Beamforming in Wireless Relay Networks

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Abstract—This paper is on relay beamforming in wireless networks, in which the receiver has perfect information of all channels and each relay knows its own channels. Instead of the commonly used total power constraint on relays and the transmitter, we use a more practical assumption that every node in the network has its own power constraint. A two-step amplify-and-forward protocol with beamforming is used, in which the transmitter and relays are allowed to adaptively adjust their transmit power and directions according to available channel information. The optimal beamforming problem is solved analytically. The complexity of finding the exact solution is linear in the number of relays. Our results show that the transmitter should always use its maximal power and the optimal power used at a relay is not a binary function. It can take any value between zero and its maximum transmit power. Also, interestingly, this value depends on the quality of all other channels in addition to the relay’s own ones. Despite this coupling fact, distributive strategies are proposed in which, with the aid of a low-rate broadcast from the receiver, a relay needs only its own channel information to implement the optimal power control. Simulated performance shows that network beamforming achieves full diversity and outperforms other existing schemes.

Index Terms—Relay network, beamforming, cooperative diversity

I. INTRODUCTION

It is well-known that due to the fading effect, the transmission over wireless channels suffers from severe attenuation in signal strength. Performance of wireless communication is much worse than that of wired communication. For the simplest point-to-point communication system, which is composed of one transmitter and one receiver only, the use of multiple antennas can improve the system capacity and reliability. Space-time coding and beamforming are among the most successful techniques developed for multiple-antenna systems during the last few decades [1], [2]. However, in many situations, due to the limited size and processing power, it is not practical for some users, especially small wireless mobile devices, to have multiple antennas. Thus, recently, wireless network communication is attracting more and more attention. A large amount of effort has been given to improve the communication by having different users in a network cooperate. This improvement is conventionally addressed as cooperative diversity and the techniques are addressed as cooperative schemes.

Many cooperative schemes have been proposed in the literature [3]–[21]. Some assume channel information at the receiver but not the transmitter and relays, for example, the noncoherent amplify-and-forward protocol in [8], [9] and distributed space-time coding in [10]. Some assume channel information at the receiving side of each transmission, for example, the decode-and-forward protocol in [8], [12] and the coded-cooperation in [13]. Some assume no channel information at any node, for example, the differential transmission schemes proposed independently in [14]–[16]. The coherent amplify-and-forward scheme in [9], [11] assumes full channel information at both relays and the receiver. But only channel phase information is used at relays. In all these cooperative schemes, the relays always cooperate on their highest powers. None of the above pioneer work allow relays to adjust their transmit powers adaptively according to channel magnitude information, and this is exactly the concern of this paper.

There have been several papers on relay networks with adaptive power control. In [22], [23], the outage capacity of networks with a single relay and perfect channel information at all nodes was analyzed. Both work assume a total power constraint on the relay and the transmitter. A decode-and-forward protocol is used at the relay, which results in a binary power allocation between the relay and the transmitter. In [24], performance of networks with multiple amplify-and-forward relays and an aggregate power constraint was analyzed. A distributive scheme for the optimal power allocation is proposed, in which each relay only needs to know its own channels and a real number that can be broadcasted by the receiver. Another related work on networks with one and two amplify-and-forward relays can be found in [25]. In [26], outage minimization of single-relay networks with limited channel-information feedback is performed. It is assumed that there is a long-term power constraint on the total power of the transmitter and the relay. In [38], we analytically solve the relay power control problem for networks with two relays and provide an iterative algorithm for networks with more than two relays. In this paper, we consider networks with a general number of amplify-and-forward relays and we assume a separate power constraint on each relay and the transmitter. The analytical power control solution for networks with any number of relays is found. Due to the difference in the power assumptions, compared to [24], analysis of this new model is more difficult and totally different results are obtained.

For multiple-antenna systems, when there is no channel information at the transmitter, space-time coding can achieve full diversity [1]. If the transmitter has perfect or partial channel information, performance can be further improved through beamforming since it takes advantage of the channel information (both direction and strength) at the transmitter side to obtain higher receive SNR [2]. With perfect channel information or high quality channel information at the transmitter, one-dimensional beamforming is proved optimal [2], [27], [28]. The more practical multiple-antenna systems with partial channel information at the transmitter, channel statistics or quantized instantaneous channel information, are
also analyzed extensively [29]–[33]. In many situations, appropriate combination of beamforming and space-time coding outperforms either one of the two schemes alone [34]–[37]. In this paper, we will see similar performance improvement in networks using network beamforming over distributed space-time coding and other existing schemes such as best-relay selection and coherent amplify-and-forward.

We consider networks with one pair of transmitter and receiver but multiple relays. The receiver knows all channels and every relay knows its own channels perfectly. A two-step amplify-and-forward protocol is used, where in the first step, the transmitter sends information and in the second step, the relays transmit. First, it is proved that the transmitter should always use its full power. Then, the optimal relay beamforming vector is found analytically. The exact solution can be obtained with a complexity that is linear in the number of relays. To perform network beamforming, we propose two distributive strategies in which a relay needs only its own channel information and a low-rate feedback from the receiver, which is common to all relays. Simulation shows that network beamforming outperforms other existing schemes. We should clarify that only amplify-and-forward is considered here. For decode-and-forward, the result may be different and it depends on the details of the coding schemes.

The paper is organized as follows. In the next section, the relay network model and the idea of network beamforming are introduced. In Section III, the network beamforming problem is solved analytically. Then in Section IV, we propose two schemes to implement the optimal relay beamforming distributively in networks. Simulated performance of network beamforming and its comparison to other schemes are shown in Section V. Section VI contains the conclusion and several future directions.

