## Chapter 7
### Solving Systems of Linear Equations and Inequalities
#### Chapter Overview and Pacing

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<td>Use systems of equations by substitution.</td>
<td>Solve systems of equations by using elimination with addition.</td>
<td>Solve systems of equations by using elimination with subtraction.</td>
<td>Solve systems of equations by using elimination with multiplication.</td>
<td>Solve systems of inequalities by graphing.</td>
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<td><strong>Follow-Up:</strong> Use a graphing calculator to solve a system of equations.</td>
<td>Solve real-world problems involving systems of equations.</td>
<td>Solve systems of equations by using elimination with subtraction.</td>
<td>Determine the best method for solving systems of equations.</td>
<td>Solve real-world problems involving systems of inequalities.</td>
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<td>1.5 (with 7-1 Preview and Follow-Up)</td>
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<td><strong>7-3</strong></td>
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<tbody>
<tr>
<td>1</td>
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<td>0.5</td>
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| **TOTAL** | 11 | 11 | 6 | 6 |

Pacing suggestions for the entire year can be found on pages T20–T21.
### Chapter Resource Manager

#### CHAPTER 7 RESOURCE MASTERS

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<td>433–446, 450–452</td>
<td>58</td>
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</table>

*Key to Abbreviations:* GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

**ELL** Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
Mathematical Connections and Background

Graphing Systems of Equations
A solution of a system of equations is the set of points that satisfy each equation in the system. Carefully graph each equation on the same coordinate plane. There is only one solution if the graphs of the lines intersect, since the intersection is at only one point. There is no solution if the lines are parallel. The lines never intersect, so no one point is common to both graphs. If the graphs are the same line, the system has an infinite number of solutions.

If there is one solution or infinitely many solutions, the system of equations is described as consistent. Systems with one solution are said to be independent, while those with infinitely many solutions are said to be dependent. If there is no solution, the system is described as inconsistent.

Substitution
It is sometimes difficult to determine the exact solution of a system of equations from a graph. Therefore, an algebraic method may be used to find the exact solution. Substitution is an algebraic method. First solve one equation for one variable in terms of the other. Substitute the expression into the other equation so that one variable is eliminated. Solve for the remaining variable. Substitute this value into either equation and solve for the other variable. The two values make up the solution of the system. They are written in the form \((x, y)\) to represent the point where the two lines intersect if graphed.

If a solution results in an identity, for example \(2 = 2\), the system has an infinite number of solutions. If the result is a false statement, such as \(5 = 3\), there is no solution. If you incur either of these situations during any part of the solution process, you may stop solving and write either infinitely many solutions or no solution.

Prior Knowledge
In Chapter 2, students learned that when you add additive inverses, the sum is always 0. They used the Addition, Subtraction, Multiplication, and Division Properties of Equality to solve equations throughout Chapter 3. Students graph linear equations in Chapter 5 and graph linear inequalities in Chapter 6.

This Chapter
Chapter 7 introduces students to systems of linear equations. They first solve the systems by graphing and then classify the systems as consistent or inconsistent, and as independent or dependent. Students also learn to apply the algebraic methods to solving the systems. These methods include substitution, elimination using addition or subtraction, and elimination using multiplication first. Students must determine which method is best for different systems. The chapter ends with students solving systems of inequalities by graphing.

Future Connections
Business analysts use systems of linear equations to determine where break-even points are and to analyze trends for predicting future events. There are not only systems of linear equations and inequalities, but also systems of all types of functions including quadratic, absolute value, and sine. These systems can mix any types of functions. The solutions of these systems are not only used in business, but also in science and other fields.
7-3 Elimination Using Addition and Subtraction

Elimination is another algebraic method used to solve systems of equations. The objective is to combine the two equations to eliminate one of the variables. If the coefficients of one variable are additive inverses of each other, use addition. If the coefficients of one variable are the same, use subtraction. Because the Addition and Subtraction Properties of Equality state that equal amounts can be added to or subtracted from each side of an equation, you can add or subtract one equation with the other. This step eliminates one variable. Then solve for the other variable. Substitute this value into either original equation to find the value of the variable that was eliminated. The two values are the solution of the system.

7-4 Elimination Using Multiplication

If the coefficients of one variable are neither additive inverses nor equal, one or both equations must be changed so that the elimination method can be applied to solve the system of equations. Change either one, or both, of the equations by applying the Multiplication Property of Equality. Every term of the equation is multiplied by the same number, or both equations are multiplied by different numbers, in order to make one pair of coefficients of a variable either additive inverses or the same. Then follow the steps for solving the system using the elimination method.

Five methods for solving systems of equations have been seen in this chapter. They are graphing, substitution, elimination using addition, elimination using subtraction, and elimination using multiplication. Graphing is used only if an estimation is needed, since it is difficult to get an exact solution. Use substitution if one of the variables has a coefficient of 1 or −1. Elimination using addition is used when one of the variables has coefficients that are additive inverses of each other, and elimination using subtraction is best used when one of the variables has the same coefficient. Apply elimination using multiplication if none of the above situations occur.

7-5 Graphing Systems of Inequalities

A solution of a system of inequalities is all the points that satisfy both inequalities. Use the methods learned in Lesson 6-6 to graph each inequality. The points that are solutions of both inequalities lie in the region where the graphs overlap, or intersect. A system of inequalities has no solution if the boundary lines are parallel and the shaded regions do not overlap. Otherwise, there are infinitely many solutions since the overlapping shaded region extends on indefinitely.

Additional mathematical information and teaching notes are available in Glencoe’s Algebra 1 Key Concepts: Mathematical Background and Teaching Notes, which is available at www.algebra1.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Graphing Systems of Equations (Lesson 18)
- Substitution (Lesson 19)
- Elimination Using Addition and Subtraction (Lesson 20)
- Elimination Using Multiplication (Lesson 21)
## Chapter 7: Solving Systems of Linear Equations and Inequalities

### TestCheck and Worksheet Builder

This networkable software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

### Key to Abbreviations

- **TWE** = Teacher Wraparound Edition
- **CRM** = Chapter Resource Masters

### Additional Intervention Resources

- The Princeton Review’s *Cracking the SAT & PSAT*
- The Princeton Review’s *Cracking the ACT*
- ALEKS

### Table: Daily Intervention and Assessment

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**Key to Abbreviations:** TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters
## Intervention Technology

**AlgePASS:** Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

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<tr>
<td>7-5</td>
<td>17 Graphing Linear Inequalities on the Coordinate Plane</td>
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**ALEKS** is an online mathematics learning system that adapts assessment and tutoring to the student’s needs. Subscribe at [www.k12aleks.com](http://www.k12aleks.com).

## Intervention at Home

**Parent and Student Study Guide**

Parents and students may work together to reinforce the concepts and skills of this chapter. (Workbook, pp. 53–58 or log on to [www.algebra1.com/parent_student](http://www.algebra1.com/parent_student))

**Log on for student study help.**

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes. 
  - [www.algebra1.com/extra_examples](http://www.algebra1.com/extra_examples)
  - [www.algebra1.com/self_check_quiz](http://www.algebra1.com/self_check_quiz)
- For chapter review, there is vocabulary review, test practice, and standardized test practice. 
  - [www.algebra1.com/vocabulary_review](http://www.algebra1.com/vocabulary_review)
  - [www.algebra1.com/chapter_test](http://www.algebra1.com/chapter_test)
  - [www.algebra1.com/standardized_test](http://www.algebra1.com/standardized_test)

For more information on Intervention and Assessment, see pp. T8–T11.

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## Reading and Writing in Mathematics

*Glencoe Algebra 1* provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

### Student Edition

- Foldables Study Organizer, p. 367
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 371, 379, 384, 390, 396)
- Reading Mathematics, p. 393
- Writing in Math questions in every lesson, pp. 374, 381, 386, 392, 398
- WebQuest, pp. 373, 398

### Teacher Wraparound Edition

- Foldables Study Organizer, pp. 367, 399
- Study Notebook suggestions, pp. 372, 379, 384, 390, 393, 396
- Modeling activities, p. 374
- Speaking activities, pp. 381, 392
- Writing activities, pp. 386, 398
- Differentiated Instruction, (Verbal/Linguistic), p. 389
- ELL Resources, pp. 366, 373, 380, 385, 389, 391, 393, 397, 399

### Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 7 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 7 Resource Masters*, pp. 407, 413, 419, 425, 431)
- Vocabulary PuzzleMaker software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*
- *Hot Words/Hot Topics* Sections 6.2–6.4, 6.6–6.8, 9.4

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Business decision makers often use systems of linear equations to model a real-world situation in order to predict future events. Being able to make an accurate prediction helps them plan and manage their businesses.

Trends in the travel industry change with time. For example, in recent years, the number of tourists traveling to South America, the Caribbean, and the Middle East is on the rise. You will use a system of linear equations to model the trends in tourism in Lesson 7-2.

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 7 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 7 test.
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1

- Graph each equation. (For review, see Lesson 4-5.)
  1. \( y = 1 \)
  2. \( y = -2x \)
  3. \( y = 4 - x \)
  4. \( y = 2x + 3 \)
  5. \( y = 5 - 2x \)
  6. \( y = \frac{1}{2}x + 2 \)

For Lesson 7-2

- Solve each equation or formula for the variable specified. (For review, see Lesson 3-8.)
  7. \( 4x + a = 6x, \) for \( x = \frac{a}{2} \)
  8. \( 8a + y = 16, \) for \( a = \frac{16 - y}{8} \)
  9. \( 7bc - d = 12, \) for \( b = \frac{120 + d}{7c} \)
  10. \( 7m + n = 2m, \) for \( q = \frac{7m + n}{2m} \)

For Lessons 7-3 and 7-4

- Simplify each expression. If not possible, write simplified. (For review, see Lesson 1-5.)
  11. \( (3x + y) - (2x + y) \)
  12. \( (7x - 2y) - (7x + 4y) \)
  13. \( (16x - 3y) + (11x + 3y) \)
  14. \( 8x - 4y + (-8x + 5y) \)
  15. \( 4(2x + 3y) - (8x - y) \)
  16. \( 3(x - 4y) + (x + 12y) \)
  17. \( 2(x - 2y) + (3x + 4y) \)
  18. \( 5(2x - y) - 2(5x + 3y) \)
  19. \( 3(x + 4y) + 2(2x - 6y) \)
  20. \( -11y \)

For Prerequisite Skills Workbook

- Exercises 1-6
- In addition to graphing as in Exercises 1-6, you may want to review slope-intercept form. These are two skills essential for students’ success in this chapter.

Tips for New Teachers

- Make this Foldable to record information about solving systems of equations and inequalities. Begin with five sheets of grid paper.

  **Step 1**  Fold
  Fold each sheet in half along the width.

  **Step 2**  Cut
  Unfold and cut four rows from left side of each sheet, from the top to the crease.

  **Step 3**  Stack and Staple
  Stack the sheets and staple to form a booklet.

  **Step 4**  Label
  Label each page with a lesson number and title.

Reading and Writing

- As you read and study the chapter, unfold each page and fill the journal with notes, graphs, and examples for systems of equations and inequalities.

Organization of Data: Visualization Journal

- After students make their visualization journal, have them label the top of each front page with a lesson number. Under the tabs of their Foldable, students take notes and define terms presented in each lesson. At the end of each lesson, ask students to design a visual (graph, diagram, picture, chart) that presents the lesson information in a concise, easy-to-study format. Encourage students to clearly label their visuals.

For Prerequisite Skills

- This section provides a review of the basic concepts needed before beginning Chapter 7. Page references are included for additional student help.
- Additional review is provided in the Prerequisite Skills Workbook, pp. 27–28.

**For Lesson**  |  **Prerequisite Skill**
---|---
7-2 | Solving Equations for a Specified Value, p. 374
7-3 | Simplifying Expressions, p. 381
7-4 | Distributive Property, p. 386
7-5 | Graphing Inequalities, p. 392
A Preview of Lesson 7-1

Copying Formulas Once the spreadsheet formulas are entered for one sales amount, those formulas can be dragged to copy for all the other sales amounts. Students must take their time creating the first formulas as any errors will be duplicated to all the other cells.

Teach

• In Excel, you create a graph by using the Chart Wizard button on the standard toolbar. If using another spreadsheet software program, consult the Help feature for instructions on creating graphs (charts).
• Remind students that spreadsheet software cannot interpret an expression such as 0.1x. Students must type the multiplication sign implied.
• Urge students not to assume that the software will apply the order of operations. Students should use parentheses to insure that operations are performed in the correct order.
• You may wish to extend the activity by discussing the pros and cons of different sales commissions. Students who can imagine themselves as salespeople can relate personally to the amounts calculated and to the process of using equations to explore pay rates.

Assess

Exercises 1–2 Students should write the correct expressions to translate the real-world scenario into an algebraic representation.
Exercise 4 Students should be able to translate between the mathematical solution and its real-world meaning.
**5-Minute Check Transparency 7-1** Use as a quiz or review of Chapter 6.

**Mathematical Background** notes are available for this lesson on p. 366C.

**How can you use graphs to compare the sales of two products?**

Ask students:
- The graph that is sloping downward represents sales of which product? cassette singles
- The graph that is sloping upward represents sales of which product? CD singles
- In what year were the sales of cassette and CD singles equal? 1996; Note: The $x$-value of 0 represents the year 1991.
- You are told to assume that the sales are linear functions. What does this mean? It means that sales either increase or decrease by the same amount every year. Also, the graphs of linear functions are straight lines.

**NUMBER OF SOLUTIONS** Two equations, such as $y = 69 - 6.9x$ and $y = 5.7 + 6.3x$, together are called a system of equations. A solution of a system of equations is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have 0, 1, or an infinite number of solutions.

- If the graphs intersect or coincide, the system of equations is said to be **consistent**. That is, it has at least one ordered pair that satisfies both equations.
- If the graphs are parallel, the system of equations is said to be **inconsistent**. There are no ordered pairs that satisfy both equations.
- Consistent equations can be **independent** or **dependent**. If a system has exactly one solution, it is independent. If the system has an infinite number of solutions, it is dependent.

### Concept Summary

<table>
<thead>
<tr>
<th>Graph of a System</th>
<th>Systems of Equations</th>
</tr>
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<tbody>
<tr>
<td>Intersecting Lines</td>
<td>Same Line</td>
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<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Number of Solutions</td>
<td>exactly one solution</td>
</tr>
<tr>
<td>Terminology</td>
<td>consistent and independent</td>
</tr>
</tbody>
</table>

**Cassette and CD Singles Sales**

<table>
<thead>
<tr>
<th>Years Since 1991</th>
<th>Sales (millions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
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<tr>
<td>3</td>
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<td>50</td>
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<td>6</td>
<td>60</td>
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<tr>
<td>7</td>
<td>70</td>
</tr>
</tbody>
</table>

Cassette singles: $y = 69 - 6.9x$

CD singles: $y = 5.7 + 6.3x$

These equations are graphed at the right.

