Strips with nails

Activity7: To verify Basic proportionality theorem using parallel line board

Activity8: To verify the theorem: Ratio of the Areas of Two Similar Triangles is Equal to the Ratio of the Squares of their corresponding sides through paper cutting.

Activity9: To verify Pythagoras Theorem by paper cutting, paper folding and adjusting (Arranging)

Activity10: Verify that two figures (objects) having the same shape (and not Necessarily the same size) are similar figures. Extend the similarity criterion to Triangles.

Activity11: To find the Average Height (in cm) of students studying in a school.

Activity12: To Draw a cumulative frequency curve (or an ogive) of less than type.

Activity13: To Draw a cumulative frequency curve (or an ogive) of more than type.
ACTIVITES (TERM-II)
(Any Eight)

Activity1: To find Geometrically the solution of a Quadratic Equation \( ax^2 + bx + c = 0 \), \( a \neq 0 \) (where \( a=1 \)) by using the method of computing the square.

Activity2: To verify that given sequence is an A.P (Arithmetic Progression) by the paper Cutting and Paper Folding.

Activity3: To verify that \( \sum n = \frac{n(n+1)}{2} \) by Graphical method

Activity4: To verify experimentally that the tangent at any point to a circle is perpendicular to the Radius through that point.

Activity5: To find the number of Tangent from a point to the circle

Activity6: To verify that lengths of Tangents Drawn from an External Point, to a circle are equal by using method of paper cutting, paper folding and pasting.

Activity7: To Draw a Quadrilateral Similar to a given Quadrilateral as per given scale factor (Less than 1)

Activity8: (a) To make mathematical instrument clinometer (or sextant) for measuring the angle of elevation/depression of an object
   (b) To calculate the height of an object making use of clinometers (or sextant)

Activity9: To get familiar with the idea of probability of an event through a double color card experiment.

Activity10: To verify experimentally that the probability of getting two tails when two coins are tossed simultaneously is \( \frac{1}{4} = 0.25 \) (By eighty tosses of two coins)

Activity11: To find the distance between two objects by physically demonstrating the position of the two objects say two Boys in a Hall, taking a set of reference axes with the corner of the hall as origin.

Activity12: Division of line segment by taking suitable points that intersects the axes at some points and then verifying section formula.

Activity13: To verify the formula for the area of a triangle by graphical method.

Activity14: To obtain formula for Area of a circle experimentally.

Activity15: To give a suggestive demonstration of the formula for the surface Area of a circus Tent.

Activity16: To obtain the formula for the volume of Frustum of a cone.
PROJECTS

Project 1: Efficiency in packing
Project 2: Geometry in Daily Life
Project 3: Experiment on probability
Project 4: Displacement and Rotation of a Geometrical Figure
Project 5: Frequency of letters/ words in a language text.
Project 6: Pythagoras Theorem and its Extension
Project 7: Volume and surface area of cube and cuboid.
Project 8: Golden Rectangle and golden Ratio
Project 9: Male-Female ratio
Project 10: Body Mass Index (BMI)
Project 11: History of Indian Mathematicians and Mathematics
Project 12: Career Opportunities
Project 13: \( \pi \) (Pie)

Project Work Assignment (Any Eight)
ACTIVITY-1

TOPIC:- Prime factorization of composite numbers.

OBJECTIVE:- To verify the prime factorization 150 in the form 
\[5^2 \times 3 \times 2\] i.e 150 = 5^2 \times 3 \times 2.

PRE-REQUISITE KNOWLEDGE:- For a prime number P, P^2 can be represented by the area of a square whose each side of length P units.

MATERIALS REQUIRED:-

i. A sheet of graph paper (Pink / Green)
ii. Colored (black) ball point pen.
iii. A scale

TO PERFORM THE ACTIVITY:-

Steps:-

1. Draw a square on the graph paper whose each side is of length 5 cm and then make partition of this square into 25 small squares as shown in fig 1.1 each small square has its side of length 1 cm.

