Reference and Practice Book

CORE-PLUS MATHEMATICS PROJECT

Course 2

Contemporary Mathematics in Context
A Unified Approach

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Contents

Introduction ................................................................. 3

Summary and Review of Course 1 ................................. 5
  Algebra and Functions ............................................. 6
  Statistics and Probability .................................... 16
  Geometry .............................................................. 26
  Discrete Mathematics .......................................... 35

Maintaining Concepts and Skills ................................. 43
  Exercise Set 1 .......................................................... 44
  Exercise Set 2 .......................................................... 46
  Exercise Set 3 .......................................................... 48
  Exercise Set 4 .......................................................... 50
  Exercise Set 5 .......................................................... 52
  Exercise Set 6 .......................................................... 54
  Exercise Set 7 .......................................................... 56
  Exercise Set 8 .......................................................... 58
  Exercise Set 9 .......................................................... 60
  Exercise Set 10 .......................................................... 62
  Exercise Set 11 .......................................................... 64
  Exercise Set 12 .......................................................... 66
  Exercise Set 13 .......................................................... 68
  Exercise Set 14 .......................................................... 70
  Exercise Set 15 .......................................................... 72
  Exercise Set 16 .......................................................... 74
  Exercise Set 17 .......................................................... 76
  Exercise Set 18 .......................................................... 78
  Exercise Set 19 .......................................................... 80
  Exercise Set 20 .......................................................... 82
Course 1 of the *Contemporary Mathematics in Context (CMIC)* series introduced important ideas and problem solving techniques from algebra and functions, geometry, statistics, probability, and discrete mathematics. Many of those concepts and skills will reappear in Course 2 units. However, in order to make use of what you’ve learned, you may need periodic reminders of key ideas and practice with the skills that put those ideas to work. This *Reference and Practice (RAP)* book includes information and exercises that should be very helpful in reviewing and polishing the mathematics that you encountered in Course 1.

This book has three main sections: **Summary and Review of Course 1**, **Maintaining Concepts and Skills**, and **Practicing for Standardized Tests**.

The first section, Summary and Review of Course 1, contains summaries of key ideas developed in each of the four Course 1 mathematical strands:

- algebra and functions
- statistics and probability
- geometry
- discrete mathematics

The examples in this section illustrate application of the above strands to specific problems. Summaries of each topic are followed by short review problem sets to *Check Your Understanding*. It is a good idea to solve the *Check Your Understanding* problems, then check your solutions against the answer key at the back of the book.
The second section, Maintaining Concepts and Skills, contains twenty sets of review exercises from the various content strands mixed together as they might be in a cumulative examination or in a real-life problem situation. These maintenance exercise sets draw primarily from material in Course 1 and should be used as periodic reviews to keep ideas from all strands fresh in your mind. Exercise Sets 1–10 review Course 1 and can be used at any time during the course. Since Exercise Sets 11–20 include some material from the first part of Course 2, they should be used any time during the second half of the course year. Additional exercise sets for maintaining Course 2 concepts and skills are included in the Teaching Resources for Course 2.

The third section, Practicing for Standardized Tests, presents ten sets of questions that draw on all content strands. The questions are presented in the form of test items similar to how they often appear in standardized tests such as state assessment tests or the Preliminary Scholastic Aptitude Test (PSAT). Use these test sets any time during the school year to become familiar with the formats of standardized tests and to develop effective test-taking strategies for performing well on such tests.

Because you will probably use this book when studying outside of regular mathematics class sessions, answers to the problems are given at the end of the book. It is often possible to learn a lot by studying worked examples and working back from given answers to the required solution process. However, it’s better to try to solve the problems on your own first, and then look at the answer key.
The four parts in this section of the Course 2 RAP book give brief summaries and illustrative examples for the key topics in algebra and functions, statistics and probability, geometry, and discrete mathematics that you studied in Course 1. As you progress through Course 2, there may be activities or problems for which you need to use previously-learned ideas that you don’t completely remember. You can use this section to refresh your understanding of those ideas.

Within each topic summary you will find brief explanations of related concepts and methods, together with worked examples that are intended as a reference. These don’t need to be studied from beginning to end. However, you should scan through this section so that you have an idea of what mathematics is reviewed and where various topics and subtopics are located. You can then refer to this section for specific information to help you when you need it.
Algebra and Functions

One of the main topics in high school mathematics is the study of relations among quantitative variables. For example, if your school needs an affordable Internet service provider for its computer network, it might consider monthly cost options like these:

- **Web-Con**: $195 plus $5 per hour of use.
- **Core-Net**: $265 plus $12 per hour of use beyond 20 hours.

You’ve begun to develop skill in the use of algebraic ideas to answer questions such as these:

- What equations will relate time used $x$ and monthly cost $y$ for each service?
- How can the two pricing options be displayed and compared by tabular or graphic displays?
- How can the school determine which is the cheaper of the two services?
- How will the school’s Internet connection bill grow if usage increases 50% each year from its present level of 500 hours per year?

The following sections review the key concepts and skills required to answer these kinds of questions and provide additional practice problems.

### 1.1 Linear Relations

The most common and important relations among variables are those that mathematicians call *linear relations*. The two Internet service provider price rules are examples of linear relations.

**EXAMPLE 1** Any linear relationship can be expressed as an equation in the form $y = a + bx$. The equation $y = 195 + 5x$ represents the Web-Con pricing policy. The Core-Net pricing policy can be expressed as $y = 265 + 12(x - 20)$ which is equivalent to $y = 25 + 12x$ for $x > 20$.

**EXAMPLE 2** The graph of a linear relation $y = a + bx$ is always a straight line with $y$-intercept $a$ and slope $b$. This graph corresponds to patterns in a table of $(x, y)$-values where $y$ changes by $b$ for every increase of 1 in $x$. The Web-Con pricing policy can be represented by tables and graphs like these:
EXAMPLE 3 ▶ The slope of a nonvertical line is the ratio \( \frac{y_2 - y_1}{x_2 - x_1} \) where \((x_1, y_1)\) and \((x_2, y_2)\) are any two points on the line. The y-intercept is the point where the graph intersects the y-axis.

The slope of the line containing \((0, 195)\) and \((20, 295)\) is 5, since \( \frac{295 - 195}{20 - 0} = \frac{100}{20} = 5 \). The y-intercept of this line is 195.

EXAMPLE 4 ▶ The constant rate of change pattern in linear relations can be expressed by an equation \( NEXT = NOW + b \) (start at \( NOW = a \)). For example, the linear relation defined by the equation \( y = 195 + 5x \) could also be given by the equation \( NEXT = NOW + 5 \) (start at 195).

Check Your Understanding 1.1

Solve the following problems to check your understanding of linear relations in equation, graph, and table form.

1. Find equations in \( y = a + bx \) and \( NEXT = NOW + b \) forms to express the following problem conditions.

   a. Valerie sells ice cream bars from a traveling truck. She is paid $25 per day plus $0.20 for each item sold. How is her daily pay related to the number of items sold in a day?

   b. To avoid a finance charge on the purchase of a $1,500 big-screen television set, a customer agrees to pay $100 per month. How is the outstanding debt \( d \) related to the number of months \( m \) for which payment has been made?
2. For each of the two linear relations in Problem 1:
   a. Identify the rate of change in a table of values.
   b. Identify the slope and the y-intercept of the graph.
3. Write equations for the lines that satisfy these conditions.
   a. Slope of 2 and y-intercept of 5
   b. Passing through the points \((-3, -1)\) and \((5, 3)\)
   c. Slope of \(-3\) and passing through the point \((0, 2)\)
4. Draw graphs of each of the following linear equations.
   a. \(y = -3 + 1.5x\)
   b. \(2x + 3y = 6\)
   c. \(y = -1.5 - 2x\)
   d. NEXT = NOW + 3 (start at 5)
5. Find an equation expressing the relationship between \(x\) and \(y\) in each table of values.
   a. \[
   \begin{array}{c|cccccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
   \hline
   y & -1 & 2 & 5 & 8 & 11 & 14 & 17 & 20 & 23 \\
   \end{array}
   \]
   b. \[
   \begin{array}{c|cccccccc}
   x & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & 12 & 10 & 8 & 6 & 4 & 2 & 0 & -2 & -4 \\
   \end{array}
   \]

### 1.2 Linear Equations and Inequalities

Many questions about linear relations require substituting specific values for variables in an expression or solving an equation or inequality to find an unknown value of a variable.

_Evaluation of linear expressions_ follows the rules for order of operations in arithmetic. As a general rule: Operations within parentheses come first; multiplication and division (from left to right) come next; addition and subtraction (from left to right) come last.
EXAMPLE 1  The equation \( y = 265 + 12(x - 20) \) gives monthly cost of the Core-Net computer service as a function of time used, when \( x > 20 \).

When \( x = 60 \),

\[
\begin{align*}
  y &= 265 + 12(60 - 20) \\
  &= 265 + 12(40) \quad \text{(Operate within parentheses first.)} \\
  &= 265 + 480 \quad \text{(Multiply next.)} \\
  &= 745 \quad \text{(Add last.)}
\end{align*}
\]

Questions that require solving linear equations are usually in the form “What value of \( x \) gives a specified value of \( y \)?” For example, the question “How much Internet time can a school get from Web-Con service for $400 per month?” requires solving the equation \( 400 = 195 + 5x \). There are several basic strategies to use in solving linear equations.

- In simple cases, you can make an estimate of the required \( x \)-value and check it to see if it works. You can use a graphing calculator to help with this estimation by producing tables of values and graphs of the underlying equations.
- You can undo the operations that produce \( y \)-values from given \( x \)-values.
- You can think of the equation as a balance and apply identical operations to both sides of the equation until an equivalent simpler form is reached, revealing the required solution.

The following examples illustrate use of these strategies.

EXAMPLE 2  The equation \( 400 = 195 + 5x \) looks quite simple, so you might begin by making some numerical estimates or using a calculator.

a. Try \( x = 50 \), \( 195 + 5(50) = 445 \) (too large).

Try \( x = 40 \), \( 195 + 5(40) = 395 \) (too small).

Try \( x = 41 \), \( 195 + 5(41) = 400 \).

b. Graph \( y = 195 + 5x \) and trace the graph for an \( x \)-value that gives \( y = 400 \).
c. Make a table of \((x, y)\)-values and look for an \(x\)-value that gives \(y = 400\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(Y_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>95</td>
</tr>
<tr>
<td>10</td>
<td>245</td>
</tr>
<tr>
<td>20</td>
<td>295</td>
</tr>
<tr>
<td>30</td>
<td>345</td>
</tr>
<tr>
<td>40</td>
<td>395</td>
</tr>
<tr>
<td>50</td>
<td>445</td>
</tr>
<tr>
<td>60</td>
<td>495</td>
</tr>
</tbody>
</table>

\(X=0\)

The costs for Web-Con service are calculated by first multiplying the number of hours of use \(x\) by 5, then adding 195. To solve for an unknown \(x\) we need to undo those operations.

If \(400 = 195 + 5x\), then \(205 = 5x\) (Subtract 195.)

then \(41 = x\) (Divide by 5.)

In some cases, the linear equations get more complex. For example, sometimes it is necessary to determine when two linear expressions involving a variable \(x\) yield the same \(y\)-value.

EXAMPLE 3

To determine the number of hours of usage for which Web-Con and Core-Net would cost the same for Internet service, you need to solve the equation \(195 + 5x = 25 + 12x\).

a. The new equation can be solved by looking at tables of values or graphs of the two cost expressions and looking for a number of hours used that gives the same cost under each plan.

Both the table and graph indicate that the cost for the two plans will be approximately the same when 24.3 hours of time are used. You can get a more accurate solution by zooming in on either the table or graph.

b. The strategy of thinking about an equation as a statement of balance applies well to this sort of equation.

If \(195 + 5x = 25 + 12x\),

Then \(170 + 5x = 12x\) (Subtract 25 from both sides.)

\(170 = 7x\) (Subtract 5\(x\) from both sides.)

\(24.3 \approx x\) (Divide both sides by 7.)
In some cases, the questions that arise about linear relationships are more accurately stated as inequalities. The strategies for solving inequalities are almost identical to those for working with equations: inspect tables and graphs, rewrite the inequality using undo operations, or perform identical operations on both sides of the inequality. The only caution to keep in mind is the effect of multiplying or dividing an inequality by a negative number—that reverses the direction of the inequality.

**Check Your Understanding 1.2**

Check your understanding by solving each of the following linear equations and inequalities in at least two ways—one using estimation, perhaps with a calculator table or graph, and another method that applies reasoning like undoing or operating on a balanced equation.

1. $3x + 24 = 63$
2. $89 \leq 159 - 4x$
3. $12 + 5x = 7x - 32$
4. $-2.5x + 9 > 4.75x - 15$
5. $\frac{2}{3}x + \frac{4}{5} = \frac{7}{8}$

**1.3 Equivalent Linear Expressions**

In many situations, the natural way to write a linear relation between two variables does not give the standard symbolic form $y = a + bx$. Instead it makes sense to write some equivalent expression. For example, the Core-Net pricing policy is most naturally written as $y = 265 + 12(x - 20)$ which is equivalent to $y = 25 + 12x$.

You can test the equivalence of two expressions informally by comparing tables and graphs of each. You can prove the equivalence by reasoning with principles that allow rewriting of given expressions by combining, expanding, and rearranging terms. The basic rules for rewriting expressions in equivalent forms are illustrated in the following example.

**EXAMPLE 1**

Terms combined by addition can be reordered to give equivalent expressions because addition is commutative. However, rearranging expressions involving subtraction must be done with care, since subtraction is not commutative.

a. $3x + 5 + 7x$ is equivalent to $5 + 3x + 7x$.

b. $3x - 5 - 7x$ is equivalent to $-5 + 3x - 7x$, but is not equivalent to $5 - 3x - 7x$. 
c. 3x + 7x is equivalent to 10x and 3x – 7x is equivalent to –4x.
d. 265 + 12(x – 20) is equivalent to 265 + 12x – 240 or 25 + 12x.

The key to combining or expanding terms is the *distributive property of multiplication over addition*: For any a, b, and c, it is always true that \((a + b)c = ac + bc\) and also \(a(b + c) = ab + ac\).

**Check Your Understanding 1.3**

Check your understanding of equivalence of linear expressions in the problems that follow.

1. Which of the following pairs of expressions are equivalent? Be prepared to explain the reasoning that led to your answers.
   a. 13x + 7x + 5 and 5 + 20x
   b. 8x – 4 + 12(x – 2) and –6 + 20x
   c. –4(3x + 5) – 9 and –29 – 12x

2. Write each of the following linear expressions in two different equivalent forms, one of which is the standard \(a + bx\) form.
   a. 4x + 72 – 9x
   b. 19 + 12(2x – 3)
   c. –7.4x + 3(x + 7) – 9.5

3. If a linear relation is expressed as an equation like \(ax + by = c\), it can be written in equivalent standard form by solving the given equation for \(y\) in terms of \(x\). Solve the following equations for \(y\), and write in the \(y = a + bx\) form.
   a. 6x + 2y = 8
   b. –12x + 3y = 15
   c. 8x – 3y = 7

**1.4 Exponential Relations**

In many important problem situations, the values of key variables increase or decrease at a *constant percent rate*. The patterns in those situations are often what mathematicians and scientists call *exponential growth* or *decay*.

**EXAMPLE 1** If a school is currently using 200 hours per year of Internet connection time but expects its usage to double each year, the future usage levels can be predicted by two kinds of equations.

a. \(y = 200(2^x)\) gives 200, 400, 800, 1,600, ...

b. \(NEXT = 2 \cdot NOW\) (start at 200) gives the same projections.
EXAMPLE 2  The doubling prediction is very optimistic. If, instead, the projection was for 25% growth each year, the exponential growth can also be modeled in two ways.

a. \( \text{NEXT} = \text{NOW} + 0.25 \cdot \text{NOW} = 1.25 \cdot \text{NOW} \) (start at 200)

b. \( y = 200(1.25^x) \)

EXAMPLE 3  Exponential decay models are useful in predicting the way that medications are metabolized in human or animal bodies. If you receive a 500 milligram shot of penicillin at 8 A.M., only 70% would remain active in your body one hour later. Only 70% of that amount would remain active after another hour passes. This pattern of decay can be modeled by equations in two ways.

a. \( \text{NEXT} = 0.70 \cdot \text{NOW} \) (start at 500)

b. \( y = 500(0.7^x) \)

The general forms of equations modeling exponential growth and decay are \( y = a(b^x) \) and \( \text{NEXT} = b \cdot \text{NOW} \) (start at \( a \)). When \( b > 1 \), the model produces exponential growth; when \( 0 < b < 1 \), the model produces exponential decay. The value of \( b \) is called the growth or decay factor. Typical graphs of growth and decay look like those in the following diagram.

Specific questions about exponential growth and decay can often be expressed as equations or inequalities. Those problems can be solved by careful inspection of calculator tables and graphs. (In Courses 3 and 4 you will learn how to solve such problems using symbolic reasoning strategies.)
EXAMPLE 4  

To predict the time when only 100 milligrams of penicillin will be active in your body, you need to solve the equation $100 = 500(0.7^x)$. The table and graph show that this time will be approximately 4.5 hours after the injection, so the solution of the equation is $x \approx 4.5$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
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<table>
<thead>
<tr>
<th>$X$</th>
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Check Your Understanding 1.4

Check your understanding of exponential models in the problems that follow.

1. Express the following exponential relations between time and amount using equations of two types: $y = a(b^x)$ and $NEXT = b \cdot NOW$ (start at $a$).

   a. If a golf ball is dropped from a height of 15 feet, it will rebound to about 60% of that height on the first bounce. What equations will predict the height on any succeeding bounce?

   b. If an experiment with bacteria causes an initial colony of 50 to double every hour, what equations will predict the population of the colony at any time?

   c. If a person’s credit card balance is now $5,000 and is being increased by interest charges of 1.5% per month, what equations will predict the size of that credit card debt at any time in the future? (Assume no payments are made.)

2. Sketch the shapes of the graphs you would expect for the three situations in Problem 1.

   a. Rebound height as a function of bounce number

   b. Bacteria population as a function of time

   c. Credit card balance as a function of time
3. Solve each of the following problems involving exponential growth and decay.

a. How long will it take for the number of bacteria in Problem 1, Part b to reach 1 million?

b. For how long will the credit card balance in Problem 1, Part c remain under $6,000?

4. Match each equation to one of the graphs in the figure.

a. \( y = 3(1.5^x) \)

b. \( y = 4 - 2x \)

c. \( NEXT = \frac{2}{3} \cdot NOW \)

d. \( NEXT = NOW + 2 \)
Making sense of patterns in data is one of the most common and important tasks in science, business, and government problem solving and decision making. For example, the following table shows the normal high temperature (˚F) for each date in the month of October in a midwestern city and the actual high temperatures (˚F) for those dates in October of 1998.

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<thead>
<tr>
<th>Date</th>
<th>Normal</th>
<th>Actual</th>
</tr>
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<tr>
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</tbody>
</table>

You’ve begun to develop skill in the use of several basic statistical techniques for organizing, displaying, and analyzing such numerical data to answer questions like these:

- What graphical displays effectively show the distribution of high temperatures in that city during October, 1998?
- What numerical summaries describe the center of the actual high temperatures experienced in that month and the variability of the actual high temperature values?
What displays or numerical summaries are useful in comparing normal high temperatures and actual high temperatures for this particular year?

How does the normal high temperature change from the beginning to the end of the month?

The following sections review the key concepts and skills for answering questions like these.

### 2.1 Data Displays

The distribution of values for a particular set of data can be displayed by common statistical plots. To show the distribution of actual high temperatures in the data given above, you could use any of the following plots.

**EXAMPLE 1** A number line plot records each temperature with an \( \times \) stacked above the corresponding temperature value on a number line. To simplify plotting, it makes sense to treat nearby data points (like 60 and 61) the same. This gives the following distribution picture.

![Number line plot example](image)

**EXAMPLE 2** A stem-and-leaf plot for two-digit data, like the actual high temperatures, lists the possible tens digits in a column stem and the actual ones digits as leaves off the stem at the appropriate level.

<table>
<thead>
<tr>
<th>1</th>
<th>2 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2 5</td>
</tr>
<tr>
<td>4</td>
<td>0 2 3 5 5 5 6 6 7 7</td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 0 6 6 9</td>
</tr>
<tr>
<td>6</td>
<td>0 1 3 5 5 8</td>
</tr>
<tr>
<td>7</td>
<td>0 5 8</td>
</tr>
<tr>
<td>8</td>
<td>0 1</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

28 represents 28°F
EXAMPLE 3  A histogram groups data into intervals and has bars above those intervals to indicate frequency of occurrences in the interval.

EXAMPLE 4  After you have created a plot of a data set, it is often important to describe the distribution in words. When describing a distribution, you should discuss maximum and minimum values, outliers, gaps, the shape—symmetric, skewed (stretched to the) right, or skewed left—the center, and the spread of the distribution. The data displayed in the plots in Examples 1 through 3 have a minimum value of 28°F and a maximum value of 81°F. There are no outliers or gaps in this data. The distribution is centered at about 50°F and is spread over a large range. The distribution has one peak and is skewed slightly to the right.

EXAMPLE 5  A box plot shows a box over the interval containing the middle half of the data, whiskers reaching to the minimum and maximum data values, and a vertical line dividing the box at the median point.

Each of these data displays is useful in showing the distribution of values in a data set. You can use back-to-back stem-and-leaf plots or parallel box plots to compare distributions of two data sets.

A scatterplot can be used to display paired data. If the variables determining each coordinate are related, the relationship will appear as a pattern in the plot.
EXAMPLE 6 ▶ This scatterplot shows pairs of temperature values (normal, actual). The line is the graph of $y = x$, where one would expect points if normal and actual high temperatures were the same.

EXAMPLE 7 ▶ This plot over time shows how the actual high temperature varied from the beginning to the end of the month observed.

Check Your Understanding 2.1

Check your understanding of data plots in the problems that follow.

1. The following data are scores from 25 students on a unit test in mathematics.

75 77 65 45 50 92 93 50 54 60 62 65 70 73 74 78 80 57 84 85 85 90 98 65 70

Construct these plots of the data:

- a. Stem-and-leaf plot
- b. Box plot
- c. Number line plot
- d. Histogram (you choose a suitable interval length)

2. The following data are scores from the same 25 students on the next unit test (students are given in the same order as above).

80 75 60 75 60 90 95 70 50 70 80 60 80 63 80 85 75 55 85 95 75 95 90 65 75

Construct a scatterplot of the pairs of scores for each student (first test, second test), draw the $y = x$ line on the plot and circle some points that
show students who improved on the second test. Place squares around points that show some students who did worse on the second test.

3. Describe the distribution displayed in the histogram below.

2.2 Numerical Data Summaries

Graphic displays convey a visual image of the distribution of values in a data set. However, it is often useful to quantify that distribution with numerical measures of center and variability. There are two common measures of center.

- The **median** is the midpoint of an ordered list of data—half the values are at or below it and half are at or above it. The *xth percentile* is the value with $x\%$ of the data at or below it.

- The **mean**, or arithmetic average, is the sum of the data values divided by the number of values in the data set. It is the balance point of the distribution.