The point at which the two graphs intersect represents the time when the sales of cassette singles equaled the sales of CD singles. The ordered pair of this point is a solution of both equations.
Building on Prior Knowledge
In Chapter 4, students learned to graph linear equations. In this lesson, they should recognize that graphing systems of equations simply involves graphing more than one linear equation on the same coordinate grid.

**NUMBER OF SOLUTIONS**

**In-Class Example**

**Reading Tip** Tell students to pay close attention to the labels on the graph. If any lines are labeled with more than one equation, then the two equations will have infinitely many solutions.

1. Use the graph to determine whether each system has no solution, one solution, or infinitely many solutions.

   **Study Tip**
   Look Back
   To review graphing linear equations, see Lesson 4-5.

   **Example 1 Number of Solutions**

   Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

   a. \(y = -x + 5\)
      \(y = x - 3\)
      Since the graphs of \(y = -x + 5\) and \(y = x - 3\) intersect, there is one solution.

   b. \(y = -x + 5\)
      \(2x + 2y = -8\)
      Since the graphs of \(y = -x + 5\) and \(2x + 2y = -8\) are parallel, there are no solutions.

   c. \(2x + 2y = -8\)
      \(y = -x - 4\)
      Since the graphs of \(2x + 2y = -8\) and \(y = -x - 4\) coincide, there are infinitely many solutions.

   **SOLVE BY GRAPHING** One method of solving systems of equations is to carefully graph the equations on the same coordinate plane.

   **Example 2 Solve a System of Equations**

   Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

   a. \(y = -x + 8\)
      \(y = 4x - 7\)
      The graphs appear to intersect at the point with coordinates (3, 5). Check this estimate by replacing \(x\) with 3 and \(y\) with 5 in each equation.

      **CHECK**
      \[y = -x + 8\]
      \[5 \triangleq -3 + 8\]
      \[5 = 5\]
      \[5 \triangleq 4(3) - 7\]
      \[5 \triangleq 12 - 7\]
      \[5 = 5\]

      The solution is (3, 5).

   b. \(x + 2y = 5\)
      \(2x + 4y = 2\)
      The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Notice that the lines have the same slope but different \(y\)-intercepts. Recall that a system of equations that has no solution is said to be inconsistent.

   **Differentiated Instruction**

   **Logical** Before students graph a system of equations, have them write the two equations in slope-intercept form and compare slopes and intercepts. Different slopes mean the lines intersect and there is one solution. Same slope, same intercept means they are the same line and there are infinite solutions. Same slope, different intercepts indicates parallel lines and no solution.
**Example 3** Write and Solve a System of Equations

**WORLD RECORDS** Use the information on Guy Delage’s swim at the left. If Guy can swim 3 miles per hour for an extended period and the raft drifts about 1 mile per hour, how many hours did he spend swimming each day?

**Words** You have information about the amount of time spent swimming and floating. You also know the rates and the total distance traveled.

**Variables** Let $s =$ the number of hours Guy swam, and let $f =$ the number of hours he floated each day. Write a system of equations to represent the situation.

**Equations**

\[
\begin{align*}
\text{The number of hours swimming} & \quad \text{plus} \quad \text{the number of hours floating} \quad \text{equals} \quad \text{the total number of hours in a day.} \\
3s + f & = 24 \\
& \quad \text{The daily miles traveled swimming} \quad \text{plus} \quad \text{the daily miles traveled floating} \quad \text{equals} \quad \text{the total miles traveled in a day.} \\
\quad f & = 44
\end{align*}
\]

Graph the equations $s + f = 24$ and $3s + f = 44$.

The graphs appear to intersect at the point with coordinates (10, 14). Check this estimate by replacing $s$ with 10 and $f$ with 14 in each equation.

**CHECK**

\[
\begin{align*}
10 + 14 & \leq 24 \\
3(10) + 14 & \leq 44 \\
24 & = 24 \checkmark \\
30 + 14 & \leq 44 \\
44 & = 44 \checkmark
\end{align*}
\]

Guy Delage spent about 10 hours swimming each day.

---

**Check for Understanding**

**Concept Check**

1. **OPEN ENDED** Draw the graph of a system of equations that has one solution at $(-2, 3)$.
2. **Determine** whether a system of equations with $(0, 0)$ and $(2, 2)$ as solutions sometimes, always, or never has other solutions. Explain.
3. **Find a counterexample** for the following statement.

   If the graphs of two linear equations have the same slope, then the system of equations has no solution.

**Guided Practice**

Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

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<td></td>
<td>14</td>
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</tbody>
</table>

4. $y = x - 4$ 
   $y = \frac{1}{3}x + 2$ **no solution**

5. $y = \frac{1}{3}x + 2$ **no solution**
   $y = \frac{1}{3}x - 2$

6. $x - y = 4$ 
   $y = x - 4$ **infinitely many**
   $y = -\frac{1}{3}x + 4$ **one solution**

**Lesson 7-1 Graphing Systems of Equations 371**
Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it. 8–13. See pp. 405A–405D for graphs.

8. \( y = -x \)  
   \( y = 2x + 1 \) one; \((0, 0)\)  
9. \( x + y = 8 \)  
   \( x - y = 2 \) one; \((5, 3)\)  
10. \( 2x + 4y = 2 \) infinitely many  
   \( 3x + 6y = 3 \)  
11. \( x + y = 4 \)  
   \( x + y = 1 \) no solution  
12. \( x - y = 2 \)  
   \( 3y + 2x = 9 \) one; \((3, 1)\)  
13. \( x + y = 2 \)  
   \( y = 4x + 7 \) one; \((-1, 3)\)

Application

14. RESTAURANTS The Rodriguez family and the Wong family went to a brunch buffet. The restaurant charges one price for adults and another price for children. The Rodriguez family has two adults and three children, and their bill was $40.50. The Wong family has three adults and one child, and their bill was $38.00. Determine the price of the buffet for an adult and the price for a child. $10.50; $6.50

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it. 23–40. See pp. 405A–405D for graphs.

23. \( y = -6 \)  
   \( 4x + y = 2 \) one; \((2, -6)\)  
24. \( x = 2 \)  
25. \( y = \frac{1}{2}x \)  
   \( 2x + y = 10 \) one; \((4, 2)\)  
26. \( y = -x \)  
   \( 3x - y = 8 \) one; \((2, -2)\)  
27. \( y = 3x - 4 \)  
   \( y = -3x - 4 \) one; \((0, -4)\)  
28. \( y = 2x + 6 \)  
   \( y = -x - 3 \) one; \((-3, 0)\)  
29. \( x - 2y = 2 \)  
   \( 3x + y = 6 \) one; \((2, 0)\)  
30. \( x + y = 2 \)  
31. \( 3x + 2y = 12 \)  
32. \( 2x + 3y = 4 \) infinitely many \(-4x - 6y = -8\)  
33. \( 2x + y = -4 \) one; \((-2, 4)\)  
   \( 3x + 2y = 6 \) no solution  
34. \( 4x + 3y = 24 \) one; \((3, 4)\)  
35. \( 3x + y = 3 \) infinitely many \(2y - 6x + 6\)  
36. \( y = x + 3 \) one; \((-1, 2)\)  
37. \( 2x + 3y = -17 \)  
   \( y = x - 4 \) one; \((-1, -5)\)  
38. \( y = \frac{2}{3}x - 5 \) infinitely many \(3y = 2x\)  
39. \( 6 - \frac{3}{2}y = x \) infinitely many \(\frac{1}{2}x + \frac{1}{3}y = 6\)  
40. \( \frac{1}{2}x + \frac{1}{3}y = 6 \) one; \((8, 6)\)

41. GEOMETRY The length of the rectangle at the right is 1 meter less than twice its width. What are the dimensions of the rectangle? 13 m by 7 m

Perimeter = 40 m
### GEOMETRY
For Exercises 42 and 43, use the graphs of $y = 2x + 6$, $3x + 2y = 19$, and $y = 2$, which contain the sides of a triangle.

42. Find the coordinates of the vertices of the triangle. $(-2, 2), (5, 2), (1, 8)$

43. Find the area of the triangle. 21 units²

### BALLOONING
For Exercises 44 and 45, use the information in the graphic at the right.

44. In how many minutes will the balloons be at the same height? 4 min

45. How high will the balloons be at that time? 70 m

### SAVINGS
For Exercises 46 and 47, use the following information.

Monica and Michael Gordon both want to buy a scooter. Monica has already saved $250 and plans to save $5 per week until she can buy the scooter. Michael has $16 and plans to save $8 per week.

46. In how many weeks will Monica and Michael have saved the same amount of money? 3 weeks

47. How much will each person have saved at that time? $40

### BUSINESS
For Exercises 48–50, use the graph at the right.

48. Which company had the greater profit during the ten years? **Widget Company**

49. Which company had a greater rate of growth? **neither**

50. If the profit patterns continue, will the profits of the two companies ever be equal? Explain. No; the graphs are parallel so the lines will never meet and there is no year when the profits will be equal.

### POPULATION
For Exercises 51–54, use the following information.

The U.S. Census Bureau divides the country into four sections. They are the Northeast, the Midwest, the South, and the West.

51. $p = 60 + 0.4t$ ★

51. In 1990, the population of the Midwest was about 60 million. During the 1990s, the population of this area increased an average of about 0.4 million per year. Write an equation to represent the population of the Midwest for the years since 1990.

52. The population of the West was about 53 million in 1990. The population of this area increased an average of about 1 million per year during the 1990s. Write an equation to represent the population of the West for the years since 1990.

53. ★

54. ★

55. ★

### CRITICAL THINKING
The solution of the system of equations $Ax + y = 5$ and $Ax + By = 20$ is $(2, −3$). What are the values of $A$ and $B$? **$A = 4, B = −4$**

### Enrichment, p. 408

**Graphing a Trip**

The distance formula, $d = rt$, is used to solve many types of problems. In this problem, the distance is the same as the length of the line segment from one vertex to another. This represents the distance traveled in miles per hour. The 30-foot length is the length of the trip.

Solve each problem.

1. Estimate the answer to the problem in the above example about how fast you are driving.

2. About 60 miles per hour

**Reading the Lesson**

**Reading the Lesson**

1. Each figure shows the graph of a system of two equations. Write the letters of the figures that contain the sides of a triangle.

   a. A system of two linear equations can have an infinite number of solutions. ★

   b. A system of equations in standard form is if there is at least one ordered pair that satisfies both equations. ★

   c. If the system of equations has exactly one solution, it is independent. ★

   d. If a system of equations has an infinite number of solutions, it is dependent. ★

**Helping You Remember**

2. Describe here how you can solve a system of equations by graphing.

   Sample answer: Graph the equations on the same coordinate plane. Locate any points of intersection.
Open-Ended Assessment

**Modeling** Model a line on a coordinate plane with string, spaghetti, or a similar item. Then ask volunteers to come up and model another line that would represent a system of equations with one solution. Do the same for systems of equations with no solutions and with infinitely many solutions.

Getting Ready for Lesson 7-2

**PREREQUISITE SKILL** Students will learn to solve systems of equations by substitution in Lesson 7-2. The process of substitution involves solving equations for a specific variable. Use Exercises 65–68 to determine your students’ familiarity with solving equations for a specific variable.

Answer

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you use graphs to compare the sales of two products?
Include the following in your answer:
• an estimate of the year in which the sales of cassette singles equaled the sales of CD singles, and
• an explanation of why graphing works.

57. Which graph represents a system of equations with no solution? **B**

58. How many solutions exist for the system of equations below? **B**

\[4x + y = 7\]
\[3x - y = 0\]

- **A** no solution
- **B** one solution
- **C** infinitely many solutions
- **D** cannot be determined

Maintain Your Skills

**Mixed Review** Determine which ordered pairs are part of the solution set for each inequality. (Lesson 6-6)

59. \[y \leq 2x, \{(1, 4), (-1, 5), (5, -6), (-7, 0)\} \{(5, -6)\}\]
60. \[y < 8 - 3x, \{(-4, 2), (-3, 0), (1, 4), (1, 8)\} \{(-4, 2), (-3, 0), (1, 4)\}\]

61. **MANUFACTURING** The inspector at a perfume manufacturer accepts a bottle if it is less than 0.05 ounce above or below 2 ounces. What are the acceptable numbers of ounces for a perfume bottle? (Lesson 6-5) \([a \mid 1.95 < n < 2.05]\)

Write each equation in standard form. (Lesson 5-5)

62. \[y - 1 = 4(x - 5)\]
63. \[y + 2 = \frac{1}{3}(x + 3)\]
64. \[y - 4 = -6(x + 2)\]

65. \[4x - y = 19\]
66. \[x - 3y = 3\]
67. \[6x + y = -8\]

**PREREQUISITE SKILL** Solve each equation for the variable specified. (To review solving equations for a specified variable, see Lesson 3-8.)

65. \[12x - y = 10x, \text{ for } y = 2x\]
66. \[6a + b = 2a, \text{ for } a = \frac{1}{4}b\]

67. \[\frac{7m - n}{q} = 10, \text{ for } q = \frac{7m - n}{10}\]
68. \[\frac{5z - s}{2} = 6, \text{ for } z = \frac{12 + s}{5t}\]
Systems of Equations

You can use a TI-83 Plus graphing calculator to solve a system of equations.

**Example**

Solve the system of equations. State the decimal solution to the nearest hundredth.

\[2.93x + y = 6.08\]
\[8.32x - y = 4.11\]

**Step 1** Solve each equation for \(y\) to enter them into the calculator.

\[
\begin{align*}
2.93x + y &= 6.08 \\
2.93x + y &= 2.93x - 2.93x \\
y &= 6.08 - 2.93x \\
8.32x - y &= 4.11 \\
8.32x - y &= 8.32x - 8.32x \\
y &= 4.11 - 8.32x \\
\end{align*}
\]

**Step 2** Enter these equations in the \(Y=\) list and graph.

**Step 3** Use the \(\text{CALC}\) menu to find the point of intersection.

**Exercises**

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. \[y = 3x - 4\]
   \[y = -0.5x + 6\] \((2.86, 4.57)\)
2. \[y = 2x + 5\]
   \[y = -0.2x - 4\] \((-4.09, -3.18)\)
3. \[x + y = 5.35\]
   \[3x - y = 3.75\] \((2.28, 3.08)\)
4. \[0.35x - y = 1.12\]
   \[2.25x + y = -4.05\] \((-1.13, -1.51)\)
5. \[1.5x + y = 6.7\]
   \[5.2x - y = 4.1\] \((1.61, 4.28)\)
6. \[5.4x - y = 1.8\]
   \[6.2x + y = -3.8\] \((-0.17, -2.73)\)
7. \[5x - 4y = 26\]
   \[4x + 2y = 53.3\] \((10.2, 6.25)\)
8. \[2x + 3y = 11\]
   \[4x + y = -6\] \((-2.9, 5.6)\)
9. \[0.22x + 0.15y = 0.30\]
   \[-0.33x + y = 6.22\] \((-2.35, 5.44)\)
10. \[125x - 200y = 800\]
    \[65x - 20y = 140\] \((1.14, -3.29)\)

**Assess**

Ask students how they can verify that their solution is correct. They should respond that substitution of values into the original equations will confirm solutions.
Focus

5-Minute Check Transparency 7-2 Use as a quiz or review of Lesson 7-1.