   Here, we observe that the area of the square having side of length 5 cm = 5^2 \times 25 = 25 cm^2

2. As shown in Fig 1.2 draw there equal squares where each square is of same size as in figure 1.1 then the total area in the fig 1.2

   = 5^2 \times 3 \times 3 \times \text{cm}^2
   = 5^2 \times 3 \times 2 \text{ cm}^2

![Fig=1.1](image1.jpg)

3. As shown in fig 1.3 draw six equal square where each square is as same size as in Fig 1.1 Here, three squares are in one row and three squares in the second row.

   We observe that the total area of six squares

   = 5^2 \times (3+3) \text{cm}^2
   = 5^2 \times 3 \times 2 \text{ cm}^2

![Fig=1.2](image2.jpg)
Also observe that the total area 
\[ = 75\text{cm}^2 + 75\text{cm}^2 = 150\text{cm}^2 \]

Hence, we have verified that

\[ 150 = 5^2 \times 3 \times 2 \]

Fig-1.3
ACTIVITY-2

TOPIC:- Ratio of the areas of two similar triangles

STATEMENT:- The ratio of the area of two similar triangle is equal to the ratio of the squares of their corresponding sides.

OBJECTIVE:- To verify the above statement through activity.

PRE-REQUISITE KNOWLEDGE:-

1. The concept of similar triangles.
2. Division of a line segment into equal parts.
3. Construction of lines parallel to given line.

MATERIAL REQUIRED:-

1. White paper sheet
2. Scale /Rubber
3. Paint box
4. Black ball point pen or pencil

TO PERFORM THE ACTIVITY:-

STEPS:-

1. On the poster paper sheet, draw two similar triangle ABC and DEF. We have the ratio of their corresponding sides same and let as have
   \[ \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{5}{3} \]
   \[ \text{i.e., } \frac{AB}{DE} = \frac{5}{3}, \quad \frac{BC}{EF} = \frac{5}{3}, \quad \frac{CA}{FD} = \frac{5}{3}, \]
   \[ \text{i.e. } \frac{DE}{AB} = \frac{3}{5}, \quad \frac{EF}{BC} = \frac{3}{5}, \quad \frac{FD}{CA} = \frac{3}{5} \]

2. Divide each side of\( \Delta ABC \) into 5 equal parts and those of\( \Delta DEF \) into 3 equal parts as shown in Fig (i) and (ii).

3. By drawing parallel lines as shown in Fig (i) and (ii), we have partition\( \Delta ABC \) into 25 smaller triangle of same size and also each smaller triangle in fig (i) has same size and as that of the smaller triangle fig (ii).

4. Paint the smaller triangle as shown in Fig (i) and (ii).

OBSERVATION:-

1. Area of\( \Delta ABC \)= Area of 25 smaller triangle in fig (i)=25 square unit
Where area of one smaller triangle in fig (i)=P (square unit)

2. Area of $\triangle DEF$=Area of a smaller triangle in Fig (ii)=9P where area of one smaller triangle in fig (ii)=P square units.

3. \[
\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{25P}{9P} = \frac{25}{9}
\]

4. \[
\frac{(AB)^2}{(DE)^2} = \frac{(AB)^2}{\left(\frac{3}{5}AB\right)^2} = \frac{25}{9} \quad \text{Similarly}
\]
\[
\frac{(BC)^2}{(EF)^2} = \frac{25}{9} \quad \text{and} \quad \frac{(CA)^2}{(FD)^2} = \frac{25}{9}
\]

5. From steps (3) and (4), we conclude that
\[
\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{(AB)^2}{(DE)^2} = \frac{(BC)^2}{(EF)^2} = \frac{(CA)^2}{(FD)^2}
\]

Hence the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
TOPIC:- Trigonometric identities.