The median is especially useful when the distribution is skewed or when there are a few outliers in the data set. **Outliers** are data points that stand apart from the overall pattern of the data.

The minimum, first quartile ($Q_1$), median, third quartile ($Q_3$), and maximum values in any data set are called the **five number summary** of that distribution. They are the values needed to construct a box plot for the data. See the box plot on page 18.

**EXAMPLE 1**

The median of actual high temperatures in the data set given on page 16 is 50.

```
28 32 35 40 42 43 45 45 45 46 46 47 47 50 50 50
50 56 56 59 60 61 63 65 65 68 70 75 78 80 81
```

The 25th percentile or lower quartile is 45 and the 75th percentile or upper quartile is 65.
EXAMPLE 2  The mean of the actual high temperature values is 54.13 since the sum of all values is 1,678 and \(1,678 \div 31 \approx 54.13\).

While it is useful to have a single number indicating the center of a distribution, it is equally important to describe the variability in the data—the tendency of the data to lie close to or far away from the center of the distribution. Variability is measured by several common statistical summaries.

- The range is the difference between the minimum and maximum values.
- The interquartile range is the difference between the lower and upper quartiles or the difference between the 25th and 75th percentile values.
- The Mean Absolute Deviation (MAD) is the average distance of the data values from the mean.

The interquartile range is relatively unaffected by a few outliers, but the MAD will be increased by introduction of such “far out” data.

EXAMPLE 3  The variability of actual high temperatures for that midwestern city in October 1998 can be summarized by:

a. Range: \(81 - 28 = 53\)

b. Interquartile range: \(65 - 45 = 20\)

c. MAD: 
\[
\left(\left|28 - 54.13\right| + \left|32 - 54.13\right| + \ldots + \left|81 - 54.13\right|\right) \div 31 \approx 11.56
\]

Check Your Understanding 2.2

Check your understanding of measures of center and variability in the problems that follow.

1. Consider again the mathematics unit test scores from “Check Your Understanding 2.1” (page 19).

   Test 1: 75 77 65 45 50 92 93 50 54 60 62 65 70 73 74 78 80 57 84 85 85 90 98 65 70

   Test 2: 80 75 60 75 60 90 95 70 50 70 80 60 80 63 80 85 75 55 85 95 75 95 90 65 75

   a. Calculate five number summaries for each set of test data.

   b. Calculate the mean score on each test.

   c. Calculate the range, interquartile range, and MAD for each set of test data.

2. Based on the data summaries you calculated in Problem 1, what comparisons of student performance on the two tests seem interesting and justified?
For many activities in life, the outcomes of any individual trial cannot be predicted, but the expected long-run relative frequency of various possibilities can be described with some confidence. Probability is the science of making such predictions, and one of the simplest effective techniques for making probability judgments is simulation.

**EXAMPLE 1** Suppose that half-time entertainment at a school basketball game involves picking 5 students at random from the crowd and giving each a chance to make a long shot and win a prize. If the crowd is about 40% girls and 60% boys, how many of each are likely to be selected?

Selection of the 5 students to take the long shot might be simulated in several ways.

a. Inspect groups of 5 numbers in a table of random digits—each occurrence of 0, 1, 2, or 3 would represent a girl chosen, and each occurrence of 4, 5, 6, 7, 8, or 9 would represent a boy.

b. A game spinner with its face divided into 5 congruent sectors numbered 1 to 5 can be used in groups of 5 spins. A 1 or 2 would represent a girl chosen and a 3, 4, or 5, a boy chosen.

c. If there are, for example, 50 students at the game, you could put 20 red chips and 30 blue chips in a bag and draw groups of 5 chips, recording the number of red ones (girls) and blue ones (boys).

These simulations are not identical, because the first two methods assume that chances of a girl or a boy being selected don’t change after some selections from the crowd have been made. If the crowd is large enough, that won’t make a significant difference; if the crowd is smaller, then the simulation in Part c is the better method.
EXAMPLE 2 Suppose the simulation in Example 1 involved 100 trials and gave the following results:

<table>
<thead>
<tr>
<th>Number of Girls Chosen</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.02</td>
</tr>
</tbody>
</table>

a. These results suggest, for example, that about 25% of the time exactly 1 girl will be selected and rarely will all 5 students selected be girls.

b. The distribution of frequencies for the number of girls chosen can be displayed by a frequency histogram as follows.

EXAMPLE 3 Suppose that your school drama club is selling deluxe pizzas to raise money for the school musical production, and each member of the club agrees to sell three pizzas. If past experience suggests that you’ll make a sale to roughly 80% of the people you contact, how many sales calls would you expect to make before reaching your quota of three pizzas sold?
a. This situation can be simulated in several ways. You could use a calculator to generate random digits from 1 to 5 and count 1 to 4 as sales. Each trial would continue until you reached 3 sales. For example, it might go 5, 1, 2, 5, 4 (no sale, sale, sale, no sale, sale).

b. If you ran 50 trials of the simulation, the number of sales calls until reaching your quota might be distributed as in the following table.

<table>
<thead>
<tr>
<th>Calls Until 3 Sales</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>22</td>
<td>0.44</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>0.28</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>0.14</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.02</td>
</tr>
</tbody>
</table>

These simulation data suggest that on average one could expect to require about \[
\frac{22(3) + 14(4) + 7(5) + 5(6) + 1(7) + 1(9)}{50}
\] or slightly more than four calls to make the three sales.

This simulation assumes that the trials are independent. That is, the probability each person you ask will buy the pizza is 0.8, no matter what has happened on the previous trials.

EXAMPLE 4  When running a simulation, you have to be able to generate random numbers in a given range. You may choose to do this using dice, spinners, random digit tables, or random number generators. The pizza selling simulation in Example 3 used random digits from 1 to 5. Using a TI calculator, the command, \(\text{int} 5 \text{ rand} + 1\), will generate a random integer from 1 to 5, inclusive.

One of the critical decisions in designing and using a simulation to estimate probabilities is how many times to repeat the simulation experiment. The law of large numbers says that as a simulation or experiment is repeated more and more times, the proportion or relative frequency of each possible outcome tends to get closer and closer to the theoretical probability of that outcome.
EXAMPLE 5 If two people flip the same fair coin—one 20 times and one 200 times—which is the more likely result: (1) between 9 and 11 heads out of 20; or (2) between 90 and 110 heads out of 200?

Both possible results include the same range of relative frequency for heads, between 0.45 and 0.55. However, with the larger number of trials one would expect a relative frequency that is closer to the true probability of 0.50. Thus, you have a better chance of getting between 90 and 110 heads in 200 trials than of getting between 9 and 11 heads in 20 trials.

In general, to estimate a probability with 95% confidence that your estimate is accurate within a margin of error \( E \) you’ll need to run \( n = 1 / E^2 \) trials in your simulation.

Check Your Understanding 2.3

Check your understanding of simulation and probability in the problems that follow.

1. Suppose that six students are to be chosen at random for interviews by a visiting team evaluating their school. Twenty percent of the students in the school are on the honor roll. Design and use an appropriate simulation to approximate the probability distribution for the number of honor students in the interview group.

2. Suppose that 40% of the households in a city are connected to cable television and a company is conducting a survey of satisfaction with that service. If you are asked to pick names at random from the city phone book and call until you reach five cable subscribers, how many calls would you expect to make, on average, before satisfying the quota?
   a. Design and run a simulation with 20 trials to get data for estimating the probability distribution of this activity.
   b. Use the data from Part a to estimate the average number of calls that would need to be made to reach the quota of five cable subscribers.

3. What random numbers will be generated by the command: int 20 rand?
   By the command: int 36 rand + 1?

4. How many trials of a simulation do you need in order to be 95% confident that the relative frequency of an outcome is within 0.05 of the actual probability of that outcome?
The spaces we live in and the objects we use for work and play have a rich variety of functional and decorative geometric shapes. In your study of 2- and 3-dimensional shapes, you have developed skill in answering questions like these:

- What are the most common geometric shapes that occur in natural and designed objects?
- How does the shape of an object affect its uses?
- How can 3-dimensional shapes be portrayed effectively in 2-dimensional diagrams?
- How can perimeter, area, and volume of shapes be measured?
- What kind of figures have various sorts of symmetry?
- What planar shapes can be used to tile a flat surface?

The answers to these questions lie in several important and widely useful geometric principles.

### 3.1 Space-Shapes and Properties

There are many types of 3-dimensional or space-shapes, from rectangular boxes to cylinders, cones, and spheres. But many of the most common space-shapes are in the families called *prisms* and *pyramids*.

**EXAMPLE 1** The cardboard packages of many food products such as cereal, pasta, or sugar are *prisms* because they have two congruent parallel faces (base and top), with edges on these faces joined by parallelograms (usually rectangles). If the base and top of a prism are congruent pentagons, the figure is called a pentagonal prism; a six-sided base and top give a hexagonal prism, and so on.
Pyramids have one polygonal base and triangular faces formed by joining each corner of the base to a single vertex point. They are named by the shape of their base.

Cylinders and cones are like prisms and pyramids but they have circular bases.

Space-shapes can be represented on flat surfaces by providing face-views—top, front, right side, or by isometric drawings.

**EXAMPLE 2** The top, front, and right side views of a rectangular prism like a cereal box can be drawn on square-grid dot paper.

**EXAMPLE 3** An isometric view from a corner of such a box can be drawn on isometric dot paper.
As you explored the various shapes in which 3-dimensional figures can be constructed, you discovered that those with triangular frames are rigid, while others are not. This principle is used in many construction projects to strengthen a shape. You also discovered some very clever ways that flat patterns or nets can be designed for space-shapes.

**EXAMPLE 4**  The following patterns of squares and equilateral triangles can be folded along edges to form a cube and a triangular pyramid.

![Cube and Triangular Pyramid Patterns](image)

---

**Check Your Understanding 3.1**

Check your understanding of space-shapes and skill in drawing by completing these problems.

1. Describe the faces in each space-shape and their relationship to each other.
   a. An octagonal pyramid  
   b. A square prism

2. Draw the following space-shapes.
   a. A square pyramid with top, front, and side views on square-grid dot paper
   b. A cube on isometric dot paper

3. Suppose the sketch at the right shows a triangular prism made by connecting 9 rods that have eyelets at each end. Will the figure be rigid? If not, what rods can be added to make it rigid?

4. Draw nets that can be folded to make these figures.
   a. A rectangular box that is $3 \times 4 \times 2$ centimeters
   b. A square pyramid that has base edges with a length of 5 centimeters
3.2 Measuring Perimeter, Area, and Volume

In designing, building, and using 2-dimensional shapes, it is often important to consider the perimeter and area and, in the case of 3-dimensional shapes, the surface area and volume.

EXAMPLE 1  The following diagram shows the top and side views of a swimming pool to be built in a hotel courtyard. The drawing is on quarter-inch grid paper with a linear scale in which each grid segment represents 3 feet in the pool.

a. To calculate the amount of tile needed to go around the edge of the pool, you need to estimate the perimeter of the pool. It is about 28 grid lengths or 84 feet.

b. To calculate the cost of a pool cover, you need to estimate the surface area of the pool. It is about 378 square feet, since each square represents an area of 9 square feet on the pool and it will take about 42 squares to cover the pool.

c. To calculate the cost of filling the pool with water, you need to estimate the volume. It looks like the average water depth is about 1.5 grid lengths or about 4.5 feet, so the volume will be about $4.5 \times 378 = 1,701$ cubic feet.

For geometric shapes with standard polygonal or circular shapes, there are formulas for calculating perimeter, area, and volume.
The measurement formulas for familiar figures are

a. Triangle: Perimeter $P = a + b + c$
   
   $\text{Area } A = \frac{1}{2}bh$

\[ \text{Diagram of a triangle} \]

b. Parallelogram: Perimeter $P = 2(a + b)$
   
   $\text{Area } A = bh$

\[ \text{Diagram of a parallelogram} \]

c. Circle: Circumference $C = 2\pi r$ or $C = \pi d$
   
   $\text{Area } A = \pi r^2$

\[ \text{Diagram of a circle} \]

d. Prism: The surface area is the sum of the areas of the faces, and the volume is the product of the area of the base and the height. In the case of a rectangular prism:
   
   $A = 2(lw + lh + wh)$
   
   $V = lwh$

\[ \text{Diagram of a rectangular prism} \]
e. Cylinder: Surface area is $A = 2\pi r^2 + 2\pi rh$
Volume $V = \pi r^2 h$

![Cylinder diagram]

In many measurement problems, the figures of interest are drawn on some sort of rectangular grid, giving coordinates of key points. One common error in measuring distances on such a grid occurs if you forget that the distance across a grid square from corner to corner is shorter than the distance along the edges.

![Grid diagram]

Straight line path from $A$ to $C$ is shorter than the path via $B$.
$AC < AB + BC$

To calculate the exact distance from corner to corner of a rectangle you need to use the Pythagorean Theorem: In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

![Pythagorean theorem diagram]

**EXAMPLE 3** The diagonal of a rectangle with side lengths of 8 cm and 15 cm is 17 cm because

$$8^2 + 15^2 = 64 + 225 = 289 = 17^2.$$
Check Your Understanding 3.2

Check your understanding and skill in working with measurement of plane figures and space-shapes by solving these problems.

1. Find the surface area and volume of a box in the shape of a square prism with a base that is 20 inches on a side and a height of 30 inches.

2. Find the surface area and volume of a cylinder that has a base with a radius of 5 centimeters and a height of 12 centimeters.

3. Find the perimeter and area for each figure pictured below.

   a. 
   
   5 cm
   13 cm
   12 cm

   b. 
   
   5 m
   4 m
   3 m

   c. 
   
   60 ft
   120 ft

4. Find the missing dimensions of the television screens pictured below.

   a. 
   
   30
   40

   b. 
   
   16.2
   27

3.3 Shapes of Plane Figures

When working with 2-dimensional figures it is often interesting and useful to look for two important properties.

- A figure can be used to tile or tessellate the entire plane if multiple copies of the figure can be laid down together to cover the plane without gaps and without overlapping.

- A figure has symmetry if there is a rigid motion—reflection, rotation, translation, or glide reflection—that maps the figure onto itself.

Symmetry and tiling are very often used together to create artistic designs.
EXAMPLE 1  The following patterns show ways that triangles, parallelograms, and regular hexagons can be used to tile the plane. In general, any triangle and any quadrilateral will tile the plane.

Among regular polygons (all sides and angles are congruent), only equilateral triangles, squares, and hexagons will give tilings, because only those polygons have interior angles with a measure that divides 360 evenly.

To find the measure of the interior angle of a regular polygon, subdivide the polygonal shape into triangles by drawing all possible diagonals from one vertex; then use the fact that the sum of the measures of the angles of a triangle is 180°. The sum of the measures of the angles of the regular hexagon shown is 4(180°) = 720°. Since each angle of a regular hexagon is the same size, the measure of one angle is 720° ÷ 6 = 120°.

EXAMPLE 2  The following sketches show figures with reflection (line), rotation, translation, and glide reflection symmetries.

Reflection symmetry
Rotation symmetry

Translation symmetry (assuming pattern extends indefinitely)

Translation symmetry (assuming pattern extends indefinitely)
Check Your Understanding 3.3

Check your understanding of tiling and symmetry by completing these problems.

1. What types of symmetry are present in these letters of the alphabet?

2. Show with sketches how each of the following figures can be used to tile a flat surface.
   a. 
   b. 
   c. 

3. If a figure is a regular $n$-gon, what is the measure in degrees of each interior angle?

4. What types of symmetry are present in the diagram below? Assume the pattern continues to the left and to the right.
In business, government, and everyday life, you often need to plan work on complex jobs—jobs that combine the efforts and time of many people and many kinds of equipment. Efficient planning of this work is often aided by use of vertex-edge graph models and algorithms for analyzing those models.

In Course 1 of Contemporary Mathematics in Context, you learned how to use graph models in answering questions like these:

- What are the most efficient routes that a city government can set up to collect money from parking meters or to plow snow off its streets?
- How can the minimum number of frequencies be assigned to radio or television stations so that they don’t interfere with each other’s broadcasts?
- How can you schedule all the tasks in a complex project like a school play or a large construction project so that the project is completed in the least amount of time?

You also saw ways that vertex-edge graphs can be used to play and analyze games that are popular around the world.

4.1 (a) Using Euler Paths to Find Efficient Routes

The first step in using graph models is to make a simple diagram of vertices and edges that represents the key features of a more complex situation. To do this, you must identify what the vertices and edges represent.

**EXAMPLE 1**

The floor plan of Buck Lodge Middle School has classrooms along several different hallways. When the fire department holds a fire drill at the school, the inspector has to walk down each hallway to see that all students are out of the building. It would be nice if the inspector could walk down each hallway just once and end up at his or her starting point.

a. The floor plan can be represented as a graph like this:
The vertices in this situation represent “intersections” and the edges represent hallways.

b. The graph does not have an Euler circuit—a sequence of edges and vertices including every edge exactly once and returning to the starting point. In fact, the graph does not even have an Euler path where the starting point does not have to be the ending point. Thus, the inspector will have to retrace steps along some halls.

c. You can plan which hallways to retrace by Eulerizing the graph, that is, adding edges that duplicate existing edges so that the resulting new graph does have an Euler circuit. It is possible to Eulerize the graph by adding the edges shown in the following diagram.

![Eulerizing Graph Diagram]

There are three key mathematical questions to consider in this problem. (1) Is there an Euler circuit in the graph? (2) If so, how can you find it? (3) If not, how can you Eulerize the graph? The answers to these questions can be determined by using the following ideas.

Fact (theorem): A connected graph has an Euler circuit if and only if every vertex has even degree. (The degree of a vertex is the number of edges that touch the vertex.) Since the original graph in this problem has several vertices with odd degree, it is easy to see that there is no Euler circuit. A connected graph has an Euler path that is not a circuit if and only if it has exactly two vertices of odd degree. The path starts at one odd-degree vertex and ends at the other.

The basic strategy for Eulerizing a graph is to add edges so that every vertex has even degree. One way to at least begin doing this is to add edges between pairs of odd-degree vertices.

Once you know that an Euler circuit exists, there are several systematic methods (algorithms) for efficiently constructing the Euler circuit.
4.1 (b) Coloring Graphs to Resolve Conflicts

In many complex problem situations, there are competing interests. Graph models provide one technique for resolving those conflicts in a systematic way.

**EXAMPLE 2** Suppose that a school chorus is making a trip to a competition and the director needs to assign the students to hotel rooms. To make sure that the trip goes well, the director doesn’t want any students to be unhappy about their roommates. So she asks each of the eight boys to list other boys with whom they would rather not share a room. The following graph shows the results. The vertices represent the boys, and an edge indicates that there is at least one student preference against rooming with the other.

![Graph showing preferences among boys for roommates](image)

The task of assigning rooms to avoid conflict is equivalent to coloring the vertices of the graph so that any two vertices that are connected by an edge have different colors.

This can be done with three colors, but no fewer. One possible 3-coloring would assign color 1 to $D$, $H$, and $B$; color 2 to $E$, $F$, $C$, and $A$; and color 3 to $G$. The colors correspond to rooms, so three rooms are needed for all eight boys. Note that you could adjust the coloring (that is, the room assignments) so that there are more comparable numbers of students in each room.

There are often several ways to color the vertices of a graph with the fewest number of colors, but no one knows an efficient algorithm that will color any graph with the fewest possible number of colors.
4.1 (c) Using Digraphs to Schedule Complex Projects

Vertex-edge graphs can also be used to organize and schedule the many tasks of a complex project in the most efficient manner. This type of problem generally uses graphs with directed edges, called directed graphs or digraphs. The vertices represent the tasks to be done, and a directed edge from one vertex to another indicates that the one task is an immediate prerequisite for the other. Once a project is represented as a digraph, there are several algorithms that can be used to schedule the tasks and determine the earliest completion time for the whole project.

**EXAMPLE 3**

When a commercial airliner lands, there are many tasks to be done before it can take off again on its next flight.

A Unloading passengers might take 15 minutes and must be done before cleaning can begin.

B Unloading baggage might take 15 minutes and must be done before loading new baggage can begin.

C The pilot must inspect the plane to see that all systems are in good condition. This must happen after passengers are unloaded and before new passengers are allowed to board. It takes about 10 minutes.

D Cleaning the plane might take 10 minutes and must be done before loading new passengers.

E Loading new baggage might take 20 minutes. It can begin as soon as the arriving baggage is off the plane.

G Loading new passengers might take 25 minutes. It cannot begin until the plane has been cleaned and inspected.

The dependence of these tasks can be seen in the following digraph.
a. A critical path for this project is a sequence of tasks, from start to finish, that has the largest total task time. That is, a critical path is the longest path through a graph. In this case, there are two such paths: SADGF and SACGF. Both have a length of 50 minutes. The length of a critical path is the least amount of time required to finish the whole project, which is called the earliest finish time. Thus, the earliest finish time for this project is 50 minutes.

b. It is important to know the earliest finish time for a project. But to actually get the project done in that shortest amount of time, you need to know how to schedule the individual tasks. In particular, it would be nice to know the earliest start time and the latest start time for each task. The earliest start times for the various tasks in this example are 0 minutes for tasks A and B; 15 minutes for C, D, and E; and 25 minutes for G. The latest start times (that would allow completion of the project in the minimal 50 minutes) are 0 minutes for A; 15 minutes for B, C, and D; 25 minutes for G, and 30 minutes for E.

c. A task on a critical path is called a critical task. A critical task is critical in the sense that you cannot get behind schedule on it without delaying the whole project. Thus, there is no slack time for critical tasks. Slack time is the difference between latest start time and earliest start time. In this example, the critical tasks are A, C, D, and G. These tasks have 0 slack time. In contrast, each of tasks B and E has a slack time of 15 minutes.

4.1 (d) Representing a Graph with a Matrix

A graph can be represented by a matrix. When analyzing large graphs or using computers to analyze graphs, a matrix representation is more effective than the vertex-edge pictorial representation. One way to represent a graph with a matrix is by using an adjacency matrix. List the vertices of the graph across the top and down the side of a matrix. Enter a number in the ith row and jth column corresponding to the number of direct connections (single edges) between vertex i and vertex j. You can then operate on the adjacency matrix to get information about the graph.
EXAMPLE 4

The graph below can be represented by the following adjacency matrix.

An example of how to analyze the graph by analyzing the adjacency matrix is to compute the sums of the rows of the matrix. The row sums of the adjacency matrix give the degrees of the vertices. So, for example, you can see that this graph does not have an Euler circuit because not all row sums are even.

Check Your Understanding 4.1

Solve the following problems to check your understanding of graph models and their uses.

1. The sketch at the right shows a map of six countries in Southeast Asia—Myanmar (MY), Thailand (TH), Malaysia (MA), Cambodia (CA), Laos (LA), and Vietnam (VN).

   a. Draw a vertex-edge graph in which vertices represent countries and edges join countries that have a common border.

   b. Determine whether there is either an Euler circuit or an Euler path for the graph in Part a. Explain what such a circuit or path would mean to someone traveling in the six countries.

   c. Determine the minimum number of colors needed to color the map so that no countries with a common border have the same color.
2. Which of the following diagrams can be traced with a pencil starting and ending at the same point and never retracing any edge? Which can be traced without retracing any edge, but perhaps not ending at the starting point?

