Quantities and Units

As scientists, we need to make quantitative statements about the physical quantities measured in our experiments. Algebra provides the language and grammar to make these statements. In this language the sentences are equations or inequalities whereas the words are symbols. A symbol may stand for a physical quantity or a number; for an operation such as addition or multiplication; or for a relationship such as ‘is equal to’ or ‘is greater than’. Often we use letters such as $x$, $t$, $m$, or $A$ to stand for physical quantities such as distance, time, mass, or area. Symbols can also be special characters such as $+$ for addition, or a combination of letters such as ‘sin’ for the sine function introduced in Chapter 4.

A physical quantity is a combination of a numerical value and a unit, for example a length of 1 m, a time of 2 s, or a mass of 70 kg, where the ‘m’ stands for meter, ‘s’ for second, and ‘kg’ for kilogram. Both are needed; if we change the unit the number changes accordingly. Many of the laws of science are expressed as simple equations relating physical quantities. A familiar example is $F = ma$ where $F$, $m$, and $a$ stand for force, mass, and acceleration. There are various systems for choosing units and conventions for how physical quantities are to be described. In this book we use the Système Internationale (SI) system of units, which has become standard for scientists and engineers throughout the world.

Figure 1.1
The authors, Mike, Steve, and Bill, holding a protein sample destined for the 500 MHz NMR spectrometer in the background. Image courtesy of Mike Aitken, Bill Broadhurst, and Steve Hladky.
1.1 Symbols, operations, relations, and the basic language of mathematics

In the language of mathematics, the words are symbols like \( x, t, m, +, \times, \div, =, > \). Symbols can stand for numbers or for physical quantities; they can indicate operations or they can state relationships like ‘is equal to’ or ‘is greater than’. You first started using many of these symbols back in primary school where you learned what \( + \) and \( \div \) mean. Even then you also used symbols to stand for unknown numbers in exercises like that shown in Figure 1.2.

You may have thought of \( h \) as just a box to tell you where to write the answer, but it can also be regarded as a symbol called ‘box’ that stands for a number whose value is not already known. The equation \( h + 3 = 8 \) tells us a relation between \( h \) and the numbers 3 and 8, and this relation allows us to solve for the value, 5, to be assigned to the variable ‘box’. That really is the crux of using algebra; it allows us to state relations before we know the actual values.

Of course the relations between our symbols are going to be a bit more complicated – but the principle behind the use of algebra is still the same.

Note that whenever algebraic expressions are typeset, the letters used in a symbol are written in italics if the symbol represents either a number or a physical quantity. By contrast plain roman type is used for symbols that represent units or labels. Typographical conventions like these are fiddly but they can be very important. For example, in the equation for the gravitational force on an object at the earth’s surface,

\[
F = mg = m \times 9.8 \text{ m s}^{-2}, \tag{EQ1.1}
\]

\( m \) and \( m \) are completely different. The italic type tells us that \( m \) stands for a physical quantity, mass, which might be expressed in kilograms; the plain roman type for the \( m \) after the 9.8 tells us it stands for the unit, meter.

Symbols and algebra can be used to express very profound notions. For instance \( E \) can represent the total energy of a chunk of matter, \( m \) its mass, and \( c \) the speed of light. Combining these with the symbol for ‘is equal to’ and the notation for raising to a power Einstein wrote
\[ E = mc^2. \]  

That bit of shorthand is a lot more compact and a lot more famous than its equivalent in English, ‘The total energy of an object is equal to its mass multiplied by the square of the speed of light.’ However, and this is the important point for now, the algebra and the English are being used to say exactly the same thing.

Now consider a very simple example. We can say in English:

‘John is thirty centimeters taller than Robert.’  

How can that be converted to an algebraic equation? First rewrite the sentence to emphasize that it is telling us something about heights:

‘The height of John is equal to the height of Robert plus thirty centimeters.’

This sentence is a bit stilted but is still correctly constructed in English. Now introduce some symbols: \( J \) to stand for ‘the height of John’, \( R \) for ‘the height of Robert’, = for ‘is equal to’, + for ‘plus’, and the abbreviation ‘cm’ for ‘centimeter’. This yields the equation

\[ J = R + 30 \text{ cm}. \]

This statement is called an equation because it says that one expression is equal to another. The expressions are symbols combined according to the grammar of algebra. Even though the algebraic grammar is rigid and restricted, the equation is still a sentence that can, with a bit of effort, be read in English.

A more sophisticated approach is to use subscripts. Let \( H \) stand for ‘height’ whereas \( J \) and \( R \) stand for ‘John’ and ‘Robert’, respectively. The subscripted symbols \( H_J \) and \( H_R \) then stand for the height of John and the height of Robert and the sentence becomes the equation

\[ H_J = H_R + 30 \text{ cm}. \]

Note that the subscripts in this case are not italic because they are labels attached to known individuals.

When reading an equation the equals sign is the verb. There are several other verbs used in elementary algebra. For instance \( H_R < H_J \) is a relation that can be read as the height of Robert is less than the height of John. The algebraic verbs you will encounter in this book are: =, is equal to; \( \neq \), is not equal to; \( \equiv \), is identical to (is always equal to for any values of the variables); \( < \), is less than; \( \leq \), is less than or equal to; \( > \), is greater than; \( \geq \), is greater than or equal to; \( \approx \), is approximately equal to (as close as we need); and \( \sim \), is similar to. There are others that are very common in mathematics, for example \( \in \) which means ‘is an element of’, but they are rarely seen in science and so will not be covered here.

Figure 1.3
The fact that John is 30 cm taller than Robert can be represented algebraically as: \( H_J = H_R + 30 \text{ cm}. \)
Using algebraic notation as in EQ1.6 is more compact than saying the same thing in plain English, and it leads directly to a solution for the height of either John or Robert in terms of the other. However, we have paid a high price for this compactness. We have introduced five symbols, an abbreviation, and strange-looking combinations of symbols and labels like $H_J$ and $H_R$. We run the risk that these symbols and expressions will be seen as just jargon, and like all jargon will get in the way of understanding. For the simple example given here this price is too high to pay — it would be much better to tell someone about the heights using plain English. However, that is to miss the main point: when the relations get a bit more complex, the English versions become so clumsy that we would just give up. We must pay the high price to use symbols and algebra because the gains are much greater. Furthermore, obeying the rules of algebra, we can safely manipulate equations to gain insights which would be beyond us by other means.

The rules of algebra will be explained more thoroughly in Chapter 2. There are not many, they are very simple, you have already met them, and they are easy to remember.

We can add zero to any expression without changing its value.
We can multiply any expression by 1 without changing its value.
We can swap the two sides of an equation.
We can add or subtract the same thing to both sides of an equation.
We can multiply both sides of an equation by the same thing.
We can divide both sides of an equation by the same thing (other than zero).
We can replace any expression by another equal expression.

Mathematics as it is taught in school deals with equations and symbols that refer to numbers. In the same spirit, Chapter 2 will review arithmetic, algebra, and solving equations. You will already know much of the information but do not skip it because the story is told in a different way using algebra and its grammar. Thus you can both refresh your understanding of numbers and arithmetic and learn how to use algebra.

The emphasis on numbers and structure in Chapter 2 is fine if you like that sort of thing. However, if that were the only use of algebra most of us as scientists simply would not bother. Why do we bother? Is not arithmetic with a calculator enough? In a sense the answer to that is given throughout the rest of this book, but something a little less sweeping should be said here at the beginning.

### 1.2 Physical quantities and physical value equations

Trying to be a scientist without being able to use basic algebra is a bit like trying to live in France without being able to speak French. Algebra is the
basic language for stating many of the fundamental relationships between the real physical quantities that we describe. In fact, if you are reading this book, it is very likely that you already know some famous equations, for instance:

\[ d = vt \]  \text{distance traveled at constant velocity equals velocity multiplied by time,}  \quad \text{(EQ1.7)}

\[ F = ma \]  \text{force equals mass multiplied by acceleration,}  \quad \text{(EQ1.8)}

and

\[ E = mc^2 \]  \text{energy equals mass multiplied by the speed of light squared.}  \quad \text{(EQ1.2)}

Here the symbols, \( d, v, t, F, \) etc. are not used to stand for numbers: they stand for \textit{physical quantities}, for example \( a \) is an acceleration, \( m \) is a mass, and \( F \) is a force. The equations that relate them are \textit{physical value equations}, not just equations relating numbers. A slightly more complicated example is the equation describing how the number of occupied (or bound) receptors, \( N_b, \) varies with the concentration of a drug, \( c, \) the total number of receptors, \( N_T, \) and the affinity constant for binding, \( K_a \)

\[ N_b = \frac{K_a c N_T}{1 + K_a c}. \quad \text{(EQ1.9)} \]

This equation, which reappears several times in the book, will be introduced fully in the next chapter. Note that \( c \) has very different meanings in the last two equations: physicists use \( c \) to stand for the speed of light but biologists almost always use it for concentration. That raises a very important point: it is absolutely crucial that you always make sure that the definitions of your symbols are clear to those who will read them.

1.3 \textbf{Physical quantities, numerical values, and units}

The symbols like \( a, t, \) and \( K_a \) in EQ1.7–EQ1.9 stand for the combination of a \textit{numerical value} and a \textit{unit}. These symbols for physical quantities are combined in physical value equations to make statements about the real world. This use of physical value equations is almost universal among scientists so it is vitally important to understand the distinction between a number and a physical quantity.