Mathematical Background notes are available for this lesson on p. 366C.

How can a system of equations be used to predict media use?

Ask students:

• Which is changing at a greater rate: the number of newspaper readers, or the number of people online? The number of people online is changing at a greater rate.

• According to the graph, in about what year will the number of hours online per person equal time spent reading newspapers? In about 2003.

• If the two equations weren’t labeled, how would you know which was which? The one with negative slope represents newspaper readers because the number of hours spent online is declining.

Vocabulary

• substitution

How can a system of equations be used to predict media use?

Americans spend more time online than they spend reading daily newspapers. If x represents the number of years since 1993 and y represents the average number of hours per person per year, the following system represents the situation.

Equation for daily newspapers: \( y = -2.8x + 170 \)
Equation for online: \( y = 14.4x + 2 \)

The solution of the system represents the year that the number of hours spent on each activity will be the same. To solve this system, you could graph the equations and find the point of intersection. However, the exact coordinates of the point would be very difficult to determine from the graph. You could find a more accurate solution by using algebraic methods.

**SUBSTITUTION** The exact solution of a system of equations can be found by using algebraic methods. One such method is called substitution.

**Algebra Activity**

Using Substitution

Use algebra tiles and an equation mat to solve the system of equations.

\( 3x + y = 8 \) and \( y = x - 4 \)

**Model and Analyze**

Since \( y = x - 4 \), use 1 positive x tile and 4 negative 1 tiles to represent y. Use algebra tiles to represent \( 3x + y = 8 \).

1. Use what you know about equation mats to solve for x. What is the value of x? 3
2. Use the \( y = x - 4 \) to solve for y. -1
3. What is the solution of the system of equations? (3, -1)

**Make a Conjecture**

4. Explain how to solve the following system of equations using algebra tiles.

\( 4x + 3y = 10 \) and \( y = x + 1 \)

5. Why do you think this method is called substitution?
**Example 1** Solve Using Substitution

Use substitution to solve the system of equations.

\[ \begin{align*}
y &= 3x \\
x + 2y &= -21\end{align*} \]

Since \( y = 3x \), substitute \( 3x \) for \( y \) in the second equation.

\[ \begin{align*}
x + 2(3x) &= -21 \\
x + 6x &= -21 \\
7x &= -21 \\
x &= -3\end{align*} \]

Use \( y = 3x \) to find the value of \( y \).

\[ \begin{align*}
y &= 3(-3) \\
y &= -9\end{align*} \]

The solution is \((-3, -9)\).

**Example 2** Solve for One Variable, Then Substitute

Use substitution to solve the system of equations.

\[ \begin{align*}
x + 5y &= -3 \\
3x - 2y &= 8\end{align*} \]

Solve the first equation for \( x \) since the coefficient of \( x \) is 1.

\[ \begin{align*}
x + 5y &= -3 \\
x &= -3 - 5y\end{align*} \]

Find the value of \( y \) by substituting \(-3 - 5y\) for \( x \) in the second equation.

\[ \begin{align*}
3x - 2y &= 8 \\
3(-3 - 5y) - 2y &= 8 \\
-9 - 15y - 2y &= 8 \\
-9 - 17y &= 8 \]

Add 9 to each side.

\[ \begin{align*}
-17y &= 17 \\
-17 &= 17 \\
y &= -1\end{align*} \]

Substitute \(-1\) for \( y \) in either equation to find the value of \( x \). Choose the equation that is easier to solve.

\[ \begin{align*}
x + 5y &= -3 \\
x + 5(-1) &= -3 \\
x - 5 &= -3 \]

Add 5 to each side.

The solution is \((2, -1)\). The graph verifies the solution.

**Teaching Tip** Explain that the purpose of the substitution method is that once you find one of the values (either \( x \) or \( y \)), you can then substitute it into either of the original equations to find the other value. In Example 1, the value of \( y \) is given in terms of \( x \) because \( y = 3x \). So, substitute \( 3x \) for \( y \) in the second equation and solve.

1. Use substitution to solve the system of equations.

\[ x = 4y \]

\[ 4x - y = 75 \]

\((20, 5)\)

**Teaching Tip** You must solve for one variable first because neither equation gives one variable in terms of the other as in Example 1. The easiest choice in Example 2 is to solve the first equation for \( x \) by subtracting \( 5y \) from both sides.

2. Use substitution to solve the system of equations.

\[ 4x + y = 12 \]

\[ -2x - 3y = 14 \]

\((5, -8)\)

**Algebra Activity**

**Materials:** algebra tiles, equation mat

- Remind students that anything they add to one side of the equation mat must also be added to the other side of the mat.
- Remind students that the purpose is to eliminate zero pairs.
- Once a value is found for \( x \), have students use that value to find \( y \).
3 Use substitution to solve the system of equations.
\[2x + 2y = 8\]
\[x + y = -2\]
no solution

4 GOLD Gold is alloyed with different metals to make it hard enough to be used in jewelry. The amount of gold present in a gold alloy is measured in 24ths called karats. 24-karat gold is \(\frac{24}{24}\) or 100% gold. Similarly, 18-karat gold is \(\frac{18}{24}\) or 75% gold. How many ounces of 18-karat gold should be added to an amount of 12-karat gold to make 4 ounces of 14-karat gold?
\[1\frac{1}{3}\] ounces of 18-karat gold and \[2\frac{2}{3}\] ounces of 12-karat gold

Example 3 Dependent System
Use substitution to solve the system of equations.
\[6x - 2y = -4\]
\[y = 3x + 2\]
Since \(y = 3x + 2\), substitute \(3x + 2\) for \(y\) in the first equation.
\[6x - 2y = -4\] First equation
\[6x - 2(3x + 2) = -4\] Distributive Property
\[6x - 6x - 4 = -4\] Combine like terms
\[-4 = -4\] Simplify.
The statement \(-4 = -4\) is true. This means that there are infinitely many solutions of the system of equations. This is true because the slope-intercept form of both equations is \(y = 3x + 2\). That is, the equations are equivalent, and they have the same graph.

In general, if you solve a system of linear equations and the result is a true statement (an identity such as \(-4 = -4\)), the system has an infinite number of solutions. However, if the result is a false statement (for example, \(-4 = 5\)), the system has no solution.

REAL-WORLD PROBLEMS Sometimes it is helpful to organize data before solving a problem. Some ways to organize data are to use tables, charts, different types of graphs, or diagrams.

Example 4 Write and Solve a System of Equations
METAL ALLOYS A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?
Let \(a\) = the number of grams of the 25% copper alloy and \(b\) = the number of grams of the 50% copper alloy. Use a table to organize the information.

<table>
<thead>
<tr>
<th>Grams of Copper</th>
<th>25% Copper</th>
<th>50% Copper</th>
<th>45% Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Grams</td>
<td>(a)</td>
<td>(b)</td>
<td>1000</td>
</tr>
<tr>
<td>25% Copper</td>
<td>0.25(a)</td>
<td>0.50(b)</td>
<td>0.45(1000)</td>
</tr>
</tbody>
</table>

The system of equations is \(a + b = 1000\) and \(0.25a + 0.50b = 0.45(1000)\). Use substitution to solve this system.

\[a + b = 1000\] First equation
\[a + b - b = 1000 - b\] Subtract \(b\) from each side.
\[a = 1000 - b\] Simplify.
\[0.25a + 0.50b = 0.45(1000)\] Second equation
\[0.25(1000 - b) + 0.50b = 0.45(1000)\] Distributive Property
\[250 - 0.25b + 0.50b = 450\] Combine like terms.
\[250 + 0.25b = 450\] Simplify.
\[250 + 0.25b - 250 = 450 - 250\] Subtract 250 from each side.
\[0.25b = 200\] Simplify.
\[0.25b = 200\] Divide each side by 0.25.
\[b = 800\] Simplify.
Lesson 7-2 Substitution

**Guided Practice**

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.

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</table>

1. \(y = 5x\) \(2x + 3y = 34\) (2, 10)
2. \(y = 2x + 3\) \(y = 4x - 1\) (2, 7)
3. \(8x + 2y = 13\) \(4x + y = 11\) no solution (13, 30)
4. \(2x + 3y = 1\) \(-3x + y = 15\) \((-4, 3)\)
5. \(3x - 2y = 12\) \(x + 2y = 6\) \((4, 2)\)
6. \(0.5x - 2y = 17\) \(2x + y = 104\) \((50, 4)\)

**Application**

10. TRANSPORTATION The Thrust SSC is the world’s fastest land vehicle. Suppose the driver of a car whose top speed is 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car? about 135.5 mi

**Alert!** Exercise 38 requires the Internet or other research materials.

**Odd/Even Assignments**

Exercises 11–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**

*Basic:* 11–33 odd, 34, 35, 39–53
*Average:* 11–33 odd, 34, 35, 37, 39–53
*Advanced:* 12–32 even, 36–49 (optional: 50–53)
*All:* Practice Quiz 1 (1–5)

**Check for Understanding**

1. Explain why you might choose to use substitution rather than graphing to solve a system of equations. Substitution may result in a more accurate solution.
2. Describe the graphs of two equations if the solution of the system of equations yields the equation \(4 = 2\). They are parallel lines.
3. OPEN-ENDED Write a system of equations that has infinitely many solutions. Sample answer: \(y = x + 3\), \(2y = 2x + 6\)

**Concept Check**

1. Explain why you might choose to use substitution rather than graphing to solve a system of equations. Substitution may result in a more accurate solution.
2. Describe the graphs of two equations if the solution of the system of equations yields the equation \(4 = 2\). They are parallel lines.
3. OPEN-ENDED Write a system of equations that has infinitely many solutions. Sample answer: \(y = x + 3\), \(2y = 2x + 6\)

**Guided Practice Key**

- Exercises 4–9
- Examples 1–3

**Application**

10. TRANSPORTATION The Thrust SSC is the world’s fastest land vehicle. Suppose the driver of a car whose top speed is 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car? about 135.5 mi

**About the Exercises**

- Organization by Objective
  - Substitution: 11–28
  - Real-World Problems: 29–37

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- Advanced: 12–32 even, 36–49 (optional: 50–53)
- All: Practice Quiz 1 (1–5)
Solving Systems of Linear Equations and Inequalities

380 Chapter 7

Reading the Lesson

One method of solving systems of equations is substitution.

Substitution

The solution of a system of two equations in two variables is the point where the two graphs intersect.

Substituting for a variable in the second equation

Substitute for the variable in the second equation.

\[ \begin{align*}
4x + 3y &= 23 \\
2x - y &= 8
\end{align*} \]

Substitute the expression for \( x \) in the second equation.

\[ 2(2x - y) - y = 8 \]

\[ 4x + 3(2 - 2x) = 23 \]

\[ 4x + 6 - 6x = 23 \]

\[ -2x = 17 \]

\[ x = -\frac{17}{2} \]

Solve the second equation for \( y \).

\[ y = 2x - 8 \]

Substitute the expression for \( x \) in the second equation.

\[ y = 2(-\frac{17}{2}) - 8 \]

\[ y = -17 - 8 \]

\[ y = -25 \]

The solution is \( (-\frac{17}{2}, -25) \).

Solving Systems of Linear Equations and Inequalities

Example 1

MX Labs needs to make 500 gallons of a 34% acid solution. The equation for the line where the plane intersects the x-axis is \( y = 0 \). If the group has 34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy? 4 lb of sunflower seeds, 12 lb of raisins.

Reading to Learn

Solve each problem.

1. The Future Teachers of America Club at Paint Branch High School is making a healthy trail mix to sell to students during lunch. The mix will have three times the number of pounds of raisins as sunflower seeds. Sunflower seeds cost $4.00 per pound, and raisins cost $1.50 per pound. If the group has $34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy?

2. All of his work was correct. Describe the graph of the system. Explain.

3. At the end of the 2002 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win?

4. All of her work was correct. Describe the graph of the system. Explain.

5. A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, when would you expect the trees to be the same height?

More About...

Tourism

Every year, millions of tourists visit the inca ruins of Machu Picchu, the spectacular ruins of the Lost City of the Incas. Source: www.about.com

380 Chapter 7 Solving Systems of Linear Equations and Inequalities

Enrichment, p. 414

Equations of Lines and Planes in Intercept Form

One form that a linear equation may take is intercept form. The constants \( a \) and \( b \) are the \( x \)- and \( y \)-intercepts of the graph.

For the plane in Exercise 1, write an equation for the line where the plane intersects the y-axis. Use intercept form.

Graph the equation \( \frac{x}{a} + \frac{y}{b} = 1 \).

In three-dimensional space, the equation of a plane takes a similar form:

\[ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \]

Here, the constants \( a, b, \) and \( c \) are the points where the plane intersects the \( x, y, \) and \( z \)-axis, respectively.

Solve each problem.

1. Graph the equation \( \frac{x}{a} + \frac{y}{b} = 1 \).

2. For the plane in Exercise 1, write an equation for the line where the plane intersects the y-axis. Use intercept form.
39. **CRITICAL THINKING**  Solve the system of equations. Write the solution as an ordered triple of the form \((x, y, z)\).

\[
\begin{align*}
2x + 3y - z &= 17 \\
y &= -3z - 7 \\
x - z &= 2 \quad (-1, 5, -4)
\end{align*}
\]

40. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson. **See margin.**

How can a system of equations be used to predict media use?

Include the following in your answer:
- an explanation of solving a system of equations by using substitution, and
- the way in which the slope of the line is related to the type of system of equations.