![Diagram a.](image1)
![Diagram b.](image2)
![Diagram c.](image3)

3. The following table lists times and prerequisites for tasks involved in opening a new store in a chain of fast food restaurants.

<table>
<thead>
<tr>
<th>Task</th>
<th>Task Time</th>
<th>Prerequisites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose Site (S)</td>
<td>90 days</td>
<td>none</td>
</tr>
<tr>
<td>Purchase Land (L)</td>
<td>30 days</td>
<td>S</td>
</tr>
<tr>
<td>Construct Building (C)</td>
<td>60 days</td>
<td>L</td>
</tr>
<tr>
<td>Hire Manager and Staff (H)</td>
<td>20 days</td>
<td>none</td>
</tr>
<tr>
<td>Train Staff (T)</td>
<td>5 days</td>
<td>L, H</td>
</tr>
<tr>
<td>Stock Food and Supplies (F)</td>
<td>2 days</td>
<td>C, H</td>
</tr>
<tr>
<td>Advertise for Opening (A)</td>
<td>14 days</td>
<td>L</td>
</tr>
<tr>
<td>Do Practice Before Opening (P)</td>
<td>2 days</td>
<td>T, F, A</td>
</tr>
</tbody>
</table>

a. Construct a digraph showing how these tasks and times fit together.
b. Identify a critical path and the earliest finish time for the project.
c. Identify the earliest start time for each task.
d. Identify the latest start time for each task.

4. Consider the graph at the right.

a. Construct an adjacency matrix for this graph.
b. Find the row sums.
c. Does this graph have an Euler path? Explain by examining the row sums. If there is an Euler path, trace it on a copy of the graph.
The following sets of exercises give you an opportunity to review the mathematical ideas and skills that you acquired in Course 1 units and early Course 2 units. They include questions that require knowledge of:

- algebra and functions
- geometry
- statistics
- probability
- discrete mathematics

Some problems combine ideas from two or more of those strands of mathematics. In each case, you will need to determine the appropriate ideas and techniques to apply.

If you need a refresher on some particular topic, look back at the reference material and examples in the first section of this guide. However, it is best to make a good effort at completing an exercise before looking for help in the reference material or in the answers that are given at the back of the book. Since practice of any skill is most effective when distributed in modest amounts throughout the school year, the practice exercises have been arranged in sets of 10 items, so you could do about one set every other week throughout the school year as companion work to your study of new topics in CMIC Course 2. Beginning with Exercise Set 11, exercise sets include some questions over material you studied in early Course 2 units. You should complete those exercises during the second half of
1. Suppose a provider of local telephone service offers to charge only $15 per month plus $0.15 per call.
   a. What equation shows how to calculate monthly bill $y$ as a function of the number of calls made $x$?
   b. What equation using NOW and NEXT shows how the monthly bill grows as each additional call is made?

2. For the telephone service described in Exercise 1:
   a. What will the monthly bill be if 45 calls are made?
   b. How many calls must have been made if one monthly bill was $30?
   c. Write and solve an algebraic inequality to answer the question “How many calls can be made and still have a monthly bill under $50?”

3. Suppose another company offers a flat monthly fee of $39.90 for unlimited local calling.
   a. For what number of calls will the flat fee give the same bill as the plan in Exercise 1?
   b. How can the question in Part a be answered using a table of values for the pricing rules?
   c. How can the question in Part a be answered by tracing a graph on your calculator?
   d. How can the question in Part a be answered by solving an equation through reasoning that does not rely on a table or graph?

4. If you graph the relation between number of calls and monthly bill for the local telephone service described in Exercise 1, what are the $y$-intercept and slope of that graph, and what do they tell about the pricing policy?

5. Suppose that the company in Exercise 1 decides to change its pricing to respond to competition. How will the pricing equation and its graph change if:
   a. The base fee is reduced to $10?
   b. The charge per call is reduced to $0.12?
6. Suppose that the telephone company described in Exercise 1 starts up with only 50 customers but hopes to double its customer list every month for two years.

   a. Write two equations showing how the customer list will grow—one in the form $y = ...$ and another in the NOW-NEXT form.
   
   b. Sketch the shape of the graph that can be expected for the equations in Part a.
   
   c. Find the number of customers predicted for the end of the first year.
   
   d. Find the time when the number of customers is first predicted to exceed 100,000.

7. In building the frame for a large painting, an artist wants to make the frame rigid by introducing a diagonal brace. If the frame is as pictured here, how long will the brace be? What is the area of the painting?

8. The following data give the percentages of the voting-age population who actually voted in the U.S. presidential elections from 1952 to 1996 (figures rounded to the nearest percent).

<table>
<thead>
<tr>
<th>Year</th>
<th>'52</th>
<th>'56</th>
<th>'60</th>
<th>'64</th>
<th>'68</th>
<th>'72</th>
<th>'76</th>
<th>'80</th>
<th>'84</th>
<th>'88</th>
<th>'92</th>
<th>'96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vote %</td>
<td>62</td>
<td>59</td>
<td>63</td>
<td>62</td>
<td>61</td>
<td>55</td>
<td>54</td>
<td>54</td>
<td>53</td>
<td>50</td>
<td>56</td>
<td>49</td>
</tr>
</tbody>
</table>

   a. Make a plot over time of these data. Describe any patterns you see.
   
   b. Calculate the mean, median, and MAD of the percentages and explain what each tells about voting percents in presidential elections.
   
   c. During the years that are shown, 18- to 20-year olds were first given the right to vote. Can you tell from patterns in the data when that might have occurred?

9. A metal component has dimensions as shown at the right.

   a. Find its volume.
   
   b. Find its surface area.

10. Use what you know about geography, or consult an atlas, to make a vertex-edge graph showing which of the following western states in the United States have borders in common: Alaska (AK), Arizona (AZ), California (CA), Idaho (ID), Nevada (NV), Oregon (OR), Utah (UT), and Washington (WA). Then determine the minimum number of colors required to color a map of these states so that no adjoining states are the same color.
1. Solve these equations for $x$ without using tables or graphs. Show the steps of your reasoning. Check the solutions in each case.
   a. $14x - 11 = 39$
   b. $25 = 4 - 3x$
   c. $6x - x = 30$
   d. $7x = 5(x - 12)$
   e. $-5x = -10 - 3x$
   f. $\frac{2}{3} - 4x = \frac{1}{2}x + 3$

2. Write equations for the lines satisfying these conditions.
   a. Has a slope of 1.5 and a $y$-intercept of –3
   b. Is parallel to the line in Part a and passes through the point (0, 2)
   c. Contains the points (0, 5) and (6, 14)

3. Write each of the following expressions in two equivalent forms.
   a. $5(2x - 7)$
   b. $4 + (3x - 9)$
   c. $7x + 5(x - 3)$
   d. $4x - (2 - 11x)$
   e. $2x + 3(5 - 6x)$
   f. $2(x - 3) - 4(3x - 5)$

4. If $y = 4(-3x + 5) - 9$, find $y$ if:
   a. $x = 10$
   b. $x = -10$
   c. $x = 0.5$
   d. $x = 0$
   e. $x = -1$
   f. $x = \frac{2}{3}$

5. Write the formula for perimeter $P$ of a rectangle with length $L$ and width $W$ in two equivalent forms and explain how you know the forms are equivalent.

6. What is the length of side $AB$ in this right triangle?

7. The box plot below shows the price in dollars of 20 models of cordless phones.
   a. What is the approximate median of the data?
   b. What are the approximate first and third quartiles of the data?
c. Use the information from Parts a and b to help prepare a description of the distribution.

8. Imagine folding a square piece of paper in half, then in half again, and then in half again, and so on. The fold marks at each stage divide the original square into a number of smaller regions.
   a. Make a table showing the number of regions for \( n = 1, 2, 3, 4, \text{ and } 5 \) folds.
   b. Write equations \((\text{NOW}-\text{NEXT} \text{ and } y = \ldots)\) showing how to calculate the number of smaller regions at any stage of the folding process.
   c. Predict the number of regions for 8 folds.

9. The sketches below show how polygonal shapes can be subdivided into triangles.
   a. How do these sketches suggest a way to find the sum of the angle measures of:
      (i) any quadrilateral?
      (ii) any pentagon?
      (iii) any hexagon?
      (iv) any \( n \)-gon?
   b. If the sum of the measures of the angles of a polygon is 1,440˚, how many angles does the polygon have?

10. The following diagram shows how a rigid roof brace can be constructed by connecting short bars in a triangulated pattern.

    a. Make a table showing the number of bars needed to make such a brace so that the bottom side has a length of 1, 2, 3, 4, or 5 bars.
    b. Write an expression showing how the number of bars required for a brace depends on the length of the bottom side.
    c. Write another expression for the number of bars required and explain how you know it is equivalent to what you came up with in Part b.
1. Suppose that a tour bus sets out on a trip of 800 kilometers and the driver sets the cruise control at 120 km/hr.
   a. What equation shows the relation between elapsed time $t$ in hours and distance remaining in the trip?
   b. Use the result of Part a to write and solve an algebraic equation that matches the question “How much time will it take to get within 100 kilometers of the end of the trip?”

2. Without using a graphing calculator, sketch graphs for these three linear equations.
   a. $y = 150 + 25x$
   b. $y = 150 - 50x$
   c. $y = -150 + 25x$

3. Find equations for the two lines on this graph, assuming standard scales on the axes.

4. The population of the United States in 1999 was approximately 270 million and growing at a rate of about 0.9% per year.
   b. Write two equations ($NOW$-$NEXT$ and $y = ...$) that can be used to project the U.S. population any number of years into the future.
   c. Estimate the time when the U.S. population will reach approximately 300 million.

5. Describe all symmetries of the following polygons.
   a. Equilateral triangles
   b. Squares
   c. Non-square rectangles
   d. Regular pentagons

6. Suppose a shipping box is in the shape of a rectangular prism with base dimensions of 20 inches by 15 inches and a height of 10 inches.
   a. Sketch the box and label the dimensions.
   b. What is the surface area of the box?
   c. What is the volume of the box?
   d. What is the length of a diagonal on the base of the box?
   e. What is the length of the longest object that can be packed in the box?
7. Consider all prisms with a rectangular base of 20 inches by 15 inches.

   a. Write an equation showing how the volume $V$ of such prisms is a function of height $h$.

   b. Find the box height that will give a volume of 4,500 cubic inches.

8. The following back-to-back stem-and-leaf plot shows the number of home runs hit by the leaders in the National and American Baseball Leagues at five-year intervals from 1920 to 1995. Use the plot to answer the questions below.

<table>
<thead>
<tr>
<th>National</th>
<th>American</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 1</td>
<td>2 4</td>
</tr>
<tr>
<td>8 2 9 7 4 3</td>
<td>1 2 3 6 6 7 7</td>
</tr>
<tr>
<td>8 7 5 1 0 0</td>
<td>0 0 1 4 9</td>
</tr>
<tr>
<td>6 2 1 5 0 1 4</td>
<td>2</td>
</tr>
</tbody>
</table>

   a. What are the five-number summaries for home runs in each league?

   b. What statistics could you cite in support of the National League as the home run leader? What could you cite to argue that the American League is the home run leader?

9. The following table shows time and prerequisites for tasks in assembling a pre-fabricated house.

<table>
<thead>
<tr>
<th>Task</th>
<th>Time</th>
<th>Prerequisites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy Land ($L$)</td>
<td>30 days</td>
<td>none</td>
</tr>
<tr>
<td>Prepare Foundation ($F$)</td>
<td>10 days</td>
<td>$L$</td>
</tr>
<tr>
<td>Order House Parts ($P$)</td>
<td>45 days</td>
<td>none</td>
</tr>
<tr>
<td>Assemble House ($A$)</td>
<td>5 days</td>
<td>$F, P$</td>
</tr>
<tr>
<td>Inspection of House ($I$)</td>
<td>1 day</td>
<td>$A$</td>
</tr>
</tbody>
</table>

   a. Make a digraph showing the relation of the tasks and times.

   b. Find the critical path and the earliest finish time for the project.

10. Solve each of the following inequalities.

   a. $3x + 12 < 54$
   b. $7x - 19 > 30$
   c. $7x - 19 \geq 3x + 13$
   d. $13 - 4x < 25$
   e. $\frac{1}{5} x + 3 \leq 9$
   f. $2(x - 3) > 4 - (x + 3)$
1. Write equations in two forms, NOW-NEXT and \( y = \ldots \), that express the relationship between data pairs in each of the following tables.

   a. \[
   \begin{array}{c|cccccccc}
   x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
   y & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20
   \end{array}
   \]

   b. \[
   \begin{array}{c|cccccccc}
   x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
   y & 6 & 12 & 24 & 48 & 96 & 192 & 384 & 768 & 1,536
   \end{array}
   \]

   c. \[
   \begin{array}{c|cccccccc}
   x & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\
   y & 8 & 2 & -4 & -10 & -16 & -22 & -28 & -34 & -40
   \end{array}
   \]

   d. \[
   \begin{array}{c|cccccccc}
   x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
   y & 32 & 16 & 8 & 4 & 2 & 1 & 0.5 & 0.25 & 0.125
   \end{array}
   \]

2. The soft drink machines in a school charge $0.60 per can. The school pays the distributor $0.40 per can. It costs $30 per month for electricity to operate the machines.

   a. Write two equivalent equations in the form \( y = \ldots \) showing how the school’s monthly profit from operating the soft drink machines \( p \) depends upon the number of cans sold \( x \).

   b. Write and solve an equation that answers the question, “How many cans of soft drinks must be sold in a month to make a profit of $100?”

   c. Explain how you could solve the equation in Part b using a table, a graph, or reasoning with the symbolic equation alone.

3. Find the perimeter and area of each shape.

   a. Triangle
   \[
   \begin{array}{c}
   \text{3} \\
   \text{9} \\
   \text{3}
   \end{array}
   \]

   b. Parallelogram
   \[
   \begin{array}{c}
   \text{14} \\
   \text{5} \\
   \text{4}
   \end{array}
   \]

   c. Semicircle
   \[
   \begin{array}{c}
   \text{20}
   \end{array}
   \]

   d. \( \frac{3}{4} \) Circle
   \[
   \begin{array}{c}
   \text{6}
   \end{array}
   \]
4. Solve each of the following equations by reasoning without the use of tables or graphs. Then check your work using a calculator-based solution strategy.
   a. $5.5x + 23 = -15$   b. $9 - 4x = 17$   c. $5 + 3x = 27 - 5x$
   d. $4(3x + 7) = 75$   e. $3(4x - 7) = 8x + 14$   f. $-4(3 - x) = 11(x + 2)$

5. Find equations for the lines through each pair of points.
   a. $(1, 2)$ and $(-3, 9)$   b. $(-5, -5)$ and $(11, 11)$   c. $(7, 1)$ and $(0, -3)$

6. Calculate:
   a. $(-8)(5) - 14$   b. $14 - (-5)(-8)$   c. $(-5)^4$
   d. $-3 - 5(-2)^2$   e. $(2^0)(2^5)$   f. $(-3)^2(-3)^3$

7. The shape at the right is a net of a solid with a curved face.
   a. Sketch the solid.
   b. Find $x$.
   c. Find the volume of the solid.
   d. Find the surface area of the solid.

8. Suppose a line with slope 1.5 goes through the point $(1, 4)$. If all the following points are on that line, what would the $y$-coordinates be?
   a. $(2, y)$   b. $(3, y)$   c. $(7, y)$   d. $(0, y)$

9. In a simulation of coin flipping, outcomes were recorded in pairs and results from 100 trials are given below.
   
<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>20 times</td>
</tr>
<tr>
<td>HT</td>
<td>27 times</td>
</tr>
<tr>
<td>TH</td>
<td>30 times</td>
</tr>
<tr>
<td>TT</td>
<td>23 times</td>
</tr>
</tbody>
</table>
   
   a. What do these results suggest about the total number of heads-up coins one would get in 100 trials of this coin-flipping activity?
   b. What total number of heads-up coins would you expect in 500 trials?

10. Sketch a graph that shows a likely pattern relating price charged and number of sales for a popular newly released movie on video.
1. Suppose a taxi company charges $2 for the first half-mile of a trip and $1.50 for each additional half-mile. What equation gives fare $F$ as a function of trip length $L$?

2. Solve the following linear equations by reasoning without use of calculator-produced tables or graphs. Show how each answer you find can be checked.
   a. $-6.5x - 32 = -12.5$
   b. $159 = 45 - 6x$
   c. $16 - 3x = 4x - 19$
   d. $22 = 2(9x - 7)$
   e. $2x + 5(3x - 4) = -10$
   f. $4 - (2x + 3) = 3(2 - 2x)$

3. Write equations for the lines satisfying these conditions.
   a. Has a slope of $\frac{2}{3}$ and a $y$-intercept of $\frac{5}{3}$
   b. Contains the points $(-2, 0)$ and $(0, -2)$
   c. Is perpendicular to the line in Part a and contains the point $(1, 1)$

4. Describe all symmetries for these letters.
   a. X
   b. A
   c. N
   d. U

5. A cylinder has a radius of 4 cm and a volume of $160\pi$ cm$^3$.
   a. What is the height?  
   b. What is the surface area?

6. On his way to school each day, Mike can walk around the park or on a diagonal from one corner to the other, as shown on this sketch.
   a. How much distance will Mike save by walking through the park?
   b. If Mike walks 150 feet per minute on average, how much time will he save by walking through the park, rather than around it?

7. If a tennis ball is dropped onto a hard surface, it should rebound to about 50% of its drop height. Suppose that a new ball is dropped from an initial height of 20 feet.
   a. What are the expected rebound heights for the first 3 bounces?
   b. What equation gives expected rebound height for any bounce number $n$?
   c. What equation shows how to calculate the rebound height for any bounce from the height of the previous bounce?
   d. Draw a sketch showing rebound height as a function of number of bounces.
8. The following table gives July weather data for some cities in the United States.

<table>
<thead>
<tr>
<th>Station</th>
<th>July High Temperature (°F)</th>
<th>July Low Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchorage, AK</td>
<td>65</td>
<td>52</td>
</tr>
<tr>
<td>Mobile, AL</td>
<td>91</td>
<td>73</td>
</tr>
<tr>
<td>Los Angeles, CA</td>
<td>84</td>
<td>65</td>
</tr>
<tr>
<td>Atlanta, GA</td>
<td>88</td>
<td>70</td>
</tr>
<tr>
<td>Des Moines, IA</td>
<td>87</td>
<td>67</td>
</tr>
<tr>
<td>Boston, MA</td>
<td>82</td>
<td>65</td>
</tr>
<tr>
<td>Helena, MT</td>
<td>85</td>
<td>53</td>
</tr>
<tr>
<td>Buffalo, NY</td>
<td>80</td>
<td>62</td>
</tr>
</tbody>
</table>

a. Calculate the median high and the median low temperatures for this sample of cities.

b. Make a scatterplot of the (high, low) data pairs, draw in the line \( y = x \), and describe where the data fall. Why is this reasonable?

c. Find a linear model relating between high and low temperatures in the given cities.

d. Use the model in Part c to predict the July low temperature in two cities: Barrow, Alaska (July high temperature 45°F) and Atlantic City, New Jersey, (July high temperature 85°F). Compare the results to the actual July low temperature for those cities: 34°F in Barrow and 65°F in Atlantic City. Comment on the differences between the predicted and actual temperatures.

9. The figure at the right is a parallelogram. What conclusions can you draw about the measurement in degrees of the labeled angles?

10. Many standardized tests use multiple choice questions with four or five answer choices. Find the probability of getting a passing grade of 70% by guessing on a ten-question multiple-choice test. Each question offers four answer choices, only one of which is correct.

a. Design a simulation that would help you find the probability.

b. Run 20 trials of the simulation and record your results in a frequency table.

c. Estimate the probability of passing the test.

d. How could you improve your estimate?
1. Write equations for the lines satisfying these conditions.
   a. Passes through the point (0, 4) with a slope of $-4.5$
   b. Contains the points (1, 2) and (5, 9)
   c. Contains the point $(-3, 8)$ and is parallel to the line with the equation $y = 0.8x - 5$

2. Explain why regular pentagons will not tessellate.

3. Solve the following equations for $x$.
   a. $5(x + 9) = -16 - x$
   b. $29 = 7x - 6$
   c. $8x + 2 = 5x + 44$
   d. $2^x = 32$
   e. $3^5 \cdot 3^x = 3^{15}$
   f. $0.5(2^x) = 8$

4. Without using a graphing calculator, graph each of the following equations.
   a. $y = x$
   b. $y = 5$
   c. $y = 10 - 1.5x$

5. The sketch at the right shows the tower of a radio station with two support wires attached.
   a. Find length $A$ and length $B$.
   b. Suppose that the radio station is able to broadcast its signal with good quality over a region with a radius of 15 miles. What is the area of that region?

6. Suppose the radio station in Problem 5 has about 5,000 listeners during the morning rush hour period. A new station manager sets a goal of increasing that number by 10% every month.
   a. Make a table giving the necessary number of listeners to meet the goal for each of the next five months.
   b. Write $NOW-NEXT$ and $y = ...$ equations to calculate the necessary number of listeners to meet the goal in any future month.
   c. If the manager’s goal for the new listeners is met each month, after how many months can the station expect to have doubled its number of listeners during the morning rush hour?
7. Write each of the following expressions in two equivalent forms.
   a. \(2(5x + 4) - 7x\)  
   b. \(-7(6x - 3) + 41x\)  
   c. \(18 - 3(2x + 4)\)  
   d. \(2(x - 5) + 5(4 - 2x)\)  
   e. \(10(x + 3) - (x - 8)\)  
   f. \(2(45 - x) + 5(15 - 2x)\)

8. The vendors at a baseball stadium are paid $20 per game and 10% of the value of the food and drinks they sell.
   a. Write an equation that expresses vendor pay \(P\) as a function of the dollar value of food sold by that vendor.
   b. Use your equation in Part a to determine the game pay for a vendor who sells $350 worth of food and drinks.
   c. How much food and drink must a vendor sell to earn a game pay of $75?

9. The following data show sales by two concession stands at a baseball stadium during the first 11 days of the season.
   
   | Stand 1 Sales ($) | 250 | 190 | 200 | 185 | 210 | 120 | 175 | 140 | 125 | 180 | 110 |
   | Stand 2 Sales ($) | 225 | 160 | 180 | 200 | 240 | 110 | 150 | 180 | 110 | 140 | 90 |

   a. Calculate the summary statistics needed and draw box plots comparing the sales data from the two stands.
   b. Write a short paragraph giving your judgment about the similarities or differences of sales at the two stands.

10. Determine if each vertex-edge graph below has
   i. an Euler path. If it does, describe the path.
   ii. an Euler circuit. If it does, describe the circuit.
1. Solve the following equations for $x$.
   
   a. $3x - 5 = 9 + 4x$  
   b. $9 + \frac{1}{2}x = 14$  
   c. $3.2 = 5x + 0.7$  
   d. $2(4x - 8) = 8x + 14$  
   e. $2(5^x) = 250$  
   f. $(-2)(-2)^x = 16$

2. The following stem-and-leaf plot shows monthly rainfall in millimeters for a city in the southern United States over a period of 24 months.