**Notation:** For a matrix $A$, $A^T$ and $A^*$ denote the transpose and Hermitian of $A$. $(\cdot, \cdot)$ indicates the inner product. $\| \cdot \|$ indicates the 2-norm. $P$ indicates the probability. $a_i$ denotes the $i$th coordinate of vector $a$ and $a_{i_1, \ldots, i_k}$ denotes the $k$-dimensional vector $[a_{i_1}, \ldots, a_{i_k}]^T$. If $a, b$ are two $R$-dimensional vectors, $a \preceq b$ means $a_i \leq b_i$ for all $i = 1, \ldots, R$. $0_R$ is the $R$-dimensional vector with all zero entries.

**II. NETWORK MODEL AND PROBLEM STATEMENT**

Consider a relay network with one transmit-and-receive pair and $R$ relays as depicted in Fig. 1. Every relay has one single antenna which can be used for both transmission and reception. Denote the channel from the transmitter to the $i$th relay as $f_i$ and the channel from the $i$th relay to the receiver as $g_i$. We assume that the $i$th relay knows its own channels $f_i$ and $g_i$ and the receiver knows all channels $f_1, \ldots, f_R$ and $g_1, \ldots, g_R$. Our results are valid for any channel statistics. We assume that for each transmission, the powers used at the transmitter and the $i$th relay are no larger than $P_0$ and $P_i$, respectively. Note that in this paper, only short-term power constraint is considered, that is, there is an upper bound on the average transmit power of each node for each transmission. A node cannot save its power to favor transmissions with better or worse channel realizations.

We use a two-step amplify-and-forward protocol. During the first step, the transmitter sends $\alpha_0 \sqrt{P_0}$ $s$. The information symbol $s$ is selected randomly from the codebook $S$. If we normalize it as $E|s|^2 = 1$, the average power used at the transmitter is $\alpha_0^2 P_0$. The $i$th relay receives

$$r_i = \alpha_0 \sqrt{P_0} f_i s + v_i,$$

where $v_i$ is the noise at the $i$th relay. We assume that it is $\mathcal{CN}(0, 1)$. During the second step, the $i$th relay sends

$$t_i = \alpha_i e^{j \theta_i} \frac{\sqrt{T_i}}{\sqrt{1 + \alpha_0^2 |f_i|^2 P_0}} r_i.$$

Note that, in contrast with the traditional amplify-and-forward protocol, in which

$$t_i = \frac{\sqrt{T_i}}{\sqrt{1 + \alpha_0^2 |f_i|^2 P_0}} r_i,$$

the coefficient $\alpha_i$ and the angle $\theta_i$ are used to adjust the transmit power and direction of Relay $i$. The average transmit power of the $i$th relay can be calculated to be $\alpha_0^2 P_i$. The received signal at the receiver can thus be calculated to be

$$x = \alpha_0 \sqrt{P_0} \left( \sum_{i=1}^{R} \frac{\alpha_i e^{j \theta_i} f_i g_i \sqrt{T_i}}{\sqrt{1 + \alpha_0^2 |f_i|^2 P_0}} \right) s + \sum_{i=1}^{R} \frac{\alpha_i e^{j \theta_i} g_i \sqrt{T_i}}{\sqrt{1 + \alpha_0^2 |f_i|^2 P_0}} v_i + w, \quad (1)$$

where $w$ is the noise at the receiver. It is also assumed to be $\mathcal{CN}(0, 1)$. The total average power in transmitting one symbol is thus no larger than $\sum_{i=0}^{R} P_i$.

Equation (1) can be rewritten as

$$x = \alpha_0 \sqrt{P_0} h x s + v,$$

where

$$x = \left[ \begin{array}{c} \alpha_1 e^{j \theta_1} \\ \vdots \\ \alpha_R e^{j \theta_R} \end{array} \right]^T$$

is the relay beamforming vector,

$$h = \left[ \begin{array}{c} f_1 g_1 \sqrt{T_1} \\ \sqrt{1 + \alpha_0^2 |f_1|^2 P_0} \\ \vdots \\ f_R g_R \sqrt{T_R} \\ \sqrt{1 + \alpha_0^2 |f_R|^2 P_0} \end{array} \right]$$

is the equivalent channel vector, and

$$v = \sum_{i=1}^{R} \frac{\alpha_i g_i e^{j \theta_i} \sqrt{T_i}}{\sqrt{1 + \alpha_0^2 |f_i|^2 P_0}} v_i + w$$

is the equivalent noise.

Our network beamforming design is thus the design of $\alpha_0$ and $x$, or equivalently, $\theta_1, \ldots, \theta_R$ and $\alpha_0, \alpha_1, \ldots, \alpha_R$, such
that the error rate of the network is the smallest. This is equivalent to maximize the receive SNR.

The system equation (2) has the same structure as the system equation of beamforming in a multiple-antenna system with \( R \) transmit antennas and one receive antenna. The later has been solved analytically and the optimal beamforming vector is \( \hat{x}_{\text{opt}} = h^*/\|h\| \). However, there are two main differences between relay networks and multiple-antenna systems. First, in networks, the relays have only noisy versions of the transmit signal. Thus, the information is corrupted by noises from both the receiver and the relays. Second, transmissions in relay networks usually take more than one hop, which results in totally different SNR or capacity calculations. We can see that both the equivalent channel vector \( h \) and the equivalent noise \( v \) in (2) have more complicated natures than those in multiple-antenna systems. Also, the noise \( v \) depends on the beamforming vector \( x \). Thus, the network beamforming problem is much more difficult.

III. ANALYTICAL BEAMFORMING RESULT

In this section, we present the analytical network beamforming result.

Since \( v_i \)'s are i.i.d. \( \mathcal{CN}(0,1) \), the values of the angles \( \theta_i \) have no effect on the noise power. To maximize the signal power, the \( R \) components of the first term in the right hand side of (1) should add coherently. Thus, an optimal choice of the angles is \( \theta_i = -\arg(f_i + arg \, g_i) \). That is, match filters should be used at relays to cancel the phases of their channels and form a beam at the receiver. With this choice, we have

\[
x = a_0 \sqrt{P_0} \left( \sum_{i=1}^{R} \frac{\alpha_i |f_i g_i| \sqrt{P_i}}{\sqrt{1 + \alpha_i^2 |f_i|^2 P_i}} \right)^8 \\
+ \sum_{i=1}^{R} \frac{\alpha_i |g_i| \sqrt{P_i}}{\sqrt{1 + \alpha_i^2 |f_i|^2 P_i}} e^{-j \arg f_i v_i} + w_2.
\] (3)

What is left of our problem is the optimal power control, i.e., the optimization of \( \alpha_0, \alpha_1, \ldots, \alpha_R \). This is also the main contribution of our work.