Consider the length of the line just above this text. If we measure its length in centimeters, we get 7.62 cm. Choosing different units of measurement the answers might be 0.0762 m, 76.2 mm, or 76 200 \( \mu \text{m} \); whereas in US or British units the answers might be 3 inches, or 0.25 feet. Thus we can write

\[ 7.62 \text{ cm} = 0.0762 \text{ m} = 76.2 \text{ mm} = 76200 \text{ \( \mu \text{m} \)} = 3 \text{ in} = 0.25 \text{ ft.} \quad \text{(EQ1.10)} \]
If you were asked the length of the line and replied just 7.62 or 0.25, that would be useless. The numerical value 7.62 is meaningful as the length of the line only when it is combined with the statement ‘when the length is measured in centimeters’. You can say ‘the length is 7.62 cm’ or you can say ‘the length expressed in centimeters is 7.62’, but one way or the other you must specify the unit. The number and unit should be thought of as one whole, the quantity. For instance, in the equation for the distance traveled by a car accelerating at a constant rate from a standing start,

\[ d = \frac{1}{2}at^2, \quad (\text{EQ1.11}) \]

when the time \( t \) is squared both the number and the unit must be squared. If \( t = 3 \) s, then \( t^2 = 9 \text{ s}^2 \).

### 1.4 Conversion of units

There are several different ways to remember how to change units. One of the safest (i.e. the easiest to get right every time) is based on a simple rule of algebra; we can always multiply anything by the number 1 without changing its value. The secret is to write the number 1 in a clever way. For example, because \( 1 \text{ kg} = 1000 \text{ g} \), \( 1000 \text{ g} = 1 \text{ kg} \), and a conversion from kilograms to grams can be written as

\[ 3.7 \text{ kg} = 3.7 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 3700 \text{ g}. \quad (\text{EQ1.12}) \]

Note that we have also used kg/kg = 1 (as anything except 0 divided by itself equals 1).

Any conversions we ever need to do are just applications of this principle. Suppose we have watched a leafcutter ant proceeding in a very businesslike manner carrying its leaf segment homeward. Because the pace was steady, the relation between the distance it moved \( d \), the velocity \( v \), and the time we watched, \( t \), is

\[ d = v \times t. \quad (\text{EQ1.13}) \]

![Figure 1.4](image.png)

A leafcutter ant bearing its leaf back to the nest. To find out how fast it is moving, we can record its position at two times and divide the distance traveled by the time taken. Image courtesy of Bristol Zoo Gardens.
The equation above relates the actual physical quantities: how fast it was traveling, the time, and the distance. If the elapsed time was 3 s and the distance traveled 36 mm then the velocity can be calculated as

\[ v = \frac{d}{t} = \frac{36 \text{ mm}}{3 \text{ s}} = 12 \text{ mm s}^{-1}. \] (EQ1.14)

The answer, which can be written as either 12 mm s\(^{-1}\) or 12 mm/s, is read as 12 millimeters per second. The unit s\(^{-1}\) or 1/s is called a reciprocal second.

The validity of the relation between distance, velocity, and time does not depend on our choice of units. For example, we might, somewhat perversely, have said that the distance traveled was 0.036 m and the time taken was 0.05 min and thus that its velocity was 0.036 m / 0.05 min = 0.72 m min\(^{-1}\).

We can check that the two versions of the velocity are indeed the same by converting units.

To convert from meters to millimeters note that 1 meter is the same thing as 1000 millimeters, that is, 1 m = 1000 mm and

**Box 1.1**

The bookkeeping required for conversion of units can be written down using the rule that anything divided by itself is just the number 1. For instance using 1000 g/1 kg = 1,

\[ 3.7 \text{ kg} = 3.7 \times 1 \text{ kg} = 3.7 \times 1000 \text{ g} = 3700 \text{ g}. \]

Alternatively the same thing can be written down using the rule that any physical quantity in an algebraic expression or equation can be replaced by another of equal value, e.g.

\[ 3.7 \text{ kg} = 3.7 \times 1 \text{ kg} = 3.7 \times 1000 \text{ g} = 3700 \text{ g} \]

in which 1 kg is replaced by 1000 g. As we normally wouldn’t bother to write down all the steps, this version is faster to write and most people would do it this way. However, this example was so easy we could do it in our heads. The extra work of using multiplication by 1 becomes increasingly worthwhile as the conversions become more complicated. For example the conversion from 120 km h\(^{-1}\) to 33.3 m s\(^{-1}\) can be written as either

\[ 120 \text{ km h}^{-1} = 120 \text{ km} \times \frac{1 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{60 \text{ min}} \]

\[ = \frac{120 \times 1000}{60} \times \frac{1 \text{ m}}{1 \text{ min}} \]

\[ = 120 \times 1000 \times \frac{1 \text{ m}}{60 \text{ s}} \]

\[ = \frac{120 \times 1000}{60 \times 60} \text{ m s}^{-1} \]

\[ \approx 33.3 \text{ m s}^{-1} \]

which, unless you were feeling very nervous, would be shortened to

\[ 120 \text{ km h}^{-1} = 120 \frac{\text{km}}{1 \text{ h}} = \frac{120 \times 1000}{3600} \frac{\text{m}}{\text{s}} \]

\[ \approx 33.3 \text{ m s}^{-1}. \]

The first method provides the security of specifying each step, the latter allows you to save time writing. Whichever way you choose to do it, the conversion does emphasize that a fast-moving car travels a long way in just a single second.
\[ \frac{1000 \text{ mm}}{1 \text{ m}} = 1. \]  
(EQ1.15)

Similarly to convert from minutes to seconds note that \(60 \text{ s} = 1 \text{ min}\) which is the same as

\[ \frac{1 \text{ min}}{60 \text{ s}} = 1. \]  
(EQ1.16)

Combining all of these and canceling out units that appear in both the numerator and the denominator (again anything except 0 divided by itself equals 1)

\[
0.72 \frac{\text{m}}{\text{min}} = 0.72 \frac{\text{m}}{\text{min}} \times \frac{1000 \text{ mm}}{1 \text{ m}} \times \frac{1 \text{ min}}{60 \text{ s}} \\
= \frac{0.72 \times 1000}{60} \frac{\text{m}}{\text{min}} \times \frac{\text{mm}}{\text{m}} \times \frac{\text{min}}{\text{s}} \\
= 12 \text{ mm s}^{-1}
\]  
(EQ1.17)

which is the same as before.

Conversion of reciprocal units sometimes causes confusion. Thus it is worthwhile looking specifically at these to show that application of the standard procedures produces sensible answers. First, look at a time expressed in milliseconds and change the units to seconds

\[ 500 \text{ ms} = 500 \text{ ms} \times \frac{1 \text{ s}}{1000 \text{ ms}} = 0.5 \text{ s}. \]  
(EQ1.18)

That makes sense. A large number, 500, of a small unit of time like a millisecond will be equal to a part, \(0.5 = \frac{1}{2}\), of a much longer unit of time like a second.

Now look at a rate constant, something that says how much happens per unit of time. Suppose this is 500 events per millisecond which is written as \(500 \text{ ms}^{-1}\). How do we convert this to events per second?

\[ 500 \text{ ms}^{-1} = \frac{500 \text{ ms}}{\text{s}} \times \frac{1000 \text{ ms}}{\text{s}} = \frac{500000}{\text{s}} = 500000 \text{ s}^{-1}. \]  
(EQ1.19)

Again this makes sense: if 500 events occur in a millisecond, then 1000-fold more must occur in the 1000-fold longer time of a second.

When we try to add terms in an equation, each of those terms must have the same unit. For example, in the US and British systems of units, the length of one foot, written as 1 ft, is defined as being equal to twelve inches, written as 12 in, and one inch is defined as being equal to 0.0254 m. How do we add 1 m and 1 ft? In this case we can convert 1 ft into the equivalent length in meters using an exact conversion before adding the two lengths:
1 m + 1 ft = 1 m + 1 ft × \frac{12 \text{ in}}{1 \text{ ft}} × \frac{0.0254 \text{ m}}{1 \text{ in}} = 1 \text{ m} + 0.3048 \text{ m} = 1.3048 \text{ m}.

(EQ1.20)

Units can sometimes protect us from silly mistakes. It does not make any sense to try to add a force to a distance – and reassuringly we cannot convert a unit for force into a unit for distance. To consider a slightly more complicated example, look at the equation given earlier for the number of occupied binding sites when a drug binds to a receptor,

\[ N_b = \frac{K_a c N_T}{1 + K_a c}. \]

(EQ1.21)

Because in the denominator the unit of the combination \(K_a c\) is added to the number 1, it must have the same unit as a number, that is none. Thus when calculating the value of the product \(K_a c\) the unit of \(K_a\) must be the reciprocal of the unit of the concentration, \(c\). For instance if we have chosen the unit of \(c\) to be \(\text{nmol l}^{-1}\) (nanomoles per liter) then, when we calculate the value of the product \(K_a c\), the unit of \(K_a\) must be \(\text{lmol}^{-1}\).