   
   
   | 5 | 2 3 8  
   | 6 | 1 2 3 4  
   | 7 | 2 5 7 7 8 9 9  
   | 8 | 2 3 4 6 7 9  
   | 9 | 0 1 5 7  
   5 | 2 represents 52 mm

   a. Find the median and mean of the data and explain what each tells about rainfall in the given city.
   b. Construct a histogram of the same data.
   c. Calculate the mean absolute deviation of the rainfall data and explain what it tells about rainfall in the given city.

3. The following diagram shows a rectangle with semicircles attached at each end.

   
   | a. Describe the symmetries of the figure.  
   | b. What is the area of the figure?  
   | c. What is the perimeter of the figure?  

4. Write the equation $y = 3(x + 2) - 5$ in three equivalent forms and explain how you know that each form is equivalent to the original.

5. Write equations for the lines satisfying these conditions.
   
   a. Contains the points $(0, -2)$ and $(4, 6)$  
   b. Has a slope of $-\frac{13}{2}$ and passes through the point $(-7, 2)$  
   c. Is a horizontal line and contains the point $(3, 8)$  
   d. Is perpendicular to the line in Part b and has a $y$-intercept of 10
6. Without use of a graphing calculator, sketch graphs of these equations.
   a. \( y = 0.5x \)  
   b. \( y = 3 + 0.5x \)  
   c. \( y = 3 - 0.5x \)  
   d. \( y = 0.5 - 3x \)  
   e. \( y = 3(0.5^x) \)  
   f. \( y = 0.5(3^x) \)

7. In a large high school, half of the students are girls and half are boys. Suppose the six students who volunteer to tutor children at the local elementary school are all girls.
   a. Describe two different simulations that one could use to estimate the probability of getting all girls if six students were selected at random from this high school.
   b. Choose one of your simulations and do 25 trials. Then use the data to estimate the probability of getting six girls.
   c. Use your simulation to estimate the average number of girls if six students had been selected at random.
   d. What assumptions did you make in the design of your simulation?

8. Suppose a cylindrical water storage tank has a radius of 10 feet and a height of 20 feet.
   a. Sketch a diagram showing this shape with its dimensions labeled.
   b. What is the volume of the tank?
   c. What is the circumference of the tank?
   d. What is the surface area of the tank?

9. Solve each inequality for \( x \).
   a. \( 5x - 24 < 19 \)  
   b. \( 7x \leq -18 + 9x \)  
   c. \( 3(x + 6) \geq 2x - 12 \)  
   d. \( 2(x - 3) > 3(4x - 1) \)

10. If a new truck costs $25,000, its trade-in value will decrease by about 20% each year after purchase.
    a. Make a table showing the trade-in value of the truck for the first three years after purchase.
    b. Write two equations in NOW-NEXT and \( y = ... \) form showing how to calculate the truck’s trade-in value for any number of years after purchase.
    c. After how many years will the value of the truck be less than $10,000?
1. For what value of $x$ will the points $(x, 3)$ and $(5, 13)$ determine a line with slope 0.8?

2. Solve the following equations for $x$.
   a. $3(x - 5) = 10 - 2x$
   b. $24 + 2x = 4 - 6x$
   c. $\frac{4}{5} (x + 9) = 16$
   d. $4(2^x) = 256$
   e. $-4(x - 3) - 11 = 3x + 8$

3. A large cereal box has these dimensions: width 3 inches, length 8 inches, height 13 inches.
   a. Sketch a diagram showing this box with its dimensions labeled.
   b. Calculate the volume of the box.
   c. Calculate the total surface area of the box.

4. Without using a graphing calculator, sketch graphs of these equations.
   a. $y = 4(1.5^x)$
   b. $y = 3(2.5^x)$
   c. $y = 4(0.5^x)$

5. An amusement park charges $5 per person for admission and $0.75 for tickets to the various rides it offers.
   a. What equation shows the cost $C$ of a trip to this park as a function of the number of ride tickets $t$ that are purchased?
   b. If a person takes $25 to the park, what is the maximum number of ride tickets that can be purchased?
   c. How can the question in Part b be expressed as an algebraic inequality to be solved?
   d. Solve the inequality in Part c.

6. Marie’s scores on the first three science tests were 68, 79, and 74. What score on the next test will give her an overall average of 75?

7. Find the area of this figure in two different ways.
8. Jose, Claudia, and Kathleen want to explore the probability that they will all have Mrs. Parks for their Math 2 teacher next year. Students are randomly assigned to teachers, and Mrs. Parks teaches one of the four sections of Math 2.

a. Design a simulation for assigning the three students to Math 2 sections.

b. Run 25 trials of your simulation and complete a copy of the frequency table on the right.

<table>
<thead>
<tr>
<th>Number of Students Who Get Mrs. Parks</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td><strong>Total Number of Trials</strong></td>
<td><strong>25</strong></td>
</tr>
</tbody>
</table>

c. Construct a histogram of your results.

d. What is the probability that all three students will get Mrs. Parks for Math 2?

9. If the angles of a triangle are related as indicated in the following sketch, what are the measures in degrees of those angles?

10. What is the earliest finish time for the project depicted in the following project digraph?
1. What are the slope, \(y\)-intercept, and equation of the line through points (1, 4) and (3, 2)?

2. Write equations for the lines satisfying these conditions.
   a. Contains the point (0, 5) and is parallel to the line with equation \(y = 2 + 0.5x\)
   b. Has a slope of \(-3\) and contains the point (5, 6)
   c. Is a horizontal line through the point (5, 6)

3. Solve each inequality for \(x\).
   a. \(5(x + 12) < 77\)
   b. \(3x - 5 \geq 6x + 22\)
   c. \(0 < 4(6 - x)\)

4. What equation relating \(x\) and \(y\) would produce the \((x, y)\) pairs in the table at the right?

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
</tbody>
</table>

5. What are all of the symmetries of the following strip pattern assuming the pattern continues indefinitely to both the left and the right?

   \[ \text{A} \quad \text{A} \quad \text{A} \quad \text{A} \quad \text{A} \quad \text{A} \]

6. In planning a post-prom party, the senior class officers at Kennedy High School got a price quotation from a local athletic club. There would be a basic charge of $450 for the facility plus $10 per student for food and drinks.
   a. Write an equation expressing total cost \(C\) as a function of the number of students \(n\) attending the planned party.
   b. The class officers decide to charge each student $15 to attend the party. What income will they get if 250 students buy tickets?
   c. What equation should be solved to find the number of tickets sold that will allow the senior class to break even on the event?
   d. What is the solution to the equation in Part c, and how is that solution shown in a graph of the cost and income relations?
7. The following table shows change in average charges for tuition and room and board at American public four-year colleges and universities from 1989 to 1998.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuition ($)</td>
<td>2035</td>
<td>2159</td>
<td>2410</td>
<td>2349</td>
<td>2537</td>
<td>2681</td>
<td>2811</td>
<td>2975</td>
<td>3111</td>
<td>3243</td>
</tr>
<tr>
<td>Room/Board ($)</td>
<td>3289</td>
<td>3425</td>
<td>3641</td>
<td>3670</td>
<td>3829</td>
<td>3990</td>
<td>3932</td>
<td>4167</td>
<td>4358</td>
<td>4530</td>
</tr>
</tbody>
</table>

a. Construct plots over time showing the average change in tuition and room and board charges for the decade from 1989 to 1998. Write a short summary of the patterns of change shown in those graphs.

b. Construct a scatterplot of the \((tuition, room/board)\) data pairs and use it to make a conclusion about the extent to which those two cost figures seem related to each other over time.

c. Explain why the mean, median, and MAD of the given tuition or room and board data are not particularly useful in thinking about the college cost situation.

8. Examine the net of a solid shown at the right.

a. Sketch the solid and find its volume.

b. What is the surface area of the solid?

c. Sketch two other possible nets for this solid.

9. Suppose a fair die is tossed many times and the results recorded. Are you more likely to observe that the number of times the result is an even number is

- Between 24 and 26 on the first 50 tosses,
- Between 240 and 260 on the first 500 tosses,
- Or, are these two outcomes equally likely?

10. The formula relating temperature in degrees Celsius \((C)\) and degrees Fahrenheit \((F)\) is:

\[
C = \frac{5}{9} (F - 32).
\]

a. What is the equivalent of 0°F in degrees Celsius?

b. What is the equivalent of 15°C in degrees Fahrenheit?
1. Solve each of the following equations for $x$.
   a. $-3(2x - 12) = 66$
   b. $\frac{2}{3}(x + 4) = 6 - x$
   c. $4x^2 = 36$
   d. $3(x - 2) = 6(x + 8)$
   e. $10(3^x) = 810$
   f. $5x^3 = 40$

2. What is the area of the shaded region in this figure?

![Diagram of two circles with a shaded region]

3. Suppose that a city’s water reservoir contains 150 million gallons of water at the start of the summer and typical usage in summer exceeds the inflow to the reservoir by 2 million gallons per day.
   a. Write two equations (NOW-NEXT and $y = \ldots$ forms) showing how to estimate the water in the reservoir after any number of days.
   b. Sketch a graph that shows the relation between time and reservoir contents.
   c. What pattern of change would be expected in a table of (day, reservoir water) data?

4. Solve the following system of equations in three different ways.
   \[ y = -\frac{2}{3}x + 4 \]
   \[ y = \frac{1}{2}x + 2 \]

5. Sketch graphs showing the general shape of a distribution with the following characteristics:
   a. Mean and median about equal
   b. Mean greater than the median
   c. Mean less than the median
6. If \( y = 4(2x - 9) + 25 \):
   a. What is the value of \( y \) when \( x = 5 \)? When \( x = -3 \)?
   b. What is the value of \( x \) when \( y = 21 \)? When \( y = -5 \)?

7. Find the perimeter and area of the shape below.

![Diagram of a shape with dimensions 5 cm, 8 cm, and 6 cm.]

8. Suppose a hospital patient receives an injection of medication that metabolizes in the blood according to the equation \( M = 200(0.8^t) \) (with \( M \) in milligrams and \( t \) in hours).
   a. What do the values 200 and 0.8 tell about the action of the medicine in the blood?
   b. What is the value of \( M \) when \( t = 3.5 \), and what does it tell about medicine action?
   c. When will only 10\% of the original dose of medication remain active in the blood?
   d. What is the general shape of the graph of the \((time, medication)\) relation?

9. For the right triangular prism shown,
   a. Find its surface area.
   b. Find its volume.

![Diagram of a right triangular prism with dimensions 26 cm, 31 cm, and 10 cm.]

10. Suppose that designers of a wild animal park want to have different animals living in natural settings, but they don’t want to put two species in the same part of the park if one is a predator of the other. Suppose this situation is modeled by a vertex-edge graph in which each species is represented by a vertex.
   a. What could edges represent?
   b. What could coloring of vertices represent?
   c. What would the minimal coloring of the graph tell about planning of the park?
1. Stellar DJ company charges a booking fee plus an hourly rate. The cost $C$ in dollars of hiring a DJ for $h$ hours can be modeled by the equation $C = 50h + 100$.

   a. How much is the booking fee? What is the hourly rate?
   
   b. How much would it cost to hire a DJ for a three-hour dance?
   
   c. For how long could you hire a DJ for $400$?

2. Solve each inequality for $x$.
   
   a. $5x - 7 > 33$  
   b. $3x + 4 \geq 7(x - 1)$  
   c. $5x - 6 < 2x + 3$
   
   d. $3(2x + 5) \leq 30$  
   e. $\frac{3}{4}x - 5 > 7$  
   f. $x^2 < 9$

3. A circle has a diameter with endpoints $(-3, 2)$ and $(4, 1)$.

   a. What are the coordinates of the center of the circle?
   
   b. What is the length of the radius of the circle?
   
   c. Write equations of two symmetry lines for this circle.

4. $A = \begin{bmatrix} -1 & 2 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 6 & 0 \end{bmatrix}$.

   
   b. Calculate $5A$.
   
   c. Calculate $AB$ and $BA$.

5. Use the data below to complete the following exercises.

   **LeKeshia's Test Scores in Ms. Rodriguez's Class**
   
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>47</td>
<td>98</td>
<td>72</td>
<td>55</td>
</tr>
<tr>
<td>88</td>
<td>72</td>
<td>100</td>
<td>96</td>
<td>72</td>
</tr>
<tr>
<td>84</td>
<td>69</td>
<td>78</td>
<td>84</td>
<td>90</td>
</tr>
</tbody>
</table>

   a. Calculate the mean and the mean absolute deviation (MAD) of the data.
   
   b. Create a box plot of the data.
   
   c. Using specific information from the box plot, explain why LeKeshia might be pleased with her performance.

6. Find the perimeter and the area of the right triangle at the right.
7. Suppose that you are planning a surprise birthday party for a friend. The tasks involved and estimates for the time to complete each task are given in the table below.

### Planning a Surprise Party

<table>
<thead>
<tr>
<th>Task</th>
<th>Task Time</th>
<th>Immediate Prerequisites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose a Date and Location (L)</td>
<td>2 days</td>
<td>none</td>
</tr>
<tr>
<td>Make the Guest List (G)</td>
<td>1 day</td>
<td>none</td>
</tr>
<tr>
<td>Plan the Menu (M)</td>
<td>1 day</td>
<td>none</td>
</tr>
<tr>
<td>Design and Send Invitations (I)</td>
<td>4 days</td>
<td>L, G</td>
</tr>
<tr>
<td>Allow Time for RSVPs (R)</td>
<td>5 days</td>
<td>I</td>
</tr>
<tr>
<td>Buy the Food and Decorations (B)</td>
<td>1 day</td>
<td>M</td>
</tr>
<tr>
<td>Prepare the Food and Decorate (P)</td>
<td>2 days</td>
<td>B</td>
</tr>
<tr>
<td>Arrange a Cover Story (C)</td>
<td>2 days</td>
<td>L</td>
</tr>
</tbody>
</table>

**a.** Draw the project digraph.

**b.** Find the critical tasks and the earliest finish time (EFT).

8. Write equations for the lines that satisfy these conditions.

**a.** Has a slope of 5 and contains the point (3, 9)

**b.** Contains the points (−2, −8) and (4, 7)

**c.** Has a slope of −0.5 and a y-intercept of 12

**d.** Is a horizontal line through the point (0, −6)

**e.** Is a vertical line through the point (8, 0)

9. Write an equation using NOW and NEXT for each of the equations in Parts a through c of Exercise 8.

10. For each transformation below, plot triangle ABC with vertices A(4, 1), B(0, −3), and C(−4, 2) on a coordinate grid, then find and plot its image.

**a.** Reflection across the x-axis

**b.** Reflection across the line y = x

**c.** 180° rotation about the origin

**d.** Translation that maps (0, 0) to (3, 5)

**e.** Size transformation with magnitude 2 and center at the origin
1. The equation modeling the monthly cost of one cellular telephone service is 
\[ C = 20 + 0.1m \], where \( C \) is the total monthly cost in dollars for \( m \) minutes of air time. A competitor’s monthly cost is \( C = 10 + 0.5m \).

a. Calculate the cost of each service when 60 minutes of air time is used. When 100 minutes of air time is used.

b. Calculate the number of minutes for which each service would cost $50.

c. For how many minutes are the monthly costs of the two services the same?

d. Under what conditions would you recommend the first service?

2. Solve each equation below for \( x \).
   a. \[ 4(x - 3) = 8 \]
   b. \[ 7(2 - x) = 3(x + 8) \]
   c. \[ 2x^2 + 5 = 23 \]
   d. \[ 5x + 5(1 - x) = x - 12 \]
   e. \[ x(x - 4) = 25 - 4x \]
   f. \[ \frac{1}{x} = 5 \]

3. Determine the slope and \( y \)-intercept of each line.
   a. The line with equation \( y = 5 - \frac{2}{3}x \)
   b. The line with graph below
   c. The line with table below
   ![Graph with line passing through points (1,1) and (3,1)]
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-17</td>
</tr>
<tr>
<td>-1</td>
<td>-9</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
   d. The line given by \( \text{NEXT} = \text{NOW} + 3.4 \) (start at 2)

4. Given the indicated two sides of right triangle \( ABC \), find the length of the third side.
   a. \( AC = 8 \text{ cm}, \ BC = 10 \text{ cm} \)
   b. \( AB = 26 \text{ m}, \ BC = 10 \text{ m} \)

5. If McBride averaged 12 points per game in the first 7 basketball games of the season, how many points per game must McBride average for the remaining 10 games to finish the season with a scoring average of 15 points per game?
6. Write each expression in a shorter form using exponents.
   a. $x^3x^4y^5$
   b. $x^7y^3(xy)^5$
   c. $(3x^4)^2$

7. Sketch plane shapes or patterns that have these given types of symmetry.
   a. Reflection symmetry only
   b. Translation symmetry only
   c. Glide reflection symmetry only

8. Seven new radio stations are planning to start broadcasting in the same region of the country. The FCC wants to assign the fewest possible number of frequencies so that no two stations interfere with one another. Stations within 500 miles of each other on the same frequency will interfere with one another. The locations of the seven stations are shown on the grid on the right.
   a. Draw a graph model to represent the situation. Indicate what the vertices and edges represent.
   b. Use graph coloring to assign as few frequencies as possible to the seven radio stations.

9. Damani received a $1,000 five-year certificate of deposit for his fifteenth birthday. If the certificate of deposit earns 6.5% interest compounded annually, what will its value be in five years?

10. Which data set in the scatterplots below has the strongest correlation? The weakest?
1. TKR Cablevision has the following rate schedule: basic cable and one pay channel costs $39.00 per month; basic cable and two pay channels costs $45.50 per month; all pay channels cost the same amount per month.

   a. How much does TKR charge for just basic cable? For each pay channel?
   b. The cable company offers a package price of $60.00 per month that includes basic cable and all of the pay channels. Under what circumstances would you benefit from this option?

2. Write equations for the lines satisfying these conditions.
   a. Has a slope of $-\frac{3}{2}$ and contains the point $(3, -2)$
   b. Contains the points $(-3, 4)$ and $(1, 2)$
   c. Has a slope of $10$ and a $y$-intercept of $-4$

3. Sam has 100 feet of fencing. Does he have enough fencing to enclose a rectangular region whose area is 630 square feet? Does he have enough fencing to enclose a circular region whose area is 630 square feet?

4. William is taking a ten-item multiple-choice quiz. Each item has five possible choices. Since he didn’t study, he chooses an answer for each question at random without reading the quiz.

   a. Describe a simulation that could be used to investigate his chances of getting various scores on the quiz.
   b. Fifty trials simulating this quiz were conducted. The frequency table below shows the number of questions William got correct in each trial.

<table>
<thead>
<tr>
<th>Questions Correct</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4</td>
<td>13</td>
<td>15</td>
<td>10</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

   Make a histogram representing these data.
   c. Use the frequency table to estimate, on average, the number of questions William will answer correctly.
   d. If at least six questions must be answered correctly to receive a passing grade, use the results of the simulation to estimate the probability that William will receive a passing grade.

5. Sketch each graph without using a calculator.
   a. $y = 6 - \frac{3}{2}x$
   b. $y = x^2 - 4$
   c. $y = \left(\frac{1}{2}\right)^x$

68 MAINTAINING CONCEPTS AND SKILLS
6. Determine whether each graph has an Euler circuit or path. If it does, find one. If it has neither a circuit nor a path, Eulerize it.

   a. 
   ![Graph](image)

   b. 
   ![Graph](image)

   c. 
   ![Graph](image)

   d. 
   ![Graph](image)

7. Calculate the perimeter of the triangle represented by the matrix at the right. 

   \[
   \begin{bmatrix}
   3 & 8 & -1 \\
   5 & -7 & 2 \\
   \end{bmatrix}
   \]

8. Write the matrix representation of the image of the triangle in Exercise 7 under each of the following transformations.

   a. 90° counterclockwise rotation about the origin

   b. Reflection across the line \( y = x \)

   c. 180° rotation about the origin

   d. Reflection across the \( x \)-axis followed by a 90° clockwise rotation about the origin

9. For 1980 to 1995, the annual sales \( S \) in thousands of dollars, of a shoe store can be modeled by 
   \[ S = -\frac{1}{2} t^2 + 3t + 42, \] 
   where \( t = 0 \) represents 1980.

   a. During which years did the store have sales of more than $50,000?

   b. During which year did the store achieve its maximum sales?

10. In the figure at the right, points \( A, \ C, \) and \( E \) are all on the same line. 
    What is the value of \( x? \)
1. Neil has accepted a new job that will pay an annual salary of $28,000 for the first year. He has the option of a 5% annual raise or a fixed annual raise of $1,500.

   a. For each option, write an equation using the words NOW and NEXT that shows how Neil’s salary will increase from year to year.

   b. For each option, write an equation in the form \( s = \ldots \) that shows the amount of Neil’s salary \( s \) after \( t \) years.

   c. For each option, make a table and draw a graph showing how Neil’s salary increases over a period of ten years.

   d. What should Neil consider in deciding which alternative to select?

2. Solve each system of equations.
   
   a. \( y = 2x - 3 \)  
   b. \( y = \frac{1}{2}x - 3 \)  
   c. \( x + 2y = 2 \)

   \[ \begin{align*}
   y &= -3x + 2 \\
   y &= 4 - \frac{2}{3}x \\
   5x - 3y &= -29
   \end{align*} \]

3. A line segment has endpoints \((1, -6)\) and \((7, 2)\). Write an equation of the perpendicular bisector of the line segment. The perpendicular bisector of a line segment is a line perpendicular to the segment and contains its midpoint.

4. The students in a high school class who baby-sat over spring break collected data on the number of hours they baby-sat and the amount they charged per hour. Their data appear in the following table.

<table>
<thead>
<tr>
<th>Charge Per Hour ($)</th>
<th>7</th>
<th>2</th>
<th>0</th>
<th>2</th>
<th>6</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Hours Worked</td>
<td>3</td>
<td>8</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

   a. Find the equation of the regression line modeling the relationship between charge and number of hours worked. Graph that line on a scatterplot of the data.

   b. Interpret the slope of the regression line in the context of these data.

   c. Determine the largest residual.

   d. How many hours would you predict a student would baby-sit if he or she charged $5 an hour?
5. Kandy’s Bar chocolate is packaged in triangular prism-shaped boxes.

a. Draw two different possible nets that could be used to manufacture a Kandy’s Bar box.

b. If the rectangular faces each measure 1.5 in. by 8 in., calculate the minimum amount of cardboard necessary to manufacture each Kandy’s Bar box and the volume of each box.

6. Solve the following equations for $x$.

a. $5x - 2(6 + x) = 6$  
   b. $5(x + 3) = 7(x - 4)$  
   c. $(x - 3)^2 = 4$

   d. $128 \left(\frac{1}{2}\right)^x = 4$  
   e. $2x^3 - 128 = 0$

7. The matrix $M$ at the right is the adjacency matrix for a directed graph $G$ with vertices $A$, $B$, $C$, and $D$.

a. Draw the directed graph $G$.

b. Use the matrix $M$ to determine the number of paths of length 2 from $B$ to $D$. Trace these paths on graph $G$.

c. Use the matrix $M$ to determine the number of paths of length 3 from $A$ to $D$. Trace these paths on graph $G$.