STATEMENT:- \( \sin^2 \theta + \cos^2 \theta = 1, 0^\circ < \theta < 90^\circ \)

OBJECTIVE: - To verify the above identity

PRE-REQUISITE KNOWLEDGE:- In a right angled triangle.

\[ \sin \theta = \frac{\text{Side opposite to angle } \theta}{\text{Hypotenuse of the triangle}} \]

\[ \cos \theta = \frac{\text{Side adjacent to angle } \theta}{\text{Hypotenuse of the triangle}} \]

MATERIAL REQUIRED:-

1. Drawing sheet
2. Black ball point pen
3. Geometry box
4. Scale

TO PERFORM THE ACTIVITY

Step:-

1. On the drawing sheet, draw horizontal ray AX.
2. Construct any arbitrary \( \angle \text{CAX} = \theta \) (say)
3. Construct AC=10 cm.
4. From C draw CB \( \perp \) AX.
5. Measure the length sides of sides AB and BC of the right angled \( \Delta \) ABC (see fig)
6. We measure that AB=8.4 cm (approx) and BC=5.4 cm (approx)

OBSERVATION

1. \( \sin \theta = \frac{\text{BC}}{\text{AC}} = \frac{5.4}{10} = 0.54 \) (Approx)
2. \( \cos \theta = \frac{\text{AB}}{\text{AC}} = \frac{8.4}{10} = 0.84 \) (approx)
3. \( \sin^2 \theta + \cos^2 \theta = (0.54)^2 + (0.84)^2 \)
\[ = 0.2916 + 0.7056 \]
\[ = 0.9972 \) (Approx)

Ie. \( \sin^2 \theta + \cos^2 \theta \) is nearly equal to 1. Hence the identity is verified.
ACTIVITY-4

Topics:- Measure of the central tendencies of a data.

STATEMENT:- We have an empirical relationship for statistical data as $3 \times \text{median} = \text{Mode} + 2 \times \text{mean}$.

OBJECTIVE :: To verify the above statement for a data.

PRE-REQUISITE KNOWLEDGE:-

Method to find central tendencies for grouped data.

MATERIAL REQUIRED:-

1. A data about the heights of students of a class and arranged in grouped form.
2. A ball point pen.
3. A scale.

TO PERFORM THE ACTIVITY:-

Step:

1. Count the number of girl students in the class. The number is 51
2. Record the data about their height in centimeter.
3. Write the data in grouped form as below:

<table>
<thead>
<tr>
<th>Height in cm</th>
<th>135-140</th>
<th>140-145</th>
<th>145-150</th>
<th>150-155</th>
<th>155-160</th>
<th>160-165</th>
<th>Total no of girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of girls</td>
<td>4</td>
<td>7</td>
<td>18</td>
<td>11</td>
<td>6</td>
<td>5</td>
<td>51</td>
</tr>
</tbody>
</table>

4. On three different sheets of paper find mean height on the sheet of paper, median height on the second and the modal height on the third sheet of paper.

5. Let us find mean by step deviation method:-

<table>
<thead>
<tr>
<th>Class of heights (in cm)</th>
<th>Frequency $p$</th>
<th>$f_i$</th>
<th>Class mark $x_i$</th>
<th>$U_1=\frac{x_i-147.5}{5}$</th>
<th>$F_i x ui$</th>
</tr>
</thead>
<tbody>
<tr>
<td>135-140</td>
<td>4</td>
<td>51</td>
<td>137.5</td>
<td>-2</td>
<td>-8</td>
</tr>
<tr>
<td>140-145</td>
<td>7</td>
<td></td>
<td>142.5</td>
<td>-1</td>
<td>-7</td>
</tr>
<tr>
<td>145-150</td>
<td>18</td>
<td></td>
<td>147.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>150-155</td>
<td>11</td>
<td></td>
<td>152.5</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>155-160</td>
<td>6</td>
<td></td>
<td>157.5</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>160-165</td>
<td>5</td>
<td></td>
<td>162.5</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>$\sum f_i = 51$</td>
<td></td>
<td></td>
<td></td>
<td>$\sum f_i u_i = 51$</td>
<td></td>
</tr>
</tbody>
</table>
Mean = a + h \times \frac{\sum f_{ui}}{\sum f_{i}} = 147.5 + 5 \times \frac{23}{51} = 147.5 + 115/51 = (147.5 + 2.255) \text{ cm} = 149.755 \text{ cm}