41. When solving the following system, which expression could be substituted for \(x\)?

\[
\begin{align*}
x + 4y &= 1 \\
x - 3y &= -9 \quad \text{B}
\end{align*}
\]

A) \(4y - 1\)  B) \(1 - 4y\)  C) \(3y - 9\)  D) \(-9 - 3y\)

42. If \(x - 3y = -9\) and \(5x - 2y = 7\), what is the value of \(x?\)  C

A) \(1\)  B) \(2\)  C) \(3\)  D) \(4\)

**Maintain Your Skills**

**Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.**  **(Lesson 7-1)**

43. \(x + y = 3\)  44. \(x + 2y = 1\)  45. \(2x + y = 3\) \(\text{infinitely many}\)

\[
\begin{align*}
x + y &= 4 \quad \text{no solution} \\
2x + y &= 5 \quad \text{one; (3, -1)} \\
4x + 2y &= 6 \quad \text{many}
\end{align*}
\]

**Graph each inequality.**  **(Lesson 6-6)**

46. \(y < -5\)  47. \(x \geq 4\)  48. \(2x + y > 6\)

49. **RECYCLING**  When a pair of blue jeans is made, the leftover denim scraps can be recycled. One pound of denim is left after making every five pairs of jeans. How many pounds of denim would be left if 250 pairs of jeans are made? **(Lesson 3-6)**  \(50\) lb

**Open-Ended Assessment**

**Speaking**  Write a system of equations on the chalkboard. Have students explain how they would check this system of equations to find out whether it has no solution, one solution, or infinitely many solutions. Then use student suggestions to solve the system.

**Getting Ready for Lesson 7-3**

**PREREQUISITE SKILL**  Students will learn to solve systems of equations by elimination using addition and subtraction in Lesson 7-3. This method includes simplifying expressions. Use Exercises 50–53 to determine your students’ familiarity with simplifying expressions.

**Assessment Options**

**Practice Quiz 1**  The quiz provides students with a brief review of the concepts and skills in Lessons 7-1 and 7-2. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

**Quiz (Lessons 7-1 and 7-2)** is available on p. 447 of the *Chapter 7 Resource Masters*.

**Answer**

40. When problems about technology involve a system of equations, the problem can be solved by substitution. Answers should include the following.

- To solve a system of equations using substitution, solve one equation for one unknown. Substitute this value for the unknown in the other equation and solve the equation. Use this number to find the other unknown.

- The number of hours will be the same about 9.8 years after 1993. That represents the year 2002.
Focus

5-Minute Check Transparency 7-3 Use as a quiz or review of Lesson 7-2.

Mathematical Background notes are available for this lesson on p. 366D.

How can you use a system of equations to solve problems about weather?

Ask students:

• If \(2n = 36\), then how many hours of darkness are there in Seward, AK, on the winter solstice? 18 hours
• Since you know the value of \(n\) is 18, how can you find the value of \(d\)? Use substitution.
• How many hours of daylight are there in Seward, AK, on the winter solstice? 6

Geography The Arctic Circle is at latitude 66.5° North and passes through northern Alaska. Because of the Earth’s tilt on its axis, the sun does not rise at the Arctic Circle on the winter solstice. Use this fact to write a system of equations for the number of hours of daylight at the Arctic Circle on the winter solstice. Use the problem in the text as an example. \(n + d = 24\) \(n - d = 24\)

Vocabulary

- elimination

How can you use a system of equations to solve problems about weather?

On the winter solstice, there are fewer hours of daylight in the Northern Hemisphere than on any other day. On that day in Seward, Alaska, the difference between the number of hours of darkness \(n\) and the number of hours of daylight \(d\) is 12. The following system of equations represents the situation.

\[n + d = 24\]
\[n - d = 12\]

Notice that if you add these equations, the variable \(d\) is eliminated.

\[
\frac{n + d}{2} = \frac{24}{2} \\
\frac{n - d}{2} = \frac{12}{2}
\]

ELIMINATION USING ADDITION Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called elimination.

Example 1 Elimination Using Addition

Use elimination to solve each system of equations.

\[3x - 5y = -16\]
\[2x + 5y = 31\]

Since the coefficients of the \(y\) terms, \(-5\) and \(5\), are additive inverses, you can eliminate the \(y\) terms by adding the equations.

\[
\begin{align*}
3x - 5y &= -16 \\
\text{}(+) 2x + 5y &= 31
\end{align*}
\]

\[
\begin{align*}
5x &= 15 \\
\frac{5x}{5} &= \frac{15}{5} \\
x &= 3
\end{align*}
\]

Divide each side by 5.

Now substitute 3 for \(x\) in either equation to find the value of \(y\).

\[
\begin{align*}
3x - 5y &= -16 \\
3(3) - 5y &= -16 & \text{First equation} \\
9 - 5y &= -16 & \text{Replace } x \text{ with } 3. \\
9 - 5y &= -16 & \text{Simplify.} \\
9 - 5y - 9 &= -16 - 9 & \text{Subtract } 9 \text{ from each side.} \\
-5y &= -25 & \text{Simplify.} \\
\frac{-5y}{-5} &= \frac{-25}{-5} & \text{Divide each side by } -5. \\
y &= 5 & \text{Simplify.}
\end{align*}
\]

The solution is \((3, 5)\).
Study Tip

Lesson 3-1.

Translate verbal sentences into equations, see Lesson 3-1.

Example 2  Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let x represent the first number and y represent the second number.

Twice one number added to another number is 18.

\[
2x + y = 18
\]

Four times the first number minus the other number is 12.

\[
4x - y = 12
\]

Use elimination to solve the system.

\[
2x + y = 18 \quad \text{Write the equations in column form and add.}
\]

\[
4x - y = 12 \quad \text{Notice that the variable } y \text{ is eliminated.}
\]

\[
6x = 30 \quad \text{Divide each side by 6.}
\]

\[
x = 5
\]

Now substitute 5 for x in either equation to find the value of y.

\[
4x - y = 12 \quad \text{Second equation}
\]

\[
4(5) - y = 12 \quad \text{Replace } x \text{ with 5.}
\]

\[
20 - y = 12 \quad \text{Simplify.}
\]

\[
20 - y - 20 = 12 - 20 \quad \text{Subtract 20 from each side.}
\]

\[
-y = -8 \quad \text{Simplify.}
\]

\[
y = 8 \quad \text{The numbers are 5 and 8.}
\]

ELIMINATION USING SUBTRACTION

Sometimes subtracting one equation from another will eliminate one variable.

Example 3  Elimination Using Subtraction

Use elimination to solve the system of equations.

\[
5s + 2t = 6
\]

\[
9s + 2t = 22
\]

Since the coefficients of the t terms, 2 and 2, are the same, you can eliminate the t terms by subtracting the equations.

\[
5s + 2t = 6 \quad \text{Write the equations in column form and subtract.}
\]

\[
(-) 9s + 2t = 22
\]

\[
-4s = -16 \quad \text{Notice that the variable } t \text{ is eliminated.}
\]

\[
-4s = -16 \quad \text{Divide each side by } -4.
\]

\[
s = 4 \quad \text{Simplify.}
\]

Now substitute 4 for s in either equation to find the value of t.

\[
5s + 2t = 6 \quad \text{First equation}
\]

\[
5(4) + 2t = 6 \quad s = 4
\]

\[
20 + 2t = 6 \quad \text{Simplify.}
\]

\[
20 + 2t - 20 = 6 - 20 \quad \text{Subtract 20 from each side.}
\]

\[
2t = -14 \quad \text{Simplify.}
\]

\[
\frac{2t}{2} = \frac{-14}{2} \quad \text{Divide each side by } 2.
\]

\[
t = -7 \quad \text{The solution is } (4, -7).
\]

www.algebra1.com/extra_examples

Lesson 7-3  Elimination Using Addition and Subtraction 383
Solving Systems of Linear Equations and Inequalities

Example 4 Elimination Using Subtraction

Multiple-Choice Test Item

If \( x - 3y = 7 \) and \( x + 2y = 2 \), what is the value of \( x \)?

- \( A \) 4
- \( B \) -1
- \( C \) (-1, 4)
- \( D \) (4, -1)

Read the Test Item
You are given a system of equations, and you are asked to find the value of \( x \).

Solve the Test Item
You can eliminate the \( x \) terms by subtracting one equation from the other.

\[
\begin{align*}
\text{First equation:} & \quad x - 3y = 7 \\
\text{Second equation:} & \quad x + 2y = 2 \\
\text{Subtract:} & \quad (x - 3y) - (x + 2y) = 7 - 2 \\
& \quad -5y = 5
\end{align*}
\]

Notice the \( x \) variable is eliminated.

\[
\begin{align*}
\text{Divide each side by -5:} & \quad \frac{-5y}{-5} = \frac{5}{-5} \\
& \quad y = -1
\end{align*}
\]

Now substitute \(-1\) for \( y \) in either equation to find the value of \( x \).

\[
\begin{align*}
\text{First equation:} & \quad x - 3y = 7 \\
& \quad x - 3(-1) = 7 \\
& \quad x + 3 = 7 \\
& \quad x = 4
\end{align*}
\]

Who is correct? Explain your reasoning.

Standardized Test Practice

Example 4 is an example of a test item in which too much information is given in some of the answer choices. Always carefully compare the answer choices with the question before marking your final answer. By doing so, you can eliminate choices C and D because the question asks for the value of \( x \), and choices C and D are solution sets.

Test-Taking Tip
Always read the question carefully. Ask yourself, “What does the question ask?” Then answer that question.

In-Class Example

In-Class Example

4 MULTIPLE CHOICE TEST ITEM

If \( 8x + y = 16 \) and \( -6x + y = -26 \), what is the value of \( y \)?

- \( A \) (3, -8)
- \( B \) 3
- \( C \) -8
- \( D \) (-8, 3)

DAILY INTERVENTION

FIND THE ERROR
Tell students to look at the original system of equations before they evaluate the students’ work. Do they need to use addition or subtraction to eliminate \( s \)?

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include examples of how to solve a system of equations by using elimination with addition or subtraction.
- include any other item(s) that they find helpful in mastering the skills in this lesson.
Use elimination to solve each system of equations. 9. \(-2 \frac{1}{2}, -2\)
4. \(x - y = 14\) 5. \(2a - 3b = -11\) 6. \(4x + y = -9\) (2,-1)
\(x + y = 20\) (17, 3) \(a + 3b = 8\) (\(-1, 3\)) \(4x + 2y = -10\)
7. \(6x + 2y = -10\) 8. \(2a + 4b = 30\) (6.5, 4.25) 9. \(-4m + 2n = 6\) \(2x + 2y = -10\) (0, -5) \(-2a - 2b = -21.5\) \(-4m + n = 8\)
10. The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers? 6, 18

Use elimination to solve each system of equations. 26. (1.75, 2.5) 27. (15.8, 3.4)
12. \(x + y = -3\) 13. \(s - t = 4\) 14. 3m - 2n = 13
\(x - y = 1\) (\(-1, -2\)) \(s + t = 2\) (3, -1) \(m + 2n = 7\) (5, 1)
15. \(-4x + 2y = 8\) 16. 3a + b = 5
\(4x - 3y = -10\) (\(-1, 2\)) \(2a + b = 10\) (\(-5, 20\)) \(2m - 7n = -14\) (7, 4)
18. \(3r - 5s = -35\) 19. 13a + 5b = -11
\(2r - 5s = -30\) (\(-5, 4\)) \(13a + 11b = 7\) (\(-2, 3\)) \(-3x + 2y = -10\) (2, -2)
21. \(6s + 5t = 1\) 22. 4x - 3y = 12
\(6s - 5t = 11\) (1, -1) \(4x + 3y = 24\) (\(\frac{1}{2}, 2\)) \(3a - 2b = 9\) (2, \(-\frac{1}{2}\))
\(\star\) 24. \(x + 5y = 7\) 25. 8a + b = 1
\(8x + 5y = 9\) (\(\frac{1}{2}, 1\)) \(8a - 3b = 3\) (\(\frac{3}{16}, \frac{1}{2}\)) \(1.44\) \(-3.24\) = -5.58
\(7.2m + 4.5m = 129.06\) 28. \(\frac{3}{5}c - \frac{1}{5}d = 9\)
\(7.2m + 6.7n = 136.54\) \(\frac{7}{5}c + \frac{1}{5}d = 11\) (\(-10, -15\)) \(\frac{5}{6}x - 2y = 18\) (24, 4)

The sum of two numbers is 48, and their difference is 24. What are the numbers? 36, 12
31. Find the two numbers whose sum is 51 and whose difference is 13. 32, 19
33. One number added to twice another number is 18. Four times the first number minus the other number is 12. Find the numbers. 6, 0
34. BUSINESS In 1999, the United States produced about 2 million more motor vehicles than Japan. Together, the two countries produced about 22 million motor vehicles. How many vehicles produced in each country?
U.S.: about 12 million vehicles, Japan: about 10 million vehicles
35. PARKS A youth group and their leaders visited Mammoth Cave. Two adults and 5 students in one van paid $77 for the Grand Avenue Tour of the cave. Two adults and 7 students in a second van paid $95 for the same tour. Find the adult price and the student price of the tour. adult: $16, student: $9

FOOTBALL During the National Football League’s 1999 season, Troy Aikman, the quarterback for the Dallas Cowboys, earned $0.467 million more than Deion Sanders, the Cowboys’ wide receiver. Together they cost the Cowboys $12.867 million. How much did each player make? Aikman: $6.867 million, Sanders: $6.200 million

www.algebra.com/self_check_quiz

Lesson 7-3 Elimination Using Addition and Subtraction 385

Enrichment, p. 420

Rozsa Peter

Rozsa Peter (1867–1970) was a Hungarian mathematician dedicated to teaching others mathematics. Her passion for teaching mathematics led her to challenge her students with non-standard problems that required creative thinking and problem-solving. One of her most famous problems is the so-called “Mathematician’s Dilemma,” which has been described as a way to explore the relationship between numbers and their properties.

By focusing on her students’ ability to think outside the box, Peter encouraged them to develop new mathematical concepts and approaches. Her innovative teaching methods led to the development of a new branch of mathematics known as “Natural Numbers,” which studies the properties of numbers based on their natural order.

Peter’s work in this field has influenced many mathematicians, including her students and colleagues. Her legacy continues to inspire new generations of mathematicians to explore the beauty and complexity of numbers.

Helping You Remember

• Use addition or subtraction to eliminate a variable in a system of equations. For example, if you have the equations:

- 2x + 3y = 10
- 4x - 6y = 16

You can eliminate the y variable by adding the two equations. However, if you use addition to eliminate the variable, you must ensure that the coefficients of the y-term are additive inverses. For instance, you can do:

- 2x + 3y = 10
- 2x - 3y = -14

Adding these equations will eliminate the y variable, resulting in 0x = -4, which gives x = -4. If the coefficients are not additive inverses, you must multiply one or both equations by a constant to make them so. For instance, if you have:

- 2x + 3y = 10
- 4x + 6y = 16

You can eliminate the y variable by subtracting the second equation from the first, resulting in 2x = 6, which gives x = 3. If the coefficients are not additive inverses, you can use substitution. However, if the coefficients of one variable are the same, you can use addition to eliminate the variable.
Using the Distributive Property.