8. Solve for $x$ and $y$.

\[
\begin{bmatrix} 2 & x \\ y & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ 8 \end{bmatrix}
\]

9. Sketch the graphs of the following equations.

a. $y = 2x - 3$  
   b. $y = -\frac{3}{5} x + 1$  
   c. $y = -5$

   d. $y - x = 4$  
   e. $x = 2$  
   f. $3x + 4y = 12$

10. A mechanic must replace a fan belt. The diagram below shows the arrangement of the fan belt and the parts it connects. Determine the length of the belt.
1. A high school choir is selling kits for plain cheese pizzas and pepperoni pizzas to raise money for their spring trip. One student submitted an order for 23 pizza kits with no indication of how many orders were for cheese and how many were for pepperoni. The student also submitted a check for $187. If the cheese pizza kits sell for $7 and the pepperoni pizza kits sell for $9, how many of each type of kit were ordered?

2. Determine the slope and \( y \)-intercept of each line.
   a. The line with equation \( 2x + 4y = 6 \)
   b. The line with the graph below
   c. The line with the table below
   
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -3 & 3 \\
   -1 & 2.3333 \\
   1 & 1.6667 \\
   3 & 1 \\
   \end{array}
   \]
   d. The line given by \( NEXT = NOW - 4.6 \) (start at 7)

3. In the figure at the right, find the measure of each angle marked with a letter.

4. Write equations for the lines that satisfy these conditions.
   a. Has a slope of 5 and contains the point \((-2, 1)\)
   b. Contains the points \((-2, 1)\) and \((3, -4)\)
   c. Is parallel to the line in Part a and contains the point \((3, -4)\)
   d. Is a vertical line that contains the point \((-12, 0)\)
   e. Is a horizontal line that contains the point \((0, 15)\)
   f. Is perpendicular to the line in Part a and contains the point \((3, -4)\)

5. Angela did yard work for her neighbors over the summer. Her weekly earnings were \$40, \$20, \$23, \$38, \$13, \$34, \$60, \$25, \$0, \$5, and \$31.\n   a. Calculate and compare the mean and the median weekly earnings for Angela. Which measure of the center better describes the data? Why?
b. Calculate the mean absolute deviation (MAD). What does this tell you about the data?

c. Calculate the interquartile range. What does this tell you about the data?

6. Write each of the following expressions in shorter form using exponents.

a. \(a \cdot a \cdot b \cdot b \cdot b \cdot (2.3) \cdot (2.3)\)
b. \(2 \cdot 2^3 \cdot a^2 \cdot a^4\)
c. \(\frac{a^3 \cdot b^4 \cdot c}{a \cdot b^3 \cdot c^5}\)
d. \((5x)^2(2x^2)^3\)
e. \((x^2y^3)^5\)
f. \(\frac{2xy^6}{(x^2y^2)^2}\)

7. Evaluate each of the following if \(r = -3, s = 5,\) and \(t = 10.\)

a. \(r^2(6 + t)\)
b. \(rs - r(s - 3)\)
c. \(t + 2(s - r)\)
d. \(10 - t + t(2r - 3s)\)

8. Consider the matrix equation: \[
\begin{bmatrix}
6 & -1 \\
4 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
31 \\
17
\end{bmatrix}.
\]

a. Write the system of linear equations that corresponds to this system.
b. By examining the symbolic form of the equations, explain why their graphs intersect in a point.
c. Solve the system of equations by the linear-combination method.
d. Solve the system of equations in a different way by reasoning with the symbolic forms themselves.

9. Solve each inequality.

a. \(\frac{1}{3} x + 5 \leq 7\)
b. \(3(x - 4) \leq 4(x + 5)\)
c. \(\frac{1}{2} x^2 > 8\)

10. A rectangle is represented by the matrix \(ABCD = \begin{bmatrix}
0 & 3 & 3 & x \\
4 & 4 & 10 & y
\end{bmatrix}\)

a. Find the coordinates of point \(D\) and verify that \(ABCD\) is a rectangle.
b. Find the image of \(ABCD\) reflected across the \(x\)-axis.
c. Find the image of \(ABCD\) rotated 270° counterclockwise about the origin.
d. What is the area of the image of \(ABCD\) under a size transformation with magnitude 4 and center at the origin?
1. Suppose that the captain of the girls varsity basketball team is a 75% free-throw shooter.
   a. Each time she is sent to the line for a free throw, what is the probability that she will miss?
   b. Describe how you would design one trial of a simulation model for 10 free throws.
   c. Conduct 20 trials of your simulation and use the results to determine, on average, how many free throws out of 10 one could expect her to make.

2. Two lines are graphed on the coordinate axes to the right. The scale on each axis is 1.
   a. Write an equation of each line.
   b. Determine the exact coordinates of the point of intersection of the two lines.

3. A model rocket is launched from a height of 4 feet with an upward velocity of 64 feet/second. Its height h in feet after t seconds is given by the equation \( h = 4 + 64t - 16t^2 \).
   a. If the manufacturer wants the parachute to come out when the rocket is at its maximum height, at what time should the parachute come out?
   b. How high will the rocket be when the parachute comes out?
   c. If the parachute does not open, how long will the rocket be in the air?

4. Sketch each pair of quadrilaterals. Describe their similarities and their differences.
   a. Square and rectangle
   b. Parallelogram and rhombus
   c. Trapezoid and parallelogram

5. A typical dress shirt box has these dimensions: 18 inches long by 12 inches wide by 3 inches deep.
   a. Draw a net for the top and bottom of a box having the dimensions listed above.
   b. Determine the amount of cardboard in square inches required to construct such a box.
   c. Do you think one would require more or less paper to wrap the box than the response given in Part b? Explain why you think so.
6. Consider the following matrices:

\[
A = \begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 1 & 3 \\ 4 & -5 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} -2 & 1 \\ 5 & 3 \end{bmatrix}.
\]

If possible complete each calculation. If not possible, explain why not.

a. \( A + D \)  
   b. \( B \cdot D \)  
   c. \( D \cdot C \)  
   d. \( A \cdot B \)  
   e. \( C \cdot A \)  
   f. \( D^2 \)

7. In each case below, a student has made an error in attempting to write an equivalent expression. Spot the reasoning error and write an explanation to help clear up the problem for the student who made the error.

a. \( 5(2x + 3) = 10x + 3 \)  
   b. \( 3x - 7x = 4x \)  
   c. \( 8 - 2(3x - 4) = -6x \)  
   d. \( 2 + 3(x + 1) = 5x + 5 \)

8. Justin operates a dog-walking service. He tries to schedule several dogs to walk together to save time, but some dogs do not get along and cannot be walked together. Justin’s part-time assistant walks six dogs: Alf, Bruno, Cookie, Duke, Einstein, and Flower. Alf cannot walk with Bruno, Cookie, or Duke. Bruno cannot walk with Alf, Cookie, or Einstein. Cookie cannot walk with Alf, Bruno, or Duke. Duke cannot walk with Alf or Cookie. Einstein cannot walk with Bruno or Flower. Flower cannot walk with Einstein. Describe and use a graph-coloring model to determine how many walks are necessary to walk all six dogs.

9. The area of the deck of a model ship is 110 cm\(^2\). The volume of the hold is 340 cm\(^3\). The scale of the model is 1:200.

   a. What is the area of the deck of the actual ship?
   b. Express your answer to Part a in m\(^2\).
   c. What is the volume of the hold of the actual ship?
   d. Express your answer to Part c in m\(^3\).

10. The endpoints of segment \( AB \) have coordinates \((-2, -3)\) and \((4, -2)\). Determine the coordinates of the midpoint of the image of line segment \( AB \) reflected across the \( x \)-axis.
1. The following table shows data from a national test of mathematical knowledge by eighth grade students in 1992 and 1996. Data are missing for some states because they did not participate in the studies in both years.

**Eighth Grade Mathematics Scores**

<table>
<thead>
<tr>
<th>State</th>
<th>1992</th>
<th>1996</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL</td>
<td>39</td>
<td>45</td>
</tr>
<tr>
<td>AZ</td>
<td>55</td>
<td>57</td>
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<td>AR</td>
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</tr>
<tr>
<td>FL</td>
<td>49</td>
<td>54</td>
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<table>
<thead>
<tr>
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<th>1992</th>
<th>1996</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>MA</td>
<td>63</td>
<td>68</td>
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<table>
<thead>
<tr>
<th>State</th>
<th>1992</th>
<th>1996</th>
</tr>
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<tbody>
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<td>67</td>
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<td>MN</td>
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<td>NE</td>
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<td>61</td>
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<tr>
<td>NC</td>
<td>47</td>
<td>56</td>
</tr>
<tr>
<td>ND</td>
<td>78</td>
<td>77</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>State</th>
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<th>1996</th>
</tr>
</thead>
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<td>60</td>
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<tr>
<td>SC</td>
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<td>48</td>
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<td>TN</td>
<td>47</td>
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<td>75</td>
</tr>
<tr>
<td>WY</td>
<td>67</td>
<td>68</td>
</tr>
</tbody>
</table>

a. Construct a scatterplot of the (1992, 1996) data pairs for each state and draw the \( y = x \) line on the plot. Then write a short summary of what the plot shows about improvement or decline in performance from 1992 to 1996.

b. Calculate the range and interquartile range for national test data in each year and explain what the results suggest about change in performance variability from 1992 to 1996.

2. An Ocean City hotel is advertising the low cost of a Weekend Getaway package. Their ad states that $195 will pay for a two-night stay and three meals, and $300 will pay for a three-night stay and five meals.

a. What is the cost for a room per night? The average cost per meal?

b. How much would it cost a couple for dinner for two on Saturday evening, an overnight stay in the hotel Saturday night, and brunch for two on Sunday?

3. Solve the following equations for \( x \).

a. \( 12x + 15 = 1,743 \)  
   b. \( 12x^2 + 15 = 1,743 \)  
   c. \( \frac{12}{x} + 15 = 1,743 \)

d. \( 12(x + 15) = 1,743 \)  
   e. \( 12^x + 15 = 1,743 \)  
   f. \( 12(x^2 + 15) = 1,743 \)

4. Sketch and calculate the surface area of a square prism with 8 cm edges on the base and a height of 12 cm.
5. Without using your calculator, sketch the graphs of the following equations.

   a. \( y = -2x + 5 \)  
   b. \( y = 5(3^x) \)  
   c. \( y = 5x^2 \)  
   d. \( y = 5(0.3^x) \)  
   e. \( 3x - 5y = 10 \)  
   f. \( y = \frac{5}{x} \)

6. Write equations for the lines satisfying these conditions.

   a. Passes through the points (2, 5) and (4, -3)
   b. Is parallel to the line with equation \( y = 10 + 4x \) and contains the point (5, -8)
   c. Is perpendicular to the line with equation \( y = 10 + 4x \) and contains point (5, -8)
   d. Is a horizontal line through the point (-10, -11)
   e. Is a vertical line through the point (-10, -11)

7. Rewrite each of the following expressions in a simpler form.

   a. \( 3^0 \)  
   b. \( 49^{\frac{1}{2}} \)  
   c. \( n^3n^5 \)  
   d. \( (2a^2b)^4 \)  
   e. \( \frac{x^8y^3}{x^2y} \)  
   f. \( \frac{15x^6y^7}{5x^2y^4} \)

8. Matrix \( A \) at the right shows the number of senior, adult, and student tickets sold during each performance of a school play. Tickets cost $2 for seniors, $3 for students, and $5 for adults.

\[
\begin{array}{ccc}
\text{Sr.} & \text{Adult} & \text{Stu.} \\
\text{Friday} & 8 & 67 & 82 \\
\text{Saturday} & 11 & 109 & 74 \\
\text{Sunday} & 23 & 71 & 65 \\
\end{array}
\]

   a. Define matrix \( B \) to represent the ticket pricing above, such that matrix multiplication will provide additional information.
   b. Perform the matrix multiplication and interpret the resultant matrix.

9. Consider the project digraph below.

   a. Prepare an immediate prerequisite table.
   b. Mark the critical path(s).
   c. Find the EFT for the project.

10. Provide an example of two transformations for which the order in which the transformations are applied does not matter. Provide an example for which order does matter. Use sketches, coordinate descriptions, or matrices to illustrate your examples.
1. The table at the right gives the number of computers per 1,000 people living in the United States for certain years.

   a. Produce a scatterplot of the data with your calculator.
   b. Find the linear regression equation for this data and graph it on your scatterplot. Then use the regression equation to predict the number of computers per 1,000 people in the United States in the year 2000.
   c. The *Eighth Annual Computer Industry Almanac* projected that in the year 2000, there would be 580 computers per 1,000 people. How does this compare to your estimate in Part b?

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Computers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>90.1</td>
</tr>
<tr>
<td>1988</td>
<td>166.0</td>
</tr>
<tr>
<td>1989</td>
<td>191.7</td>
</tr>
<tr>
<td>1991</td>
<td>245.4</td>
</tr>
<tr>
<td>1992</td>
<td>266.9</td>
</tr>
<tr>
<td>1993</td>
<td>296.6</td>
</tr>
<tr>
<td>1994</td>
<td>329.2</td>
</tr>
<tr>
<td>1995</td>
<td>364.7</td>
</tr>
</tbody>
</table>

   Source: *Information Please Almanac, 1997.*

2. Solve the following equations for $x$.
   a. $\left(\frac{2}{3}\right)^x = \frac{3}{2}$
   b. $\sqrt[3]{x} = 8$
   c. $3^x = \frac{1}{9}$
   d. $25^x = 1$
   e. $3(4^x) = 12,288$
   f. $3x^{\frac{1}{2}} = 27$

3. Consider the quadrilateral $PQRS$ with vertices at $P(4, 6)$, $Q(-3, 6)$, $R(-3, -2)$, and $S(4, -2)$.
   a. What type of quadrilateral is $PQRS$? Explain your reasoning.
   b. Determine the area of quadrilateral $PQRS$.

4. A copy machine will enlarge a copy to at most 120% of its original size. How many enlargements are necessary to enlarge a logo by a scale factor of about 2?

5. A Washington, D.C. high school marching band is taking a bus to Boston for a national competition. Two alternate routes each have a total distance of 460 miles. One route has a maximum speed limit of 55 mph, while the other route has a maximum speed limit of 65 mph. The bus averages the maximum speed limit on its way to Boston.

   a. Suppose that construction on the return trip slows the flow of traffic by 15 mph. For which route would you expect the greater delay due to construction?
   b. Calculate the travel times with and without construction for each route and compare your results to the prediction you made in Part a.
6. Determine the slope and \( y \)-intercept of each line.

a. The line with equation \( 5x - 2y = 7 \)

b. The line with graph below
c. The line with table below

\[
\begin{array}{c|c}
 x & y \\
-4 & -21 \\
-2 & -16 \\
0 & -11 \\
2 & -6 \\
\end{array}
\]
d. The line given by \( NEXT = NOW - 2.1 \) (start at 4)

7. Find the surface area and volume of a triangular prism with base edges of length 5 cm, 5 cm, and 8 cm, and a height of 10 cm.

8. Describe the symmetry of the images below, indicating the line(s) of reflection symmetry and the angle(s) of rotational symmetry.

a. 

b. 

c. 

d. 

9. The adjacency matrix of a graph is given below.

\[
\begin{array}{cccccc}
 & A & B & C & D & E \\
A & 0 & 1 & 0 & 0 & 1 \\
B & 1 & 0 & 1 & 0 & 1 \\
C & 0 & 1 & 0 & 1 & 0 \\
D & 0 & 0 & 1 & 0 & 1 \\
E & 1 & 1 & 0 & 1 & 0 \\
\end{array}
\]

a. Does the corresponding graph have an Euler circuit? An Euler path? How can you tell without drawing the graph?

b. Draw and label a graph corresponding to the adjacency matrix. Trace an Euler circuit or Euler path if there is one.

c. How many paths of length 3 are there from \( A \) to \( B \)? List them.

10. Jesse wants to buy a used car for $1,200. She has two options for payment.

Option One: She can borrow the $1,200 from her parents and repay them $40 a month for three years.

Option Two: She can finance the car for three years at a 14% annual interest rate.

Which option will cost Jesse the least amount of money? Explain your reasoning.
1. The chance that a newborn baby will be a boy is about 0.5. Suppose that in one large hospital, about 20 babies are born each day. In a smaller hospital nearby, about 5 babies are born each day.

a. In which hospital would you expect more days in which 40% or fewer of the babies born are boys? Explain your reasoning.

b. Describe how to conduct one trial of a simulation model to find the number of boys born on one day in the larger hospital. Describe a similar model for the smaller hospital.

c. Conduct 20 trials of the simulation for each hospital. Are the results of your simulations different from your conjecture in Part a? What should you conclude?

2. Solve the following systems of equations.

   a. \(3x + 2y = 5\) 
   b. \(y = 4 - 7x\) 
   c. \(3x + y = 3\) 

   \(-2x + y = -1\) 
   \(y = 13 + 5x\) 
   \(x - 5y = 9\)

3. The volume of a circular cylinder with a height of 6 cm is \(294\pi\) cubic centimeters. Find the radius of the base and the surface area of the cylinder.

4. Steve manages two pizza delivery places. He needs to place one order for a three-month supply of pizza boxes for both locations. The matrices below show the number of deep dish pizzas and thin crust pizzas delivered by each location in one month. Different types of boxes are used for deep dish pizzas and thin crust pizzas.

<table>
<thead>
<tr>
<th>Location A</th>
<th>Location B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sm Med Lg</td>
<td>Sm Med Lg</td>
</tr>
<tr>
<td>Deep Dish</td>
<td>86 75 26</td>
</tr>
<tr>
<td>Thin Crust</td>
<td>65 73 81</td>
</tr>
</tbody>
</table>

   How many boxes of each size and type should Steve order for the next three months? Explain your reasoning.
5. A local park operates a swimming area during the summer. Admission to the swimming area is $2.00 per day. Area residents can purchase a season pass for $20 which entitles its owner to a reduced admission of $0.50 per day.

a. Write an equation to model the total cost $C$ for $n$ days of admission to the swimming area under each plan.

b. For how many days is the total cost the same for each plan?

c. How would you advise someone trying to decide between the two plans?

6. Triangle $ABC$ is represented by the matrix on the right. \[
\begin{bmatrix}
1 & -3 & 3 \\
-2 & 1 & 4
\end{bmatrix}
\]

a. Find the perimeter of triangle $ABC$.

b. Find the area of triangle $ABC$.

7. Solve each inequality for the indicated variable.
   
   a. $-12 + 11m \leq 54$
   b. $7y - 27 \geq 4y$
   c. $7(r + 8) < 3(r + 12)$
   d. $8c - (c - 5) > c + 17$

8. Solve each of the following equations for $x$.
   
   a. $9x^2 = 25$
   b. $x^2 + 2x - 15 = 0$
   c. $-6 = 2x^2 + 13x$

9. A line contains the points $(3, -2)$ and $(-6, 1)$.

a. Write an equation for the line.

b. Write an equation of the line which contains the point $(3, -2)$ and is perpendicular the line in Part a.

c. Write an equation of the line which contains the point $(1, 6)$ and is parallel to the line in Part a.

10. Determine whether or not the image of a figure under each of the following transformations will be the same size and shape, the same shape, or neither the same shape nor size as its preimage. Identify the type of transformation in each case.

   a. $(x, y) \rightarrow (-y, x)$
   b. $(x, y) \rightarrow (2x, y)$
   c. $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$
   d. $(x, y) \rightarrow (-x, -y)$
1. You are planning a graduation party. You find a brochure for Budget Caterers which lists the following prices:

<table>
<thead>
<tr>
<th></th>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Guests</td>
<td>$305</td>
<td>$275</td>
</tr>
<tr>
<td>50 Guests</td>
<td>$430</td>
<td>$475</td>
</tr>
</tbody>
</table>

a. Assuming a linear relationship between the cost and the number of guests, write an equation that describes the cost in terms of the number of guests for each option.

b. Use your equations to calculate the cost of inviting 40 guests for each option.

c. For how many guests is the cost the same for both options?

d. If cost is the most important factor in choosing between the options, explain how you determine your decision.

2. Solve each of the following equations for $x$.

   a. $2x - 3 - 3(x + 4) = 14$
   b. $2^x = \frac{1}{4}$
   c. $x^3 + 15 = 79$
   d. $\frac{12}{x} = 28$
   e. $-3x^2 + 89 = 56$
   f. $2x^2 - 7 = 5x$

3. Match each correlation coefficient with the appropriate plot.

   | $-0.8$ | $0.9$ | $0.5$ |

   Plot 1 | Plot 2 | Plot 3

4. Suppose the makeup of a drug is such that one hour after a dose is administered into an individual’s blood stream, 80% remains active.

   a. Does it follow that one-half hour after administration of the same dose to the same person, 90% will remain active? Explain your reasoning.

   b. Estimate the half-life of the drug.

   c. Suppose the drug should be readministered when less than 20% of the drug is active. When should the second dose be administered?
5. Write each of the following expressions in its simplest form.
   a. \( \sqrt{81} \)  
   b. \( \sqrt{32} \)  
   c. \( \sqrt{200} \)  
   d. \( \sqrt{5 \cdot \sqrt{30}} \)  
   e. \( 3\sqrt{14} \cdot 5\sqrt{2} \)

6. Suppose that each of the dimensions of a cardboard box is doubled. What is the effect on the perimeter of the base of the box? On the surface area of the box? On the volume of the box?

7. Jamie wants to purchase a stereo system that costs $400. He has $100 now and will be able to save $20 each week. The stereo model has been discontinued, and the price will be reduced by 10% each week until it is sold. After how many weeks will Jamie have saved enough money to buy the stereo system?

8. Consider the project digraph below.

   ![Project Digraph]

   Schedule this project by finding the EFT.

9. Determine the slope and the length of each side of the quadrilateral formed by the vertices with coordinates \( A(0, 4), B(3, 2), C(2, -2), \) and \( D(-4, 2). \) What type of quadrilateral is \( ABCD? \) Explain your reasoning.

10. Sketch a graph showing the general shape of each of the general equations below.
    a. \( y = a(b^x), \) \( a > 0 \) and \( b < 1 \)  
    b. \( y = a + bx, \) \( a > 0 \) and \( b < 0 \)  
    c. \( y = ax^2, \) \( a > 0 \)  
    d. \( y = ax^3, \) \( a > 0 \)  
    e. \( y = \frac{a}{x}, \) \( a > 0 \)  
    f. \( NEXT = NOW + 4 \)  
    g. \( NEXT = 1.18NOW \)
As you work closely with your classmates and teachers on a daily basis, they will have a good idea of what you know and are able to do with respect to the mathematics you are studying this year. However, your school district or state department of education may ask you to take tests that they design to measure the achievement of all students, classes, or schools in the district or state. Colleges also use such external *standardized* tests like the PSAT (Preliminary Scholastic Aptitude Test) to compare the knowledge of different students who will soon be applying for admission or scholarships.