Converting between all of these different units is a major nuisance. Thus wherever possible it makes excellent sense for everyone to agree to use the same ones. Furthermore it helps a lot if everyone agrees on a standard set of abbreviations, for example \(\text{m}, \text{cm}, \text{mm}, \text{\mu m}, \text{km}, \text{etc.}\) However, even with such an agreed list, you still have to convert units – it is just that the numbers in most of the conversion factors are powers of ten.

### 1.5 SI units

The only serious candidate for an agreed list of units is the international system, SI. Because the French got there first, the definitive documents for the international treaties are all in French.

<table>
<thead>
<tr>
<th>base quantity</th>
<th>name</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>meter</td>
<td>\text{m}</td>
</tr>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>\text{kg}</td>
</tr>
<tr>
<td>time</td>
<td>second</td>
<td>\text{s}</td>
</tr>
<tr>
<td>electric current</td>
<td>ampere</td>
<td>\text{A}</td>
</tr>
<tr>
<td>thermodynamic temperature</td>
<td>kelvin</td>
<td>\text{K}</td>
</tr>
<tr>
<td>amount of substance</td>
<td>mole</td>
<td>\text{mol}</td>
</tr>
<tr>
<td>luminous intensity</td>
<td>candela</td>
<td>\text{cd}</td>
</tr>
</tbody>
</table>

The SI system is based on a few base units such as the meter for distance and the second for time. The complete list is given in Table 1.1. You will immediately recognize most of the units because they are taught in school. All physical quantities we can measure can be expressed using combinations of these seven base units. For example, within the SI system, volumes can be measured in cubic meters, \(\text{m} \times \text{m} \times \text{m}\), written \(\text{m}^3\); velocity, distance divided by time, in \(\text{m} / \text{s} = \text{m} \text{s}^{-1}\); concentrations, amount / volume, in
mol / m³ = mol m⁻³; and the association constant between a drug and a receptor, which has the units of 1/ concentration, in m³ mol⁻¹. Some examples of derived units are given in Table 1.2.

Some of these derived quantities are assigned special names, because it is useful to think of these combinations of units together. For instance, force is sufficiently important that it is measured in its own unit, the newton, with symbol N. However, because 1 N is defined as that force which would accelerate a 1 kg mass at 1 m s⁻²,

\[ 1 \text{ N} = 1 \text{ kg} \times 1 \text{ m} \text{ s}^{-2} = 1 \text{ kg m} \text{ s}^{-2}, \quad (\text{EQ1.22}) \]

the symbol N is just a synonym for kg m s⁻². Note that the first letter of the symbol used for an official SI unit is written in upper case if the symbol is named for a person; for example, we write ‘N’ because the newton was named after Sir Isaac Newton. By contrast, the first letter of the name of the SI unit is written in lower case, which allows us to distinguish whether we mean the name of the unit, newton, or the name of the person, Newton. Note that ‘Celsius’ by itself is not an SI unit, so this convention does not apply in ‘degree Celsius’.

Some other examples of special names for derived units are given in Table 1.3. Energy is even more important than force in chemistry and biology, so it should come as no surprise that it too has its own derived unit called the joule, with symbol J. The definition of the joule is the work done if a force of 1 N moves an object through 1 m,

\[ 1 \text{ J} = 1 \text{ N} \times 1 \text{ m} = 1 \text{ N m} = 1 \text{ kg m}^2 \text{ s}^{-2}. \quad (\text{EQ1.23}) \]

Both joules and newtons illustrate part of the reason that combinations of units are named. The equation 1 J = 1 N m tells us the relation between force and work very nicely: work equals force times distance. The message is a lot less obvious if we write 1 J = 1 kg m² s⁻².

<table>
<thead>
<tr>
<th>derived quantity</th>
<th>derived unit</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>square meter</td>
<td>m²</td>
</tr>
<tr>
<td>volume</td>
<td>cubic meter</td>
<td>m³</td>
</tr>
<tr>
<td>speed, velocity</td>
<td>meter per second</td>
<td>m s⁻¹</td>
</tr>
<tr>
<td>acceleration</td>
<td>meter per second squared</td>
<td>m s⁻²</td>
</tr>
<tr>
<td>mass density</td>
<td>kilogram per cubic meter</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>amount of substance</td>
<td>mole per cubic meter</td>
<td>mol m⁻³</td>
</tr>
<tr>
<td>mass fraction</td>
<td>kilogram per kilogram, which may be represented by the number 1</td>
<td>kg kg⁻¹ = 1</td>
</tr>
<tr>
<td>mole fraction</td>
<td>amount of substance per amount of all substances present</td>
<td>mol mol⁻¹ = 1</td>
</tr>
<tr>
<td>per cent</td>
<td>per cent</td>
<td>%</td>
</tr>
</tbody>
</table>
Note that in the SI convention the symbol ‘%’ stands for ‘multiply by the number 0.01’. For instance saying that a mass fraction is 10 % means that the actual mass fraction is $10 \times 0.01 = 0.1$.

The quantities we deal with are not always similar in size to the base units, so one important way in which new units are derived is by considering multiples and fractions. It is always possible to create a unit which is 1000 times bigger or 1/1000th the size of any named unit by using the standard prefixes (see Table 1.4). Note these are never combined. Thus you do not use things like ‘millimicro’: instead you use nano.

SI units are correct and consistent. In many cases they are also convenient, but not always. There is a long and noble tradition of powerful groups forcing exceptions to be made. One of these exceptions is the liter. If you have ever seen a cubic meter of water you will know that it is not a practical unit for work in the laboratory; after all, its mass is 1000 kg! The liter on the other hand is very convenient, as every shopper will know. The SI committees wanted to dispose of the liter as a unit of volume because originally it was defined in terms of the volume occupied by a kilogram of water at a certain temperature rather than in terms of the meter. There was a good practical reason for this choice; it was a lot easier to load 1 kg of water into a

### Table 1.4 SI prefixes

<table>
<thead>
<tr>
<th>symbol</th>
<th>prefix</th>
<th>factor</th>
<th>symbol</th>
<th>prefix</th>
<th>factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>yotta</td>
<td>$10^{24}$</td>
<td>d</td>
<td>deci</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Z</td>
<td>zetta</td>
<td>$10^{21}$</td>
<td>c</td>
<td>centi</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>E</td>
<td>exa</td>
<td>$10^{18}$</td>
<td>m</td>
<td>milli</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>P</td>
<td>peta</td>
<td>$10^{15}$</td>
<td>μ</td>
<td>micro</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>T</td>
<td>tera</td>
<td>$10^{12}$</td>
<td>n</td>
<td>nano</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>G</td>
<td>giga</td>
<td>$10^{9}$</td>
<td>p</td>
<td>pico</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>M</td>
<td>mega</td>
<td>$10^{6}$</td>
<td>f</td>
<td>femto</td>
<td>$10^{-15}$</td>
</tr>
<tr>
<td>k</td>
<td>kilo</td>
<td>$10^{3}$</td>
<td>a</td>
<td>atto</td>
<td>$10^{-18}$</td>
</tr>
<tr>
<td>h</td>
<td>hecto</td>
<td>$10^{2}$</td>
<td>z</td>
<td>zepto</td>
<td>$10^{-21}$</td>
</tr>
<tr>
<td>da</td>
<td>deka</td>
<td>$10^{1}$</td>
<td>y</td>
<td>yocto</td>
<td>$10^{-24}$</td>
</tr>
</tbody>
</table>

Examples: 1 nm is 1 nanometer = $10^{-9}$ m; 1 mV is 1 millivolt = $10^{-3}$ V.

The only exception to the rules for using prefixes occurs for mass, where the base unit is kg, 1 gram is 1 g, and the base for all other multiples is g – for example, mg and μg.
volumetric flask and put a mark on the neck than it was to make a box exactly 1 dm (one decimeter) on each side. However, this sort of definition wreaks havoc with the logical structure of units. In fact, as long as you stick with this sort of definition, 1 l is almost certain not to be exactly 1 dm³ because we do not know the exact density of water. This bothered some physicists but the difference was much too small for biologists to worry about and they stuck by the liter. The SI committees finally gave in and allowed the liter as a unit – but only as a proper unit of volume redefined to be exactly 1 dm³:

\[ 1 \text{ l} = 1 \text{ dm}^3 = 10^{-3} \text{ m}^3. \]  

(EQ1.24)

Table 1.5 lists other derived units still in common use.

<table>
<thead>
<tr>
<th>derived quantity</th>
<th>name</th>
<th>symbol for the units</th>
<th>expression in terms of SI units</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td>liter</td>
<td>l</td>
<td>1 dm³ or 10⁻³ m³</td>
</tr>
<tr>
<td>pressure</td>
<td>atmosphere</td>
<td>atm</td>
<td>1.013 × 10⁵ Pa</td>
</tr>
<tr>
<td>pressure</td>
<td>millimeters of mercury</td>
<td>mmHg</td>
<td>133 Pa</td>
</tr>
<tr>
<td>pressure</td>
<td>centimeters of water</td>
<td>cmH₂O</td>
<td>98 Pa</td>
</tr>
</tbody>
</table>

The meaning of ‘one liter’ is unambiguous and clear but the same cannot be said for the symbol for the unit. In most of the world the symbol is a lowercase l leading to ml, µl, nl, etc. In the USA the symbols are L, mL, µL, nL, etc. The uppercase L has the advantage that it doesn’t look like the number one. By contrast in many fonts, the shapes of the number 1 and the lowercase letter l are similar, sometimes even identical. So one liter becomes 1 l which can look a bit odd. However, the general principle in SI is clear: liter is not the name of a person so the symbol for liter should be lower case. In this book we follow most of the world in using the standard convention, lower case.

