Concepts of Elementary Mathematics
Enduring Understandings & Essential Questions

First through Fourth Grade Content Areas:
Numbers and Operations
Algebra
Geometry
Measurement
Data Analysis and Probability

Above all, students should feel enchanted by the beauty of mathematics, empowered by their increasing abilities, and captivated by new concepts.

- Annette Raphel
Enduring Understandings

Numbers can represent quantity, position, location, & relationships.

Essential Questions

How is math relevant to me?
- relate and apply concepts & skills to daily life

What do numbers convey?
- identify amount (cardinal)
- name position (ordinal)
- indicate location (nominal)

How can numbers be expressed, ordered, and compared?
- read and write positive numbers, decimals, and fractions
- sequence numbers: zero through one billion
- compare quantities as more, less, equal (>, <, =)

Numbers can be classified by attributes.

What are properties of whole numbers?
- understand odd and even
- identify prime and composite
- create figurate numbers (ex: square, triangular)

Counting finds out the answer to “how many” in objects/sets.

What are different ways to count?
- count all
- count on
- count back
- skip count
- count groups

What are efficient ways to count?
- count up (or back) from largest number
- count sets of items
- count to/using landmark numbers (ex: 10, 25, 50, 100)
**Enduring Understandings**

**Grouping (unitizing) is a way to count, measure, and estimate.**

**Essential Questions**

**In what ways can items be grouped?**
- recognize common clusters (ex: tallies, dozens)
- create groups for efficiency (ex: 10s, 100s, etc.)
- make exchanges for unit of higher value (ex: 10 pennies for 1 dime)

**How do units within a system relate to each other?**
- understand relationships between and among:
  - **time:** minute, hour, day, week, month, year, decade, century
  - **money:** penny, nickel, dime, quarter, dollar
  - **length:** inch, foot, yard, mile; millimeter, centimeter, decimeter, meter, kilometer
  - **volume:** fluid ounce, cup, pint, quart, gallon
  - **weight:** ounce, pound, ton; gram, kilo, metric ton

**Place value is based on groups of ten.**

**How does the position of a digit in a number affect its value?**
- identify place value positions
- add & subtract multiples of 10, 100, 1000
- multiply & divide by powers of 10

**In what ways can numbers be composed and decomposed?**
- use expanded notation (ex: 349 = 300+40+9)
- rename numbers (ex: 83 = 70+13, 8 tens + 3 ones = 7 tens + 13 ones)

**How are place value patterns repeated in numbers?**
- understand pattern of 10s as structure of our number system
- identify patterns in digits and periods when numbers are sequenced

**How can place value properties aid computation?**
- apply single digit facts to multi-digit computation (ex: use 5x5 for 50x50)
- regroup numbers for efficiency

**Estimation is a way to get an approximate answer.**

**What are strategies to make a reasonable estimate?**
- determine possible range for answer
- use anchoring, partitioning, and visualization techniques
- round quantities to find approximate sum, difference, product, quotient
NUMBER AND OPERATIONS

Enduring Understandings

Computation involves taking apart and combining numbers using a variety of approaches;

Flexible methods of computation involve grouping numbers in strategic ways;

Proficiency with basic facts aids estimation and computation of larger and smaller numbers.

Essential Questions

What are different models of and models for addition and subtraction?
- join and/or separate items
- interpret diagrams: part-part-total, start-change-end, open number lines
- understand and generate variety of number models

What questions can be answered using addition and/or subtraction?
- problem solve in context

What are efficient methods for finding sums and differences?
- use identity property of addition and subtraction
- apply tens partners; doubles & near-double neighbors; see 9, think 10
- demonstrate fluency with addition and subtraction facts within 20
- split addends or subtrahends for ease of multi-digit computation
- regroup 1s, 10s, 100s for multi-digit addition and subtraction
- add any series of five-digit addends
- subtract any five-digit minuend with a smaller subtrahend
- use and examine algorithms for efficiency: partial sum, regrouping, trade first

What are different models of and models for multiplication and division?
- make equal sets/groups, create fair shares
- represent with objects, diagrams, arrays, area models
- identify multiplication patterns
- understand remainders

What questions can be answered using multiplication and division?
- problem solve in context

What are efficient methods for finding products and quotients?
- use identity and zero properties of multiplication and division
- apply doubling/halving concepts to multiplication (ex: 16 x 5 is half of 16 x 10)
- demonstrate fluency with multiplication facts of factors 0-12
- identify factors/divisors of a number and multiples of a number
- multiply any whole number by a two-digit factor
- divide any whole number by a two-digit divisor and interpret the remainder
- use and examine algorithms: partial product & partial quotient, and traditional
Enduring Understandings

Essential Questions

How do the four operations related to one another?
- construct fact families for addition/subtraction and multiplication/division
- recognize inverse relationships: addition/subtraction and multiplication/division
- understand whole number multiplication as repeated addition
- comprehend whole number division as repeated subtraction
- apply commutative, associative and distributive properties

How are parenthesis used in numeric expressions?
- indicate operation to be performed first in written expression containing more than two terms and different operations

What computation tools are best suited to which circumstances?
- determine when to use objects, number line, number chart, mental math, pencil/paper, calculator

Fractions, decimals, and percents express a relationship between two numbers.