From (3), the receive SNR can be calculated to be

\[
\alpha_0 P_0 \left( \sum_{i=1}^{R} \frac{\alpha_i |f_i g_i| \sqrt{P_i}}{\sqrt{1 + \alpha_i^2 |f_i|^2 P_i}} \right)^2 \\
1 + \sum_{i=1}^{R} \frac{\alpha_i^2 |g_i|^2 P_i}{\sqrt{1 + \alpha_i^2 |f_i|^2 P_i}}.
\]

It is an increasing function of \( \alpha_0 \). Therefore, the transmitter should always use its maximal power, i.e., \( \alpha_0 = 1 \). The receive SNR is thus:

\[
P_0 \left( \sum_{i=1}^{R} \frac{\alpha_i |f_i g_i| \sqrt{P_i}}{\sqrt{1 + \alpha_i^2 |f_i|^2 P_i}} \right)^2 \\
1 + \sum_{i=1}^{R} \frac{\alpha_i^2 |g_i|^2 P_i}{\sqrt{1 + \alpha_i^2 |f_i|^2 P_i}}.
\]

To show the importance of adaptive relay power control, we give a simple two-relay network example found by computer simulation. Let \( P_0 = P_1 = P_2 = 10 \)dB and the channel values are \( |f_1| = 0.4729, \ |f_2| = 1.6420, \ |g_1| = 2.3517, \) and \( |g_2| = 0.7297 \). In Table I, the receive SNR values of different schemes are shown as well as their values of \( (\alpha_1, \alpha_2) \). We can see that in this example, the receive SNR of network beamforming is about 68\% and 52\% larger than those of no relay power control (both relays use full power) and the better relay selection, respectively.

Now let us work on the SNR maximization. We first introduce some notation to help the presentation. With a slight abuse of notation, we define our vector of unknown power coefficients as

\[
x = [\alpha_1 \ldots \alpha_R]^T.
\]

Also define

\[
b = \left[ \frac{|f_1 g_1| \sqrt{P_1}}{\sqrt{1 + |f_1|^2 P_1}} \ldots \frac{|f_R g_R| \sqrt{P_R}}{\sqrt{1 + |f_R|^2 P_R}} \right],
\]

\[
a = \left[ \frac{|g_1| \sqrt{P_1}}{\sqrt{1 + |f_1|^2 P_1}} \ldots \frac{|g_R| \sqrt{P_R}}{\sqrt{1 + |f_R|^2 P_R}} \right],
\]

and \( A = \text{diag}(a) \), where \( \text{diag}(a) \) is the diagonal matrix whose \( i \)th diagonal entry is \( a_i \). Let

\[
\Lambda = \{ v \in \mathbb{R}^R | 0 \leq v \leq a \}
\]

and for \( i = 1, \ldots, R \) and \( 1 \leq i_1 < \cdots < i_k \leq R \), let

\[
\Lambda_{i_1, \ldots, i_k} = \{ v \in \mathbb{R}^k | 0 \leq v \leq a_{i_1, \ldots, i_k} \}.
\]

With the transformation \( y = Ax \), or equivalently, \( x = A^{-1} y \), we have

\[
\text{SNR} = P_0 \frac{\langle b, x \rangle^2}{1 + ||Ax||^2} = P_0 \frac{\langle c, y \rangle^2}{1 + ||y||^2},
\]

where

\[
c = A^{-T} b = [ |f_1| \ldots |f_R| ]^T.
\]

The receive SNR maximization problem is thus equivalent to

\[
\max_{y} \frac{\langle c, y \rangle^2}{1 + ||y||^2} \quad \text{s.t.} \quad y \in \Lambda.
\] (4)

The difficulty of the problem lies in the shape of the feasible set. If \( y \) is constrained on a hypersphere, that is, \( ||y|| = r \), the solution is obvious at least geometrically. Given that \( ||y|| = r \),

\[
\frac{\langle c, y \rangle^2}{1 + ||y||^2} = \frac{r^2 ||c||^2}{1 + r^2 \cos^2 \varphi},
\]

where \( \varphi \) is the angle between \( c \) and \( y \). The optimal solution should be the vector which has the smallest angle with \( c \). This provides us an idea to solve the problem. For any given \( r \), we first fix the length of \( y \) as \( r \) and find the optimal direction of \( y \). Then, we can solve the problem by finding the optimal length. The feasible interval for the length of \( y \) is \( [0, ||a||] \).