6. Let us find median of the data:

<table>
<thead>
<tr>
<th>Class of height (in cm)</th>
<th>Frequency number of girls</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>135-140</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>140-145</td>
<td>7</td>
<td>11 = cf</td>
</tr>
<tr>
<td>145-150</td>
<td>18 = f</td>
<td>29</td>
</tr>
<tr>
<td>150-155</td>
<td>11</td>
<td>40</td>
</tr>
<tr>
<td>155-160</td>
<td>6</td>
<td>46</td>
</tr>
<tr>
<td>160-165</td>
<td>5</td>
<td>51</td>
</tr>
<tr>
<td>Total</td>
<td>n\sum f_{i} = 51</td>
<td></td>
</tr>
</tbody>
</table>

\[ n/2 = 25.5 \]

we have median class (145-150) it gives \( l = 145, h = 5, f = 18, cf = 11 \)

\[
\text{median} = l + \left\{ \frac{n - cf}{f} \right\} \times h = 145 + \left\{ \frac{25.5 - 11}{18} \right\} \times 5
\]

\[
= 145 + 14.5 \times 5
\]

\[
= 145 + 4.028
\]

\[
= 149.028 \text{ cm}
\]

7. Let us find mode of the data:

<table>
<thead>
<tr>
<th>Class of heights (in cm)</th>
<th>FREQUENCY (No of Girls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>135-140</td>
<td>4</td>
</tr>
<tr>
<td>140-145</td>
<td>7 = f1</td>
</tr>
<tr>
<td>145-150</td>
<td>18 = fm</td>
</tr>
<tr>
<td>150-155</td>
<td>11 = f2</td>
</tr>
<tr>
<td>155-160</td>
<td>6</td>
</tr>
<tr>
<td>160-165</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>51</td>
</tr>
</tbody>
</table>

Modal class is 145-150
Thus \( l = 145, h = 5, fm = 18, f1 = 7, f2 = 11 \)

\[
\text{Mode} = H \left\{ \frac{fm - f1}{2 fm - f1 - f2} \right\} \times h = 145 + \left\{ \frac{18 - 7}{36 - 7 - 11} \right\} \times 5
\]

\[
= 145 + 55/18 = 145 + 3.055
\]

\[
= 148.055 \text{ cm}
\]

8. CONCLUSION:

Mean = 149.755, median = 149.028 and mode = 148.055

\[
3 \times \text{median} = \text{mode} + 2 \times \text{mean (Approx)}
\]

Thus we have verified that \( 3 \times \text{median} = \text{mode} + 2 \times \text{mean (Approx)} \)
TOPIC : Angle of Elevation

OBJECTIVE : To find the angle of elevation of the sun at a particular time on a sunny day.

PRE-REQUISITE KNOWLEDGE: knowledge of trigonometric ratios.

MATERIAL REQUIRED :
1. A metre rod
2. Measuring tape
3. Table for tangent of angles .

TO PERFORM THE ACTIVITY:

STEPS :
1. On the particular sunny day at the given time, put the metre rod on the level ground with one end on the ground and the other vertically upward.
2. Measure the length of the shadow of the metre rod from the beginning to the end. Let the length of the shadow be 58cm = 0.58m.
3. The length of the metre rod = 1m or 100cm.