- **37.** Let \( x \) represent the number of years since 2000 and \( y \) represent population in billions. Write an equation to represent the population of China. \( y = 0.0048x + 1.28 \)

- **38.** Write an equation to represent the population of India.

- **39.** Use elimination to find the year when the populations of China and India are predicted to be the same. What is the predicted population at that time? \( 2048; 1.51 \text{ billion} \)

**38.** \( y = 0.0104x + 1.01 \)

**40.** **CRITICAL THINKING** The graphs of \( Ax + By = 15 \) and \( Ax - By = 9 \) intersect at \((2, 1)\). Find \( A \) and \( B \). \( A = 6, B = 3 \)

**41.** **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 405A–405D.

How can you use a system of equations to solve problems about weather? Include the following in your answer:

- an explanation of how to use elimination to solve a system of equations, and

**42.** If \( 2x - 3y = -9 \) and \( 3x - 3y = -12 \), what is the value of \( y \)? \( B \)

\( A \) \(-3\) \hspace{1cm} \( B \) \(1\) \hspace{1cm} \( C \) \((-3, 1)\) \hspace{1cm} \( D \) \((1, -3)\)

**43.** What is the solution of \( 4x + 2y = 8 \) and \( 2x + 2y = 2 \)? \( C \)

\( A \) \((-2, 3)\) \hspace{1cm} \( B \) \((3, 2)\) \hspace{1cm} \( C \) \((3, -2)\) \hspace{1cm} \( D \) \((12, -3)\)

**Standardized Test Practice**

**Mixed Review** Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. \( \text{(Lesson 7.2)} \)

**44.** \( y = 5x \) 
\( x + 2y = 22 \)

\((2, 10)\)

**45.** \( x = 2y + 3 \) 
\( 3x + 4y = -1 \)

\((1, -1)\)

**46.** \( 2y - x = -5 \) 
\( 4y - 3x = -1 \)

\((-9, -7)\)

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it. \( \text{(Lesson 7.3)} \)

**47.** \( x - y = 3 \) \hspace{1cm} \text{one; \((1, -2)\)}

**48.** \( 2x - 3y = 7 \) \hspace{1cm} \text{no solution.}

**49.** \( 4x + y = 12 \) 
\( 3y + 7 = 2x \)

\(x = 3 - \frac{1}{4}y\)

**50.** Write an equation of a line that is parallel to the graph of \( y = \frac{5}{4}x - 3 \) and passes through the origin. \( \text{(Lesson 5.6)} \)

\( y = \frac{5}{4}x \)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Use the Distributive Property to rewrite each expression. \( \text{(To review the Distributive Property, see Lesson 1.5)} \)

**51.** \( 23x + 4y \)

\(6x + 8y\)

\(12a - 30b\)

\(6m - 9n\)

\(-20t + 10s\)

**Teacher to Teacher**

Lou Jane Tynan, Sacred Heart Model School, Louisville, KY

“Many students will forget to distribute the negative sign over the entire equation being subtracted in Example 3. I require my students to change the signs of the equation being subtracted and then add the two equations.”

\[
5s + 2t = 0 \quad \Rightarrow \quad 5s + 2t = 0 \\
(-9s + 2t = 22) \quad \Rightarrow \quad (+) -9s - 2t = -22
\]
Elimination Using Multiplication

What You’ll Learn

• Solve systems of equations by using elimination with multiplication.
• Determine the best method for solving systems of equations.

How can a manager use a system of equations to plan employee time?

The Finneytown Bakery is making peanut butter cookies and loaves of quick bread. The preparation and baking times for each are given in the table below.

For these two items, the management has allotted 800 minutes of employee time and 900 minutes of oven time. If \( c \) represents the number of batches of cookies and \( b \) represents the number of loaves of bread, the following system of equations can be used to determine how many of each to bake.

\[
egin{align*}
20c + 10b &= 800 \\
10c + 30b &= 900
\end{align*}
\]

ELIMINATION USING MULTIPLICATION Neither variable in the system above can be eliminated by simply adding or subtracting the equations. However, you can use the Multiplication Property of Equality so that adding or subtracting eliminates one of the variables.

Example 1 Multiply One Equation to Eliminate

Use elimination to solve the system of equations.

\[
egin{align*}
3x + 4y &= 6 \\
5x + 2y &= -4
\end{align*}
\]

Multiply the second equation by \(-2\) so the coefficients of the \( y \) terms are additive inverses. Then add the equations.

\[
\begin{align*}
3x + 4y &= 6 \\
5x + 2y &= -4 & \text{Multiply by } -2
\end{align*}
\]

Multiply the second equation by \(-2\) so the coefficients of the \( y \) terms are additive inverses. Then add the equations.

\[
\begin{align*}
3x + 4y &= 6 \\
5x + 2y &= -4 & \text{Multiply by } -2
\end{align*}
\]

\[
\begin{align*}
(+)-10x - 4y &= 8 \\
-7x &= 14 & \text{Add the equations.}
\end{align*}
\]

\[
\begin{align*}
\frac{-7x}{-7} &= \frac{14}{-7} \\
x &= -2 & \text{Divide each side by } -7.
\end{align*}
\]

Now substitute \(-2\) for \( x \) in either equation to find the value of \( y \).

\[
\begin{align*}
3x + 4y &= 6 & \text{First equation} \\
3(-2) + 4y &= 6 & \text{Multiply by } -2 \\
-6 + 4y &= 6 & \text{Simplify.} \\
\end{align*}
\]

\[
\begin{align*}
-6 + 4y + 6 &= 6 + 6 & \text{Add 6 to each side.} \\
4y &= 12 & \text{Simplify.} \\
\frac{4y}{4} &= \frac{12}{4} & \text{Divide each side by } 4. \\
y &= 3 & \text{The solution is } (-2, 3).
\end{align*}
\]

Lesson 7-4 Elimination Using Multiplication 387

Workbook and Reproducible Masters

Chapter 7 Resource Masters

• Study Guide and Intervention, pp. 421–422
• Skills Practice, p. 423
• Practice, p. 424
• Reading to Learn Mathematics, p. 425
• Enrichment, p. 426
• Assessment, p. 448

Parent and Student Study Guide Workbook, p. 56
School-to-Career Masters, p. 14

Transparencies

5-Minute Check Transparency 7-4 Use as a quiz or review of Lesson 7-3.

Mathematical Background notes are available for this lesson on p. 366D.

How can a manager use a system of equations to plan employee time?

Ask students:

• Explain what the first equation in the system of equations represents. The first equation represents preparation time because the cookies take 20 minutes to prepare, the bread takes 10 minutes to prepare, and the bakery has allotted 800 minutes of employee time for preparation.

• Explain what the second equation in the system of equations represents. The second equation represents baking time because the cookies take 10 minutes to bake, the bread takes 30 minutes to bake, and the bakery has allotted 900 minutes of oven time for preparation.

Focus

Interactive Chalkboard

Resource Manager
2 Teach

ELIMINATION USING MULTIPLICATION

In-Class Examples

1 Use elimination to solve the system of equations.
   \[ 2x + y = 23 \]
   \[ 3x + 2y = 37 \quad (9, 5) \]

2 Use elimination to solve the system of equations.
   \[ 4x + 3y = 8 \]
   \[ 3x - 5y = -23 \quad (-1, 4) \]

For some systems of equations, it is necessary to multiply each equation by a different number in order to solve the system by elimination. You can choose to eliminate either variable.

**Example 2** Multiply Both Equations to Eliminate

Use elimination to solve the system of equations.
\[ 3x + 4y = -25 \]
\[ 2x - 3y = 6 \]

**Method 1** Eliminate \( x \).

\[
\begin{align*}
3x + 4y &= -25 \\
2x - 3y &= 6
\end{align*}
\]

Multiply by 2. \( \frac{3}{2} \)

\[ 6x + 8y = -50 \]
\[ 2 \cdot 17y = 12 \]

Add the equations.

\[
\begin{align*}
17y &= -68 \\
y &= -4
\end{align*}
\]

Divide each side by 17.

Now substitute -4 for \( y \) in either equation to find the value of \( x \).

\[
\begin{align*}
2x - 3y &= 6 \\
2x - 3(-4) &= 6 \\
2x &= -6 \\
x &= -3
\end{align*}
\]

The solution is \((-3, -4)\).

**Method 2** Eliminate \( y \).

\[
\begin{align*}
3x + 4y &= -25 \\
2x - 3y &= 6
\end{align*}
\]

Multiply by 3. \( \frac{4}{3} \)

\[ 9x + 12y = -75 \]
\[ 2 \cdot 7x = -51 \]

Add the equations.

\[
\begin{align*}
17x &= -51 \\
x &= -3
\end{align*}
\]

Divide each side by 17.

Now substitute -3 for \( x \) in either equation to find the value of \( y \).

\[
\begin{align*}
2x - 3y &= 6 \\
2(-3) - 3y &= 6 \\
-6 - 3y &= 6
\end{align*}
\]

Add 6 to each side.

\[
\begin{align*}
-3y &= 12 \\
\frac{-3y}{-3} &= \frac{12}{-3} \\
y &= -4
\end{align*}
\]

The solution is \((-3, -4)\), which matches the result obtained with Method 1.
DETERMINE THE BEST METHOD

You have learned five methods for solving systems of linear equations.

<table>
<thead>
<tr>
<th>Concept Summary</th>
<th>Solving Systems of Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>The Best Time to Use</td>
</tr>
<tr>
<td>Graphing</td>
<td>to estimate the solution, since graphing usually does not give an exact solution</td>
</tr>
<tr>
<td>Substitution</td>
<td>if one of the variables in either equation has a coefficient of 1 or $-1$</td>
</tr>
<tr>
<td>Elimination Using Addition</td>
<td>if one of the variables has opposite coefficients in the two equations</td>
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<tr>
<td>Elimination Using Subtraction</td>
<td>if one of the variables has the same coefficient in the two equations</td>
</tr>
<tr>
<td>Elimination Using Multiplication</td>
<td>if none of the coefficients are 1 or $-1$ and neither of the variables can be eliminated by simply adding or subtracting the equations</td>
</tr>
</tbody>
</table>

Example 3 Determine the Best Method

Determine the best method to solve the system of equations. Then solve the system.

\[
\begin{align*}
4x - 3y &= 12 \\
x + 2y &= 14
\end{align*}
\]

- For an exact solution, an algebraic method is best.
- Since neither the coefficients of $x$ nor the coefficients of $y$ are the same or additive inverses, you cannot use elimination using addition or subtraction.
- Since the coefficient of $x$ in the second equation is 1, you can use the substitution method. You could also use elimination using multiplication.

The following solution uses substitution. Which method would you prefer?

\[
\begin{align*}
x + 2y &= 14 & \text{Second equation} \\
x + 2y - 2y &= 14 - 2y & \text{Subtract 2y from each side.} \\
x &= 14 - 2y & \text{Simplify.} \\
4x - 3y &= 12 & \text{First equation} \\
4(14 - 2y) - 3y &= 12 & x = 14 - 2y \\
56 - 8y - 3y &= 12 & \text{Distributive Property} \\
56 - 11y &= 12 & \text{Combine like terms.} \\
56 - 11y - 56 &= 12 - 56 & \text{Subtract 56 from each side.} \\
-11y &= -44 & \text{Simplify.} \\
\frac{-11y}{-11} &= \frac{-44}{-11} & \text{Divide each side by } -11. \\
y &= 4 & \text{Simplify.} \\
x + 2y &= 14 & \text{Second equation} \\
x + 2(4) &= 14 & y = 4 \\
x + 8 &= 14 & \text{Simplify.} \\
x + 8 - 8 &= 14 - 8 & \text{Subtract 8 from each side.} \\
x &= 6 & \text{Simplify.}
\end{align*}
\]

The solution is $(6, 4)$. 

www.algebra1.com/extra_examples
4 TRANSPORTATION  A fishing boat travels 10 miles downstream in 30 minutes. The return trip takes the boat 40 minutes. Find the rate of the boat in still water. 17.5 mi/h

Practice/Apply

Exercises 13  Odd/Even Assignments

• Determine the Best Method: 27–43
• Odd/Even Assignments: Exercises 13–38 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–39 odd, 40, 44–57
Average: 13–39 odd, 40, 41, 43–57
Advanced: 14–42 even, 43–53 (optional: 54–57)
All: Practice Quiz 2 (1–5)

Study Notebook

Have students—
• add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
• include examples of how to solve a system of equations by using elimination with multiplication.
• add the definitions/examples of Vocabulary Builder worksheets for the vocabulary terms to their study notebook, in this lesson.

Source: World Book Encyclopedia

Check for Understanding

Concept Check

1. Explain why multiplication is sometimes needed to solve a system of equations by elimination. See margin.

2. OPEN ENDED Write a system of equations that could be solved by multiplying one equation by 5 and then adding the two equations together to eliminate one variable. Sample answer: 3x + 2y = 5, 4x – 10y = –6

3. Describe two methods that could be used to solve the following system of equations. Which method do you prefer? Explain. See pp. 405A–405D.
   a – b = 5
   2a + 3b = 15

Guided Practice

Use elimination to solve each system of equations.

4. 2x – y = 6  5. x + 5y = 4
   3x + 4y = −2  3x − 7y = −10 (−1, 1)
   (2, −2)

6. 4x + 7y = 6  7. 4x + 2y = 10.5
   6x + 5y = 20 (5, −2)  2x + 3y = 10.75 (1.25, 2.75)

Determine the best method to solve each system of equations. Then solve the system.

8. 4x + 3y = 19  9. 3x − 7y = 6
   3x − 4y = 8  3x + 7y = 4 elimination (+); (2, 0)
   elimination (×); (4, 1)

10. y = 4x + 11  11. 5x − 2y = 12 elimination (−);
    3x − 2y = −7 substitution; (−3, −1)

Answer

1. If one of the variables cannot be eliminated by adding or subtracting the equations, you must multiply one or both of the equations by numbers so that a variable will be eliminated when the equations are added or subtracted.
12. BUSINESS The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners purchase? **6 two-seat tables, 11 four-seat tables**

**Application**

- **Example** He attempted.
- **Percentage. He made 475 in the NBA in free-throw practice.**

**Practice and Apply**

**Hone Your Skills**

**For Exercises** | See Examples
--- | ---
13–26 | 1, 2
27–38 | 3
39–43 | 4

**Extra Practice**

- See page 836.