Due to the shape of the region, we need to decompose this interval into \( R \) smaller ones. This requires a relay ordering as follows.
Since $P(a_i > 0) = 1$ and $P(c_i > 0) = 1$, we assume that $a_i > 0$ and $c_i > 0$. Define
\[ \phi_j = \phi(f_j, g_j, P_j) = \frac{c_j}{a_j} = \frac{|f_j| \sqrt{1 + |f_j|^2 P_0}}{|g_j| \sqrt{P_j}}, \] (5)
for $i = 1, \ldots, R$ and, for the sake of presentation, define $\phi_{R+1} = 0$. Order $\phi_j$ as
\[ \phi_{\tau_1} \geq \phi_{\tau_2} \geq \cdots \geq \phi_{\tau_R} \geq \phi_{\tau_{R+1}}. \] (6)
$(\tau_1, \tau_2, \ldots, \tau_R, \tau_{R+1})$ is thus an ordering of $(1, 2, \ldots, R, R + 1)$ and $\tau_{R+1} = R + 1$. Define
\[ r_0 = 0, \]
\[ r_1 = \phi_{\tau_1}^{-1} \|e\| = \sqrt{\phi_{\tau_1}^{-2} \|c_{\tau_1}, \ldots, \tau_R\|^2 + a_{\tau_1}^2}, \]
\[ r_2 = \phi_{\tau_2}^{-2} \|c_{\tau_2}, \ldots, \tau_R\|^2 + a_{\tau_2}^2 = \phi_{\tau_2}^{-2} \|c_{\tau_2}, \ldots, \tau_R\|^2 + \sum_{i = 1}^{2} a_{\tau_i}^2, \]
\[ \vdots \]
\[ r_{R-1} = \phi_{\tau_{R-1}}^{-2} \|c_{\tau_{R-1}, \tau_R}\|^2 + \sum_{i = 1}^{R-2} a_{\tau_i}^2, \]
\[ r_R = \phi_{\tau_R}^{-2} \|c_{\tau_R}\|^2 + \sum_{i = 1}^{R-1} a_{\tau_i}^2 = \|a\|. \]
Since $\phi_{\tau_{j-1}} \geq \phi_{\tau_j}$, we have $r_{j-1} \leq r_j$ for $j = 1, \ldots, R$. Thus, the interval $[0, \|a\|]$ is decomposed into the following $R$ intervals:
\[ [0, \|a\|] = [r_0, r_1] \cup [r_1, r_2] \cup \cdots \cup [r_{R-1}, r_R]. \]
Let $\Gamma_i = [r_i, r_{i+1}]$. The optimization problem in (4) is equivalent to
\[ \max_{i = 1, \ldots, R} \max_{r \in \Gamma_i} \frac{1}{1 + r^2} \left( \max_{\|y\| = r} \langle c, y \rangle \right)^2. \]
It can be solved through the following three steps.

- **Step 1:** For a given length $r$ in $\Gamma_i$, find the optimal direction of $y \in A$, i.e., solve the inner product maximization problem:
\[ z^{(i,r)} = \max_{\|y\| = r} \langle c, y \rangle. \] (7)

- **Step 2:** Find the optimal length inside $\Gamma_i$, i.e., solve the ith subproblem:
\[ y^{(i)} = \max_{r \in \Gamma_i} \frac{1}{1 + r^2} \left( \max_{\|y\| = r} \langle c, y \rangle \right)^2 \]
(8)
by solving
\[ \max_{r \in \Gamma_i} \frac{\langle c, z^{(i,r)} \rangle^2}{1 + r^2}. \]

- **Step 3:** Search over the $R$ solutions of the $R$ subproblems and find the optimal solution for the whole problem.

The following lemmas and theorem solve each step and thus the whole problem.

**Lemma 1:** $z^{(i,r)}_r = a_j$ for $j = \tau_1, \ldots, \tau_i$, or equivalently, $z_{\tau_1, \ldots, \tau_i} = a_{\tau_1, \ldots, \tau_i}$.
**Proof:** We prove this lemma by contradiction. Assume that $z^{(i,r)}_r < a_j$ for some $j \in \{\tau_1, \ldots, \tau_i\}$. We first show that there exists an $\ell \in \{\tau_{i+1}, \ldots, \tau_R\}$ such that $\frac{z^{(i,r)}_r}{c_j} < \frac{z^{(i,r)}_\ell}{c_j}$. Assume that $\frac{z^{(i,r)}_j}{c_j} \geq \frac{z^{(i,r)}_\ell}{c_j}$ for all $m \in \{\tau_{i+1}, \ldots, \tau_R\}$. We have $z^{(i,r)}_m \leq c_m \frac{z^{(i,r)}_j}{c_j} < c_m \frac{a_j}{c_j} = c_m \phi_j^{-1}$. Thus,
\[ \|z^{(i,r)}\| = \sqrt{\sum_{m=1}^{i} (z^{(i,r)}_m)^2} + \sum_{m=i+1}^{R} (z^{(i,r)}_m)^2 < \sqrt{\sum_{m=1}^{i} a^2_m + \sum_{m=i+1}^{R} c^2_m \phi_j^{-2}} \]
\[ = \phi_j^{-2} \|c_{\tau_1, \ldots, \tau_R}\|^2 + \sum_{m=1}^{i} a^2_m \]
\[ \leq \phi_j^{-2} \|c_{\tau_1, \ldots, \tau_R}\|^2 + \sum_{m=1}^{i} a^2_m \]
because of (6).
This contradicts $\|z^{(i,r)}\| = r \in \Gamma_i$. Thus, there exists an $\ell \in \{\tau_{i+1}, \ldots, \tau_R\}$ such that $\frac{z^{(i,r)}_\ell}{c_j} < \frac{z^{(i,r)}_\ell}{c_j}$.
Define another vector $z'$ as $z'_j = z^{(i,r)}_j + \delta$, $z'_m = \sqrt{(z^{(i,r)}_m)^2 - 2\delta z^{(i,r)}_m - \delta^2}$, and $z'_m = z^{(i,r)}_m$ for $m \neq i, \ell$, where
\[ 0 < \delta < \min \left\{ 2c_j \left( 1 + \frac{c^2_j}{c^2_l} \right)^{-1} \left( \frac{z^{(i,r)}_j}{c_j} - \frac{z^{(i,r)}_\ell}{c_j} \right), \right. \]
\[ \left. \sqrt{(z^{(i,r)}_j)^2 + (z^{(i,r)}_\ell)^2} - z^{(i,r)}_j, a_j - z^{(i,r)}_j \right\}. \]
Since we have assumed that $z^{(i,r)}_j < a_j$, it is clear that $z'_j < a_j$ and have just proved that $\frac{z^{(i,r)}_\ell}{c_j} < \frac{z^{(i,r)}_\ell}{c_j}$. Such $\delta$ is achievable. To contradict the assumption that $z^{(i,r)}$ is the optimal, it is enough to prove the following two items:

1) $z'$ is a feasible point, i.e., $\|z'\| = r$ and $z' \in A$,
2) $\langle c, z^{(i,r)} \rangle < \langle c, z' \rangle$.