How can fractions be modeled, compared, and ordered?
- understand fractions as division of two whole numbers
- identify fractions as part of a whole, part of a set, part of an area, and locations on the number line
- read and write symbolic notation for unit fractions and common fractions
- recognize and name equivalent fractions
- order fractions, including improper fractions (ex:3/2) and mixed numbers (ex:1½)

How are common fractions, decimals and percents alike and different?
- compare fraction and decimal models using pictures, objects and numbers
- identify and describe relationship between fractions and decimals
- relate fractions and decimals to percents
- recognize common equivalent fractions, decimals and percents

How is computation with rational numbers similar and different to whole number computation?
- add and subtract fractional numbers with like denominators
- add and subtract decimals using standard algorithms
ALGEBRA

Enduring Understandings

Patterns can be found in many forms. Where are patterns in nature, architecture, music, words, and numbers? - identify patterns in daily life and through integrated classroom studies

Patterns can grow and repeat. What is the repeating and/or increasing unit in the pattern? - identify and continue numerical sequences - identify and continue figure patterns

Patterns can be generalized. What strategies can be used to continue a sequence? - express pattern as a rule (ex: frame & arrows, what’s my rule?) - use a rule to determine unknowns (ex: complete a function table)

How does finding patterns help in counting and/or computation? - problem solve in context

Mathematical expressions represent relationships. How is an equation like a balance scale? - understand equal to mean both sides of the number model have an equivalent value

How can relationships be expressed symbolically? - use and interpret notation: >, <, =, ≠

Number patterns and relationships can be represented using variables. Why are variables used? - express generalizations (ex: $N+1=1+N$) - interpret formulas (ex: area = length x width)

What strategies can be used to solve for unknowns? - apply models to determine what is known (ex: part-part-whole) - work backwards to identify missing number
GEOMETRY

Enduring Understandings

Objects can be described and compared using their geometric attributes.

Points, lines and planes are the foundation of geometry.

Essential Questions

How are geometric properties used to solve problems in everyday life?
  - problem solve in context

How can plane and solid shapes be described?
  - understand properties of polygons (ex: closed, straight, flat/2-dimensional)
  - identify quadrilaterals (ex: square, rectangle, parallelogram, rhombus, trapezoid)
  - relate 3-dimensional objects to 2-dimensional counterparts (ex: cube to square)
  - classify angles as right, acute and obtuse
  - compare polyhedrons by edges, vertices, and shape of faces
  - identify parallel and perpendicular lines and figures

How are geometric figures constructed?
  - use straight edge, ruler, compass and technology
  - identify and label points, lines, line segments, and rays
  - identify and label the vertex of an angle

What strategies can be used to verify symmetry and congruency?
  - create model (ex: trace shape on paper: cut, fold, or place on top of shape)
  - measure
  - use mirror

How will a shape look when rotated, reflected, and/or translated?
  - practice slides, flips, and turns with variety of objects
  - predict the results of transforming two-dimensional shapes

How is the location of a point on a grid described? How is this useful?
  - find and record coordinates on grid
  - relate to mapping skills

Transforming an object does not affect its attributes.

Ordered pairs show an exact location on a coordinate plane.
# MEASUREMENT

## Enduring Understandings

- Objects have distinct attributes that can be measured.
- Standard units provide common language for communication measurements.
- The choice of measurement tools depends on the measurable attribute and the degree of precision desired.

## Essential Questions

### What types of problems are solved with measurement?
- Problem solve in context

### What are tools of measurement and how are they used?
- Tell time with clocks
- Calculate elapsed time
- Describe temperature with thermometers
- Determine length/height with rulers and measuring tapes
- Measure weight with variety of scales
- Find area of rectangles
- Calculate perimeter of polygons

### What is the purpose of standard units of measurement?
- Record with customary and metric units
- Communicate measurements

### How do units within a system relate to each other?
- Understand the relationships between and among units;
- Identify equivalent periods of unit within a system;
- Carry out conversions with units of:
  - **Time:** minute, hour, day, week, month, year, decade, century
  - **Money:** penny, nickel, dime, quarter, dollar
  - **Length:** inch, foot, yard, mile; millimeter, centimeter, decimeter, meter, kilometer
  - **Volume:** fluid ounce, cup, pint, quart, gallon
  - **Weight:** ounce, pound, ton; gram, kilo, metric-ton

### When is an estimate more appropriate than an actual measurement?
- Use of judgment in context

### What strategies help estimate measurements?
- Select appropriate “anchor” or benchmark for measurement
- Apply multiplicative reasoning (ex: anchor measurement x frequency of anchor)
Data Analysis and Probability

Enduring Understandings

The quality of the question used impacts the data collected and the validity of the results.

Essential Questions

How can information be gathered, recorded, and organized?
- design investigate to address pertinent question or problem
- determine wording and/or categories to generate fair, unbiased information
- collect data via observations, surveys, samplings, experiments

How does the type of data influence the choice of display?
- understand purpose of different graphs
- create bar graph, chart/table, circle graph/pie chart, glyph
line plot, line graph, pictograph, Venn Diagram

What aspects of a graph help people understand and interpret the data easily?
- read and write parts of a graph: title, labels on x-axis and y-axis,
- understand different type of scales
- create and interpret keys/legends

What kinds of questions can and can not be answered from a graph?
- understand data landmarks: minimum & maximum value, range
- find measures of center: mode, median, mean
- interpret salient points and key relationships in the data set

How is the probability of an event determined and described?
- experience activities with random generators (ex: dice, spinners)
- test predictions
- develop vocabulary to express probable outcomes
- represent probability by a number from 0 to 1
- understand impact of “law of large numbers” on any single outcome

Sources:
About Teaching Mathematics K-8 (Burns, 2007)
Everyday Mathematics (University of Chicago, 2007)
Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000)
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