Most conversions of units follow the general pattern illustrated above. (A huge compilation of conversion factors can be found in Appendix B of SP811 available from the US National Institute of Standards and Technology (NIST) in either html or pdf format at http://physics.nist.gov/Pubs/. ) There are, however, a few conversions that cannot be done by the standard methods. One of these is between different temperature scales.

Temperature is special because the different scales in common use differ not only in the size of the basic unit but also in the point defined as zero. Originally, the Fahrenheit scale set the temperature of the freezing point of water to 32 °F and the boiling point of water to 212 °F whereas the Celsius temperature scale set these values to 0 °C and 100 °C. Although it may seem a good thing to have a nice round number of degrees between these two important temperatures, to say that the temperature is zero at the freezing point of water is still completely arbitrary. A much more natural choice exists. The random movements of atoms and molecules increase with temperature. That also works in reverse and the random movements come to a stop at approximately −273.15 °C. If we shift the zero point of our temperature scale to this point, then, and only then, we get the very convenient result that the energy of the random movements, the thing we are actually measuring, is proportional to the temperature. The kelvin scale does just that. With zero now at its correct position, called absolute zero, and the size of the degree as defined by Celsius, water freezes at 273.15 K and boils at 373.15 K. Notice
that there is no degree sign ‘°’ before the K for temperatures given in the kelvin scale.

When used in a physical value equation, temperature should always be expressed using kelvins. The physical quantity is absolute temperature and only the kelvin scale puts zero at the right place. Note that the Celsius scale is now a derived temperature scale within SI, and is defined in such a way that 0 °C is set to be exactly 273.15 K. The precise definitions of the kelvin and Celsius scales can be found on the internet at sites such as http://physics.nist.gov/cuu/Units/kelvin.html.

There are simple formulae to convert between temperatures measured on the Celsius, Fahrenheit, and kelvin scales. Suppose that we indicate the numerical value on the Celsius scale of a temperature \( T \) as \( \{ T \}_C \), with similar notation for the Fahrenheit and kelvin scales, then:

\[
\begin{align*}
\{ T \}_C &= \{ T \}_K - 273.15, \\
\{ T \}_F &= 9\{ T \}_C / 5 + 32, \\
\{ T \}_C &= 5(\{ T \}_F - 32) / 9.
\end{align*}
\] (EQ1.25, EQ1.26)

Amount versus mass

There are two sensible ways to measure how much of something we have: count up the number or determine the mass. In SI, mass is properly called exactly that: mass. However, because we almost always determine the mass of an object by weighing it, mass is often a little carelessly called weight. Strictly, the weight of an object is the force exerted on that object by gravity so the proper units for weight are the units of force. At the surface of the earth a 1 kg mass weighs 9.8 N. That is completely straightforward and logical, but you (and I) would always say that the mass weighs 1 kg. Whoever said life was logical? The use of ‘weight’ to mean either mass or weight, depending on the context, is so well entrenched that we are stuck with it.

In SI units ‘amount of a substance’ essentially means a count. The ultimate, intuitive measurement of amount is the count of the individuals present. When the number is relatively small, as in the number of frogs in a pond, this is exactly what is used. However, in a scientific measurement the number of entities present is often very, very large. Thus one entity, as in one individual ion or atom or molecule, is not a very convenient unit for specifying amount. The unit which has been chosen is the mole, with the symbol for the unit written as mol. One mole is the amount of substance that contains as many elementary entities as there are atoms in 12 g of carbon-12 (\(^{12}\text{C}\)).

The number of elementary entities per mole has to be determined experimentally. It is known as Avogadro’s constant in English and the Loschmidt constant in German. Its officially recognized value in March 2007 was (6.022 141 79 ± 0.000 000 30) \( \times 10^{23} \) mol\(^{-1}\) (scientific notation for numbers, for example 6 \( \times 10^{23} \), is explained in Section 2.6). Avogadro’s constant allows us to convert from an amount in moles to a pure number equal to the number of individuals. Choosing to use a weird quantity that must be determined experimentally as the basis for amount may seem fairly peculiar,
but in fact it is very convenient because it makes it easy to calculate the mass of 1 mol of any chemical substance. All we need to know is the chemical formula and thus the relative molecular mass.

The relative molecular mass of a substance, $M_r$, is the average mass per molecule divided by 1/12th of the mass of an atom of $^{12}$C. As such $M_r$ is a pure number. The mass of 1 mol of substance, properly called the molar mass but often called the molecular mass or molecular weight, is then $M_r \times 1 \text{ g mol}^{-1}$. Whatever you call it, the mass you start from in practical calculations is the mass of a mole of the substance, not that of a single molecule. In general, if $n$ is the amount of the substance and $m$ is the mass,

$$m = M_r \times 1 \text{ g mol}^{-1} \times n.$$  \hspace{1cm} (EQ1.27)

Consider the example of glucose, $C_6H_{12}O_6$. The relative molecular mass could be calculated simply from this formula as $(6 \times 12) + (12 \times 1) + (6 \times 16) = 180$ if we take 12, 1, and 16 as the relative atomic masses of carbon, hydrogen, and oxygen. More accurately, the natural mixture of carbon isotopes has an average relative atomic mass of 12.011; likewise, we can find more accurate values for the average relative atomic masses of hydrogen and oxygen. With these more accurate values, the answer to five significant figures is 180.16. So 180.16 g of glucose contains 1 mol of glucose. The mole is the obvious choice for specifying amount in chemistry and the molecular aspects of biology.

If you never read American medical or physiology textbooks, skip this paragraph. If you do, there is one additional unit for amount, the equivalent, with symbol Eq, that you are very likely to encounter. Equivalents can be used to state the amount of ions present. In effect this unit tells you that instead of stating how many moles of ions are present, you are stating the number of moles of unit charges. Thus 1 mmol of Na$^+$ ions, one charge per ion, is 1 mEq of Na$^+$ whereas 1 mmol of Ca$^{2+}$ ions, two charges per ion, is 2 mEq of Ca$^{2+}$. (Equivalents were introduced because they made charge balance calculations simpler. Thus in 1 mmol of CaCl$_2$ the 1 mmol of Ca$^{2+}$ balances not 1 mmol but 2 mmol of Cl$^-$. By contrast, using milliequivalents, this becomes 2 mEq Ca$^{2+}$ balancing 2 mEq of Cl$^-$, which at least to some eyes looks a bit tidier.) The unit, equivalent, was rooted in the physical chemistry of the 1930s but has largely been abandoned by chemists. It has been excluded from SI.

## 1.6 Concentration

Amount tells us how much or how many. Concentration tells us how crowded things are. Applied to a solution in a basic, primitive sense,

$$\text{concentration} = \frac{\text{amount of solute}}{\text{volume of solution}}.$$ \hspace{1cm} (EQ1.28)

Note that if we are dealing with gases, then we talk about space rather than solution. By far the most common unit for concentrations in biology, molar with symbol M, is defined as the concentration of a solution containing 1 mol
of solute for each liter of solution volume. Concentrations based on solution volume are popular partly because they correspond to the most intuitive definition but also because they are very convenient when preparing dilutions (see Section 1.7 Dilutions and doses).

For many purposes, it is far more important to know the concentration rather than the amount. The basis for this is a large topic in physical chemistry. For example: consider two chambers each containing potassium chloride and separated by a membrane as shown in Figure 1.5. We want to know how often potassium ions will be striking the membrane from each side. Now imagine that we double the amount present in each chamber by doubling the size of the chambers but keep the same composition of the fluid and the same membrane. If we keep the concentration the same, then the number of potassium ions close to the membrane will be the same, so the number hitting the membrane will be the same. In other words, if we keep the concentration constant, things like rates of reaction at the surface, rates of diffusion across the membrane from one chamber to the other, and numbers of molecules evaporating each second from each unit of surface area, will all be the same. They do not depend on the amount of potassium chloride present in the chamber: they depend on the concentration. Also, very importantly, as rates of reaction within solutions depend on collisions between the reactants, these rates depend on how many particles are close at hand, i.e. again it is the concentrations not the total amounts in the container that determine how many reactions occur per unit volume of solution (see example in the End of Chapter Questions).

**Units for concentrations**

The basic idea of concentration is clearly indicated in EQ1.28. However, you will encounter many variations on the theme. More generally we can measure how crowded the solute is in a solution by comparing some indication of how much solute we have relative to how much there is of everything else, namely

\[
\text{concentration} = \frac{\text{some measure of solute}}{\text{some measure of the solvent, solution, or space}}.
\]

(EQ1.29)

The different units for concentrations of solutions correspond to different ways of specifying how much of the solute is dispersed in how much solvent or solution. There are three measures of the solute, mass, amount, or volume, that we can choose and the same three choices for measuring the extent of either the total solution or just the solvent. Fortunately, of the 18 \((3 \times 3 \times 2)\) possible combinations, only the five combinations defined and described in Table 1.6 are commonly used. Each of these has circumstances where it is particularly convenient. However, there is a major downside of having more than one: we have to learn how to convert between them.