29. substitution; (2, 6)
30. substitution; no solution
31. elimination (+); (8, 4/3)
32. substitution; infinitely many solutions
33. elimination (−); no solution

**More About . . .**

In the 2000–2001 season, Kobe Bryant ranked 16th in the NBA in free-throw percentage. He made 475 of the 557 free throws that he attempted.

**Source:** NBA

The solution is **1.**

**Basketball**

In the 2000–2001 season, Kobe Bryant ranked 16th in the NBA in free-throw percentage. He made 475 of the 557 free throws that he attempted.

**Source:** NBA

Visit **www.algebra1.com/data_update** to learn more.

**Critical Thinking**

The solution of the system **4x + 5y = 2** and **6x − 2y = b** is (3, a). Find the values of **a** and **b.** **a = −2,** **b = 22**

**Careers**

Mrs. Henderson discovered that she had accidentally reversed the digits of a test and shorted a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score without looking at his test. What was his actual score on the test? **95**

Enrichment, p. 426

**George Washington Carver and Percy Julian**

In 1909, George Washington Carver and Percy Julian became the first African American inventors to hold a U.S. patent. Carver’s inventions included new uses for peanuts and sweet potatoes. Julian’s inventions included new uses for soybeans. Both men were known for their creativity and their ability to think outside the box.

**More About . . .**

- **More About . . .**
- **Basketball**

**Online Research**

**Data Update**

**What are the current statistics for Kobe Bryant and other players?** Visit **www.algebra1.com/data_update** to learn more.

**Read to Learn Mathematics, p. 425**

**ELL**

**Pre-Activity**

- How can a manager use a system of equations to plan employee time? Read the introduction to Lesson 7-4 on the top of page 387 in your textbook.
- Can systems of equations be solved by substitution with addition or elimination? Explain. No. Both variables have coefficients that are additive inverses or equal.

**Reading the Lesson**

- Could elimination by multiplication be used to solve the system shown below?
  
  \[ \begin{align*}
  2x + 3y &= 7 \\
  4x - 5y &= 11
  \end{align*} \]
  
  Yes, you can multiply the first equation by 2 to be the coefficients of the x terms additive inverses.

- Will substitution be used to solve the system shown below?
  
  \[ \begin{align*}
  3x + 2y &= 8 \\
  x - y &= 5
  \end{align*} \]
  
  No, substitute into the second equation. Sample answer: You can multiply the first equation by 2 to eliminate y by addition.

**Helping You Remember**

- If you are given a system of equations to solve by elimination, does your solution show when you will need to multiply one or both equations by a number? Sample answer: If both equations involve coefficients of the same variable that are additive inverses, you will need to multiply by a number. If both equations have coefficients that involve variables, you will need to use multiplication.
4 Assess

Open-Ended Assessment

Speaking Using the Concept Summary from p. 389, write on the board a list of the different methods for solving systems of equations. For each method, have a volunteer explain in his or her own words when it is appropriate to use the method.

Getting Ready for Lesson 7-5

PREREQUISITE SKILL Students will learn to graph systems of inequalities in Lesson 7-5. Students must be comfortable graphing inequalities before beginning this lesson because otherwise they will find it very confusing to have different shaded regions. Use Exercises 54–57 to determine your students’ familiarity with graphing inequalities.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 7-3 and 7-4. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lesson 7-4) is available on p. 448 of the Chapter 7 Resource Masters.

Answers

54. 

55. 

56. 

57. 

42. NUMBER THEORY The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 less than the original number. Find the original number. 86

43. TRANSPORTATION Traveling against the wind, a plane flies 2100 miles from Chicago to San Diego in 4 hours and 40 minutes. The return trip, traveling with a wind that is twice as fast, takes 4 hours. Find the rate of the plane in still air. 475 mph

44. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 469–470.

How can a manager use a system of equations to plan employee time?

Include the following in your answer:

• a demonstration of how to solve the system of equations concerning the number of cookies and bread, and

• an explanation of how a restaurant manager would schedule oven and employee time.

45. If 5x + 3y = 12 and 4x – 5y = 17, what is the value of y? A

46. Determine the number of solutions of the system x + 2y = –1 and 2x + 4y = –2. D

Standardized Test Practice

45. If 5x + 3y = 12 and 4x – 5y = 17, what is the value of y? A

46. Determine the number of solutions of the system x + 2y = –1 and 2x + 4y = –2. D

Maintain Your Skills

Mixed Review

Use elimination to solve each system of equations. (Lesson 7-3)

47. x + y = 8
   x = 4
   (2, 6)

48. 2x + s = 5
   r – s = 1
   (2, 1)

49. x + y = 18
   x + 2y = 25
   (11, 7)

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 7-2)

50. 2x + 3y = 3
   x = –3y
   (3, –1)

51. x + y = 0
   3x + y = –8
   (–4, 4)

52. x – 2y = 7
   –3x + 6y = –21
   infinitely many

53. CAREERS A store manager is paid $32,000 a year plus 4% of the revenue the store makes above quota. What is the amount of revenue above quota needed for the manager to have an annual income greater than $45,000? (Lesson 6-3)

54. 54–57. See margin.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Graph each inequality. (To review graphing inequalities, see Lesson 6-6.)

54. y ≥ x – 7
55. x + 3y ≥ 9
56. –y ≤ x
57. –3x + y ≥ –1

Practice Quiz 2

Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

1. 5x + 4y = 2
   3x – 4y = 14
   (2, –2)

2. 2x – 3y = 13
   2x + 2y = –2
   (5, 3)

3. 6x – 2y = 24
   3x + 4y = 27
   no solution

4. 5x + 2y = 4
   10x + 4y = 9

5. The price of a cellular telephone plan is based on peak and nonpeak service. Kelsey used 45 peak minutes and 50 nonpeak minutes and was charged $27.75. That same month, Mitch used 70 peak minutes and 30 nonpeak minutes for a total charge of $36. What are the rates per minute for peak and nonpeak time? (Lesson 7-4) $0.45; $0.15
Making Concept Maps

After completing a chapter, it is wise to review each lesson’s main topics and vocabulary. In Lesson 7-1, the new vocabulary words were system of equations, consistent, inconsistent, independent, and dependent. They are all related in that they explain how many and what kind of solutions a system of equations has.

A graphic organizer called a concept map is a convenient way to show these relationships. A concept map is shown below for the vocabulary words for Lesson 7-1. The main ideas are placed in boxes. Any information that describes how to move from one box to the next is placed along the arrows.

Concept maps are used to organize information. They clearly show how ideas are related to one another. They also show the flow of mental processes needed to solve problems.

Reading to Learn

Review Lessons 7-2, 7-3, and 7-4. 1–4. See margin.
1. Write a couple of sentences describing the information in the concept map above.
2. How do you decide whether to use substitution or elimination? Give an example of a system that you would solve using each method.
3. How do you decide whether to multiply an equation by a factor?
4. How do you decide whether to add or subtract two equations?
5. Copy and complete the concept map below for solving systems of equations by using either substitution or elimination.

Answers

1. There are two types of systems of equations, consistent and inconsistent. Consistent equations have one or more solutions and inconsistent equations have no solutions. If consistent equations have one solution, they are called independent. If consistent equations have infinite solutions, they are called dependent.

2. Use substitution if an expression of one variable is given or if the coefficient of a variable is 1. Use elimination if both equations are written in standard form.

Sample answers:
- system to solve using substitution: \( y = 3x + 3 \)
- system to solve using elimination: \( 4x + 3y = 9 \)
- \( 5x + 2y = 6 \)
- \( 6x - y = 10 \)

3. Multiply by a factor if neither variable has the same or opposite coefficients in the two equations.

4. Add if the variables have opposite coefficients in the two equations. Subtract if one of the variables has the same coefficients in the two equations.
**Focus**

- **5-Minute Check Transparency 7-5** Use as a quiz or review of Lesson 7-4.
- **Mathematical Background** notes are available for this lesson on p. 366D.

**Building on Prior Knowledge**

In Chapter 6, students learned to graph linear inequalities. In this lesson, they should recognize that graphing systems of inequalities simply involves graphing more than one inequality on the same coordinate grid and then shading the regions that overlap.

**How can you use a system of inequalities to plan a sensible diet?**

*Ask students:*
- Does Joshua need to eat the exact same amount of Calories and fat each day? Explain. **No,** but his Calorie and fat intake should be within the green area of the graph.
- Suppose Joshua eats 2600 Calories one day, but only 55 grams of fat. Is this an appropriate intake of Calories and fat? How do you know? It is **not** appropriate because it does not fall within the green section of the graph.

**Definition**

- **system of inequalities**

**SYSYSTEMS OF INEQUALITIES** To solve a system of inequalities, you need to find the ordered pairs that satisfy all the inequalities involved. One way to do this is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection, or overlap, of the graphs.

**Example 1 Solve by Graphing**

Solve the system of inequalities by graphing.

\[
\begin{align*}
  y &< -x + 1 \\
  y &\leq 2x + 3
\end{align*}
\]

The solution includes the ordered pairs in the intersection of the graphs of \( y < -x + 1 \) and \( y \leq 2x + 3 \). This region is shaded in green at the right. The graphs of \( y = -x + 1 \) and \( y = 2x + 3 \) are boundaries of this region. The graph of \( y = -x + 1 \) is dashed and is **not** included in the graph of \( y < -x + 1 \). The graph of \( y = 2x + 3 \) is included in the graph of \( y \leq 2x + 3 \).

**Example 2 No Solution**

Solve the system of inequalities by graphing.

\[
\begin{align*}
  x - y &< -1 \\
  x - y &> 3
\end{align*}
\]

The graphs of \( x - y = -1 \) and \( x - y = 3 \) are parallel lines. Because the two regions have no points in common, the system of inequalities has no solution.
You can use a TI-83 Plus to solve systems of inequalities.

**Graphing Calculator Investigation**

**Graphing Systems of Inequalities**

To graph the system \( y \geq 4x - 3 \) and \( y \leq -2x + 9 \) on a TI-83 Plus, select the SHADE feature in the DRAW menu. Enter the function that is the lower boundary of the region to be shaded, followed by the upper boundary. (Note that inequalities that have \( > \) or \( \geq \) are lower boundaries and inequalities that have \( < \) or \( \leq \) are upper boundaries.)

**Think and Discuss 2, 4.** See pp. 405A–405D.

1. To graph the system \( y \leq 3x + 1 \) and \( y \geq -2x - 5 \) on a graphing calculator, which function should you enter first? \( y \geq -2x - 5 \)

2. Use a graphing calculator to graph the system \( y \leq 3x + 1 \) and \( y \geq -2x - 5 \).

3. Explain how you could use a graphing calculator to graph the system \( 2x + y \geq 7 \) and \( x - 2y \geq 5 \).

4. Use a graphing calculator to graph the system \( 2x + y \geq 7 \) and \( x - 2y \geq 5 \).

**REAL-WORLD PROBLEMS**

In real-life problems involving systems of inequalities, sometimes only whole-number solutions make sense.

**Example 3 Use a System of Inequalities to Solve a Problem**

**COLLEGE** The middle 50% of first-year students attending Florida State University score between 520 and 620, inclusive, on the verbal portion of the SAT and between 530 and 630, inclusive, on the math portion. Graph the scores that a student would need to be in the middle 50% of FSU freshmen.

**Words** The verbal score is between 520 and 620, inclusive. The math score is between 530 and 630, inclusive.

**Variables** If \( v \) = the verbal score and \( m \) = the math score, the following inequalities represent the middle 50% of Florida State University freshmen.

**Inequalities**

The verbal score is between 520 and 620, inclusive.

\[ 520 \leq v \leq 620 \]

The math score is between 530 and 630, inclusive.

\[ 530 \leq m \leq 630 \]

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. However, since SAT scores are whole numbers, only whole-number solutions make sense in this problem.

**REAL-WORLD PROBLEMS**

A college service organization requires that its members maintain at least a 3.0 grade point average, and volunteer at least 10 hours a week. Graph these requirements.
EMPLOYMENT Jamail mows grass after school but his job pays only $3 per hour. He has been offered another job as a library assistant for $6 per hour. Because of school, his parents allow him to work at most 15 hours per week. How many hours can Jamail mow grass and work in the library and still make at least $60 per week?

Point Power

Jamail can work any combination of hours in the darker shaded area of the graph.

Check for Understanding

Concept Check

1. See margin for sample answer.

2. Determine which of the following ordered pairs represent a solution of the system of inequalities graphed at the right.
   
   a. (3, 1) yes  b. (−1, −3) no
   c. (2, 3) yes  d. (4, −2) yes
   e. (3, −2) no  f. (1, 4) no

3. Kayla; the graph of $x + 2y = −2$ is the line representing $x + 2y = 2$ and the region above it.

Who is correct? Explain your reasoning.

Example 4 Use a System of Inequalities

AGRICULTURE To ensure a growing season of sufficient length, Mr. Hobson has at most 16 days left to plant his corn and soybean crops. He can plant corn at a rate of 250 acres per day and soybeans at a rate of 200 acres per day. If he has at most 3500 acres available, how many acres of each type of crop can he plant?

Let $c =$ the number of days that corn will be planted and $s =$ the number of days that soybeans will be planted. Since both $c$ and $s$ represent a number of days, neither can be a negative number. The following system of inequalities can be used to represent the conditions of this problem.

$$c \geq 0$$

$$s \geq 0$$

$$c + s \leq 16$$

$$250c + 200s \leq 3500$$

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. This region is shown in green at the right. Only the portion of the region in the first quadrant is used since $c \geq 0$ and $s \geq 0$.

Any point in this region is a possible solution. For example, since (7, 8) is a point in the region, Mr. Hobson could plant corn for 7 days and soybeans for 8 days. In this case, he would use 15 days to plant 250(7) or 1750 acres of corn and 200(8) or 1600 acres of soybeans.
Solve each system of inequalities by graphing. 4–9. See pp. 405A–405D.

4. \( x > 5 \)
5. \( y > 3 \)
6. \( y \leq -x + 3 \)
7. \( x + y \geq 4 \)
8. \( y + 3x < 6 \)
9. \( x - 2y \leq 2 \)

Application

**HEALTH**

For Exercises 10 and 11, use the following information.

Natala walks and jogs at least 3 miles every day. Natala walks 4 miles per hour and jogs 8 miles per hour. She only has a half-hour to exercise. **10.** See pp. 405A–405D.