External standardized tests usually present assessment tasks in formats that can be easily scored to produce simple percent-correct ratings of your knowledge. If you want to perform well on such standardized tests, it helps to have some practice with test items in multiple-choice formats. The following ten sets of multiple-choice tasks have been designed to give you that kind of practice and to offer some strategic advice in working on such items. You will find helpful *Test Taking Tips* at the end of each of the practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. Which of the following coordinates for point Q will guarantee that line PQ has slope 2.5?
   - (a) Q(1, 2.5)
   - (b) Q(1, 4.5)
   - (c) Q(2, 1)
   - (d) Q(2.5, 3)
   - (e) Q(4, 1)

2. The following figure is made up of a rectangle with a semicircle attached to both ends. The area in square meters of the figure is
   - (a) 22 + 4π
   - (b) 28 + 4π
   - (c) 22 + 8π
   - (d) 22 + 16π
   - (e) 28 + 16π

3. Solve \( \frac{2}{3}x + 5 = \frac{1}{2}(x + 4) \) for x.
   - (a) x = −18
   - (b) x = −11
   - (c) x = −6
   - (d) x = −1
   - (e) x = 18

4. The median of the test scores displayed in the stem-and-leaf plot below is:
   - (a) 68
   - (b) 72
   - (c) 75
   - (d) 76
   - (e) 78

5. If $1,000 is invested at 8% annual interest, in about how many years will the balance double?
   - (a) 2
   - (b) 6
   - (c) 9
   - (d) 12
   - (e) 13
6. Which of the following expressions are equivalent to $12x$?

I. $7 + 5x$  
II. $12x^2 - x$  
III. $7x + 5x$

(a) I only  
(b) I and II  
(c) II only  
(d) III only  
(e) All of them

7. What is the degree measure of each interior angle of a regular pentagon?

(a) $36^\circ$  
(b) $72^\circ$  
(c) $108^\circ$  
(d) $120^\circ$  
(e) $144^\circ$

8. Which of the lines shown is the graph of $y = \frac{1}{2}x - 2$? The scale on each axis is 1.

(a) $l$  
(b) $m$  
(c) $p$

(d) $q$  
(e) None of them

9. What is the equation of the line through the points $(4, 1)$ and $(8, 3)$?

(a) $y = \frac{1}{2}x - 1$  
(b) $y = \frac{1}{2}x + \frac{7}{2}$

(c) $y = -\frac{1}{2}x + 7$  
(d) $y = 2x - 7$

(e) $y = 2x - 13$

10. Which configuration of bridges would allow people to tour the city by beginning at a point on land, walking across each bridge exactly once, and returning to the starting point?

(a)  
(b)  
(c)  
(d)  
(e)
Test Taking Tip

Work backwards from choices.

On multiple-choice tests, if you know how to solve a problem and are confident you can do it accurately and reasonably quickly, then that is the way to proceed. If you are unsure of how to solve it, then an alternative strategy is to work backwards by testing various answer choices to see which one is correct.

Example  Look back at Item 9 on page 87. To use this strategy, test choice (a): \( y = \frac{1}{2} x - 1 \). Coordinates of points on a line must satisfy the equation of the line. Substituting \( x = 4 \) gives \( \frac{1}{2} (4) - 1 = 1 \) which is the \( y \)-coordinate of the first point.

Substituting \( x = 8 \) gives \( \frac{1}{2} (8) - 1 = 3 \) which is the \( y \)-coordinate of the second point. So, the answer is (a).

■ Find, if possible, another test item in the practice set for which this strategy might be helpful. Try it.

■ Keep this strategy in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. If \( a = 2 \), \( b = 5 \), and \( c = 1 \), then \( 3a + 2(b + c) \) is equal to:
   (a) 14
   (b) 17
   (c) 18
   (d) 48
   (e) 204

2. What is the length of the missing side of the triangle below?
   (a) 22
   (b) 24
   (c) 26
   (d) 34
   (e) 38

3. Suppose that 60% of an area’s population subscribes to an Internet provider, and telemarketers must call phone numbers from the area at random until they reach 5 customers. A simulation of 20 trials is used to estimate how many calls it will take each telemarketer to reach 5 customers. The data from the trials is shown in the table below.

<table>
<thead>
<tr>
<th>Number of Calls to Reach 5</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>0.10</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0.20</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.35</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

These simulated data suggest that on average one could expect to require about how many calls to find five customers?
   (a) 6
   (b) 7
   (c) 8
   (d) 10
   (e) 12
4. The solution of the inequality $6 - 3r > 15$ is:
   (a) $r < -3$
   (b) $r > -3$
   (c) $r < 3$
   (d) $r > 3$
   (e) $-3 < r < 3$

5. The length of a rectangle is three more than its width. If the width is represented by $w$, which expression represents the area of the rectangle?
   (a) $w^2 + 3w$
   (b) $w^2 + 3$
   (c) $4w + 6$
   (d) $2w + 3$
   (e) $8w$

6. What is the slope of the line through the points $(−2, 4)$ and $(10, 0)$?
   (a) $-3$
   (b) $-2$
   (c) $\frac{1}{3}$
   (d) $\frac{1}{3}$
   (e) $\frac{1}{2}$

7. If a $2,000$ bicycle depreciates $20\%$ each year, for how many years will the bicycle be worth more than $500$?
   (a) $0$
   (b) $3$
   (c) $4$
   (d) $6$
   (e) $8$
8. What is the area in square units of the trapezoid below?

(a) 86
(b) 240
(c) 360
(d) 390
(e) 455

9. What is the equation of the line through the point \((10, 1)\) with slope \(\frac{2}{5}\)?

(a) \(y = \frac{2}{5} x + \frac{48}{5}\)
(b) \(y = \frac{2}{5} x + 1\)
(c) \(y = \frac{2}{5} x - 3\)
(d) \(y = \frac{2}{5} x - \frac{6}{5}\)
(e) \(y = -3x + \frac{2}{5}\)

10. Which of the following graphs shows that the price \(p\) of a product decreases as the supply \(s\) of the product increases?

(a)  
(b)  
(c)  
(d)  
(e)  
Test Taking Tip

If a diagram is not provided for a geometry problem, draw and label one.

Example  Look back at Item 5 on page 90. To use this strategy, draw and label a rectangle as shown below.

\[ A = w(w + 3) = w^2 + 3w. \]

So, the answer is (a).

Find another test item in the practice set for which this strategy might be helpful. Try it.

Keep this strategy in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. A traveling fair charges $5 for admission and $1 per ride. Suppose you go to the fair with $13. Which inequality represents the number of rides \( r \) that you can afford?
   (a) \( 5r + 1 \leq 13 \)
   (b) \( 5r + 1 \geq 13 \)
   (c) \( 5 + r \leq 13 \)
   (d) \( 5 + r \geq 13 \)
   (e) None of these

2. What is the area in square units of the shaded region in the figure below?
   (a) \( 8 - 2\pi \)
   (b) \( 2r(3 - 2\pi) \)
   (c) \( 4r(2\pi - \pi) \)
   (d) \( 8r(1 - 2\pi r) \)
   (e) \( 2r^2(4 - \pi) \)

3. What is the slope of the line passing through the points \((1, 2)\) and \((-3, 9)\)?
   (a) \( -\frac{11}{2} \)
   (b) \( -\frac{11}{4} \)
   (c) \( -\frac{4}{7} \)
   (d) \( -\frac{2}{11} \)
   (e) \( \frac{7}{2} \)

4. What is the mean of the degree measures of the angles of a triangle?
   (a) \( 45^\circ \)
   (b) \( 60^\circ \)
   (c) \( 90^\circ \)
   (d) \( 180^\circ \)
   (e) It will depend on the triangle.
5. Lita earns $250 per week plus 8% commission on her sales. She must earn at least $300 each week to cover her expenses. What is the minimum amount of sales each week that will enable her to cover her expenses?

(a) $62.50  
(b) $68.75  
(c) $625.00  
(d) $687.50  
(e) $3,750.00

6. What is the equation of the line passing through the point (3, −6) and having slope $\frac{4}{3}$?

(a) $y = -\frac{4}{3}x - 6$  
(b) $y = -\frac{4}{3}x - 5$  
(c) $y = -\frac{4}{3}x - 2$  
(d) $y = -\frac{4}{3}x + 11$  
(e) $y = -\frac{4}{3}x + 2$

7. Which figure cannot be used to tile a plane?

(a) A triangle  
(b) A regular hexagon  
(c) A trapezoid  
(d) A quadrilateral  
(e) A regular octagon

8. Which equation is not an example of a linear equation?

(a) $2x + 5y = 3$  
(b) $y = -10 + 2x$  
(c) $5 = 3x$  
(d) $y = \frac{1}{x} + 4$  
(e) $2^2 + 3x = y$
9. Which of the following is equivalent to $a^6$?
   (a) $a^2a^3$
   (b) $a^2a^4$
   (c) $a^2 + a^3$
   (d) $a^2 + a^4$
   (e) $2(a^3)$

10. Which graph could represent the equation $y = 4 - 3x$?
Test Taking Tip

Replace variables with numbers.

When comparing expressions, it is sometimes helpful to replace each variable with an easy-to-use but not special number. If you find one number for which the expressions are not the same, then the expressions are not equivalent.

Example  Look back at Item 9 on page 95. To use this strategy, replace $a$ with 2.

Then $a^6 = 64$.

For choice (a): $2^2 \cdot 2^3 = 4 \cdot 8 = 32$. $32 \neq 64$

For choice (b): $2^2 \cdot 2^4 = 4 \cdot 16 = 64$.

Explain why choices (c), (d), and (e) are not correct choices. So, the answer is (b).

Find, if possible, another test item in the practice set for which this strategy might be helpful. Try it.

Keep this strategy in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. What is the slope of the line through the points \((-1, -2)\) and \((4, 1)\)?

   (a) \(-3\)
   (b) \(\frac{3}{5}\)
   (c) \(\frac{1}{3}\)
   (d) \(\frac{3}{5}\)
   (e) \(\frac{5}{3}\)

2. What is the area in square units of the trapezoid below?

   (a) 12
   (b) 14
   (c) \(14 + 2\sqrt{2}\)
   (d) 20
   (e) 24

3. The expression \(4 + 8(t - 2)\) is equivalent to:

   (a) \(12t - 2\)
   (b) \(12t - 24\)
   (c) \(8t - 12\)
   (d) \(8t + 2\)
   (e) \(8t + 20\)

4. When \(2x - 3y = 6\) is solved for \(y\), which equation results?

   (a) \(-3y = 6 - 2x\)
   (b) \(y = -2 - 2x\)
   (c) \(y = -2 - \frac{2}{3}x\)
   (d) \(y = -2 + \frac{2}{3}x\)
   (e) \(y = 2 - \frac{2}{3}x\)
5. The solution of the inequality \( 6d + 10 < 28 \) is

(a) \( d < -3 \)
(b) \( d > -3 \)
(c) \( -3 < d < 3 \)
(d) \( d < 3 \)
(e) \( d > 3 \)

6. The number of respondents to a survey that rated their health in each of four categories is given in the following table.

<table>
<thead>
<tr>
<th>Condition of Health</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
<td>437</td>
</tr>
<tr>
<td>Good</td>
<td>697</td>
</tr>
<tr>
<td>Fair</td>
<td>258</td>
</tr>
<tr>
<td>Poor</td>
<td>69</td>
</tr>
<tr>
<td>Total</td>
<td>1,461</td>
</tr>
</tbody>
</table>

About what percent of the respondents rated their health as at least “Good”?

(a) 20%
(b) 30%
(c) 50%
(d) 70%
(e) 80%

7. What is the equation of the line parallel to \( y = \frac{2}{3}x - 7 \) that passes through the point (6, 3)?

(a) \( y = \frac{2}{3}x - 1 \)
(b) \( y = \frac{3}{2}x - 6 \)
(c) \( y = \frac{2}{3}x + 4 \)
(d) \( y = \frac{3}{2}x + \frac{3}{2} \)
(e) \( y = \frac{2}{3}x - 4 \)
8. A man-made lake is initially stocked with 500 fish. The population is expected to increase about 20% each year. If the lake can’t support more than 2,000 fish, in about how many years will the lake become overpopulated with fish?

(a) 3
(b) 8
(c) 15
(d) 20
(e) It will never become overpopulated with fish.

9. What is the length, in units, of the missing side of the triangle below?

(a) 2
(b) 6
(c) 11
(d) 12
(e) 17

10. Which graph best represents the 15% annual depreciation in value $v$ over time $t$ of an object?

(a)  
(b)  
(c)  
(d)  
(e)  
Test Taking Tip

Break complex geometric shapes into simpler shapes if a particular formula cannot be remembered.

Example  Look back at Item 2 on page 97. To use this strategy, draw a perpendicular segment in the diagram as shown below.

Then calculate the areas of the rectangle and right triangle and add: $10 + \frac{1}{2} (4) = 12$ square units. The answer is (a).

■ Find, if possible, another test item in the practice set for which this strategy might be helpful. Try it.

■ Keep this strategy in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. For what value of $x$ will the line passing through $(-1, 4)$ and $(x, 7)$ have a slope of $\frac{1}{3}$?
   (a) $-2$
   (b) $0$
   (c) $2$
   (d) $8$
   (e) $10$

2. If the perimeter of $\triangle ABC$ is 30 cm, then what is the length in centimeters of the longest side?
   (a) $3$
   (b) $8$
   (c) $10$
   (d) $12$
   (e) $16$

3. The solution of the inequality $\frac{t + 2}{5} \geq 1$ is:
   (a) $t \geq -5$
   (b) $t \leq -1$
   (c) $t \leq 3$
   (d) $t \geq 3$
   (e) $t \geq 5$

4. If $x + y = 12$, $y + z = 20$, and $x + z = 4$, what is the average (mean) of $x$, $y$, and $z$?
   (a) $4$
   (b) $6$
   (c) $12$
   (d) $18$
   (e) It cannot be determined from the information given.
5. Which system of equations has more than one solution?

(a) \( y = 2(x + 3) \)
    \( y = 2x - 7 \)

(b) \( y = 3(x - 1) \)
    \( y = \frac{1}{3} x - 3 \)

(c) \( y = 4(x + 2) \)
    \( y = 8 + 4x \)

(d) \( 2x + 3y = 6 \)
    \( 8x + 12y = 6 \)

(e) None of these

6. In which distribution would you expect the mean to be less than the median?

(a) 

(b) 

(c) 

(d) 

(e) All are possible.

7. The cost of a catered event can be modeled by a linear equation. The cost is $1,050 for 50 guests and $1,550 for 75 guests. How much will dinner cost if 125 guests attend?

(a) $2,550.00

(b) $2,575.00

(c) $2,583.33

(d) $2,600.00

(e) $2,625.00
8. What is the value of $x$ in the figure below?

(a) 10°
(b) 15°
(c) 30°
(d) 50°
(e) 150°

9. At Best Videos, the regular price for used videos is $d$ dollars. How many videos can be purchased for $x$ dollars when Best Videos is having a clearance sale and all used videos are on sale at 40% off the regular price?

(a) $x/0.6d$
(b) $x/0.4d$
(c) $0.6xd$
(d) $\frac{x}{0.4d}$
(e) $\frac{x}{0.6d}$

10. If the angles of a quadrilateral are in the ratio 1:2:3:3, what is the degree measure of the smallest angle?

(a) 20
(b) 40
(c) 60
(d) 80
(e) 120
Memorize important facts and formulas.

Some standardized tests provide a list of commonly used area and volume formulas and facts such as: The measure in degrees of a straight angle is 180°, or the sum of the measures in degrees of the angles of a triangle is 180°. Standardized tests are timed tests often allowing an average of one minute per question. You can save precious time on these tests if you memorize facts such as those above.

Example Look back at Item 8 on page 103. By being able to quickly recall the above two facts about angles, you can reason that the unlabeled angle in the triangle has degree measure 30 since 30 + 150 = 180.

Then, since \( x + 2x + 30 = 180 \), \( 3x = 150 \) and \( x = 50° \) (Choice d).

- Find, if possible, another test item in the practice set for which having memorized these facts about angles would be helpful.
- Look back at previous practice sets and make a list of facts and formulas that were frequently needed and memorize them.
- Keep these facts and formulas in mind and add others to your list as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. The inequality \( 5z - 4 > 2z + 8 \) is equivalent to which inequality?
   (a) \( z < 1 \)
   (b) \( z > 1 \)
   (c) \( z < 4 \)
   (d) \( z > 4 \)
   (e) \( z > 12 \)

2. What is the approximate length in inches of the diagonal of a rectangle that is 25 inches long by 20 inches wide?
   (a) 15
   (b) 22
   (c) 27
   (d) 32
   (e) 45

3. Suppose a new car is purchased for $22,000, and its value is predicted to decrease by 15% each year. Which equation gives the car’s value \( V \) in thousands of dollars \( t \) years after its purchase?
   (a) \( V = 22(-0.15^t) \)
   (b) \( V = 22(-0.15t) \)
   (c) \( V = 22(0.85^t) \)
   (d) \( V = -0.15(22^t) \)
   (e) \( V = 18.7^t \)

4. What is the equation of the line that contains the point \((4, -2)\) and has a slope of \(\frac{5}{2}\)?
   (a) \( y = \frac{5}{2}x - 12 \)
   (b) \( y = \frac{5}{2}x - 8 \)
   (c) \( y = \frac{5}{2}x - 1 \)
   (d) \( y = \frac{5}{2}x + 9 \)
   (e) \( y = \frac{5}{2}x + 12 \)
5. What is the area in units of the rectangle shown in the figure below? The top of the rectangle is tangent to the curve with equation \( y = 15x^2 - 3x + 7 \).

(a) \(-2.74\)
(b) \(2.74\)
(c) \(4.11\)
(d) \(4.47\)
(e) \(4.90\)

6. The box plot below shows the price in dollars of twenty models of cordless phones.

Which of the following statements is not true?

(a) The median price is about $32.
(b) There are more cordless phones that cost more than $36 than cost less than $27.
(c) The upper quartile is about $36.
(d) The least expensive model is about $20.
(e) About 25% of the phones cost more than $36.

7. The following formula relates temperature in degrees Celsius \( (C) \) and temperature in degrees Fahrenheit \( (F) \):

\[
C = \frac{5}{9} (F - 32)
\]

What is the equivalent of \(15^\circ C\) in degrees Fahrenheit?

(a) \(\frac{85}{9}\)
(b) 9
(c) 15
(d) 59
(e) 85
8. The base of a rectangular box measures 3.5 inches by 7.5 inches. What is the height, in inches, of the box if its volume is approximately 300 cubic inches?
   (a) 11.5  
   (b) 13.5  
   (c) 23.25 
   (d) 27    
   (e) 289

9. What is the value of $x$ if the area of the triangle is $\frac{1}{4}$ the area of the square?
   (a) $\sqrt{2}$  
   (b) $2 \sqrt{2}$  
   (c) 4 
   (d) 8    
   (e) $8 \sqrt{2}$

10. If $x^2 = 25$ and $y^2 = 16$, what is the smallest possible value for $(x + y)^2$?
    (a) $-81$  
    (b) $-1$  
    (c) 1    
    (d) 9    
    (e) 81
Test Taking Tip

Answer the easy questions first; then answer the more difficult ones.

Most standardized tests have a time limit, so you should carefully budget your time. Some questions will be easier than others. If after reading a question you immediately know how to do it, then you should do it right away. If you are not sure, try to get started on it; perhaps by using one of the previous test taking tips. If you have not made any progress within 30 seconds or so, circle the item and go on to the next question. If there is still time left when you get to the end of the test, go back to the items you circled.

■ Look back over the items in this practice set. Which ones were you able to answer in a minute or less? If you would have been using this test taking tip, which items would you have circled as items to come back to if time remained?

■ Keep this strategy in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. The circle below shows which languages students in Greenwood Valley’s Headstart Classes primarily speak. If there are 120 students in the headstart program and there are twice as many Polish-speaking students as there are Chinese-speaking students, how many Polish-speaking students are there?
   (a) 10
   (b) 20
   (c) 30
   (d) 40
   (e) 60

2. The cost of ordering custom T-shirts can be modeled by a linear equation. If 50 T-shirts cost $360 and 100 T-shirts cost $685, how much will 80 T-shirts cost?
   (a) $520
   (b) $548
   (c) $555
   (d) $576
   (e) $600

3. The area of a circle is $36\pi$ square units. What is its circumference?
   (a) $6\pi$ units
   (b) $12\pi$ units
   (c) $18\pi$ units
   (d) $36\pi$ units
   (e) $72\pi$ units

4. A jar contains 30 marbles: 10 red, 5 white, and 15 blue. If you randomly remove one marble at a time, what is the minimum number of marbles that you must remove to be certain that you have one of each color?
   (a) 3
   (b) 16
   (c) 21
   (d) 26
   (e) 27
5. Which expression describes the perimeter of the rectangle below?
(a) $10x^4 + 6$
(b) $5x + 3$
(c) $5x^2 + 3$
(d) $(3x + 2)(2x + 1)$
(e) $10x + 6$

6. Which expression is equivalent to $5 + 2(3n + 1)$?
(a) $21n + 1$
(b) $6n + 6$
(c) $21n + 7$
(d) $11n + 2$
(e) $6n + 7$

7. What is the degree measure of the smallest angle of rotational symmetry of a regular pentagon?
(a) 36
(b) 54
(c) 72
(d) 90
(e) 108

8. Musicexpress.com is offering Grammy-nominated CDs for $7.95 each plus a shipping/handling charge of $14.95 for the entire order. How many CDs could you purchase with a $100 gift certificate?
(a) 4
(b) 6
(c) 10
(d) 11
(e) 12
9. If $3^x + 2 = 243$, what is the value of $2^x + 4$?
   (a) 3
   (b) 12
   (c) 24
   (d) 64
   (e) 128

10. About what percentage of the students scored between 70 and 90 on the test depicted in the box plot below?

(a) 40
(b) 50
(c) 75
(d) 90
(e) Cannot be determined from information given
Test Taking Tip

Use the table-building capability of your calculator to aid in reasoning with complicated function rules.

Sometimes algebraic expressions on standardized tests may be unfamiliar to you or intimidating because of their complexity. In these cases, thinking about the meaning of the symbols and using the table-building capability of your calculator may be helpful.

Example  Look back at Item 9 on page 111. To use this strategy, enter $3^x + 2$ in the $y = \text{menu}$ of your graphing calculator and produce a table. Be careful to use parentheses correctly.

Scanning the table, you see that when $x = 3$, $3^x + 2 = 243$.

So, $2^x + 4 = 2^3 + 4 = 2^7 = 128$. The answer is (e).

Find, if possible, another test item in the practice set for which this strategy might be helpful. Try it.

Keep this strategy in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. Which is the graph of the solution set of the inequality \(-21 < 3t\)?

(a)  
(b)  
(c)  
(d)  
(e)  

2. Suppose a square has the same perimeter as the triangle below. How many units greater is the area of the square than the area of the triangle?

(a) 6  
(b) 10  
(c) 12  
(d) 18  
(e) 20  

3. Which equation is a model of exponential decay?

(a) \(y = \frac{1}{5} (2^x)\)  
(b) \(y = 100(0.7^x)\)  
(c) \(y = 200(1.08^x)\)  
(d) \(y = 3\left(\frac{10^x}{7}\right)\)  
(e) None of these  

4. What is the area in square units of the isosceles right triangle below?

(a) 2  
(b) 8  
(c) 16  
(d) 32  
(e) 64
5. For custom T-shirts, one company charges a $25 art fee plus $7 per T-shirt, another charges a $50 art fee plus $6.50 per T-shirt. For how many T-shirts would the total cost be the same from both companies?

