Course Overview
Introduction to Flight Dynamics
Math Preliminaries

At the End of the Course,
you should be able to:

- Understand aircraft configuration aerodynamics, performance, stability, and control
- Estimate an aircraft’s aerodynamic characteristics from geometric and inertial properties
- Analyze linear and nonlinear dynamic systems
- Recognize airplane modes of motion and their significance
- Compute aircraft motions
- Appreciate historical development of aviation
Syllabus, First Half

- Introduction, Math Preliminaries
- Point Mass Dynamics
- Aerodynamics of Airplane Configurations
- Cruising Flight Performance
- Gliding, Climbing, and Turning Performance
- Wind Tunnel and Flight Testing
- Nonlinear, 6-DOF Equations of Motion
- Aircraft Control Devices and Systems
- Linearized Equations of Motion
- Longitudinal Dynamics
- Lateral-Directional Dynamics

Details, reading, homework assignments, and references at http://blackboard.princeton.edu/

Syllabus, Second Half

- Analysis of Linear Systems
  - Time Response
  - Transfer Functions and Frequency Response
  - Root Locus Analysis
- Advanced Problems in Longitudinal Dynamics
- Advanced Problems in Lateral-Directional Dynamics
- Flying Qualities Criteria
- Maneuvering at High Angles and Rates
- Aeroelasticity and Fuel Slosh
- Problems of High Speed and Altitude
- Atmospheric Hazards to Flight
Your interested in MAE 331 because ...?

Text and References

- **Science, Engineering, and Math:**
  - [http://pup.princeton.edu/titles/7909.html](http://pup.princeton.edu/titles/7909.html)

- **Case Studies, Historical Context**

- **Supplemental reference**
Quick Quizzes
First 5 Minutes of 10 Classes

- One question about the lectures and reading assignments from the previous week
- Largely qualitative but may require simple calculations
- Be sure to bring a pencil, paper, and calculator to class

Homework Assignments

- All assignments will be done in groups of 2 or 3 students
- Team members for each assignment will be different
  - chosen using a spreadsheet and random number generator (TBD)
- Each member of each team will receive the same grade as the others
Flight Tests Using Balsa Glider and Cockpit Flight Simulator

- Flight envelope of full-scale aircraft simulation
  - Maximum speed, altitude ceiling, stall speed, ...
- Performance
  - Time to climb, minimum sink rate, ...
- Turning Characteristics
  - Maximum turn rate, ...

Assignment #1
due: September 19, 2014

- Document the physical characteristics and flight behavior of a balsa glider
  - Everything that you know about the physical characteristics of the glider
  - Everything that you know about the flight characteristics of the glider
- 2-person team, joint write-up
• Can determine height, range, velocity, flight path angle, and pitch angle from sequence of digital photos (QuickTime)
Reading Assignments

• **All students are expected to do all reading assignments before class**

• **Reading for Case Studies and Historical Context, *Airplane Stability and Control***

  • 10-minute synopses by groups of 3 students
    • Principal subject and scope of the chapter
    • Technical ideas needed to understand the chapter
    • When did the events covered in the chapter occur?
    • Three main “takeaway points” or conclusions
    • Three most surprising or remarkable facts

• **1st synopsis: Sept 23rd, team members TBD**
Goals for Airplane Design

- Shape of the airplane determined by its purpose
- Safety, handling, performance, functioning, and comfort
- Agility vs. sedateness
- Control surfaces adequate to produce needed moments (i.e., torques)
- Center of mass location
  - too far forward increases unpowered control-stick forces
  - too far aft degrades static stability

Configuration Driven By The Mission and Flight Envelope
Inhabited Air Vehicles

Uninhabited Air Vehicles (UAV)
Introduction to Flight Dynamics

Airplane Components

- Vertical stabilizer ("fin")
- Rudder
- Elevator
- Horizontal stabilizer
- Aileron
- Flap
- Fuselage
- Wing
- Landing gear
Airplane Rotational Degrees of Freedom

Airplane Translational Degrees of Freedom
Phases of Flight

Flight of a Paper Airplane
Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

- Equations of motion integrated numerically to estimate the flight path
- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop

Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop
Assignment #2

- Compute the trajectory of a balsa glider
- Computer simulation of the equations of motion
- Compare to the actual flight of the glider (Assignment #1)
- Similar to the flight of a paper airplane
- 2-person team assignment

Gliding Flight

(a) Unaccelerated glide conditions.
Configuration Aerodynamics

(b) Glide aerodynamic characteristics.

Math Preliminaries
Dynamic Systems

Dynamic Process: Current state depends on prior state
- $x = \text{dynamic state}$
- $u = \text{input}$
- $w = \text{exogenous disturbance}$
- $p = \text{parameter}$
- $t$ or $k = \text{time or event index}$

Observation Process: Measurement may contain error or be incomplete
- $y = \text{output (error-free)}$
- $z = \text{measurement}$
- $n = \text{measurement error}$

• All of these quantities are multidimensional
• They can be expressed as vectors

Notation for Scalars and Vectors

• **Scalar**: usually lower case: $a, b, c, \ldots, x, y, z$

  $$a = 12; \quad b = 7; \quad c = a + b = 19; \quad x = a + b^2 = 12 + 49 = 61$$

• **Vector**: usually bold or with underbar: $\mathbf{x}$ or $\mathbf{x}$
  - Ordered set
  - Column of scalars
  - Dimension = $n \times 1$

\[
\mathbf{a} = \begin{bmatrix} 2 \\ -7 \\ 16 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}
\]
Matrices and Transpose