10. Draw a graph of the possible amounts of time she can spend walking and jogging.

11. List three possible solutions. **Sample answers:** walk: 15 min, jog: 15 min; walk: 10 min, jog: 20 min; walk: 5 min, jog: 25 min

**Guided Practice**

**GUIDED PRACTICE KEY**

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</table>

**Guided Practice**

12. \( x < 0 \)
13. \( y \geq 2 \)
14. \( y - x \geq 2 \)
15. \( x \geq -4 \)
16. \( y \leq 3 \)
17. \( y - x < 3 \)
18. \( x + y < 1 \)
19. \( y - x > 3 \)
20. \( x - 2y > 2 \)
21. \( x + 2y < 4 \)
22. \( y - 2x > 4 \)
23. \( y > x \)
24. \( y \geq 1 \)

Write a system of inequalities for each graph.

27. \( y \leq x \), \( y > x - 3 \)
28. \( y \leq -\frac{2}{3}x + 2 \), \( y > x - 3 \)

**Visual Artist**

Visual artists create art to communicate ideas. The work of fine artists is made for display. Illustrators and graphic designers produce art for clients, such as advertising and publishing companies.

**Online Research**

For information about a career as a visual artist, visit: www.algebra1.com/careers

29. See pp. 405A–405D.
30. **Sample answers:** 2 light, 8 dark; 6 light, 8 dark; 7 light, 4 dark

www.algebra1.com/self_check_quiz

---

**Study Guide and Intervention, p. 427 (shown) and p. 428**

**System of Inequalities**

**Objective:** Solve a system of linear inequalities graphically.

**Example:** Solve the system by graphing.

**Steps:**
1. Graph each inequality.
2. Shade the solution set of each inequality.
3. The solution set of the system of inequalities is the set of all ordered pairs that satisfy both inequalities.
4. **Problem:** Solve the system by graphing.

---

**Practice and Apply**

Practice 29–31, 32, 34
33–35
12–28 1–2

**Guided Practice**

9 1, 2

**Practice**

9 1, 2

**Enrichment**

**Lesson 7-5**

**Enrichment, p. 432**

**Describing Regions**

The shaded region inside the triangle can be described with a system of three inequalities:

\( y < 3x + 1 \)

\( y > \frac{1}{3}x - 3 \)

\( y > 2x - 31 \)

Write systems of inequalities to describe each region. You may need to divide a region into triangles or quadrilaterals.

---

**Reading to Learn Mathematics, p. 431**

**Pre-Activity**

How can you use a system of inequalities to plan a sensible diet?

1. **Read the introduction to Lesson 7-5 on the top of page 405.**
2. **List three possible combinations of eating out and walking that meet Diego’s goals.**

---

**Reading the Lesson**

Write the inequality symbols that you need to get a system whose graph looks like one given. Use 4, 3, 2, or 1.

---

**Helping You Remember**

5. Draw a line on a coordinate plane to separate the region to the right of the line from the other region. The set of all points on one side of the line is the solution set. Note that the line is not in the solution set.

---

**Lesson 7-5**

**Graphing Systems of Inequalities**
About the Exercises...
Organization by Objective
• Systems of Inequalities: 12–28
• Real-World Problems: 29–31, 33–35

Odd/Even Assignments
Exercises 12–28 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 36–38 require a graphing calculator.

Assignment Guide
Basic: 13–23 odd, 29–32, 35, 39–49
Advanced: 12–28 even, 32–49

Open-Ended Assessment
Writing Have students describe in words the method for determining whether to shade above or below a line when graphing an inequality, and how to determine the common solutions when graphing a system of inequalities.

Assessment Options
Quiz (Lesson 7-5) is available on p. 448 of the Chapter 7 Resource Masters.

Answers
33. Furniture Manufacturing

36. Graphing a system of inequalities.

34. List three possible solutions. Sample answers: 8 desks, 10 tables; 6 desks, 12 tables; 4 desks, 14 tables

35. Writing in Math Answer the question that was posed at the beginning of the lesson. See pp. 405A–405D.

How can you use a system of inequalities to plan a sensible diet?
Include the following in your answer:
• two appropriate Calorie and fat intakes for a day, and
• the system of inequalities that is represented by the graph.

Graphing Systems of Inequalities
Use a graphing calculator to solve each system of inequalities. Sketch the results.

36. Which system of inequalities is represented by the graph?

37. Which ordered pair does not satisfy the system \( x + 2y < 5 \) and \( 3x - y < -2 \)?

38. Which system of inequalities is represented by the graph?

39. What system of inequalities is represented by the graph?

40. Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

41. \( 2x + 3y = 1 \) \( 42. 5x - 2y = -3 \)
\( 4x - 5y = 13 \) \( 43. -3x + 2y = 12 \)
\( 2x - 3y = -13 \) \( 44. 6x - 2y = 4 \)
\( 3x + 6y = -9 \) \( 45. 2x + 5y = 13 \)
\( -2, -1 \) \( 46. -2x - 3y = -18 \)
\( -1, 3 \) \( 47. \frac{1}{3}x - \frac{11}{3} \)
\( 3, 3 \)

48. Write an equation of the line that passes through each point with the given slope.
(Lesson 5-4)

47. \( (4, -1), m = 2 \) \( 48. (1, 0), m = -6 \)
(5, -2), \( m = \frac{1}{3} \)

398 Chapter 7 Solving Systems of Linear Equations and Inequalities

4 Assess

4. Graphing Calculator

36. Which system of inequalities is represented by the graph?

37. Which ordered pair does not satisfy the system \( x + 2y < 5 \) and \( 3x - y < -2 \)?

38. Which system of inequalities is represented by the graph?

39. What system of inequalities is represented by the graph?

40. Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

41. \( 2x + 3y = 1 \) \( 42. 5x - 2y = -3 \)
\( 4x - 5y = 13 \) \( 43. -3x + 2y = 12 \)
\( 2x - 3y = -13 \) \( 44. 6x - 2y = 4 \)
\( 3x + 6y = -9 \) \( 45. 2x + 5y = 13 \)
\( -2, -1 \) \( 46. -2x - 3y = -18 \)
\( -1, 3 \) \( 47. \frac{1}{3}x - \frac{11}{3} \)
\( 3, 3 \)

48. Write an equation of the line that passes through each point with the given slope.
(Lesson 5-4)

47. \( (4, -1), m = 2 \) \( 48. (1, 0), m = -6 \)
(5, -2), \( m = \frac{1}{3} \)

398 Chapter 7 Solving Systems of Linear Equations and Inequalities

4. Graphing Calculator

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37. Which ordered pair does not satisfy the system \( x + 2y < 5 \) and \( 3x - y < -2 \)?

38. Which system of inequalities is represented by the graph?

39. What system of inequalities is represented by the graph?

40. Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

41. \( 2x + 3y = 1 \) \( 42. 5x - 2y = -3 \)
\( 4x - 5y = 13 \) \( 43. -3x + 2y = 12 \)
\( 2x - 3y = -13 \) \( 44. 6x - 2y = 4 \)
\( 3x + 6y = -9 \) \( 45. 2x + 5y = 13 \)
\( -2, -1 \) \( 46. -2x - 3y = -18 \)
\( -1, 3 \) \( 47. \frac{1}{3}x - \frac{11}{3} \)
\( 3, 3 \)

48. Write an equation of the line that passes through each point with the given slope.
(Lesson 5-4)

47. \( (4, -1), m = 2 \) \( 48. (1, 0), m = -6 \)
(5, -2), \( m = \frac{1}{3} \)

398 Chapter 7 Solving Systems of Linear Equations and Inequalities
Choose the correct term to complete each statement.

1. If a system of equations has exactly one solution, it is (dependent, independent).
2. If the graph of a system of equations is parallel lines, the system is (consistent, inconsistent).
3. A system of equations that has infinitely many solutions is (dependent, independent).
4. If the equations in a system have the same slope and different intercepts, the graph of the system is (intersecting lines, parallel lines).
5. If a system of equations has the same slope and intercepts, the system has (exactly one, infinitely many) solution(s).
6. The solution of a system of equations is (3, –5). The system is (consistent, inconsistent).

**Lesson-by-Lesson Review**

### 7-1 Graphing Systems of Inequalities

**Concept Summary**

<table>
<thead>
<tr>
<th>Graph of a System</th>
<th>Intersecting Lines</th>
<th>Same Line</th>
<th>Parallel Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Solutions</td>
<td>exactly one solution</td>
<td>infinitely many</td>
<td>no solutions</td>
</tr>
<tr>
<td>Terminology</td>
<td>consistent and independent</td>
<td>consistent and dependent</td>
<td>inconsistent</td>
</tr>
</tbody>
</table>

**Example**

Graph the system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

3x + y = -4
6x + 2y = -8

When the lines are graphed, they coincide. There are infinitely many solutions.

**Exercises**

Graph each system of equations. Then determine whether the system of equations has one solution, no solution, or infinitely many solutions. If the system has one solution, name it. **See Example 2 on page 370. 7-10. See margin for graphs.**

7. x - y = 9  
   x + y = 11

8. 9x + 2 = 3y  
   y - 3x = 8

9. 2x - 3y = 4  
   6y = 4x - 8

10. 3x - y = 8  
    one; (10, 1)  
    no solution  
    infinitely many  
    one; (2, -2)

**Foldables Study Organizer**

Have students look through the chapter to make sure they have included examples in their Foldables for each method of solving systems of equations they learned.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.
**Answers**

7. ![Graph of x + y = 11]

8. ![Graph of y - 3x = 8]

9. ![Graph of 8y = 4x - 8]

10. ![Graph of -3x - y = 8]

---

**7-2 Substitution**

**Concept Summary**

- In a system of equations, solve one equation for a variable, and then substitute that expression into the second equation to solve.

**Example**

Use substitution to solve the system of equations.

\[ y = x - 1 \]
\[ 4x - y = 19 \]

Since \( y = x - 1 \), substitute \( x - 1 \) for \( y \) in the second equation.

\[ 4x - (x - 1) = 19 \quad y = x - 1 \]
\[ 4x - x + 1 = 19 \quad \text{Distributive Property} \]
\[ 3x + 1 = 19 \quad \text{Combine like terms.} \]
\[ 3x = 18 \quad \text{Subtract 1 from each side.} \]
\[ x = 6 \quad \text{Divide each side by 3.} \]

Use \( y = x - 1 \) to find the value of \( y \).

\[ y = 6 - 1 \quad x = 6 \]
\[ y = 5 \quad \text{The solution is (6, 5).} \]

**Exercises** Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solutions or infinitely many solutions. See Examples 1–3 on pages 377 and 378.

11. \( 2m + n = 1 \)
   \( m - n = 8 \)
   \( (3, -5) \)

12. \( x = 3 - 2y \)
   \( 2x + 4y = 6 \)
   \( (2, 2) \)

13. \( 3x - y = 1 \)
   \( 2x + 4y = 3 \)
   \( (1.5, 0) \)

14. \( 0.6m - 0.2n = 0.9 \)
   \( n = 4.5 - 3m \)
   \( \text{infinitely many solutions} \)

---

**7-3 Elimination Using Addition and Subtraction**

**Concept Summary**

- Sometimes adding or subtracting two equations will eliminate one variable.

**Example**

Use elimination to solve the system of equations.

\[ 2m - n = 4 \]
\[ m + n = 2 \]

You can eliminate the \( n \) terms by adding the equations.

\[ 2m - n = 4 \quad \text{Write the equations in column form and add.} \]
\[ (+) m + n = 2 \]
\[ 3m = 6 \quad \text{Notice the variable \( n \) is eliminated.} \]
\[ m = 2 \quad \text{Divide each side by 3.} \]
Now substitute 2 for \( m \) in either equation to find \( n \).

\[
\begin{align*}
\text{Second equation} & \quad m = 2 \\
\text{Subtract 2 from each side.} & \quad n = 0 \\
\text{Simplify.} & \quad \text{The solution is } (2, 0).
\end{align*}
\]

**Exercises**  Use elimination to solve each system of equations.

See Examples 1–3 on pages 382 and 383.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + 2y = 6 )</td>
<td>( x - 3y = -4 )</td>
<td>((2, 2))</td>
</tr>
<tr>
<td>( 2m - n = 5 )</td>
<td>( 2m + n = 3 )</td>
<td>((2, -1))</td>
</tr>
<tr>
<td>( 3x - y = 11 )</td>
<td>( x + y = 5 )</td>
<td>((4, 1))</td>
</tr>
<tr>
<td>( 3x + 1 = -7y )</td>
<td>( 6x + 7y = 0 )</td>
<td>((1, -2))</td>
</tr>
</tbody>
</table>

**7-4 Elimination Using Multiplication**

**Concept Summary**

- Multiplying one equation by a number or multiplying each equation by a different number is a strategy that can be used to solve a system of equations by elimination.
- There are five methods for solving systems of equations.

<table>
<thead>
<tr>
<th>Method</th>
<th>The Best Time to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphing</td>
<td>to estimate the solution, since graphing usually does not give an exact solution</td>
</tr>
<tr>
<td>Substitution</td>
<td>if one of the variables in either equation has a coefficient of 1 or (-1)</td>
</tr>
<tr>
<td>Elimination Using Addition</td>
<td>if one of the variables has opposite coefficients in the two equations</td>
</tr>
<tr>
<td>Elimination Using Subtraction</td>
<td>if one of the variables has the same coefficient in the two equations</td>
</tr>
<tr>
<td>Elimination Using Multiplication</td>
<td>if none of the coefficients are 1 or (-1) and neither of the variables can be eliminated by simply adding or subtracting the equations</td>
</tr>
</tbody>
</table>

**Example**

Use elimination to solve the system of equations.

\[
\begin{align*}
x + 2y &= 8 \\
3x + y &= 1.5
\end{align*}
\]

Multiply the second equation by \(-2\) so the coefficients of the \( y \) terms are additive inverses. Then add the equations.

\[
\begin{align*}
x + 2y &= 8 \\
3x + y &= 1.5 \\
\text{Multiply by } -2 & \quad \text{Multiply by } -2 \\
-5x &= 5 \\
-6x - 2y &= -3 & \quad \text{Add the equations.} \\
-5x - 5 &= -5 \\
x &= -1 & \quad \text{Divide each side by } -5. \\
& \quad \text{Simplify.}
\end{align*}
\]

(continued on the next page)
Study Guide and Review

Answers

27. [Graph showing the solution to the system of equations: \(-2x + 2y = -12\) and \(y = x + 4\).]

28. [Graph showing the solution to the system of inequalities: \(y = 2x + 1\) and \(y = -x - 1\).]

29. [Graph showing the solution to the system of inequalities: \(2x + y = 9\) and \(x + 11y = -6\).]

30. [Graph showing the solution to the system of inequalities: \(x = 1\) and \(y = x + 3\).]

Answers (page 403)

4. [Graph showing the solution to the system of inequalities: \(y = x + 2\) and \(y = 2x + 7\).]

5. [Graph showing the solution to the system of inequalities: \(x + 2y = 11\) and \(x = 14 - 2y\).]