OBSERVATION:
1. If \( \theta \) be the angle of elevation of the sun at the given moment, then we have the following figure on a sheet of paper by taking a suitable scale.
2. From the right angle \( \triangle OMP \) drawn in figure, we have
   \[
   \tan \theta = \frac{MP}{OM} = \frac{100}{58} = 1.724 \text{ (approx.)}
   \]
   \[
   \tan \theta = \sqrt{3} \text{ (approx.)}
   \]
   i.e. \( \tan \theta = \tan 60^\circ \)
   \[
   \theta = 60^\circ
   \]

Hence, the required angle of elevation of the sun is 60°. For better result, we can take the help of the table of tangent of angles.
ACTIVITY – 6

TOPIC - Probability of events of a random experiment.

STATEMENT: For an event E of a random experiment, \( P(\text{not } E) = 1 - P(E) \).

OBJECTIVE: To verify the above statement by tossing three coins of different denominations simultaneously for head and tail. Event E happens if we get at least two heads and the event not-E happens if we do not get two or more than two heads.

PRE-REQUISITE KNOWLEDGE:
1. Probability of an event: \( \frac{\text{Number of outcome which favour the happening of the event } E}{\text{Total number of outcome}} \)
2. Event not-E happens when the outcome is not favourable for the event E to happen.

TO PERFORM THE ACTIVITY:

STEPS:
1. Take three fair coins of different denominations and toss these coins simultaneously.
2. We imaging about the possible outcomes as below.
   - HHH, HHT, HTH, THH, HTT, TTH, TTT
   - i.e. there can be 8 possible outcomes
   - favourable outcomes to the event E are
   - HHH, HHT, HTH, THH
   - Then \( P(E) = \frac{4}{8} = \frac{1}{2} \)
   - Now, favourable outcomes to the event not-E are HTT, THT, TTH, TTT
   - Then \( P(\text{not-}E) = 1 - \frac{1}{2} = 1 - P(E) \)
3. Repeating above random experiment, we record the observation of 20 trials as below:
   - Number of Heads:
     |   | 0 | 1 | 2 | 3 |
     |---|---|---|---|---|
     |   | 4 | 7 | 5 | 4 |
   - Number of times out of 20 trials :
4. From table in step 3, we observe that for 2 heads or for 3 heads, the event E happens i.e. there are 5+4=9 chances out of 20 which favour E
   - Thus, we have \( P(E) = \frac{9}{20} \)
   - Also we observe that for 0 head or for 1 head the event not-E happens. There are 4+7=11 chances out of 20 which favour not-E.
   - So, \( P(\text{not-}E) = \frac{11}{20} = 1 - \frac{9}{20} = 1 - P(E) \).
(REAL NUMBERS)
Answer the following questions
1. What is a lemma?
2. State Euclid’s Division Lemma?
3. What does HCF stand for?
4. Give the full form of LCM.
5. State Euclid’s division algorithm.

ORAL TEST
(REAL NUMBERS)
Answer the following questions:
1. Euclid’s division algorithm is a technique to compute the __________ of two given positive integers.
2. HCF(124, 24) is __________.
3. “Every composite number can be expressed(factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occurs”. The above statement is called __________.
4. For any two positive integers a and b, a x b = HCF(a, b) x ________
5. If a number cannot be written in the form p/q, where p and q are integers and q ≠ 0, then it is called __________

(REAL NUMBERS)
Answer the following questions:
1. What is a quadratic polynomial?
2. What is the degree of a quadratic polynomial?
3. What are the zeros of a polynomial?
4. What is the shape of curve of a quadratic polynomial graph?
5. State remainder theorem.

ORAL TEST
1. Every solution (x, y) of a linear equation in two variables, ax+by+c = 0 corresponds to a ____ on the line representing the equation, and vice versa.
2. If the pair of linear equations in two variables have only one common point on both the lines, then we have a ________ solution.
3. A pair of equations which has no solution is called a/an ________ pair of linear equations.
4. Half the perimeter of a rectangular garden, whose length is 4 m more than its width is 36 m. The dimension of the garden are ________ and __________.
5. A pair of linear equations in two variables can be represented and solved by the graphical method and _______ method.