**Molarity**

In biology the most commonly used of all units for concentration is molar. This is not an officially sanctioned SI unit but its meaning is precise in terms of units that are:
Such concentrations expressed in mol l\(^{-1}\) are often referred to as molarities but more often just as ‘concentrations’. Using molarity is convenient and relatively harmless, and all efforts by the ‘standardizers’ to stamp it out have so far failed. Perhaps someday this unit will follow the liter and be allowed in from the cold.

Concentration in terms of the amount of solute is what we want whenever we are considering reactions or even just diffusion. Concentration in terms of (amount of solute)/(volume of solution), namely molarity, is just what we want whenever we want to use a pipette to deliver a known amount of solute.

**Molality**

In most laboratories, mass can be measured much more accurately than volume. Thus in the most careful work, it is very useful to refer concentration not to the volume of the solution but rather to the mass of solvent. The reason for choosing mass of solvent rather than mass of solution is considered in the next section. The SI unit for concentration, defined as amount of solute divided by the mass of solvent, is mol kg\(^{-1}\). The obsolete name for this unit was molal, with symbol m. This unit and its symbol have largely disappeared from use, which is just as well because m is already overworked as meter and \(m\) as mass. However, concentrations measured in mol kg\(^{-1}\) are still often referred to as molalities. A non-SI unit, mol (liter of cell water)\(^{-1}\), has been used in cell physiology to mean essentially the same as mol kg\(^{-1}\) because to sufficient accuracy the mass of 1 l of cell water is 1 kg. In this case the SI unit does exactly what is wanted and is easier to write so the non-SI unit should be abandoned.
**Molarity versus molality**

In a relatively dilute solution such as 0.15 M NaCl, the concentration can be expressed as either 0.15 M or 0.15 mmol kg\(^{-1}\) because almost all of the solution is solvent and 1 l of solution will contain very close to 1 kg of solvent. If these numerical values are so similar, why bother with the distinction between molarity and molality? The answer is, of course, that they are not always close – molarity and molality differ if *either* some of the solutes in the solution are sufficiently large or the solution is sufficiently concentrated that the solutes occupy a significant proportion of the volume. Normally, the solutions we deal with outside of cells are relatively dilute and it really does not make any difference which type of concentration we use. However, things are very different when we come to consider the solution inside a cell.

Cells contain lots of large solutes, like proteins and nucleic acids, that are in solution but nevertheless occupy a lot of space that is not available for the solvent or for the smaller solutes (see Figure 1.6). Perhaps the simplest example is the red blood cell where 30% of the volume is occupied by hemoglobin. The rest of the cell interior, 70% of the volume, is occupied by a solution with fairly normal properties. For instance sugars, alcohols, and small ions like K\(^+\) and Cl\(^-\) dissolve in the cell water of a red cell much as they do in a simple solution. Thus at equilibrium the concentrations of these solutes inside and outside are the same if we express them as molalities. However, because the volume of the red cell exceeds that of the cell water by a factor of 100/70, when we divide the amount of solute by the cell volume instead of the water volume we will get a concentration (a molarity) that is only 70% as large as expected inside the cell. That might lead us to conclude,
incorrectly, that somehow the solutes interact very differently with water inside the cell than outside it.

Molality and molarity also differ for very concentrated solutions because in these concentrated solutions even small solutes like glucose, Na\(^+\), and Cl\(^-\) can occupy a large proportion of the volume. Very concentrated solutions are rarely encountered with living cells but they are commonplace in any laboratory that uses a technique known as density gradient centrifugation.

**Mass concentration**

Stating concentrations in terms of the mass of solute can be very convenient when we want to make up a solution. To prepare a 1 mg ml\(^{-1}\) solution, you might weigh out 10 mg of the solute, and make it up to 10 ml by adding the solvent. In practice, when making up a solution with much less solute than solvent, it is accurate enough to just add 10 ml of solvent using a pipette. Now compare this with what we have to do to make up a 1 mM solution. We need to convert from amount, how many millimoles, to mass, how many milligrams, so that we can weigh out the solute using our balance. If the relative molecular mass of glucose is 180.16, the molar mass is 180.16 g mol\(^{-1}\) and the concentration in terms of mass of a 1 mM solution will be

\[
1 \text{ mmol l}^{-1} \times 180.16 \text{ g/mol} \times 1 \text{ mol/1000 mmol} = 180.16 \text{ mg l}^{-1}.
\]

To prepare 10 ml of this we thus need 180.16 mg l\(^{-1}\) \(\times\) 0.01 l = 1.8016 mg. So, we need to know the relative molecular mass, we have a lot more calculation to do, and we end up with a number that is difficult to remember. Nevertheless, because we usually want to end up with a known molarity, these are conversions that must be mastered.

In general, all you do to convert from concentration expressed as mol l\(^{-1}\) to concentration expressed as g l\(^{-1}\) is multiply by the relative molecular mass. It is very easy – but it does still take time and when you are in a hurry it is one more step you can get wrong. These are the main reasons why recipe books for making up solutions in the laboratory often use mass concentrations measured in mg ml\(^{-1}\) or g l\(^{-1}\) rather than amount concentrations measured in M or mM.

**Concentrations expressed as percentages**

Many find that it is more convenient to express concentrations in terms of percentages than in terms of any of the basic units we have considered so far. However, carelessness in the use of percentages has caused a great deal of confusion because it is often not made clear what is a percentage of what. In SI, the symbol ‘%’ stands for multiply by 0.01. Thus the only quantities, usually ratios, that can be expressed as a percentage are those with no units. Thus the ratio of mass of solute to mass of solution and the ratio of volume of solute to volume of solution can properly be expressed as percentages, often written by biologists as % (w/w) and % (v/v) respectively. Unfortunately, in older literature, a 1% solution often means a solution containing 1 g solute per 100 ml of solution. Use of % (w/v) is not permitted in SI because a mass cannot be a fraction or a percentage of a volume. It is indefensible to use 1 % to mean 1 g (100 ml)\(^{-1}\). It would be best even to avoid 1 % (w/v); use
10 g l⁻¹ or 10 mg ml⁻¹ instead. Nevertheless such terminology continues to be used. Table 1.7 summarizes the various ways in which concentrations are expressed using percentages.

Having defined the various units for concentrations, the challenge now is to understand how to convert between them.

**Conversion between molarity and g l⁻¹**

How much NaCl must we weigh out to prepare a solution with concentration \( c = 0.15 \) M? This is probably the simplest conversion and, fortunately, it is the only one you need to do frequently. To calculate the mass/volume concentration we need to know that the relative molecular mass is \( M_r = 58.45 \). The mass required for a volume of solution \( V \) is then (see Eq.1.27)

\[
m = M_r \times 1 \text{ g mol}^{-1} \times c \times V
\]

(EQ1.30)

and

\[
m/V = 0.15 \text{ mol l}^{-1} \times 58.45 \text{ g mol}^{-1} = 8.77 \text{ g l}^{-1}.
\]

(EQ1.31)

In practice, it is often more convenient to write this as 8.77 mg ml⁻¹. Whichever, you now choose a convenient volume and mass so that the mass you need to weigh is large enough for your balance while the volume of solution is small enough for your bottle.

<table>
<thead>
<tr>
<th>Table 1.7 Concentrations expressed as percentages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>designation</strong></td>
</tr>
<tr>
<td>% by weight</td>
</tr>
<tr>
<td>% by volume</td>
</tr>
<tr>
<td>%(w/v)</td>
</tr>
</tbody>
</table>
Conversion between molarity and molality

This is perhaps the most complicated conversion you will encounter because you need to convert between volume of solution and mass of solvent. Thus, as an intermediate step, you need to know both the mass of the solute and the mass of the solvent in a liter of the solution. To find this you need to know the density of the solution, i.e. its total mass per unit volume.

Call the molarity of the solution \( c \), its volume \( V \), the density of the solution \( \rho \), and the molality \( b \). The amount of solute present is then \( c \times V \). The mass of the solution is \( m_{\text{solution}} = \rho \times V \), the mass of solute present is \( m_{\text{solute}} = c \times V \times M_r \times 1 \text{ g mol}^{-1} \), and by difference the mass of the solvent is

\[
m_{\text{solvent}} = m_{\text{solution}} - m_{\text{solute}} = V \times (\rho - c \times V \times M_r \times 1 \text{ g mol}^{-1}).
\]  
(EQ1.32)

The molality can now be calculated directly as

\[
b = \frac{\text{(amount of solute)}}{\text{(mass of solvent)}} = \frac{c \times V}{(\rho - c \times V \times M_r \times 1 \text{ g mol}^{-1})V} \quad \text{(EQ1.33)}
\]

Thus, for 0.1500 M NaCl, which has a density of 1.0064 kg l\(^{-1}\), the molality is

\[
b = \frac{0.1500 \text{ mol l}^{-1}}{1.0064 \text{ kg l}^{-1} - 0.15 \text{ mol l}^{-1} \times 58.45 \times 1 \text{ g mol}^{-1}} \quad \text{(EQ1.34)}
\]

\[
= \frac{0.1500 \text{ mol l}^{-1}}{(1.0064 - 0.00877) \text{ kg l}^{-1}} = 0.1504 \text{ mol kg}^{-1}.
\]

That confirms what we said before. For relatively dilute solutions the numerical values of the molarity in mol l\(^{-1}\) and the molality in mol kg\(^{-1}\) are almost the same. Accurate conversion between them requires you to know the density of the solution – which requires delving into esoteric sources like the *Handbook of Physics and Chemistry* or the International Critical Tables.