(a) 12  
(b) 25  
(c) 50  
(d) 100  
(e) 375

6. If $\frac{1}{x} - y = 3$, then $x =$

(a) $y + 3$  
(b) $y - 3$  
(c) $\frac{1}{y + 3}$  
(d) $y - \frac{1}{3}$  
(e) $1 + \frac{3}{y}$

7. What is the length of segment $PS$?

(a) 5  
(b) 12  
(c) $\sqrt{153}$  
(d) $\sqrt{222}$  
(e) $\sqrt{231}$

8. The perimeter of a rectangle is 12 yards. If its length is 4 yards, what is its width in yards?

(a) 2  
(b) 3  
(c) 4  
(d) 6  
(e) 8
9. Juan’s scores on the first three of four 100-point tests were 72, 83, and 81. What is the lowest score he can earn on the fourth test to achieve an average of at least 80?

(a) 59  
(b) 79  
(c) 80  
(d) 81  
(e) 84

10. Which graph best represents the value $v$ over time $t$ of a certificate of deposit earning 8% annual interest?

(a)  
(b)  
(c)  
(d)  
(e)
Test Taking Tip

Know the Pythagorean Theorem and how to use it.

The Pythagorean Theorem is one of the most widely used relationships in elementary mathematics. It should already appear on the list of facts to be memorized that you prepared at the end of Practice Set 5. If it is not on your list, add it now.

Example  Look back at Item 4 on page 113. Since the triangle is an isosceles right triangle, the legs are equal in length. So, if \( x \) represents the length of each leg, then \( x^2 + x^2 = 64 \). So, \( 2x^2 = 64 \), \( x^2 = 32 \), and \( x = \sqrt{32} \).

![Diagram of a right triangle with legs of length \( x \) and hypotenuse of length 8.]

The area of the triangle is \( \frac{1}{2} \sqrt{32}(\sqrt{32}) = \frac{1}{2} (32) = 16 \) square units (Choice c).

- Look back at Practice Sets 1 through 8 and identify the items for which the Pythagorean Theorem was the key idea in solving the problems.
- Keep the Pythagorean Theorem in mind as you work on future practice sets.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. Jeremy transferred from a first period Earth Science class to a fourth period Earth Science class. Because of his transfer, the class test average was lowered in both classes. What must be true in order for this to occur?

   (a) Jeremy’s test average was below both class averages.

   (b) Jeremy’s test average was below the original class average and above the new class average.

   (c) Jeremy’s test average was above the original class average and below the new class average.

   (d) Jeremy’s test average was above both class averages.

   (e) This situation could not occur.

2. Imagine that the strip pattern below extends indefinitely to the left and to the right.

   ![Pattern Image]

   Which type of symmetry is evident in this pattern?

   (a) Translation

   (b) Reflection

   (c) Glide reflection

   (d) Rotation

   (e) No symmetry

3. The midpoint of the segment whose endpoints have coordinates (3, −2) and (−6, 1) is:

   (a) (4.5, −1.5)

   (b) (−1.5, −.5)

   (c) (−1.5, −1.5)

   (d) (−4.5, 1.5)

   (e) (−4.5, −1.5)
4. The simplest form of \((-3xy^2)^2 (x^2y)^3\) is
   (a) \(-9x^{12}y^{12}\)
   (b) \(-9x^8y^7\)
   (c) \(9x^4y^{12}\)
   (d) \(-3x^4y^7\)
   (e) \(9x^8y^7\)

5. A triangle has side lengths of 4, 10, and 12 units. What is the perimeter, in units, of a similar triangle with 30 units as the longest side?
   (a) 26
   (b) 44
   (c) 65
   (d) 70
   (e) 80

6. Which equation describes the relationship between \(x\) and \(y\) in the table below?
   (a) \(y = 0.5^x\)
   (b) \(\text{NEXT} \div \text{NOW} = 0.5\)
   (c) \(y = 2^{-x}\)
   (d) \(y = \frac{1}{2^x}\)
   (e) All of the above

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>16</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
</tbody>
</table>

7. Find the solution to the following system of equations.
   \[
   \begin{align*}
   5x + 6y &= 18 \\
   -5x + 7y &= 21
   \end{align*}
   \]
   (a) (1, 3)
   (b) (0, 3)
   (c) (0, 0)
   (d) (1, 0)
   (e) No solution
8. Which of the following statements is not true?

(a) If all the vertices of a graph are even, then the graph has an Euler circuit.
(b) If a graph has an Euler circuit, then all the vertices of the graph are even.
(c) If all the vertices of a graph are even, then the graph has an Euler path.
(d) If a graph has an Euler path, then all the vertices of the graph are even.
(e) All are true.

9. Assume a linear relationship. If a stand sells about 40 ice cream bars when the maximum daily temperature is 70°F and sells about 65 bars when the maximum daily temperature is 85°F, how many ice cream bars would you predict would be sold on a day with a maximum temperature of 100°F?

(a) 80
(b) 85
(c) 90
(d) 95
(e) 100

10. Arielle had $240 at the start of the new year and has been steadily saving about $25 each month. Her brother Shawn had $320 at the start of the new year and has been steadily saving about $20 each month. They have agreed to continue their savings plan together until Arielle has at least $325.99—the price of the new stereo system she wants. At the current rate, how much will Shawn have saved when Arielle reaches her goal?

(a) $340
(b) $360
(c) $380
(d) $400
(e) $420
Test Taking Tip

Be careful in applying proportional reasoning to linear relationships.

Standardized tests often include items that can easily be solved using proportions. The key is to compare the same units in the same order.

Example  Look back at Item 9 on page 119. Since a linear relationship is assumed, if \( n \) represents the number of ice cream bars predicted to be sold on a 100° day, then

\[
\frac{65 - 40}{85 - 70} = \frac{n - 40}{100 - 70}.
\]

So, \( \frac{n - 40}{30} = \frac{25}{15} \), \( n - 40 = 50 \), or \( n = 90 \). The answer is (c).

Note that \( \frac{40}{70} \neq \frac{65}{85} \). Be careful!

■ Find another test item in this practice set which can be solved using proportional reasoning.

■ Keep this caution in mind as you work on the next practice set and in your future work.
Solve each problem. Then record the letter that corresponds to the correct answer.

1. The table below shows summary statistics for the number of hours per week that several students spent on homework over a 10-week period.

<table>
<thead>
<tr>
<th>Hours Spent on Homework</th>
<th>Mean</th>
<th>Median</th>
<th>Mode</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student A</td>
<td>16</td>
<td>16</td>
<td>7</td>
<td>26</td>
<td>4</td>
</tr>
<tr>
<td>Student B</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>Student C</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Student D</td>
<td>15</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>Student E</td>
<td>14</td>
<td>14</td>
<td>13</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Which student was most consistent in the number of hours studying from week to week?

(a) Student A
(b) Student B
(c) Student C
(d) Student D
(e) Student E

2. If each of the dimensions of a rectangular box are doubled, then how much more material will be required to construct the new box?

(a) Twice as much
(b) Four times as much
(c) Six times as much
(d) Eight times as much
(e) It depends on the original dimensions of the box.

3. Which is an equivalent form of $8a^2b(3a + 2ab^3)$?

(a) $24a^2b + 16a^2b^3$
(b) $11a^3b + 2ab^3$
(c) $24a^3b + 2ab^3$
(d) $24a^3b + 16a^3b^4$
(e) $11a^2b + 10a^2b^3$
4. Which equation describes the relationship between $x$ and $y$ in the table below?

(a) $y = 8x - 2$
(b) $NEXT = NOW + 4$
(c) $y = 8x - 6$
(d) $NEXT = 4NOW - 2$
(e) None of the above

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$-14$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-6$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2$</td>
</tr>
<tr>
<td>$3$</td>
<td>$10$</td>
</tr>
</tbody>
</table>

5. Which graph below represents the solution of the inequality $5 - 2x < 3$?

(a) [Graph A]
(b) [Graph B]
(c) [Graph C]
(d) [Graph D]
(e) [Graph E]

6. Which expression represents the measure in degrees of the third angle in the triangle below?

(a) $176 - 13x$
(b) $13x - 4$
(c) $184 - 13x$
(d) $13x - 176$
(e) $176 - 7x$

7. If $x, y,$ and $z$ are consecutive integers and $x > y > z$, then $(x - y)(x - z)(y - z) =

(a) $-4$
(b) $-2$
(c) $1$
(d) $2$
(e) $4$
8. If 2 less than 3 times a certain number is the same as 4 more than the product of 6 and 3, what is the number?

(a) \(-\frac{20}{3}\)
(b) \(\frac{20}{3}\)
(c) \(\frac{22}{3}\)
(d) 8
(e) 10

9. What are the coordinates of the point of intersection of the lines with equations \(x - 2y = 6\) and \(3x + y = 4\)?

(a) (2, -2)
(b) (2, 2)
(c) (-2, 2)
(d) (-2, -2)
(e) None of the above

10. What is the length of arc \(AB\) in the diagram below?

(a) 10
(b) 24
(c) \(10\pi\)
(d) \(24\pi\)
(e) \(60\pi\)
Test Taking Tip

For general problem situations, create and analyze a specific example.

When you create a specific example for a general problem situation, you can test the given choices against your example.

Example  Look back at Item 2 on page 121. To use this strategy, consider a rectangular box with the indicated dimensions.

The surface area of this box is \(2(2) + 2(6) + 2(3) = 22\) square units.

If the dimensions are doubled, the box is \(2 \times 4 \times 6\). The surface area is \(2(8) + 2(24) + 2(12) = 88\) square units.

The surface area of the new box is four times as great. The correct choice is (b).

■ Find, if possible, another test item in this practice set for which the strategy might be helpful. Try it.

■ Keep this strategy in mind as you work on future problems of this type.
Solutions

Check Your Understanding ........................................ 126
Exercise Sets ........................................................... 134
Practice Sets for Standardized Tests .............................. 166
Solutions to Check Your Understanding

Check Your Understanding 1.1, pp. 7–8

1. a. Let $x$ represent the number of items sold and $y$ represent Valerie’s pay. Then
   
   $$y = 25 + 0.20x$$  

   and $\text{NEXT} = \text{NOW} + 0.20$ (start at 25).
   
   b. $d = 1,500 - 100m$ and $\text{NEXT} = \text{NOW} - 100$ (start at 1,500).

2. a. Valerie’s pay as a function of items sold increases at a rate of $0.20$ per sale. The
   outstanding debt on the television set as a function of payments made decreases
   at a rate of $100$ per month.
   
   b. The slope of the pay graph will be $0.20$ and the y-intercept is $25$. The slope of the
   debt graph is $-100$, and the y-intercept is $1,500$.

3. a. $y = 5 + 2x$ or $\text{NEXT} = \text{NOW} + 2$ (start at 5)
   
   b. $y = 0.5 + 0.5x$ or $\text{NEXT} = \text{NOW} + 0.5$ (start at 0.5)
   
   c. $y = 2 - 3x$ or $\text{NEXT} = \text{NOW} - 3$ (start at 2)

4. a. $y = -3 + 1.5x$  

   b. $2x + 3y = 6$

   ![Graph](image1)

   ![Graph](image2)

   ![Graph](image3)

   ![Graph](image4)

   c. $y = -1.5 - 2x$
   
   d. $\text{NEXT} = \text{NOW} + 3$ (start at 5)

5. a. $y = 5 + 3x$ or $\text{NEXT} = \text{NOW} + 3$ (start at 5)
   
   b. $y = 4 - 2x$ or $\text{NEXT} = \text{NOW} - 2$ (start at 4)
Check Your Understanding 1.2, p. 11
1. $x = 13$
2. $x \leq 17.5$
3. $x = 22$
4. $x < \frac{24}{7.25} \approx 3.31$
5. $x = \frac{7}{8} - \frac{4}{5} \div \frac{2}{3} = 0.1125$

Check Your Understanding 1.3, p. 12
1. a. Equivalent    b. Not equivalent    c. Equivalent

2. In addition to the standard form, there are many possible alternative equivalent forms.
   a. $72 - 5x$
   b. $-17 + 24x$
   c. $11.5 - 4.4x$

3. a. $y = 4 - 3x$
   b. $y = 5 + 4x$
   c. $y = -\frac{7}{3} + \frac{8}{3}x$

Check Your Understanding 1.4, pp. 14–15
1. a. (1) $y = 15(0.6^x)$ and (2) $\text{NEXT} = 0.6 \cdot \text{NOW}$ (start at 15)
   b. (1) $y = 50(2^x)$ and (2) $\text{NEXT} = 2 \cdot \text{NOW}$ (start at 50)
   c. (1) $y = 5,000(1.015^x)$ and (2) $\text{NEXT} = 1.015 \cdot \text{NOW}$ (start at 5,000)

2. a. Rebound height as a function of bounce number
   b. Bacteria population as a function of time

![Graphs](15(0.6^x) and 50(2^x))
c. Credit card balance as a function of time

3. a. About 14.3 hours  
   b. 12 months

4. a. Graph (iii)  
   b. Graph (ii)  
   c. Graph (i)  
   d. Graph (iv)

Check Your Understanding 2.1, pp. 19–20

1. a. 4 | 5
   5 0 0 4 7
   6 0 2 5 5 5
   7 0 0 3 4 5 7 8
   8 0 4 5 5
   9 0 2 3 8
   4 | 5 = 45

   b. Unit Test Scores

   c. Unit Test Scores

   d. Histogram answers will vary depending on the width chosen for the bars. Here is an example with intervals of length 10 and x-axis scale from 40 to 100.
2. At the right is a scatterplot of the pairs of scores for each student (first test, second test) and the \( y = x \) line. Points below the line show students who did worse on the second test than the first. Points above the line show students who did better on the second test.

3. The minimum value is 1 and the maximum value is 26. There are two gaps in the data, one from 12 to 16 and the other from 19 to 26. The data value 26 is an outlier. The distribution is spread out and skewed to the right. The data value 1 has the greatest frequency and in general the frequencies decrease as the data values increase. The center of the distribution is about 4.

**Check Your Understanding 2.2, p. 21**

1. a. Test 1: Min = 45, \( Q_1 = 61 \), Median = 73, \( Q_3 = 84.5 \), Max = 98  
   Test 2: Min = 50, \( Q_1 = 64 \), Median = 75, \( Q_3 = 85 \), Max = 95  
   b. Test 1: 71.88; Test 2: 75.32  
   c. Test 1: Range = 53, interquartile range = 23.5, MAD = 11.96  
   Test 2: Range = 45, interquartile range = 21, MAD = 10.12

2. Although the various statistics for the two test score distributions are quite similar, it does appear that scores on Test 1 were a bit lower on average and had a bit greater variation from the mean.

**Check Your Understanding 2.3, p. 25**

1. To simulate a selection of 6 students from a school with a probability of 0.2 of any selected student being on the honor roll, you could generate random numbers from 1, 2, 3, 4, 5 and let 1s represent honor students (1 out of 5 is 20%). If you do many trials of this simulation, you should get a distribution that looks something like the data in this table:

<table>
<thead>
<tr>
<th>Number of Honor Students</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability Estimate</td>
<td>0.25</td>
<td>0.40</td>
<td>0.25</td>
<td>0.08</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2. a. This can be simulated by generating random numbers from 1 through 10 and letting 1, 2, 3, and 4 represent success in reaching a cable subscriber.
   
b. The average number of calls required to reach five subscribers should be about 20.

3. Command `int 20 rand` will generate integers from 0 to 19. The command `int 36 rand +1` will generate integers from 1 to 36.
4. To reach the desired accuracy you would need to run 400 trials of your simulation:
\[ 1 \div 0.05^2. \]

**Check Your Understanding 3.1, p. 28**

1. a. An octagonal pyramid has a base that is an octagon and eight triangular faces formed by joining the vertices of the base to the vertex of the pyramid.

   b. A square prism has base and top that are congruent squares and four rectangular sides formed by joining vertices of those squares in order. In a right prism, the lines joining base and top are perpendicular to the planes of both squares.

2. Drawing space-shapes:

   a. ![Square Pyramid](image)

   b. ![Cube Viewed from a Corner and Above](image)

3. The figure as drawn will not be rigid. Adding diagonal cross braces to two of the three rectangular faces will make the space-shape rigid.

4. a. A rectangular box that is $3 \times 4 \times 2$ centimeters can be done with many possible nets. Here is one possibility:

   b. Again, there are many possibilities of which the following diagram is one:
Check Your Understanding 3.2, p. 32

1. \( A = 3,200 \text{ in.}^2; V = 12,000 \text{ in.}^3 \)

2. \( A = 170\pi \text{ cm}^2; V = 300\pi \text{ cm}^3 \)

3. a. \( P = 50 \text{ cm} \)
   \( A = 144 \text{ cm}^2 \)
   b. \( P = 13 + \sqrt{39} \text{ m} \)
   \( A = 16 \text{ m}^2 \)
   c. \( P = 120 + 60\pi \text{ ft} \)
   \( A = 3,600 + 900\pi \text{ ft}^2 \)

4. a. 50
   b. 21.6

Check Your Understanding 3.3, p. 34

1. a. Reflection symmetry across the vertical line through the top of the A
   b. Reflection symmetry across vertical and horizontal lines
      Half-turn rotation symmetry about the center point of the cross bar
   c. Half-turn rotation symmetry about the center of the diagonal bar
   d. Reflection symmetry across horizontal and vertical lines
      Half-turn rotation symmetry about the point where the diagonals cross

2. Shown below are partial sketches indicating how the figures can be used to tile the plane.
   a. Triangles can be paired to make parallelograms.
   b. Quadrilaterals can be used as tiles. At any vertex you can assemble copies of each angle of the quadrilateral, adding to 360°
   c. 

3. If a figure is a regular \( n \)-gon, the measure in degrees of each interior angle is
   \[
   \frac{180(n - 2)}{n} = 180 - \left( \frac{360}{n} \right).
   \]

4. Reflection symmetry across a vertical line between the two identical figures;
   Reflection symmetry across a horizontal line through the middle of the strip;
   Half-turn rotation symmetry with center any point on the horizontal midline;
   Translation symmetry and glide reflection symmetry
Check Your Understanding 4.1, pp. 40–41

1. a. 

b. There is not an Euler circuit because vertices \( MA \) and \( CA \) have odd degree. There is an Euler path that begins at one of \( MA \) or \( CA \) and ends at the other. This means that it is not possible to cross all borders exactly once and end up in the same country in which you began. However it is possible to cross each border exactly once provided you start in either \( MA \) or \( CA \) and end in the other.

c. This graph can be colored with three colors: Color 1: \( MA, LA \); Color 2: \( MY, CA \); Color 3: \( TH, VN \).

2. a. This diagram can be traced without retracing any edge, but because it has two odd degree vertices, the path will not have the same beginning and ending points.

b. Because all vertices have even degree, this diagram can be traced starting and ending at the same point.

c. Since this diagram has more than two vertices with odd degree, it cannot be traced without retracing an edge.

3. a. 

b. The earliest finish time for the project is 184 days. The critical path is \( ST-S-L-C-F-P-FN \).
c–d. | Task | Earliest Start Time | Latest Start Time |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L</td>
<td>90 days</td>
<td>90 days</td>
</tr>
<tr>
<td>C</td>
<td>120 days</td>
<td>120 days</td>
</tr>
<tr>
<td>H</td>
<td>0</td>
<td>157 days</td>
</tr>
<tr>
<td>T</td>
<td>20 days</td>
<td>177 days</td>
</tr>
<tr>
<td>F</td>
<td>180 days</td>
<td>180 days</td>
</tr>
<tr>
<td>A</td>
<td>120 days</td>
<td>168 days</td>
</tr>
<tr>
<td>P</td>
<td>182 days</td>
<td>182 days</td>
</tr>
</tbody>
</table>

14. a. 

\[
\begin{bmatrix}
A & B & C & D & E & F \\
0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 2 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 2 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}
\]

b. Row A: 2; row B: 4; row C: 5; row D: 4; row E: 5; row F: 2

c. There is an Euler path because exactly two of the row sums are odd. One such path is \textit{CABCDBEFDECE}.
Solutions to Exercise Sets

Exercise Set 1, pp. 44–45

1. a. $y = 15 + 0.15x$
   b. $\text{NEXT} = \text{NOW} + 0.15$ (start at 15)

2. a. $21.75$
   b. 100 calls
   c. $15 + 0.15x \leq 50$
      \[ x \leq 233.33 \]
      Up to 233 calls can be made.

3. a. 166 calls
   b. Make tables of values for both plans and look for the $x$-values that give the same $y$-value for both functions.
   c. Draw graphs of both functions and trace to the point of intersection. The $x$-coordinate of the point of intersection is the answer to the question.
   d. Set the equations for the two functions equal to each other ($15 + 0.15x = 39.90$) and use symbolic reasoning.

4. slope: 0.15; $y$-intercept: 15
   The slope indicates that the charge per call is 15 cents and the $y$-intercept indicates that the charge without any calls is $15.

5. a. The new equation is $y = 10 + 0.15x$ or $\text{NEXT} = \text{NOW} + 0.15$ (start at 10). The graph will be parallel to the original one and 5 units below it. The $y$-intercept will be 10.
   b. The new equation is $y = 15 + 0.12x$ or $\text{NEXT} = \text{NOW} + 0.12$ (start at 15). The $y$-intercepts of the two graphs are the same. But the graph of this equation will rise more slowly because the slope is 0.12 instead of 0.15.

6. a. $y = 50(2^x)$; $\text{NEXT} = 2 \cdot \text{NOW}$ (Start at 50)
   b. c. 204,800 customers  
   d. 11 months
7. Length of brace: 30.23 inches; Area of painting: 425 square inches

8. a. 

Generally, the percentage voting has been decreasing since 1956, which had the highest percentage, 63%, of these years. There was a particularly large drop from 1968 to 1972. For some reason, 1992 was an exception to the downward trend.

b. Mean: 56.5%  The average percentage of the voting-age population that voted during this time period was 56.5%.

Median: 55.5%  In half of the elections, more than 55.5% of the voting-age population voted.

MAD: 4.08%  The average deviation of the data from the mean of 56.5% was 4.08%. We can say that in a typical year, the percentage voting was 56.5 ± 4.08.

c. From the data it looks like it probably happened between 1968 and 1972. From 1972 on, the percent of the population that voted was lower than during the years prior to 1972. In fact, the 26th Amendment to the constitution, which lowered the voting age to 18, was ratified on July 1, 1971.

9. a. Volume: 1,080 cubic units   b. Surface area: 954 square units

10. Four colors are needed to color this map.
Exercise Set 2, pp. 46–47

1. a. \( x = \frac{25}{7} \)  
   b. \( x = -7 \)  
   c. \( x = 6 \)  
   d. \( x = -30 \)  
   e. \( x = 5 \)  
   f. \( x = \frac{-14}{27} \) 

2. a. \( y = -3 + 1.5x \)  
   b. \( y = 2 + 1.5x \)  
   c. \( y = 5 + \frac{3}{2}x \) 

3. Responses may vary. 
   a. \( 5(2x) - 5(7) \)  
      \( 10x - 35 \)  
   b. \( 4 + 3x - 9 \)  
      \( 3x - 5 \)  
   c. \( 7x + 5x - 15 \)  
      \( 12x - 15 \)  
   d. \( 4x - 2 + 11x \)  
      \( 15x - 2 \)  
   e. \( 2x + 15 - 18x \)  
      \( 15 - 16x \)  
   f. \( 2x - 6 - 12x + 20 \)  
      \( -10x + 14 \) 

4. a. \( y = -109 \)  
   b. \( y = 131 \)  
   c. \( y = 5 \)  
   d. \( y = 11 \)  
   e. \( y = 23 \)  
   f. \( y = 3 \) 

5. \( P = 2L + 2W \) or \( P = 2(L + W) \) The distributive property tells us these forms are equivalent. 