- **Matrix**: usually bold capital or capital: \( \mathbf{F} \) or \( \mathbf{F} \)
  - Dimension = \((m \times n)\)

\[
x = \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \quad A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}
\]

  - **Transpose**: interchange rows and columns

\[
x^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \quad A^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}
\]

Multiplication

- Operands must be conformable
- Multiplication of vector by scalar is associative, commutative, and distributive

\[
a \mathbf{x} = \mathbf{x}a = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix}
\]

\[
a(x + y) = (x + y)a = (ax + ay)
\]

\[
\dim(x) = \dim(y)
\]

\[
a \mathbf{x}^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix}
\]

- Could we add \((\mathbf{x} + \mathbf{a})\)?
- Only if \(\dim(x) = (1 \times 1)\)
Addition

• Conformable vectors and matrices are added term by term

\[
\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix}; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix}
\]

\[
\mathbf{x} + \mathbf{z} = \begin{bmatrix} a + c \\ b + d \end{bmatrix}
\]

Inner Product

• Inner (dot) product of vectors produces a scalar result

\[
\mathbf{x}^T \mathbf{x} = \mathbf{x} \cdot \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

\[
= (x_1^2 + x_2^2 + x_3^2)
\]

• Length (or magnitude) of vector is square root of dot product

\[
= (x_1^2 + x_2^2 + x_3^2)^{1/2}
\]
Vector Transformation

- **Matrix-vector product** transforms one vector into another
- **Matrix-matrix product** produces a new matrix

\[ y = Ax = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \]

\[(n \times 1) = (n \times m)(m \times 1)\]

Derivatives and Integrals of Vectors

- **Derivatives and integrals of vectors** are vectors of derivatives and integrals

\[ \frac{dx}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} \]

\[ \int x \, dt = \begin{bmatrix} \int x_1 \, dt \\ \int x_2 \, dt \\ \int x_3 \, dt \end{bmatrix} \]
Matrix Inverse

Transformation  \[ \mathbf{x}_2 = \mathbf{A}\mathbf{x}_1 \]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_2 =
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_1
\]

Inverse Transformation  \[ \mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_2 \]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_1 =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_2
\]

Matrix Identity and Inverse

- **Identity matrix**: no change when it multiplies a conformable vector or matrix

\[
\mathbf{I}_3 =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
y = \mathbf{I}\mathbf{y}
\]

- **A non-singular square matrix** multiplied by its inverse forms an identity matrix

\[
\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}
\]

\[
\mathbf{A}\mathbf{A}^{-1} =
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
Mathematical Models of Dynamic Systems are Differential Equations

Continuous-time dynamic process:
Vector Ordinary Differential Equation

\[
\frac{dx(t)}{dt} = f[x(t), u(t), w(t), p(t), t]
\]

Output Transformation

\[
y(t) = h[x(t), u(t)]
\]

Measurement with Error

\[
z(t) = y(t) + n(t)
\]

Next Time:

Learning Objectives
Point-Mass Dynamics
Aerodynamic/Thrust Forces

Reading:
Flight Dynamics
Introduction, 1-27
The Earth's Atmosphere, 29-34
Kinematic Equations, 38-53
Forces and Moments, 59-65
Introduction to Thrust, 103-107
Supplemental Material

Flight Dynamics Book and Computer Code

• All programs are accessible from the Flight Dynamics web page
  – http://www.princeton.edu/~stengel/FlightDynamics.html
• ... or directly
• Errata for the book are listed there
• 6-degree-of-freedom nonlinear simulation of a business jet aircraft (MATLAB)
  – http://www.princeton.edu/~stengel/FDcodeB.html
• Linear system analysis (MATLAB)
• Paper airplane simulation (MATLAB)
• Performance analysis of a business jet aircraft (Excel)
  – http://www.princeton.edu/~stengel/Example261.xls
Helpful Resources

- Web pages
  - http://blackboard.princeton.edu/
  - http://www.princeton.edu/~stengel/MAE331.html
- Princeton University Engineering Library (paper and online)
- NACA/NASA pubs
  - http://ntrs.nasa.gov/search.jsp

Course Learning Objectives
(Accreditation Board for Engineering and Technology)

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<thead>
<tr>
<th>Course Learning Objectives</th>
<th>ABET Criterion</th>
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<tbody>
<tr>
<td>Understanding of the dynamics and control of aircraft.</td>
<td>a</td>
</tr>
<tr>
<td>Ability to estimate aerodynamic coefficients and stability derivatives from aircraft geometry and flight envelope.</td>
<td>a, c</td>
</tr>
<tr>
<td>Facility in analyzing mathematical descriptions of the rigid-body motions of flying vehicles.</td>
<td>a</td>
</tr>
<tr>
<td>Ability to estimate the performance, stability, and control characteristics of aircraft.</td>
<td>b</td>
</tr>
<tr>
<td>Development of appreciation for flight-testing methods and results.</td>
<td>b, k</td>
</tr>
<tr>
<td>Ability to apply systems-engineering approach to the analysis, design, and testing of aircraft.</td>
<td>b, c</td>
</tr>
<tr>
<td>Demonstration of ability to work in multidisciplinary teams.</td>
<td>d</td>
</tr>
<tr>
<td>Demonstration of computational problem-solving, through thorough knowledge, application, and development of analytical software.</td>
<td>e, k</td>
</tr>
<tr>
<td>Appreciation of the historical context within which airplanes have evolved to present-day configurations.</td>
<td>f, h, i, j</td>
</tr>
<tr>
<td>Competence in presenting ideas.</td>
<td>g</td>
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