From the definition of $z'$, we have
\[ \|z'\|^2 = (z'_j)^2 + (z'_\ell)^2 + \sum_{m \neq j, \ell} (z'_m)^2 \]
\[ = (z^{(i,r)}_j + \delta)^2 + (z^{(i,r)}_\ell)^2 - 2\delta z^{(i,r)}_j - \delta^2 + \sum_{m \neq j, \ell} (z^{(i,r)}_m)^2 \]
\[ = \|z^{(i,r)}\|^2 = r^2. \]
Since $0 < \delta < a_j - z^{(i,r)}_j$, we have $0 < z'_j < a_j$. Also, since $0 < \delta < \sqrt{(z^{(i,r)}_j)^2 + (z^{(i,r)}_\ell)^2 - z^{(i,r)}_j}$, we can easily prove that $z'_\ell = \sqrt{(z^{(i,r)}_\ell)^2 - 2\delta z^{(i,r)}_\ell - \delta^2} > 0$ and $z'_\ell < z^{(i,r)}_\ell < a_j$. Thus, $z' \in A$. The first item has been proved. For the second item, since $\delta < 2c_j \left( 1 + \frac{c^2_j}{c^2_l} \right)^{-1} \left( \frac{z^{(i,r)}_j}{c_j} - \frac{z^{(i,r)}_\ell}{c_j} \right)$, we
have
\[
\left(1 + \frac{c_j^2}{c_i^2}\right) \delta^2 < 2c_i \left(z_{j,i}^{(r)} - z_{j,i}^{(s)}\right) \delta = 2 \left(z_{j,i}^{(r)} c_j - z_{j,i}^{(s)} c_{j,i}\right) \Rightarrow \left(z_{j,i}^{(r)}\right)^2 + \frac{c_j^2 \delta^2 - 2c_j z_{j,i}^{(r)} \delta}{c_i^2} < \left(z_{j,i}^{(s)}\right)^2 - 2z_{j,i}^{(s)} \delta - \delta^2 = z_i^2
\]
\[
\Rightarrow c_j z_{j,i}^{(r)} - c \delta < c z_{j,i}^{(s)}
\]
\[
\Rightarrow \langle c, z_{j,i}^{(r)} \rangle < \langle c, z \rangle.
\]

\begin{proof}
\end{proof}

\textbf{Lemma 2:} \(z_{j,i}^{(r)} = \lambda^{(i,r)} c_j\) for \(j = \tau_{i+1}, \ldots, \tau_R\), or equivalently, \(z_{j,i}^{(r)} = \lambda^{(i,r)} c_{\tau_{i+1}, \ldots, \tau_R}\). where
\[
\lambda^{(i,r)} = \frac{\sqrt{y^2 - \sum_{m=1}^{i} a_{m,i}^2}}{\|c_{\tau_{i+1}, \ldots, \tau_R}\|}.
\]

\textbf{Proof:} From Lemma 1, \(z_{j,i}^{(r)} = a_j\) for \(j = \tau_1, \ldots, \tau_i\). Thus, (7) can be written as
\[
\max_{\|y\| = \Gamma} \sum_{m=1}^{i} a_{m,i} c_{m,i} + \langle c_{\tau_{i+1}, \ldots, \tau_R}, y_{\tau_{i+1}, \ldots, \tau_R}\rangle
\]
\[
= \sum_{m=1}^{i} b_{m,i} + \max_{\|y\| = \Gamma} \langle c_{\tau_{i+1}, \ldots, \tau_R}, y_{\tau_{i+1}, \ldots, \tau_R}\rangle.
\]

It is obvious that
\[
\langle c_{\tau_{i+1}, \ldots, \tau_R}, y_{\tau_{i+1}, \ldots, \tau_R}\rangle \leq \langle c_{\tau_{i+1}, \ldots, \tau_R}, \lambda^{(i,r)} c_{\tau_{i+1}, \ldots, \tau_R}\rangle
\]
for all \(\|y\| = \Gamma\). In other words, to maximize the inner product, \(y_{\tau_{i+1}, \ldots, \tau_R}\) should have the same direction as \(c_{\tau_{i+1}, \ldots, \tau_R}\). Thus, we only need to show that this direction is feasible for \(r \in \Gamma_i\). This is equivalent to show that \(\lambda^{(i,r)} c_{\tau_{i+1}, \ldots, \tau_R} \in \Lambda_{\tau_{i+1}, \ldots, \tau_R}\) for any \(r \in \Gamma_i\).

We can easily prove that
\[
r \in \Gamma_i \Rightarrow \lambda^{(i,r)} \in \Omega_i,
\]
where \(\Omega_i = [\phi^{-1}_{\tau_{i+1}}, \phi^{-1}_{\tau_i}]\) for \(i = 0, \ldots, R - 1\). Thus, for any \(r \in \Gamma_i\) and \(j = \tau_{i+1}, \ldots, \tau_R\), we have
\[
0 \leq \lambda^{(i,r)} c_j \leq \phi^{-1}_{\tau_{i+1}} c_j \leq \phi^{-1}_{\tau_i} c_j = a_j.
\]

Hence, \(\lambda^{(i,r)} c_{\tau_{i+1}, \ldots, \tau_R} \in \Lambda_{\tau_{i+1}, \ldots, \tau_R}\). Combining Lemma 1 and Lemma 2, we have
\[
z_{j,i}^{(r)} = \begin{cases} a_j & j = \tau_1, \ldots, \tau_i \\ \lambda^{(i,r)} c_j & j = \tau_{i+1}, \ldots, \tau_R \end{cases}
\]
and thus
\[
\max_{\|y\| = \Gamma} \langle c, y \rangle = \sum_{m=1}^{i} b_{m,i} + \lambda^{(i,r)} \|c_{\tau_{i+1}, \ldots, \tau_R}\|. \quad (10)
\]

We have solved the inner product maximization problem of Step 1. The solutions of the \(R\) subproblems can thus be obtained.