6. [Graph showing the solution to the system of inequalities: \(2y - 10 = -6x\) and \(3x + y = 5\).]

Exercises

Use elimination to solve each system of equations.

19. \(x - 5y = 0\)
   \(2x - 3y = 7\)
   \((5, 1)\)

20. \(x - 2y = 5\)
   \(3x - 5y = 8\)
   \((-9, -7)\)

21. \(2x + 3y = 8\)
   \(x - y = 2\)
   \((\frac{4}{5}, \frac{4}{5})\)

22. \(-5x + 8y = 21\)
   \(10x + 3y = 15\)
   \((3, 3)\)

Determine the best method to solve each system of equations. Then solve the system.

23. \(y = 2x\)
   \(x + 2y = 8\)
   Substitution; \(\left(\frac{3}{5}, \frac{3}{5}\right)\)

24. \(9x + 8y = 7\)
   \(18x - 15y = 14\)
   Elimination \((\times)\); \(\left(\frac{7}{9}, 0\right)\)

25. \(3x + 5y = 2x\)
   \(x + 3y = y\)
   Substitution; \((0, 0)\)

26. \(2x + y = 3x - 15\)
   \(x + 5 = 4y + 2x\)
   Substitution; \((13, -2)\)

7-5 Graphing Systems of Inequalities

Concept Summary

Graph each inequality on a coordinate plane to determine the intersection of the graphs.

Example

Solve the system of inequalities.

\[ x \geq -3 \]
\[ y \leq x + 2 \]

The solution includes the ordered pairs in the intersection of the graphs \(x \geq -3\) and \(y \leq x + 2\). This region is shaded in green. The graphs of \(x \geq -3\) and \(y \leq x + 2\) are boundaries of this region.

Exercises

Solve each system of inequalities by graphing.

27. \(y < 3x\)
   \(x + 2y \geq -21\)

28. \(y > -x - 1\)
   \(y \leq 2x + 1\)

29. \(2x + y < 9\)
   \(x + 11y < -6\)

30. \(x \geq 1\)
   \(y + x \leq 3\)
Choose the letter that best matches each description.

1. a system of equations with two parallel lines  \( \text{c} \)
2. a system of equations with at least one ordered pair that satisfies both equations  \( \text{a} \)
3. a system of equations may be solved using this method  \( \text{b} \)

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.  4–6.  See margin for graphs.

4. \[ \begin{align*} y &= x + 2 \\
y &= 2x + 7 \end{align*} \] one; \((-5, -3)\)
5. \[ \begin{align*} x + 2y &= 11 \\
x &= 14 - 2y \end{align*} \] no solution
6. \[ \begin{align*} 3x + y &= 5 \\
2y - 10 &= -6x \end{align*} \] infinitely many

Use substitution or elimination to solve each system of equations.

7. \[ \begin{align*} 2x + 5y &= 16 \\
5x - 2y &= 11 \end{align*} \] \((3, 2)\)
8. \[ \begin{align*} y + 2x &= -1 \\
y - 4 &= -2x \end{align*} \] no solution
9. \[ \begin{align*} 2x + y &= -4 \\
5x + 3y &= -6 \end{align*} \] \((-6, 8)\)
10. \[ \begin{align*} y &= 7 - x \\
x - y &= -3 \end{align*} \] \((2, 5)\)
11. \[ \begin{align*} x &= 2y - 7 \\
y - 3x &= -9 \end{align*} \] \((5, 6)\)
12. \[ \begin{align*} x + y &= 10 \\
x - y &= 2 \end{align*} \] \((6, 4)\)
13. \[ \begin{align*} 3x - y &= 11 \\
x + 2y &= -36 \end{align*} \] \((-2, -17)\)
14. \[ \begin{align*} 3x + y &= 10 \\
x - 2y &= 16 \end{align*} \] \((4, -2)\)
15. \[ \begin{align*} 5x - 3y &= 12 \\
-2x + 3y &= -3 \end{align*} \] \((3, 1)\)
16. \[ \begin{align*} 2x + 5y &= 12 \\
x - 6y &= -11 \end{align*} \] \((1, 2)\)
17. \[ \begin{align*} x + y &= 6 \\
3x - 3y &= 13 \end{align*} \] \((5, \frac{5}{3})\)
18. \[ \begin{align*} 3x + \frac{1}{3}y &= 10 \\
2x - \frac{5}{3}y &= 35 \end{align*} \] \((5, -15)\)

19. **NUMBER THEORY**  The units digit of a two-digit number exceeds twice the tens digit by 1. Find the number if the sum of its digits is 10.  \(37\)

20. **GEOMETRY**  The difference between the length and width of a rectangle is 7 centimeters. Find the dimensions of the rectangle if its perimeter is 50 centimeters.  \(16 \text{ cm by 9 cm}\)

Solve each system of inequalities by graphing.  21–23.  See pp. 405A–405D.

21. \[ \begin{align*} y &> -4 \\
y &< -1 \end{align*} \]
22. \[ \begin{align*} y &\leq 3 \\
y &> -x + 2 \end{align*} \]
23. \[ \begin{align*} x &\leq 2y \\
2x + 3y &\leq 7 \end{align*} \]

24. **FINANCE**  Last year, Jodi invested \$10,000, part at 6% annual interest and the rest at 8% annual interest. If she received \$760 in interest at the end of the year, how much did she invest at each rate?  \$2000 at 6%, \$8000 at 8%.

25. **STANDARDIZED TEST PRACTICE**  Which graph represents the system of inequalities \( y > 2x + 1 \) and \( y < -x - 2 \)?  \(\text{D}\)

**Portfolio Suggestion**

**Introduction**  Have you ever found that your preference for completing a task may be different from those of others?

**Ask Students**  Select one of the systems of equations from this chapter that could be solved by various methods. Demonstrate how to solve it using some of the methods you learned in this chapter. Write some pros and cons for using each method. Which method do you prefer, and why?

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404 Chapter 7 Solving Systems of Linear Equations and Inequalities

Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the value of x in 4x - 2(x - 2) - 8 = 0?  
   (Lesson 3-4)  B
   
   A  -2  B  2
   C  5  D  6

2. Noah paid $17.11 for a CD, including tax. If the tax rate is 7%, then what was the price of the CD before tax?  
   (Lesson 3-5)  C
   
   A  $10.06  B  $11.98  C  $15.99  D  $17.04

3. What is the range of f(x) = 2x - 3 when the domain is [3, 4, 5]?  
   (Lesson 4-3)  B
   

4. Jolene kept a log of the numbers of birds that visited a bird feeder over periods of several hours. On the table below, she recorded the number of hours she watched and the cumulative number of birds that she saw each session. Which equation best represents this data set shown in the table?  
   (Lesson 4-8)  D

<table>
<thead>
<tr>
<th>Number of hours, x</th>
<th>Number of birds, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
</tr>
</tbody>
</table>

   A  y = x + 5  B  y = 3x + 3
   C  y = 3x + 5  D  y = 4x + 2

5. Which equation describes the graph?  
   (Lesson 5-3)  C

   A  3y - 4x = -12  B  4y + 3x = -16
   C  3y + 4x = -12  D  3y + 4x = -9

6. Which equation represents a line parallel to the line given by y = -3x + 6?  
   (Lesson 5-6)  B

   A  y = -3x + 4  B  y = 3x - 2
   C  y = \frac{1}{3}x + 6  D  y = -\frac{1}{3}x + 4

7. Tamika has $185 in her bank account. She needs to deposit enough money so that she can withdraw $230 for her car payment and still have at least $200 left in the account. Which inequality describes d, the amount she needs to deposit?  
   (Lesson 6-1)  D

   A  (185 - 230) \geq 200  B  185 - 230d \geq 200
   C  185 + 230 + d \geq 200  D  185 + d - 230 \geq 200

8. The perimeter of a rectangular garden is 68 feet. The length of the garden is 4 more than twice the width. Which system of equations will determine the length \( \ell \) and the width \( w \) of the garden?  
   (Lesson 7-2)  B

   A  2\ell + 2w = 68  B  2\ell + 2w = 68
   \ell = 4 - 2w  C  4 - 2w  D  2\ell + 2w = 68
   2\ell - w = 4  E  2\ell - w = 4

   W = \ell + 4

9. Ernesto spent a total of $64 for a pair of jeans and a shirt. The jeans cost $6 more than the shirt. What was the cost of the jeans?  
   (Lesson 7-2)  C

   A  $26  B  $29
   C  $35  D  $58

10. What is the value of \( y \) in the following system of equations?  
    (Lesson 7-3)  C

      3x + 4y = 8  
      3x + 2y = -2

      A  -2  B  4
      C  5  D  6
Part 3  Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

A  the quantity in Column A is greater,
B  the quantity in Column B is greater,
C  the two quantities are equal, or
D  the relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>3(x)</th>
<th>9(x^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column A</td>
<td>C</td>
<td>(Lesson 1-1)</td>
</tr>
</tbody>
</table>

| 16.        | 3\(x\)   | 9\(x^2\) |
| Column A   | C        | (Lesson 1-1) |

| Slope of line A: \(y = \frac{1}{3}x + 1\)
| Slope of line B: \(y = -2x + 5\) |
| Column B   | A        | (Lesson 5-4) |

| 17.        | the slope of the line that contains \((2, 4)\) and \((-1, 3)\) | the slope of the line that contains \((-2, 1)\) and \((5, 3)\) |
| Column A   | A        | (Lesson 5-4) |

| 18.        | \(x - 3y = 11\) | \(3x + y = 13\) |
| Column B   | B        | (Lesson 7-4) |

| 19.        | \(3x - 2y = 19\) | \(5x + 4y = 17\) |
| Column A   | A        | (Lesson 7-4) |

Part 4  Open Ended

Record your answers on a sheet of paper. Show your work.

20. The manager of a movie theater found that Saturday’s sales were $3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost $7.50, and children’s tickets cost $4.50. (Lesson 7-2) a-b. See margin.

a. Write equations to represent the number of tickets sold and the amount of money collected.
b. How many of each kind of ticket were sold? Show your work. Include all steps.

Answers

20a. \(A + C = 650\),
7.5\(A\) + 4.5\(C\) = 3675
20b. \(A + C = 650\)
\(A + C - C = 650 - C\)
\(A = 650 - C\)
7.5\(A\) + 4.5\(C\) = 3675
4875 - 7.5\(C\) + 4.5\(C\) = 3675
4875 - 3\(C\) = 3675
4875 - 3\(C\) - 4875 = 3675 - 4875
-3\(C\) = -1200
-3\(C\) = -1200
\(-3\) = 3
\(C = 400\)

\(A = 650 - C\)
\(A = 650 - 400\) or \(A = 250\)
250 adult tickets and 400 child tickets

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Page 367, Chapter 7 Getting Started

1. Sample answer:

2. Always; if the system of linear equations has 2 solutions, their graphs are the same line and there are infinitely many solutions.

3. Sample answer: The graphs of the equations $x + y = 3$ and $2x + 2y = 6$ have a slope of $-1$. Since the graphs of the equations coincide, there are infinitely many solutions.
Elimination can be used to solve problems about meteorology if the coefficients of one variable are the same or are additive inverses. Answers should include the following.

- The two equations in the system of equations are added or subtracted so that one of the variables is eliminated. You then solve for the remaining variable. This number is substituted into one of the original equations, and that equation is solved for the other variable.

\[
\begin{align*}
\frac{n}{2} + d &= 24 \\
\frac{1}{2}x + \frac{1}{3}y &= 6
\end{align*}
\]

**Write the equations in column form and add.**

\[
\begin{align*}
\frac{2n}{2} &= 36 \\
n &= 18
\end{align*}
\]

**Notice the \( d \) variable is eliminated.**

\[
\begin{align*}
\frac{2n}{2} &= 36 \\
n &= 18 \\
18 + d &= 24 \\
n &= 18 \\
18 + d - 18 &= 24 - 18
\end{align*}
\]

**Subtract 18 from each side.**

\[
\begin{align*}
18 + d - 18 &= 24 - 18 \\
d &= 6
\end{align*}
\]

**Simplify.**

On the winter solstice, Seward, Alaska, has 18 hours of nighttime and 6 hours of daylight.
3. Sample answer: (1) You could solve the first equation for $a$ and substitute the resulting expression for $a$ in the second equation. Then find the value of $b$. Use this value for $b$ and one of the equations to find the value of $a$. (2) You could multiply the first equation by 3 and add this new equation to the second equation. This will eliminate the $b$-term. Find the value of $a$. Use this value for $a$ and one of the equations to find the value of $b$. See student’s work for their preference and explanation.

44. By having two equations that represent the time restraints, a manager can determine the best use of employee time. Answers should include the following.

- $20c + 10b = 800$  
  $10c + 30b = 900$  
  $20c + 10b = 800$  
  $10c + 30b = 900$  
  $20c + 200 = 800$  
  $20c + 200 = 800$  
  $20c = 600$  
  $20c = 600$  
  $c = 30$

- In order to make the most of the employee and oven time, the manager should make assignments to bake 30 batches of cookies and 20 loaves of bread.

Page 395, Graphing Calculator Investigation

2. $[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

4. $[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Pages 396–398, Lesson 7-5

4. $\begin{align*}
y &= 4 \\
x &= 5
\end{align*}$

5. $\begin{align*}
y &= 3 \\
x &= 2 \\
y &= -x + 4
\end{align*}$

6. $\begin{align*}
y &= x + 3 \\
x &= 2y + 6
\end{align*}$

7. $\begin{align*}
y &= -2x - 1 \\
x &= 0 \\
y &= 3 \\
y &= x + 3
\end{align*}$

8. $\begin{align*}
y &= 3x - y = 4 \\
y &= 2y + x = 6 \\
y &= 0 \\
y &= 3
\end{align*}$

9. $\begin{align*}
y &= 2x + y = 4 \\
x &= 3x + 4y = 12
\end{align*}$

10. Natasha’s Daily Exercise

12. $\begin{align*}
y &= 0 \\
x &= 0 \\
y &= x
\end{align*}$

13. $\begin{align*}
x &= -4 \\
y &= 0 \\
y &= -1
\end{align*}$

14. $\begin{align*}
y &= x - 1 \\
x &= x
\end{align*}$

15. $\begin{align*}
x &= 2 \\
y &= -2 \\
y &= x + 3
\end{align*}$
By graphing a system of equations, you can see the appropriate range of Calories and fat intake. Answers should include the following.

- Two sample appropriate Calorie and fat intakes are 2200 Calories and 60 g of fat and 2300 Calories and 65 g of fat.
- The graph represents $2000 \leq c \leq 2400$ and $60 \leq f \leq 75$.

Additional Answers for Chapter 7