### 1.7 Dilutions and doses

In virtually every branch of biology you need to add chemicals to solutions to produce effects. In biochemistry this might be an inhibitor to change the rate of a reaction. In pharmacology the substance added is called a drug and the amount added the dose. If the volume to which the doses are added can be defined, the doses are chosen to produce known concentrations.

Much of pharmacology is based on experimental determination of dose–response curves using *in vitro* assay systems that produce some measurable response to the drug of interest. It is an intriguing fact that the muscle of the ileum, the longest part of the small intestine, has receptors for a huge variety
of interesting chemicals. Some say if there is a drug receptor in the brain you will also find it somewhere in the wall of the gut. However true that may be, a lot has been learned from applying drugs to lengths of ileum and noting how much they contract. As indicated in Figure 1.7, the length of gut is suspended in a bath of known volume, for example 10 ml, with one end tethered and the other attached to a transducer which detects changes in length. The experiments measure the shortening produced by various concentrations of the drug, for instance acetylcholine, and how these responses are altered by other drugs, for instance atropine. Typically a concentration–response curve (see Figure 1.8) for acetylcholine might be determined using a range of concentrations from 10 nM to 50 μM acetylcholine in the absence of atropine and compared with those measured in the presence of a low concentration, for example 1 nM, of atropine. How the bath concentrations of acetylcholine are achieved is considered in the exercises at the end of this chapter. Here we will consider how to add 1 nM atropine. ‘Adding a concentration of a drug to a solution’ means you add the amount of the drug needed to produce the stated concentration.

Suppose we have cells or a tissue suspended in 10 ml of a buffered solution (often called ‘the buffer’) and we want to add atropine (see Figure 1.9), relative molecular mass 289, to achieve a concentration of 1 nM. What mass of drug must be added? The answer is (10 ml = 10⁻² l):

\[ 1 \text{ nM} = 10^{-9} \text{ mol} \times 10^{-2} \text{ l} \times \frac{289 \text{ g}}{\text{mol}} = 2.89 \times 10^{-9} \text{ g} = 2.89 \text{ ng}. \]  

(EQ1.35)

(Scientific notation for numbers in which 10⁻⁹ is used as a shorthand for 1/1 000 000 000 is discussed in Section 2.6.) Routinely, we can easily weigh out 10 mg to 1 % accuracy, but 2.89 ng is not something that is exactly easy to handle; in fact you probably could not see it on the end of a spatula. So we will need to add the required amount of drug by pipetting a small quantity of a concentrated stock solution into our final solution. How do we prepare this stock solution and what volume of the stock must we add? To consider the volume first, the smallest amount we can add is limited by the accuracy of our pipettes. Modern pipettes (like the one in Figure 1.10) can deliver as little as 0.5 μl but to achieve 1 % accuracy it is best to handle volumes of 10 μl or greater. The largest volume of stock we can add is limited by the amount of its solvent we can tolerate in the final solution. If the drug is dissolved in a

---

**Figure 1.7**
Organ bath apparatus for measuring contractions of a segment of ileum. Drugs in concentrated stock solutions can be added to the bathing solution by simple pipetting. The dilution factor is just the volume added divided by the final volume present.

---

**Figure 1.8**
Shortening of a 4 cm length of guinea-pig ileum in response to a range of concentrations of acetylcholine. Note that in order to display the data for the high concentrations, 10, 20, and 50 μM, the data points for the low concentrations are plotted very close to each other.
solvent like ethanol we would normally ensure that the volume pipetted was less than 0.1 % of the final volume. This limit can be increased to 1 % if the solvent is water but if we must add an even larger volume then the stock we add must have been prepared by adding drug to a solution with the same concentrations of everything else, namely ions, sugars, etc., as the final solution. The solution with all of these ‘ordinary’ components is often called the buffer solution or even just ‘the buffer’. Strictly, of course, the ‘buffer’ is the pH buffer which is almost always included in the solution. In effect the buffer solution can then be thought of as the solvent for the drug.

Pipetting 10 µl of our stock solution to a final volume of 10 ml will produce a 1000-fold dilution of the stock, thus the concentration of atropine in the stock solution must be 1 µM so that the final concentration can be 1 nM. Preparing a 1 µM stock solution is still problematic. To make up 10 ml we would need to weigh out 2.89 µg (1000 times more than before), which is still too small for most laboratory balances. So the procedure has to be a little more complicated: we must prepare an initial stock at a higher concentration, say 1 mM; use that to prepare a second, working stock 1000 times more dilute; and finally use the second stock to add the atropine to the experimental solution.

The principle behind making dilutions is simple; we add a small volume of a concentrated solution of drug to a volume of solvent or buffer solution. The key point is that the amount of drug in the pipetted volume equals the amount of drug in the total volume of the final solution. If \( V_p \) is the volume pipetted, \( V_f \) is the final volume, \( c_p \) is the concentration in the stock solution, and \( c_f \) is the concentration in the final solution (same units), then the amount of drug is just

\[
\text{amount} = c_p \times V_p = c_f \times V_f.
\]

(EQ1.36)

The concentrations can be either amount/volume or mass/volume, but they must both be expressed the same way. The ratio of the concentrations is just the inverse of the ratio of the volumes (you can think of inverting a ratio as turning it upside down)

\[
\frac{c_f}{c_p} = \frac{V_p}{V_f}.
\]

(EQ1.37)

It is important to remember that the ratio is of the pipetted volume to the final volume. To achieve a dilution of 10-fold, i.e. to reduce the concentration to 1/10th, we might pipette 100 µl into 900 µl of buffer solution to achieve a final volume of 1 ml. Dilutions greater than 1000-fold can be prepared by making up large volumes of final solution, but it is usual to do it in stages with smaller volumes as in the example above.

In microbiology it is common to want a series of dilutions where successive solutions differ in concentration by just twofold. One quick way to produce these is the method of doubling dilutions, a procedure for producing a series of dilutions in which each has a concentration half that of its predecessor. Pick a convenient volume for the final solutions, here we will use 0.5 ml. We add 0.5 ml of the buffer solution once to each of the tubes in the series except
the first. Then we add 0.5 ml of the original stock in buffer to the first tube twice. Now we take 0.5 ml from the first tube, add it to the second and mix. The concentration in the second tube is now half that in the first. Next we take 0.5 ml from the second tube and mix it into the third. The concentration in the third tube is now $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$th. If then we take 0.5 ml of the third solution and mix it into the fourth, the concentration in the fourth tube is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$th and so on. This procedure is simple and can be repeated as often as needed. It has several advantages. Only one pipette and tip are required, the volume pipetted is always the same, and the accuracy of the dilutions is limited only by the reproducibility, sometimes called the precision, of the pipette and not by its absolute accuracy. Equally importantly, the simplicity of the procedure helps us to avoid mistakes. For instance the tube from which we must take the 0.5 ml is always the one that has 1 ml in it. All we need to remember while pipetting is which tube is next for the length of time it takes to move the pipette tip from one to the other. This sounds silly but when doing really tedious tasks any trick that helps to prevent errors is very welcome. (Why doubling? Probably because we keep doubling the volume of successive tubes.)

### Box 1.2

Acetylcholine is to be added to the bathing solution for a length of ileum at concentrations 1, 2, 5, 10, 20, 100, 200, 500, 1000, 2000, or $5000 \times 10^{-8}$ M. The drug is available as a 10 mM solution in water. What other stocks in water should be prepared?

The intermediate stock solutions will also be in water – so it will be acceptable for the volume of stock pipetted into the final solution to be as high as 1% of 10 ml = 10 μl. The smallest volume we can pipette accurately is 10 μl. Thus for the solution with the least drug we will want a stock such that the smallest volume we can pipette accurately, 10 μl, into 10 ml will give the correct concentration. The ratio of these volumes is 1 : 1000 so the ratio of the concentration in the most dilute stock to the final concentration must be 1000 : 1; that is, the concentration must be $1 \times 10^{-8}$ M $\times 10^3 = 10$ μM. Using additions between 10 and 100 μl this stock can be used to produce 1, 2, 5, and $10 \times 10^{-8}$ M. 10 μM is a 1000-fold dilution of the 10 mM stock. A 100-fold dilution of the 10 mM stock will suffice for 20, 50, and $100 \times 10^{-8}$ M, a 10-fold dilution for 200, 500, and $1000 \times 10^{-8}$ M and the full-strength stock can be used for 2000 and $5000 \times 10^{-8}$ M. So, to cover the entire range of concentrations, we need to make three dilutions, 10-fold, 100-fold, and 1000-fold. Each of these could be prepared in a single step from the original stock or they could be prepared using three 10-fold serial dilutions.

### 1.8 Numerical value equations

So far in this chapter almost all of the equations have been physical value equations that relate real physical quantities. These are often just what we want, especially when writing down equations that are to be applied to many situations. However, you will have noticed that when using these physical value equations it is necessary to write out the units whenever specific values are included. That can get really tedious. There is another way to write equations that can greatly reduce the number of times we need to write out units. These are numerical value equations and you will see a lot of them in Chapters 5 to 8.