\textbf{Lemma 3:} For \(i = 1, \ldots, R - 1\), define
\[
\lambda_i = \frac{1 + \sum_{m=1}^{i} a_{m,i}^2}{\sum_{m=1}^{i} b_{m,i}}.
\]

The solution of Subproblem 0 is \(y^{(0)} = \phi^{-1}_{\tau+1} c\). For \(i = 1, \ldots, R - 1\), the solution of Subproblem \(i\), \(y^{(i)}\), is defined as
\[
y^{(i)} = \begin{cases} a_j & j = \tau_1, \ldots, \tau_i \\ \min \{\lambda_i, \phi^{-1}_{\tau_{i+1}}\} c_j & j = \tau_{i+1}, \ldots, \tau_R \end{cases}. \quad (11)
\]

\textbf{Proof:} From (10), Subproblem \(i\) is equivalent to the following 1-dimensional optimization problem:
\[
\max_{\lambda \in \Omega_i} \frac{\sum_{m=1}^{i} b_{m,i} + \|c_{\tau_{i+1}, \ldots, \tau_R}\|^2}{\lambda^2}.
\]

When \(i = 0\), (12) is equivalent to \(\max_{\lambda \in \Omega_0} \frac{\|c\|^2}{1 + \|\|c\|\|^2} \lambda^2\). Since \(\frac{\|c\|^2}{1 + \|\|c\|\|^2}\lambda^2\) is an increasing function of \(\lambda\), its maximum is at \(\lambda = \phi^{-1}_{\tau_1}\).

For \(i = 1, \ldots, R - 1\), define
\[
\xi_i(\lambda) = \frac{\sum_{m=1}^{i} b_{m,i} + \|c_{\tau_{i+1}, \ldots, \tau_R}\|^2}{\lambda^2}.
\]

We have,
\[
\frac{\partial \xi_i}{\partial \lambda} = \frac{2 \left(\sum_{m=1}^{i} b_{m,i} + \|c_{\tau_{i+1}, \ldots, \tau_R}\|^2\right)}{\left(1 + \sum_{m=1}^{i} a_{m,i}^2 + \|c_{\tau_{i+1}, \ldots, \tau_R}\|^2\right)^2} \cdot \frac{\partial \Omega_i}{\partial \lambda}.
\]

Thus, \(\frac{\partial \xi_i}{\partial \lambda} > 0\) if \(\lambda < \lambda_i\) and \(\frac{\partial \xi_i}{\partial \lambda} < 0\) if \(\lambda > \lambda_i\). So, if \(\lambda_i \leq \phi^{-1}_{\tau_{i+1}}\), the optimal solution is reached at \(\lambda = \lambda_i\). Otherwise, the optimal solution is reached at \(\lambda = \phi^{-1}_{\tau_{i+1}}\).

From (9), Subproblem \(i\) is solved at \(y^{(i)}\) as defined in (11).

Now, we are ready for solution of the relay power control problem in (4).

\textbf{Theorem 1:} Define \(x^{(i)}\) as
\[
x^{(i)} = \begin{cases} 1 & j = \tau_{i+1}, \ldots, \tau_R \\ \phi^{-1}_{\tau_{i+1}} & j = \tau_{i+1}, \ldots, \tau_R \end{cases}. \quad (13)
\]

The solution of the SNR optimization is \(x^{(i_0)}\), where \(i_0\) is the smallest \(i\) such that \(\lambda_i < \phi^{-1}_{\tau_{i+1}}\).

\textbf{Proof:} First, since \(\phi_{\tau+1} = \phi^{-1}_{\tau+1} = \phi^{-1}_{\tau+1} = \infty\). Thus, \(i_0\) exists. Also, since \(\lambda_{i_0} < \phi^{-1}_{\tau_{i+1}}\), and \(\phi^{-1}_{\tau_{i+1}}\) decreases with \(j\), we have \(x^{(i)} \leq 1\) for \(j = \tau_{i+1}, \ldots, \tau_R\). This means that \(x^{(i_0)}\) is in the feasible region of the optimization problem.

Denote
\[
\eta(y) = \frac{\langle c, y \rangle^2}{1 + \|y\|^2}.
\]

Note that \(\|y^{(0)}\| = r_1\). Since \(r_1 \in \Gamma_1\), \(y^{(0)}\) is also a feasible point of Subproblem 1. Thus, \(\eta(y^{(0)}) \leq \eta(y^{(1)})\) due to the optimality of \(y^{(1)}\) in Subproblem 1. This means that there is no need to consider Subproblem 0. For \(i = 1, \ldots, R - 2\), if \(\lambda_i \geq \phi^{-1}_{\tau_{i+1}}\),
\[
y^{(i)} = \begin{cases} a_j & j = \tau_1, \ldots, \tau_i \\ \phi^{-1}_{\tau_{i+1}} c_j & j = \tau_{i+1}, \ldots, \tau_R \end{cases}.
\]
\[ \| \mathbf{y}^{(i)} \| = \sqrt{\phi_{r_{i+1}}^2 \| \mathbf{c}_{r_{i+1}, \ldots, r_R} \|^2 + \sum_{j=1}^{i} \lambda_j^2} = r_{i+1}. \]

Since \( r_{i+1} \in \Gamma_{i+1} \), \( \mathbf{y}^{(i)} \) is a feasible point of Subproblem \( i + 1 \). Thus, \( \eta(\mathbf{y}^{(i)}) \leq \eta(\mathbf{y}^{(i+1)}) \) due to the optimality of \( \mathbf{y}^{(i+1)} \) in Subproblem \( i + 1 \). This means that there is no need to consider Subproblem \( i + 1 \). Thus, we only need to check those \( \mathbf{y}^{(i)} \)'s with \( \lambda_i < \phi_{r_{i+1}}^{-1} \), and find the one that results in the largest receive SNR. From the definition in (13), this is the same as to check those \( \mathbf{x}^{(i)} \)'s with \( \lambda_i < \phi_{r_{i+1}}^{-1} \).