**QUIZ**

*(Triangles)*

1. What is SAS similarity criterion?
2. What is the relationship between congruency and similarity of figures?
3. What is the criteria for the similarity of two triangles?
4. For what types of triangles is Pythagoras theorem applicable?
5. What is the another name of Basic Proportionality Theorem?

**ORAL TEST**

1. All _______ triangles are similar(equilateral/ isosceles/Scalene)
2. The longest side of a right angled triangle is called ________.
3. In a __________ the square of the hypotenuse is equal to the sum of squares of the other two sides.
4. In the given figure, if DE \| BC, then the value of x is _________

5. State whether the following quadrilateral are similar or not.

![Quadrilateral Diagram]

**QUIZ**

*(Introduction to Trigonometry)*

1. What is trigonometry?
2. What are trigonometric ratios of an acute angle in a right triangle?
3. From the figure find the value of cos A.

![Right Triangle Diagram]

4. Write the trigonometric ratios of 60°.
5. Evaluate tan 70° / cot 20°.

**ORAL TEST**

1. In a right triangle ABC, right angles at B, sin A = ______.
2. Sec(90o – A) = __________
3. Sec² A - __________ = 1 , for 0° ≤ A ≤ 90°.
4. If cot θ= 7/8, then (1+ sin θ)(1 – sin θ)/(1 + cos θ)(1 – cos θ)
5. (1 – tan² 45°)/( 1+tan² 45°) = __________
QUIZ
(STATISTICS)
1. Name the measures of central tendency.
2. What is cumulative frequency?
3. How will you represent the cumulative frequency distribution graphically?
4. How will you find the median of a grouped data graphically with the help of one ogive?
5. How will you find the median of a grouped data graphically with the help of both ogives (i.e. of the less than type and of more than type)?

ORAL TEST
1. __________ is the sum of the values of all the observations divided by the total number of observations.
2. Class mark = _____ /2.
3. The formula for finding the mean using the step deviation method is __________.
4. The formula for finding the mode in a grouped frequency distribution is __________.
5. The formula for finding the median of grouped data is ____________.

FORMATIVE ASSESSMENT
QUIZ
1. Define the fundamental theorem of arithmetic.
2. Define euclid’s division lemma.
3. What is a quadratic polynomial.
4. What is the relationship between zeros and coefficients of a quadratic polynomial.
5. Give the condition for a pair of linear equations to be inconsistent.

ORAL TEST
1. For any two positive integers a and b, HCF(a,b) x LCM(a, b) = __________
2. 5 – √3 is a/an __________ number.
3. A polynomial of degree 3 is called a _______ polynomial.
4. A quadratic polynomial having the sum and product of its zeroes respectively 5 and 6 is _______.
5. All _______ triangles are similar. (equilateral/isosceles/scalene).

QUIZ
QUADRATIC EQUATION
1. What is a quadratic equation?
2. How many roots can a quadratic equation have?
3. Give the formula for finding the roots of ax² + bx + c = 0 (a≠0)
4. Give the nature of roots of the equation ax² + bx + c = 0 (a≠0)
5. Find the nature of the roots of the equation 3x² – 2x +1/3 =0

ORAL TEST
1. A real number α is said to be a root of the quadratic equation ax² + bx + c = 0 , if aα² + bα + c = ________.
2. A quadratic equation ax² + bx + c = 0 has two roots, if b² – 4ac > 0.
3. The quadratic equation 3x² – 4√3x + 4 = 0 has two _______ roots.
4. The roots of a quadratic equation 2x² – 7x +3 = 0 are _______ and _______.
5. Two numbers whose sum is 27 and product is 182 are _______ and _______.

QUIZ
(ARITHMETIC PROGRESSIONS)
1. What is an A.P.?
2. What is meant by common difference in an A.P.?
3. What is the formula for the nth term of an A.P.?
4. What is the formula for the sum of first n terms of an A.P.?
5. What is the formula for the sum of first n natural numbers?