You will have met the equation $F = mg$ where $m$ is the mass of an object, $F$ is the force due to the gravity of the earth, and $g$ is the acceleration due to
gravity. The constant $g$ is not a fundamental constant of physics. It depends on exactly where we are: the further we are from the center of the earth the smaller it is. If our interest were calculating satellite orbits we would need to worry about how $g$ varies with position. However, to two significant figures it is the same everywhere on the earth’s surface and can be written as $9.8 \text{ m s}^{-2}$. So we can write

$$F = m \times (9.8 \text{ m s}^{-2}).$$

This equation is the subject of one of the more interesting legends in physics, that Galileo dropped different masses from the Leaning Tower of Pisa (see Figure 1.11) and discovered that they all fell with the same acceleration, which according to this equation is just $9.8 \text{ m s}^{-2}$. EQ1.38 is a physical value equation; $F$ and $m$ represent the actual force and the actual mass. Thus if the mass is 70 kg

$$F = 70 \text{ kg} \times 9.8 \text{ m s}^{-2}$$

$$= 686 \text{ N}.$$  

(EQ1.39)

(The standard 70 kg man weighs 686 N at the surface of the earth.)

We can convey exactly the same information using a numerical value equation by writing the following:

$$F = m \times 9.8$$

(EQ1.40)

where $F$ is the force measured in newtons and $m$ is the mass measured in kilograms. Here, in contrast to the equations before, $F$ and $m$ are pure numbers. This numerical value equation correctly relates the numerical values of the force and mass only when the physical quantities have been expressed using a particular choice of units. If we change the units we have to change the numerical constant.

Consider again our standard man with mass 70 kg. We can define a symbol, $m$, to represent the mass of an individual and for this man $m = 70 \text{ kg}$. However, we could also define a different symbol, let us call it {$m$}, by saying the mass of an individual is {$m$} kg. So defined, {$m$} is just a number that gives the

---

**Figure 1.11**
Leaning Tower of Pisa. According to legend, Galileo dropped objects of different masses while hanging over the edge and determined that they fell with the same acceleration. This is the prediction from Newton’s second law, $F = ma$, and the law of gravitation near the earth’s surface, $F = mg$, because these imply $a = g$. Image courtesy of M4rvin under Creative Commons Attribution–Share Alike 2.0 Generic.
numerical value of the mass when it is expressed in kilograms; \( \{m\} \) itself has no units. Now what happens if we want to express the mass in grams? The physical quantity is not affected by our choice of units, thus we write

\[
m = 70 \text{ kg} = 70,000 \text{ g}.
\]

(EQ1.41)

However, the numerical value most definitely does change. If the mass is expressed in grams rather than kilograms, \( \{m\} \) becomes 70,000! Hopefully, this is clear. Now to add confusion, most of the time we do not bother to write the curly brackets! You have got to work out from context whether a symbol like \( m \) is referring to the physical quantity, 70 kg = 70,000 g, or to the numerical value which would be 70 if mass is being expressed in kilograms or 70,000 if it is being expressed in grams.

How do you know whether a symbol represents a physical quantity or a numerical value? First, look for the definition of the symbol. In the paragraph just above, the indication that \( \{m\} \) is a numerical value (in addition to the dead giveaway of the curly brackets) is that it is immediately followed by the units! The combination of the numerical value and the units describes the physical quantity. By contrast, as \( m \) is defined at the start of this section as ‘the mass of an object’ then it must be a physical quantity, a mass. What do you do if the definition is not immediately to hand? You look for clues in the context. Look at the equation and its surrounding text. EQ1.38 must be a physical value equation because the constant is written with its units, 9.8 m s\(^{-2}\). By contrast, with EQ1.40 you are given two clues that it is a numerical value equation: the 9.8 is written without units and the units needed to interpret \( F \) and \( m \) are given immediately after the equation.

Why do we use numerical value equations? Why put up with the complications of having two sorts of equation to deal with? This may be made clearer by the following example.

Suppose we have measured the rate of a reaction as a function of the concentration of the substrate. The measurements can be reported in a table, as in Table 1.8, or in a graph, as in Figure 1.12. In the SI convention, entries listed in a table or plotted on a graph should be pure numbers. We get the number by dividing the physical quantity by a standard reference value. Here the concentrations have been divided by 1 mM and the rates by 1 mol s\(^{-1}\). Ignoring the units, the numbers in this table, the numerical values, can be fitted with a smooth curve (see Figure 1.12) with equation

**Table 1.8 Rate of reaction versus concentration**

<table>
<thead>
<tr>
<th>( C / \text{mM} )</th>
<th>( R / (\text{mol s}^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>0.017</td>
</tr>
<tr>
<td>5</td>
<td>0.079</td>
</tr>
<tr>
<td>10</td>
<td>0.147</td>
</tr>
<tr>
<td>15</td>
<td>0.202</td>
</tr>
<tr>
<td>20</td>
<td>0.246</td>
</tr>
<tr>
<td>25</td>
<td>0.277</td>
</tr>
<tr>
<td>30</td>
<td>0.296</td>
</tr>
<tr>
<td>35</td>
<td>0.303</td>
</tr>
<tr>
<td>40</td>
<td>0.298</td>
</tr>
<tr>
<td>45</td>
<td>0.281</td>
</tr>
<tr>
<td>50</td>
<td>0.253</td>
</tr>
</tbody>
</table>

**Figure 1.12**

Rate of reaction versus concentration. The points are taken from Table 1.8. The smooth curve is a plot of EQ1.42.
$R = 0.0171C - 0.000241C^2$.  \hspace{1cm} (EQ1.42)

$C$ and $R$ in Table 1.8 and Figure 1.12 represent the physical quantities concentration and rate. In the numerical value equation EQ1.42, these symbols have different meanings. In this equation they represent the numerical values obtained when the concentration and rate are divided by particular reference values, 1 mM for concentration and 1 mol s$^{-1}$ for the rate of reaction. Any other choice of reference values would require different values of the constants in the numerical value equation. For instance, if we wanted to express the concentrations with unit M instead of mM (using reference value 1 M instead of 1 mM), the equation would have to be rewritten as

$R = 17.1C - 241C^2$.  \hspace{1cm} (EQ1.43)

As a check, if the physical value of the concentration is 0.02 M = 20 mM, either EQ1.42 with $C = 20$ or EQ1.43 with $C = 0.02$ tells us that $R \approx 0.25$, but if we use the wrong numerical value for $C$ in either equation we get the wrong answer!

It is of course possible to describe the data in Table 1.8 using a physical value equation where now $R$ and $C$ have the same meanings as in Table 1.8 and Figure 1.12.

$R = (0.0171 \text{ mol s}^{-1} \text{ mM}^{-1}) \times C - (0.000241 \text{ mol s}^{-1} \text{ mM}^{-2}) \times C^2$.  \hspace{1cm} (EQ1.44)

This equation has the advantage that it remains valid for any choice of units; or, to put it another way, before we can use it to calculate actual values the equation forces us to change units if $C$ and $R$ are initially expressed in units other than mM and mol s$^{-1}$. Thus in a sense this equation is safe; it forces us to sort out the units. However, because EQ1.44 is so much messier, most people would just make sure they got the reference values right before deciding on the values of $C$ and $R$. Numerical value equations often save a lot of writing (compare EQ1.43 and EQ1.44), but they leave sorting out the units entirely to you. It is a little like removing the crash barriers on a twisting mountain road. The bad news is you will frequently encounter both physical value and numerical value equations (and you will probably have to work out just what somebody meant when they have not given clear clues). The good news is both are very useful and a wise choice of which to use can make life a lot easier.

Advice:

1. We **must** state the units close-at-hand if we use a numerical value equation.
2. If the equation states a general relation or a law, something we might want to try to put into words, we should use symbols for the physical quantities and a physical value equation.
3. It is much easier for most of us to keep track of units if we use symbols that stand for the physical quantities rather than the numerical values.
4. If the equation states an empirical relation, where the constants are represented by their values rather than by symbols, we are probably better off using numerical value equations.

Although we may sometimes curse units because they can take so much time to write, it is often worth the effort just to make sure that you have all the factors in the right places. If the units come out right at the end of the calculation, then there is a good chance that we have done things correctly. If they come out wrong, then we know there must be an error, so we have to go back and find the mistake.

**Box 1.3**

In official documentation an unambiguous convention is needed to distinguish between physical quantities and numerical values of those quantities. Sometimes this extra notation actually aids understanding and an example crept into the discussion of temperature (see Eq1.25 and Eq1.26) because we thought it helped. Using the same curly-bracket notation for numerical values the physical value equation,

\[ F = m \times (9.8 \text{ m s}^{-2}) \]

becomes

\[ \{F\}_N = \{m\}_kg \times 9.8 \]

where \( \{F\}_N = F / 1 \text{ N} \) and \( \{m\}_kg = m / 1 \text{ kg} \).

Similarly we could rewrite Eq1.42 as

\[ \{R\}_\text{mol s}^{-1} = 0.0171 \{C\}_\text{mM} - 0.000241 \{C\}^2_{\text{mM}}. \]

The main reason to use numerical value equations is to save the tedium of writing units. Most people, including us, decide that the tedium of writing all the brackets is still too great. You will rarely see the curly brackets, even in this book.