Now, we prove that \( \lambda_{i+1} < \phi_{r_{i+2}}^{-1} \) if \( \lambda_i < \phi_{r_{i+1}}^{-1} \). First, from \( \lambda_i < \phi_{r_{i+1}}^{-1} \), we have

\[ 1 + \sum_{m=1}^{i+1} a_{r_m}^2 \sum_{m=1}^{i+1} b_{r_m} < \phi_{r_{i+1}}^{-1}. \]

Thus, we only need to check those \( \mathbf{x}^{(i)} \)'s for \( i_0 \leq i \leq R \) and find the one causing the largest receive SNR. From previous discussion, \( i_0 \geq 1 \).

Define \( SNR_i = \frac{(b, \mathbf{x}^{(i)})^2}{1 + \| \mathbf{c}_{r_{i+1}, \ldots, r_R} \|^2 \lambda_i^2} \). Now, we prove that \( SNR_i > SNR_{i+1} \) for \( i_0 \leq i \leq R \). From the proof of Lemma 3, we have

\[ SNR_i = \frac{\left( \sum_{m=1}^{i} b_{r_m} + \| \mathbf{c}_{r_{i+1}, \ldots, r_R} \| \lambda_i \right)^2}{1 + \sum_{m=1}^{i+1} a_{r_m}^2 + \sum_{m=1}^{i+1} b_{r_m}^2 \lambda_i^2} \]

\[ = \sum_{m=1}^{R} \lambda_i^{-1} \left( \sum_{m=1}^{i} b_{r_m} \right)^2 + \sum_{m=1}^{i+1} a_{r_m}^2 \lambda_i^{-1} \left( \sum_{m=1}^{i+1} b_{r_m} \right)^2 \]

\[ = SNR_{i+1} + \frac{b_{r_{i+1}}^2}{\phi_{r_{i+1}}^2} + \frac{\left( \sum_{m=1}^{i} b_{r_m} \right)^2}{1 + \sum_{m=1}^{i+1} a_{r_m}^2} \lambda_i^{-1} \left( \sum_{m=1}^{i+1} b_{r_m} \right)^2 \]

\[ > SNR_{i+1}. \]

Thus, the optimal power control vector that maximizes the receive SNR is \( \mathbf{x}^{(i_0)} \).

**IV. DISTRIBUTED SCHEMES FOR NETWORK BEAMFORMING**

It is natural to expect the power control at relays to undergo an on-or-off scenario: a relay uses its full power if its channels are good enough and otherwise not to cooperate at all. Our result shows otherwise. The optimal power used at a relay can be any value between 0 and its maximal power. In many situations, a relay should use a fraction of its power, whose value is determined not only by its own channels but all others’ as well. This is because every relay has two effects on the transmission. For one, it helps the transmission by forwarding the information, while for the other, it harms the transmission by forwarding noise as well. Its transmit power has a non-linear effect on the powers of both the signal and the noise, which makes the optimization solution not an on-or-off one, not a decoupled one, and, in general, not even a differentiable function of channel coefficients.

As shown in Theorem 1, the fraction of power used at relay \( j \) satisfies \( \alpha_j = 1 \) for \( j = \tau_1, \ldots, \tau_{i_0} \) and \( \alpha_j = \lambda_{i_0} \phi_j \) for \( j = \tau_{i_0+1}, \ldots, \tau_R \). Thus, the \( i_0 \) relays whose \( \phi_j \)'s are the largest use their full power. Since \( i_0 \geq 1 \), there is at least one relay that uses its full power. This tells us that the relay with the largest \( \phi \) always uses its full power. The remaining \( R - i_0 \) relays whose \( \phi_j \)'s are smaller only use parts of their power. For \( j = \tau_{i_0+1}, \ldots, \tau_R \), the power used at the \( j \)th relay is

\[ \alpha_j^2 P_j = \lambda_{i_0} \phi_j^2 P_j = \lambda_{i_0} |f_j/g_j|^2 (1 + |f_j|^2 P_0), \]

which is proportional to \( |f_j/g_j|^2 (1 + |f_j|^2 P_0) \) since \( \lambda_{i_0} \) is a constant for each channel realization. Although \( P_j \) does not appear explicitly in the formula, it affects the decision of whether the \( j \)th relay should use its full power. Actually, in determining whether a relay should use its maximal power, not only do the channel coefficients and power constraint at this relay account, but also other channel coefficients and power constraints. The power constraint of the transmitter, \( P_0 \), plays a roll as well.

Due to these special properties of the optimal power control solution, it can be implemented distributively with each relay knowing only its own channel information. In the following, we propose two distributed strategies. One is for networks with a small number of relays, and the other is more economical in networks with a large number of relays.

The receiver, which knows all channels, can solve the power control problem. When the number of relays, \( R \), is small, the receiver broadcasts \( R \) bits \((m_1, \ldots, m_R)\) and the coefficient \( \lambda_{i_0} \). The binary symbol \( m_j \) is to indicate whether Relay \( j \) should use its full power. If \( m_j = 1, \) Relay \( j \) uses its full power to transmit during the second step. Otherwise, it uses power \( \lambda_{i_0} |f_j/g_j|^2 (1 + |f_j|^2 P_0) \). The number of required feedback bits is \( R + B_1 \) where \( B_1 \) is the amount of bits needed in broadcasting the real number \( \lambda_{i_0} \). This number increases linearly in the network size.

The other scheme is to have the receiver broadcast two real numbers: \( \lambda_{i_0} \) and a real number \( d \) that satisfies \( \phi_{\tau_{i_0}} > d > \phi_{\tau_{i_0+1}} \). Relay \( j \) calculates its own \( \phi_j \). If \( \phi_j > d \), Relay \( j \) uses its full power. Otherwise, it uses power \( \lambda_{i_0} |f_j/g_j|^2 (1 + |f_j|^2 P_0) \). The number of bits needed for this
feedback is $2B_1$. Thus, when $R$ is large, this strategy needs less bits of feedback compared to the first one.