ORAL TEST
1. The common difference of a sequence of multiples of 7 is ________.
2. The difference of consecutive terms in an A.P. is always ________.
3. The sum of first 20 natural numbers is ________.
4. The sum of first eight odd natural numbers is ________.
5. The sum of first ten even natural numbers is ________.

QUIZ
(Coordinate geometry)
1. What is abscissa?
2. What is ordinate?
3. What is distance formula?
4. What is the distance of a point p(x,y) from origin?
5. Give the section formula.

ORAL TEST
1. If the area of a triangle is 0 square units, then its vertices are ________.
2. The area of a triangle whose vertices are (1, -1), (-4, 6) and (-3, -5) is ________ square units.
3. The distance between the points (-5, 7) and (-1, 3) is ________ units.
4. ________ has been developed as an algebraic tool for studying geometry of figures.
5. The distance between the points (a,b) and (-a, -b) is ________ units.

QUIZ
(Some applications of trigonometry or heights and distance)
1. Why trigonometry was invented? Give its uses.
2. What is the line of sight?
3. What is the angle of elevation?
4. What is the angle of depression?
5. What is a theodolite?

ORAL TEST
1. The other name of clinometer is ________.
2. If height of clinometer is 1 m, distance between object and clinometer is 40m and angle of elevation of object is 45°, then the height of object is ________.
3. A tower stands vertically on the ground. From the point on the ground, which is 25m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. The height of the tower is ________.
4. The angles of elevation of the top of a tower from two points at distances a and b from the base and on the same straight line with it are complementary. The height of the tower is ________.
5. A ladder 15m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is ________.

QUIZ
(CIRCLES)
1. Define tangent to a circle.
2. How many tangent(s) is/are there at a point of circle?
3. How many tangent can be drawn to a circle from a point outside the circle?
4. Define length of a tangent.
5. What is the relation between the lengths of tangents drawn from an external point to a circle?
ORAL TEST
1. A tangent to a circle intersects it in ___________ point(s).
2. A line intersecting a circle in two points is called a ___________.
3. A circle can have _________ parallel tangents at the most.
4. The common point of a tangent to a circle and the circle is called __________.
5. The tangent at any point of a circle is __________ to the radius through the point of contact.

QUIZ
(Constructions)
1. What is scale factor?
2. How will you draw a tangent at a point of a circle?
3. How will you locate the centre of a circle, if it is not given?
4. How many tangents can be drawn from a point outside the circle?
5. Is it possible to draw a tangent from a point inside a circle?

ORAL TEST
1. To divide a line segment AB in the ratio m:n (m, n are positive integers), draw a ray AX so that \( \angle BAX \) is an acute angle and then mark point on ray AX at equal distances such that the minimum number of these points is _______
2. To draw a pair of tangents to a circle which are inclined to each other at an angle of 45°, it is required to draw tangents at the end point of those two radii of the circle, the angle between which is _______
3. To divide a line segment AB in the ration 4:5, a ray AX is drawn first such that \( \angle BAX \) is an acute angle and them points A1, A2, A3... are located at equal distance on the ray AX and the point B is joined to _______
4. To construct a triangle similar to a given \( \triangle ABC \) with its sides 3/5 of the corresponding sides of \( \triangle ABC \), first draw a ray BX such that \( \angle CBX \) is an acute angle and X lies on the opposite side of A with respect to BC. To locate points B1, B2, B3, ____ on BX at equal distances and next step is to join ______ to _______.
5. State ‘True’ or ‘False’
   a. By geometrical construction, it is possible to divide a line segment in the ratio 3+\( \sqrt{5} \): 3-\( \sqrt{5} \).
   b. A pair of tangents can be drawn from a point P to a circle of radius 4.5 cm situated at a distance of 4 cm from the centre.
   c. By geometrical construction, it is possible to divide a line segment in the ratio \( \sqrt{5} : 1/\sqrt{5} \).
   d. A pair of tangents can be constructed to a circle inclined at an angle of 175°.
   e. From a point P outside the circle we can draw only one tangent.
   f. We cannot locate the centre of a circle if it is not given.