### 1.9 Scalar and vector physical quantities

So far, we have discussed each physical quantity in terms of its unit and numerical value. A physical quantity that is completely specified by the unit and a single numerical value is called a scalar. These are what we will be considering in the rest of this book. If we do not say otherwise, physical quantity means scalar physical quantity. However, if you think for a minute about physical quantities like force, velocity, or acceleration, you will realize that we need to specify not only their magnitudes but also their directions. To specify a force completely takes not one magnitude but three, which we would often choose to be values along three perpendicular axes, \( x \), \( y \), and \( z \). A physical quantity that requires a direction as well as a magnitude is called a vector. (The word vector also has other meanings in pathology and molecular biology; it can mean an agent that spreads a disease or an agent used to insert genetic material into a cell.)

Physicists need to use vector quantities like force, acceleration, and electric field in almost everything they do, but biologists need vector quantities only rarely so we will say very little more about them.
Presenting Your Work

QUESTION

Stock solutions for preparation of a ‘physiological’ saline.

It is common practice to make up and store concentrated stock solutions so that working buffer solutions can be prepared quickly by simple dilutions. Complete the table below to indicate (i) how much of each substance should be weighed out to prepare the stocks and (ii) the volumes to be added to prepare 250 ml of final solution in a volumetric flask. Note that ‘how much’ in this context means how much mass.

<table>
<thead>
<tr>
<th>substance</th>
<th>$M_r$</th>
<th>$C_{stock}$ / mM</th>
<th>$V_{stock}$ / ml</th>
<th>mass of substance to prepare stock $m_{stock}$ / g</th>
<th>$C_{final}$ / mM</th>
<th>volume to pipette to prepare 250 ml of buffer $V_p$ / ml</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaCl</td>
<td>58.45</td>
<td>1000</td>
<td>1000</td>
<td>140</td>
<td>140</td>
<td>approximately†: mass to add‡: $m_{glu}$ =</td>
</tr>
<tr>
<td>KCl</td>
<td>74.56</td>
<td>500</td>
<td>50</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>MgCl$_2$·6H$_2$O</td>
<td>203.33</td>
<td>500</td>
<td>10</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>CaCl$_2$ (anhydrous)</td>
<td>110.99</td>
<td>500</td>
<td>20</td>
<td>1.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Hepes (pK 7.49)</td>
<td>238.3</td>
<td>100</td>
<td>500</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>NaOH</td>
<td>40.01</td>
<td>1000</td>
<td>100</td>
<td>$\sim$5†</td>
<td>$\sim$5†</td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>glucose</td>
<td>180.16</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
†The NaOH would in fact be added dropwise to titrate the final solution to pH 7.4 (or whatever is desired). Because the pK of Hepes is 7.49, slightly less than half as much NaOH must be added as Hepes.
‡You can assume for this relatively dilute solution that all of the volumes must add up to 250 ml.
§Glucose solutions grow bacteria unless sterilized. It is thus common practice not to use a glucose stock solution but rather to add glucose as solid when making up the final solution.
For NaCl  
amount of NaCl in stock = \( C_{stock} \times V_{stock} = 1 \) mol  
mass of NaCl in stock = amount \( \times M_r = 58.45 \text{ g} \)  
volume to pipette = \( (\text{amount in buffer}) / C_{stock} \)  
= \( 35 \text{ mmol} / 1000 \text{ mmol}^{-1} = 35 \text{ ml} \)

<table>
<thead>
<tr>
<th>substance</th>
<th>( M_r )</th>
<th>( C_{stock} ) / mM</th>
<th>( V_{stock} ) / ml</th>
<th>mass of substance to prepare stock ( m_{stock} ) / g</th>
<th>( C_{final} ) / mM</th>
<th>volume to pipette to prepare 250 ml of buffer ( V_p ) / ml</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaCl</td>
<td>58.45</td>
<td>1000</td>
<td>1000</td>
<td>58.45</td>
<td>140</td>
<td>35</td>
</tr>
<tr>
<td>KCl</td>
<td>74.56</td>
<td>500</td>
<td>50</td>
<td>1.864</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>MgCl(_2)-6H(_2)O</td>
<td>203.33</td>
<td>500</td>
<td>10</td>
<td>1.017</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>CaCl(_2) (anhydrous)</td>
<td>110.99</td>
<td>500</td>
<td>20</td>
<td>1.10</td>
<td>1.5</td>
<td>0.75</td>
</tr>
<tr>
<td>Hepes (pK 7.49)</td>
<td>238.3</td>
<td>100</td>
<td>500</td>
<td>11.91</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>NaOH</td>
<td>40.01</td>
<td>1000</td>
<td>100</td>
<td>4.001</td>
<td>~5</td>
<td>1.25</td>
</tr>
<tr>
<td>water</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>185.75</td>
</tr>
<tr>
<td>glucose</td>
<td>180.16</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>10</td>
<td>0.4504 g</td>
</tr>
</tbody>
</table>
End of Chapter Questions

Basic
1. In the British system of units, length is measured in feet and inches. One foot is defined as twelve inches (1 ft = 12 in), whereas one inch is defined as 1 in = 25.4 mm. Suppose that a leafcutter ant travels at 2.4 ft min⁻¹; what is its speed in mm s⁻¹ to three significant figures?

2. Convert the following percentages into decimal numbers using the SI convention that % stands for ‘multiply by the number 0.01’:
   (a) 45 %
   (b) 2 %
   (c) 0.00374 %
   (d) 237 %

3. Convert the following into percentages:
   (a) 0.67
   (b) 0.00000024
   (c) 1/5
   (d) 5.671

4. (a) Express 20 °C in Fahrenheit and kelvin.
   (b) Express 98.6 °F in Celsius and kelvin.
   (c) The temperature in outer space is said to be about 4 K. What is this temperature in Celsius and Fahrenheit?

5. If 180.16 g of glucose contains 1 mol of glucose calculate how many glucose molecules there are in 1 pg. Take Avogadro’s constant (Loschmidt constant) to be 6.022142 × 10²³ mol⁻¹. Give your answer to the nearest million.

6. Find the molality of a 1 % (w/w) solution of glucose. (180.16 g of glucose contains 1 mol.)

7. Find the molarity of a 0.05 % solution of NaCl given that the relative molecular mass of NaCl (one Na⁺ ion and one Cl⁻ ion) is 58.45. What assumption do you need to make?

8. The molar mass of sucrose is 342.3 g mol⁻¹ and the density of a 2.555 M solution of sucrose is 1.2887 kg l⁻¹. What is the concentration of sucrose expressed as % (w/w)? What is the concentration expressed as molality (mol kg⁻¹)?

Intermediate
9. The flux of bicarbonate ions across the membrane of a cultured brain endothelial cell has been reported as 1.25 pmol cm⁻² s⁻¹. Express this in terms of the derived SI unit for flux, mol m⁻² s⁻¹.

10. Write out a protocol for producing a series of 100-fold dilutions with final volumes close to 1 ml.

11. The initial rate of a bimolecular reaction (i.e. a reaction involving 2 reactants) depends on the number of collisions between the reactants. How much larger will the total amount of product produced per second be if we:
   (a) double the volume of the solution keeping its composition the same;
   (b) double the volume of the solution keeping the amounts of the reactants present constant;
   (c) double the concentration of one reactant, keeping the concentration of the other and the volume constant;
   or
   (d) double the concentrations of both reactants while keeping the volume constant?

12. In a simple bimolecular reaction
   \[ A + B \xrightleftharpoons{\kappa}{\kappa} C \]
   the initial rate of formation of C per unit volume, \( R_{\text{initial}} \), is given by
   \[ R_{\text{initial}} = k [A]_{\text{initial}} [B]_{\text{initial}} \]
   where \([A]_{\text{initial}}\) means initial concentration of A. For a 100 ml reaction flask, 1 µM initial concentrations of A and B, and \( k = 2 \times 10^3 \text{ M}^{-1} \text{s}^{-1} \), calculate the total initial rate of formation of C.

13. What is the mass of a hydrogen molecule expressed in kilograms?

14. If the charge on a proton is \( 1.6 \times 10^{-19} \text{ C} \) what is the sum of the charges on the positive ions in one liter of a 1 M NaCl solution?
Advanced

15. You are provided with a 60% (w/w) solution of cesium chloride.

The relative molecular mass of CsCl is 168.37 and the specific gravity of a 3 M solution is 1.385.

How would you prepare:
(a) 15 g of a 20% (w/w) solution;
(b) 15 g of a solution whose molality is 5 mol kg\(^{-1}\);
(c) 10 ml of a 3 M solution? (You do not have access to a 10 ml volumetric flask.)

Hint: In general, you should first calculate how much CsCl you need in the final solution – then determine how to obtain this from the stock. With a 60% (w/w) stock is it more appropriate for ‘how much’ to mean amount or mass?

16. What considerations are taken into account when deciding on the concentrations and volumes of stock solutions? Devise a choice of stock concentrations for NaCl, KCl, MgCl\(_2\), CaCl\(_2\), and Hepes such that 10 ml of each stock solution must be added to make up 250 ml of the final buffer solution described in Presenting Your Work.