Networks with an aggregate power constraint $P$ on relays were analyzed in [24]. In that case, with the same notation as in Section III, $P_j = P$ and $\sum_{j=1}^R \alpha_j \leq 1$. The optimal relay power allocation solution is

$$\alpha_j = \frac{|f_j g_j| \sqrt{1 + |f_j|^2 P_j}}{\sqrt{\sum_{m=1}^R |f_m g_m|^2 (1 + |f_m|^2 P_m)/P + 1}}.$$

$\alpha_j$ is a function of its own channels $f_j, g_j$ only and an extra coefficient $c = \sqrt{\sum_{m=1}^R |f_m g_m|^2 (1 + |f_m|^2 P_m)/P}$, which is the same for all relays. Therefore, this power allocation can be done distributively with the extra knowledge of one single coefficient $c$, which can be broadcasted by the receiver. In our case, every relay has a separate power constraint. This is a more practical assumption in sensor networks since every sensor or wireless device has its own battery power limit. The power control solutions of the two cases are totally different.

If relay selection is used and only one relay is allowed to cooperate, it can be proved easily that we should choose the relay with the highest

$$h_j = h(f_j, g_j, P_j) = \frac{P_j |f_j g_j|^2}{1 + |f_j|^2 P_0 + |g_j|^2 P_j}.$$

We call $h$ the relay selection function since a relay with a larger $h$ results in a higher receive SNR. While all relays are allowed to cooperate, the concepts of the best relay and relay selection function are not clear. Since the power control problem is a coupled one, it is hard to measure how much contribution a relay has. As discussed before, in network beamforming, a relay with a larger $\phi$ does not necessarily use a larger power or has more contribution. But we can conclude that if $\phi_k > \phi_l$, the fraction of power used at Relay $k$, $\alpha_k$, is no less than the fraction of power used at Relay $l$, $\alpha_l$. It is worth to mention that in network beamforming, relays with large enough $\phi$’s use their full power no matter what their full power is. Actually, it is not hard to see that if at one time channels of all relays are good, every relay should use its full power.

V. Simulation Results

In this section, we show simulated performance of network beamforming and compare it with performance of other existing schemes. Figures 2 and 3 show performance of networks with Rayleigh fading channels and the same power constraint on the transmitter and relays. In other words, $f_i, g_i$ are $CN(0,1)$ and $P_0 = P_1 = \ldots = P_R = P$. The horizontal axis of the figures indicates $P$. In Fig. 2, simulated block error rates of network beamforming with optimal power control are compared to those of best-relay selection, Larsson’s scheme [24] with total relay power $P$, Alamouti distributed space-time coding [10], [40], and amplify-and-forward without power control (every relay uses its maximal power) in a 2-relay network. The information symbol $s$ is modulated as BPSK. We can see that network beamforming outperforms all other schemes. It is about 0.5dB and 2dB better than Larsson’s scheme and best-relay selection, respectively. With perfect channel knowledge at relays, it is 7dB better than Alamouti distributed space-time coding, which needs no channel information at relays. Amplify-and-forward with no power control only achieves diversity 1, distributed space-time coding achieves a diversity slightly less than two, while best-relay selection, network beamforming, and Larsson’s scheme achieve diversity 2. Fig. 3 shows simulated performance of a 3-relay network under different schemes. Similar diversity results are obtained. But for the 3-relay case, network beamforming is about 1.5dB and 3.5dB better than Larsson’s scheme and best-relay selection, respectively.

In Fig. 4, we show performance of a 2-relay network in which $P_0 = P_1 = P$ and $P_2 = P/2$. That is, the transmitter and the first relay have the same power constraint while the second relay has only half the power of the first relay. The channels are assumed to be Rayleigh fading channels. In Fig. 5, we show performance of a 2-relay network whose channels have both fading and path-loss effects. We assume that the distance between the first relay and the transmitter/receiver is 1, while the distance between the second relay and the transmitter/receiver is 2. The path-loss exponent [39] is assumed to be 2. We also assume that the transmitter and
relays have the same power constraint, i.e., $P_0 = P_1 = P_2 = P$. In both cases, distributed space-time coding does not apply, and Larsson’s scheme applies for the second case only. So, we compare network beamforming to best-relay selection and amplify-and-forward with no power control only. Performance of Larsson’s scheme is shown in Fig. 5 as well. Both figures show the superiority of network beamforming to other schemes.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we propose the novel idea of beamforming in wireless relay networks to achieve both diversity and array gain. The scheme is based on a two-step amplify-and-forward protocol. We assume that each relay knows its own channels perfectly. Unlike previous works in network diversity, the scheme developed here uses not only the channels’ phase information but also their magnitudes. Match filters are applied at the transmitter and relays during the second step to cancel the channel phase effect and thus form a coherent beam at the receiver, in the mean while, optimal power control is performed based on the channel magnitudes to decide the power used at the transmitter and relays. The power control problem for networks with any numbers of relays is solved analytically. The solution can be obtained with a complexity that is linear in the number of relays. The power used at a relay depends on not only its own channels nonlinearly but also all other channels in the network. In general, it is not even a differentiable function of channel coefficients. Simulation with Rayleigh fading and path-loss channels show that network beamforming achieves full diversity while amplify-and-forward without power control achieves diversity 1 only. Network beamforming also outperforms other cooperative strategies.

We have just scratched the surface of a brand-new area. There are a lot of ways to extend and generalize this work. First, it is assumed in this work that relays know their channels perfectly, which is not practical in many networks. Network beamforming with limited and delayed feedback from the receiver is an important issue. Second, the relay network probed in this paper has only one pair of transmitter and receiver. When there are multiple transmitter-and-receiver pairs, an interesting problem is how relays should allocate their powers to aid different communication tasks. Finally, the two-hop protocol can be generalized as well. For a given network topology, one relevant question is how many hops should be taken to optimize the criterion at consideration, for example, error rate or capacity.

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