QUIZ
(AREAS RELATED TO CIRCLES)
1. What is circumference of a circle? Give its formula.
2. Name the great Indian mathematician who gave an approximate value of \( \pi \).
3. Give the formula for the area of a circle of radius r cm.
4. Give the formula for area of a sector of a circle having radius r and measuring an angle \( \theta \) at the centre.
5. How will you find the area of a segment of a circle?

ORAL TEST
1. If the area of a circle is 154 cm\(^2\), then its perimeter is ________.
2. Area of the largest triangle that can be inscribed in a semicircle of radius r is ________.
3. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is ________.
4. If the areas of two circles are equal, then their circumferences are ________.
5. The circles which have the same centre are called ________ circles.
QUIZ
(SURFACE AREAS AND VOLUMES)
1. A cone of height 24cm and radius of base 6cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.
2. A shuttle cork used for playing badminton has the shape of the combination of which basic solids?
3. What is a frustum of a right circular cone?
4. Does a frustum have two circular ends with equal radii?
5. Give the formula for the volume of the frustum of a cone.

ORAL TEST
1. A plumbline(sahul) shown in the figure is the combination of a ________ and a cone.

2. If the radii of the circular ends of a conical bucket which is 45cm high, are 28cm and 7cm then the capacity of the bucket is _______ cm$^3$.
3. The volume of the solid formed by joining two basic solids will actually be the _____ of the volumes of the constituents.
4. The curved surface area of the frustum of a cone is _________, where $l=\sqrt{h^2 + (r_1 - r_2)^2}$
5. If two cubes each of volumes 64cm$^3$ are joined end to end then the surface area of the resulting cuboid is ________.

QUIZ
(PROBABILITY)
1. Define the theoretical probability of an event E.
2. What is the probability of a sure event?
3. What is an elementary event?
4. What are complementary events?
5. One card is drawn from a well shuffled deck of 52 cards. Calculate the probability that the card will be a king.

ORAL TEST
1. The probability of an impossible event is ________.
2. The probability of an event lies between ________ and __________.
3. The sum of the probabilities of all the elementary events of an experiment is ________.
4. A die is thrown once, the probability of getting a prime number is ________.
5. Two coins are tossed simultaneously. The probability of at most one tail is ________.
1. Catching Fish
   If five fishermen catch 5 fishes in 5 minutes, how long will it take fifty fishermen to catch fifty fish?

2. Look at the Division
   One day professor Agarwal went to the blackboard and demonstrated to his astonished class that one half of eight was equal to three! What did the professor do?

3. How Big
   Can you guess how big the number : ninth power nine?

4. Counting Street Lights
   On two sides of a street, there are 35 street lights, each one is at a distance of 30 metres from the other. The street lights on one side are arranged so that each lamp fills a gap between the two other street lights on the opposite. How long is the street?

5. Who covered more distance
   Two friends Vijay and Ajay walk with constant speed of 100m/min. Vijay takes rest for 1 min after walking 100metres while Ajay takes rest for 3 min after walking 300 metres on a square path of side 400m. Both of them start from the same corner in opposite direction. Who covered more distance and when they meet?

6. The missing Six
   Place the six numbers below into empty circles, so that both the equation are true. Use each number once and only once.

   \[
   \begin{array}{ccc}
   1 & 2 & 3 \\
   4 & 5 & 7 \\
   \hline
   + & = & \\
   - & = & 
   \end{array}
   \]

7. Magic Triangle
   Place the numbers 4 through 9 in the circles in such a way that every side of the triangle add up to 21.

8. Add up
   Here is an equilateral triangle. Add another equilateral triangle to it in such a way that you get five equilateral triangles.
9. Magic Sticks
   Just by moving one stick, make another equation.

\[
\begin{array}{c}
7 + 5 = 12
\end{array}
\]

10. Identical Four
    Divide the adjoining figure into four identical pieces.

---XXX---