6. \( \sqrt{136} = 11.66 \) 

7. a. \$150 \)  
   b. \( Q_1 = $100; Q_3 = $175 \)  
   c. The minimum price of these phones is approximately \$45\) and the maximum price is \$200\). The median price is \$150\). Half of the phones cost at least \$150\). The most expensive 25% of the phones cost at least \$175\). The prices for the cheapest 25% range from \$45\) to \$100\). The distribution is skewed left showing that most phones are in the more expensive categories. 

8. a. 

<table>
<thead>
<tr>
<th>Folds</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regions</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

   b. \( y = 2^x \) 

   \( NEXT = 2 \cdot NOW \) (start at 1) 

   c. 256 regions 

9. a. (i) There are two triangles so the sum of the angle measures is \(2(180^\circ) = 360^\circ\). 
   
   (ii) There are three triangles so the sum of the angle measures is \(3(180^\circ) = 540^\circ\). 

136 solutions
(iii) There are four triangles so the sum of the angle measures is $4(180^\circ) = 720^\circ$.

(iv) There are $n - 2$ triangles so the sum of the angle measures is $(n - 2)180^\circ$.

b. 10 angles

10. a.

<table>
<thead>
<tr>
<th>Bottom Side Length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bars</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
</tr>
</tbody>
</table>

b–c. Equations for Parts b and c will vary.

$$B = 3 + 4(n - 1) = -1 + 4n \text{ or } B = 3n + n - 1 = 4n - 1$$

Each of these describes the same situation and are equivalent by using the distributive and commutative laws, or by combining like terms.

**Exercise Set 3, pp. 48–49**

1. a. $D = 800 - 120t$

   b. $800 - 120t \leq 100$ has solution $t \geq 5.83$. So it will take about 5.83 hours (about 5 hours and 50 minutes) to get within 100 kilometers of the destination.

2. The graphs will look approximately like this:

3. a. $y = 3 + 2x$

   b. $y = -1 - \frac{1}{3}x$

4. a. 2000: 272.4 million; 2001: about 274.9 million; 2002: about 277.4 million

   b. $y = 270(1.009^x)$, $x$ years after 1999; NEXT = 1.009 • NOW (start at 270 in 1999).

   c. About the year 2011

5. a. Reflection symmetry across each line from a vertex perpendicular to the opposite side; rotation symmetry of 120° and 240°

   b. Reflection symmetry across the two diagonals and lines that are the perpendicular bisectors of the pairs of opposite sides; rotation symmetry of 90°, 180°, and 270°
c. Reflection symmetry across lines that are the perpendicular bisectors of the pairs of opposite sides; rotation symmetry of 180°

d. Reflection symmetry across each line from a vertex perpendicular to the opposite side; rotation symmetry of 72°, 144°, 216°, and 288°

6. a. The box can be sketched in a variety of ways. One view from slightly above a corner is

```
10
20
15
```

b. 1,300 square inches   c. 3,000 cubic inches   d. 25 inches

e. Depending on its width, the object must have a length less than 26.9 inches.

7. a. \( V = 300h \)   b. 15 inches

8. a. American League: Min = 24, \( Q_1 = 34.5 \), Median = 38.5, \( Q_3 = 46.5 \), Max = 54
National League: Min = 15, \( Q_1 = 35.5 \), Median = 40, \( Q_3 = 47.5 \), Max = 56

b. The National League has the greater record (56), the greater 25th percentile point (35.5), the greater median (40), and the greater 75th percentile point (47.5). The American League does not have a minimum as low as the National League.

9. a. A task-prerequisite digraph might look something like this:

```
S 0
\( L \) 30 \( F \) 10
\( P \) 45 \( A \) 5 \( I \) 1 \( E \) 0
```

b. The critical path would be SPAIE and earliest finish time is 51 days.

10. a. \( x < 14 \)   b. \( x > 7 \)   c. \( x \geq 8 \)

d. \( x > -3 \)   e. \( x \leq 30 \)   f. \( x > \frac{7}{3} \)

Exercise Set 4, pp. 50–51

1. a. \( y = 4 + 2x \) or \( NEXT = NOW + 2 \) (start at 4)
b. \( y = 6(2^x) \) or \( NEXT = 2 \cdot NOW \) (start at 6)

c. \( y = 8 - 3x \) or \( NEXT = NOW - 3 \) (start at 8)

d. \( y = 32(0.5^x) \) or \( NEXT = 0.5 \cdot NOW \) (start at 32)

2. a. Two forms are: \( y = -30 + 0.20x \) and \( y = 0.60x - 0.40x - 30 \)

b. \(-30 + 0.20x = 100 \) when \( x = 650 \) cans.

c. To solve by table or graph, enter \( y_1 = 0.20x - 30 \) in the “\( y = \)” list and trace tables or graphs looking for a value of \( x \) that gives \( y = 100 \). To solve by reasoning with the symbolic equation, your work might look like this:

\[-30 + 0.20x = 100 \] is equivalent to \( 0.20x = 130 \) or \( x = 130 ÷ 0.20 = 650. \)

3. a. Perimeter: approximately 25.7 units; Area: 18 square units

b. Perimeter: approximately 40.8 units; Area: 70 square units

c. Perimeter: \( 20 + 10\pi \approx 51.4 \) units; Area: \( 50\pi \approx 157.1 \) square units

d. Perimeter: \( 12 + 9\pi \approx 40.3 \) units; Area: \( 27\pi \approx 84.8 \) square units

4. a. \( x = -\frac{76}{11} \)  
b. \( x = -2 \)  
c. \( x = 2.75 \)

d. \( x = \frac{47}{12} \)  
e. \( x = \frac{35}{4} = 8.75 \)  
f. \( x = -\frac{34}{7} \)

5. a. \( y = \frac{15}{4} - \frac{7}{4}x \)  
b. \( y = x \)  
c. \( y = -3 + \frac{4}{7}x \)

6. a. \(-54 \)  
b. \(-26 \)  
c. \(625 \)

d. \(-23 \)  
e. \(32 \)  
f. \(-243 \)

7. a. 

b. \( x = 2\pi \approx 6.28 \)

c. Volume: \( 10\pi \approx 31.4 \) cubic units

d. Surface area: \( 20 + 14\pi \approx 64 \) square units

8. a. \( (2, 5.5) \)  
b. \( (3, 7) \)  
c. \( (7, 13) \)  
d. \( (0, 2.5) \)

9. a. Based on the given results, one would expect 97 heads-up coins.

b. Based on the experimental results, one would expect \( 5 \cdot 97 \) or 485 heads-up coins.
Exercise Set 5, pp. 52–53

1. There are several possible equivalent rules for this relationship. One would be \( F = 2 + 1.5(2L - 1) \); another would be \( F = 0.5 + 3L \); another would be \( F = 2 + 3(L - 0.5) \) where \( L \) is distance in miles. There are still others that are algebraically equivalent to the given examples. All assume that distance is at least one-half mile and that fare is charged as a continuous variable. If the fare is charged on a "half-mile or any part thereof" basis, it is not possible to give a simple linear expression for the fare charge rule. You will need to round up to the nearest half-mile and then calculate the fare.

2. a. \( x = -3 \)  
   b. \( x = -19 \)  
   c. \( x = 5 \)  
   d. \( x = 2 \)  
   e. \( x = \frac{10}{17} \)  
   f. \( x = \frac{5}{4} \)

3. a. \( y = \frac{5}{3} + \frac{2}{3}x \)  
   b. \( y = -2 - x \)  
   c. \( y = -1.5x + 2.5 \)

4. a. Reflection symmetry through vertical and horizontal lines and half-turn symmetry  
   b. Reflection symmetry through a vertical line  
   c. Half-turn symmetry  
   d. Reflection symmetry through a vertical line

5. a. 10 cm  
   b. 112\pi

6. a. About 434 feet  
   b. About 2.9 minutes (2 minutes and 54 seconds)

7. a. 10 feet, 5 feet, and 2.5 feet  
   b. \( h = 20(0.5^n) \)  
   c. Rebound height = 0.5 \cdot \text{Previous height}  
   d. [Graph showing height vs. number of bounces]

140 Solutions
8. a. Median high temperature is 84.5°; median low temperature is 65°.

b. The data will all fall below the $y = x$ line. This is reasonable since the high temperature will be greater than the low temperature. With the exception of one outlier, there appears to be quite a tight linear pattern to the data. The outlier is not Anchorage, Alaska; it is Helena, Montana, which has a very large gap between high and low temperatures in July.

c. A linear model that relates low $y$ to high $x$ in these cities would have equation something like $y = 5.02 + 0.71x$.

d. Your predictions will depend on your equation in Part c. Using the model in Part c, one gets these predictions: Barrow, Alaska 37°; this compares quite favorably to the actual low of 34°; Atlantic City, New Jersey 65°, right on the average low temperature there. The predicted temperatures are very close to the actual temperatures because the data clusters closely about a line.

9. $b = 120°; a = c = d = 60°$

10. a. Any simulation with two possible outcomes—one three times as likely as the other—will work. For example, use random numbers from 0, 1, 2, and 3 and count 0 as a correct guess and 1, 2, and 3 as errors. A spinner divided into sectors of 90° and 270° will work also. You could also flip two coins and count HH as success and HT, TH, and TT as failures.

b. Simulation results will vary.

c. To estimate the probability of passing, find the percentage of trials for which there were seven or more correct guesses. (This is likely to be zero.)

d. The estimate could be improved by running a greater number of trials.
Exercise Set 6, pp. 54–55

1. \( a. \ y = 4 - 4.5x \quad b. \ y = \frac{1}{4} + \frac{7}{4}x \quad c. \ y = 0.8x + 10.4 \)

2. Regular pentagons do not tessellate because each interior angle has a measure of \(108^\circ\), and \(108^\circ\) is not a factor of \(360^\circ\).

3. \( a. \ x = -\frac{61}{6} \quad b. \ x = 5 \quad c. \ x = 14 \)
   \( d. \ x = 5 \quad e. \ x = 10 \quad f. \ x = 4 \)

4. 

5. \( a. \ A = 130; B = 64 \quad b. \ 225\pi \text{ square miles or about 707 square miles} \)

6. \( \begin{array}{|c|c|c|c|c|c|} \hline \text{Month (m)} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline \text{Listeners (L)} & 5,000 & 5,500 & 6,050 & 6,655 & 7,321 & 8,053 \\ \hline \end{array} \)

   \( b. \ \text{NEXT} = 1.1 \cdot \text{NOW} \text{ or } \text{NEXT} = \text{NOW} + 0.1 \cdot \text{NOW} \text{ (start at 5,000)} \)

   \( L = 5,000(1.1^m) \)

   \( c. \ \text{After about 7.3 months} \)

7. Equivalent forms may vary. Here are some:
   \( a. \ 10x + 8 - 7x \quad b. \ -42x + 21 + 41x \quad -x + 21 \)
   \( c. \ 18 - 6x - 12 \quad d. \ 2x - 10 + 20 - 10x \quad -8x + 10 \)
   \( e. \ 10x + 30 - x + 8 \quad f. \ 90 - 2x + 75 - 10x \quad -12x + 165 \)

8. \( a. \ P = 20 + 0.1x \text{ where } x \text{ is the dollar value of food sold.} \)

   \( b. \ P = 55 \)

   \( c. \$550 \text{ worth of food and drink} \)

142 solutions
9. a. Five number summaries for the stands are

Stand 1:  Min = 110; Q₁ = 125; Median = 180; Q₃ = 200; Max = 250
Stand 2:  Min = 90; Q₁ = 110; Median = 160; Q₃ = 200; Max = 240

b. The sales at the two stands have approximately the same variation. However, Stand 1 seems to sell slightly more than Stand 2, because the minimum, lower quartile, and median of Stand 1 are all higher than those of Stand 2.

10. a. (i) Does not have an Euler path.
   (ii) Does not have an Euler circuit.

b. (i) Any Euler path must start at either T or X and end at the other. One possible circuit is TSYXTVWX.
   (ii) Does not have an Euler circuit.

Exercise Set 7, pp. 56–57

1. a. \( x = -14 \)  
   b. \( x = 10 \)  
   c. \( x = 0.5 \)  
   d. no solution  
   e. \( x = 3 \)  
   f. \( x = 3 \)

2. a. Median = 78.5 indicates that half the months had more than 78.5 mm of rainfall and half the months had less; Mean \( \approx 76.4 \) indicates that the total rainfall was the same as if 76.4 mm had fallen each month.

b. A histogram with x-axis scale of 10 millimeters will look like this:

c. The MAD is 10.65. This says that a typical month will have rainfall that is more than 10 millimeters away from the mean of 76.4 millimeters.
3. a. Reflection symmetry across a vertical line and a horizontal line
   Half-turn symmetry about the intersection of the two lines of symmetry

   b. $4\pi + 28$ square meters
   c. $4\pi + 14$ meters

4. There are many possible equivalent forms of this equation. Three equivalent equations are $y = 3x + 6 - 5$, $y = 3x + 1$, and $-3x + y = 1$.

5. a. $y = -2 + 2x$  b. $y = -89 - 13x$  c. $y = 8$  d. $y = \frac{1}{13}x + 10$

6. The graphs will look like this on unit grids.

   a–d. 
   e–f. 

7. a. You could simulate this event in many ways: One would be to roll a fair die and count even results as a selection of a boy and odd results as a selection of a girl. Collect sets of six such tosses and count the number of times all six represent girls. You could flip a coin and count heads as a girl and tails as a boy. Flip in groups of six and see how often all turn up heads.
   
   b. Simulations will vary. Probability estimates will depend on simulation results.
   
   c. You should find the total number of girls in all 25 of your trials and divide that by 25 to get the average number of girls in a group of six students chosen at random.
   
   d. You are assuming that it is equally likely that a girl or a boy volunteers.

8. a. One sketch from the front and slightly above the cylinder will look like this:

   b. $2,000\pi$ cubic feet  c. $20\pi$ feet  d. $600\pi$ square feet
9. a. $x < 8.6$  
   b. $x \geq 9$  
   c. $x \geq -30$  
   d. $x < -0.3$

10. a. 

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value ($1,000$)</td>
<td>25</td>
<td>20</td>
<td>16</td>
<td>12.8</td>
</tr>
</tbody>
</table>

b. $\text{NEXT} = 0.8 \cdot \text{NOW}$ or $\text{NEXT} = \text{NOW} - 0.2 \cdot \text{NOW}$ (start at 25)

$$y = 25(0.8^n)$$ where $n$ is time in years and $y$ is trade-in value in thousands of dollars.

c. After about 5 years, or after about 4.2 years if we assume continuous depreciation

Exercise Set 8, pp. 58–59

1. $x = -7.5$

2. a. $x = 5$  
   b. $x = -2.5$  
   c. $x = 11$  
   d. $x = 6$  
   e. $x = -1$

3. a. The box is drawn like the one at the right. It is viewed from above and at an angle to one corner.

   b. $V = 312$ cubic inches

   c. $A = 334$ square inches

4. 

5. a. $C = 5 + 0.75t$  
   b. 26 tickets  
   c. $5 + 0.75t \leq 25$  
   d. $t \leq 26.7$

6. 79

7. $A = 360$ square units

8. a. Simulations will vary. Using the random numbers 0, 1, 2, and 3, let 0 represent being assigned to Mrs. Parks and 1, 2, and 3 represent being assigned to some other teacher.
b–c. Tables and histograms will vary.
d. To get the probability, divide the number of trials that indicated all three students getting Mrs. Parks by 25.

9. \( x = 25.7^\circ, 2x = 51.4^\circ, 4x = 102.9^\circ \)

10. The critical path is \( SBDEGF \) indicating an earliest finish time of 19 time units.

**Exercise Set 9, pp. 60–61**

1. Slope: \(-1\); y-intercept: 5; \( y = 5 - x \)

2. a. \( y = 5 + 0.5x \)  
   b. \( y = 21 - 3x \)  
   c. \( y = 6 \)

3. a. \( x < \frac{17}{5} \)  
   b. \( x \leq -9 \)  
   c. \( x < 6 \)

4. \( y = 9 + 2x \)

5. The given pattern has bilateral symmetry about vertical lines through the center of any A, half-turn symmetry about any point midway between two of the As, translation symmetry in a left or right direction, and glide reflection symmetry with glide right or left and reflection about a horizontal line.

6. a. \( C = 450 + 10n \)  
   b. $3,750  
   c. \( 15n = 450 + 10n \)

   d. The solution to the break-even equation is \( n = 90 \); it would be located at the intersection of the graphs of \( y_1 = 15n \) and \( y_2 = 450 + 10n \).

7. a. The plots over time for tuition and room and board charges will look like this:

Both graphs show a steady increase as time passes. Linear equations match the patterns quite well. For tuition, the line of best fit has the equation \( y = 2,034 + 133x \), meaning tuition increased at an annual rate of $133 per year. For room and board charges, the line of best fit has the equation \( y = 3,303 + 129x \), meaning that those expenses increased at an annual rate of $129 per year. In both cases, we have used \( x \) to represent "years since 1989."
b. A scatterplot of the \((tuition, room/board)\) data pairs looks like this:

![Scatterplot](image)

It shows a fairly strong linear connection between the two variables.

c. The common measures of center and spread in a data distribution don’t tell very meaningful information in this case because what is of interest is the pattern of change in the situation over time. Mean, median, and MAD are used to describe a fixed population on the basis of a sample.

8. a. Volume: \(16\sqrt{33} \approx 91.9\) cubic units

![Volume](image)

b. The surface area of the solid is \(88 + 12\sqrt{33} \approx 156.9\) square units.

![Surface Area](image)

9. The probability that the number of even results on 500 tosses will be between 240 and 260 is greater than the probability that the number of even results on 50 tosses will be between 24 and 26 due to the Law of Large Numbers.

10. a. \(-17.8\)°C  
    b. 59°F
Exercise Set 10, pp. 62–63

1. a. $x = -5$ b. $x = 2$ c. $x = 3$ or $x = -3$
   d. $x = -18$ e. $x = 4$ f. $x = 2$

2. $200 - 50\pi$ square units

3. a. $NEXT = NOW - 2$ (start at 150) and $y = 150 - 2x$, for $y$ in million gallons and $x$ in days.
   b. The graph will look roughly like this:
   ![Graph Image]
   c. In a table of $(day, \, reservoir \, water)$ data, each increase of one day would have a corresponding decrease of 2 (million) in supply.

4. You can use graphs, tables, or matrices, set the two equations equal to each other or use linear combinations to solve this system of equations. The solution is $x = \frac{12}{7}$, $y = \frac{20}{7}$.

5. The distributions with given characteristics can have many different specific shapes, but will have the following general traits:
   a. The mean and median are approximately equal when the distribution graph is symmetric about a vertical line through that common value. For example:
   ![Graph Image]
   b. The mean is greater than the median when the distribution is skewed to the right (longer tail in that direction). For example:
   ![Graph Image]
   c. The mean is less than the median when the distribution is skewed to the left. For example:
6. a. \( y = 29, y = -35 \) \hspace{1cm} b. \( x = 4, x = \frac{3}{4} \)

7. Perimeter: 32 cm; Area: 48 square centimeters

8. a. 200 is the initial dose in milligrams; 0.8 indicates that the decay rate is 20% of the active medication each hour.

b. \( M \approx 91 \) This indicates that after 3.5 hours there will be only 91 milligrams of medication active in the blood.

c. There will be only 20 milligrams of the medication active in the blood after approximately 10.3 hours or about 10 hours and 18 minutes.

d. The graph will look roughly like this:

9. a. Surface area: 2,100 cm\(^2\) \hspace{1cm} b. Volume: 3,720 cm\(^3\)

10. a. Edges could connect species that are not "friendly."

b. Colors would represent different natural settings.

c. The minimum number of colors would indicate the number of separate enclosures required.

**Exercise Set 11, pp. 64–65**

1. a. The booking fee is $100 and the hourly rate is $50.

b. $250 \hspace{1cm} c. 6 hours

2. a. \( x > 8 \) \hspace{1cm} b. \( x \leq 2.75 \) \hspace{1cm} c. \( x < 3 \)

\hspace{1cm} d. \( x \leq 2.5 \) \hspace{1cm} e. \( x > 16 \) \hspace{1cm} f. \( -3 < x < 3 \)

3. a. \((0.5, 1.5)\) \hspace{1cm} b. \( \sqrt{12.5} \approx 3.54 \)

\hspace{1cm} c. Any line that contains the center is a symmetry line. Two such lines are \( x = 0.5 \) and \( y = 1.5 \).
4. a. \[ A + B = \begin{bmatrix} 3 & 0 \\ 10 & 0 \end{bmatrix} \] and \[ A - B = \begin{bmatrix} -5 & 4 \\ -2 & 0 \end{bmatrix} \] b. \[ 5A = \begin{bmatrix} -5 & 10 \\ 20 & 0 \end{bmatrix} \] c. \[ AB = \begin{bmatrix} 8 & 2 \\ 16 & -8 \end{bmatrix} \] and \[ BA = \begin{bmatrix} -12 & 8 \\ -6 & 12 \end{bmatrix} \]

5. a. Mean: 80; MAD: \[ \frac{2}{3} \] b. To make a box plot of the data you need a five number summary:

   Minimum = 47, Q_1 = 72, Median = 84, Q_3 = 95, Maximum = 100
   The box plot will look like this:

   ![Box Plot]

   c. Her median score is in the mid-80s and 75% of her scores are above 70. Other observations might be made as well, including one score of 100%.

6. Perimeter: 40 cm; Area: 60 square centimeters

7. a. 

    ![Diagram]

   b. The critical tasks are L, I, and R, and the earliest finish time is 11 days.

8. a. \[ y = 5x - 6 \]  
   b. \[ y = \frac{5}{2} x - 3 \]  
   c. \[ y = -0.5x + 12 \]  
   d. \[ y = -6 \]  
   e. \[ x = 8 \]  

9. a. \[ NEXT = NOW + 5 \] (start at -6)  
   b. \[ NEXT = NOW + 2.5 \] (start at -3)  
   c. \[ NEXT = NOW - 0.5 \] (start at 12)

150 SOLUTIONS
10.  
<table>
<thead>
<tr>
<th></th>
<th>A'</th>
<th>B'</th>
<th>C'</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>(4, −1)</td>
<td>(0, 3)</td>
<td>(−4, −2)</td>
</tr>
<tr>
<td>b.</td>
<td>(1, 4)</td>
<td>(−3, 0)</td>
<td>(2, −4)</td>
</tr>
<tr>
<td>c.</td>
<td>(−4, −1)</td>
<td>(0, 3)</td>
<td>(4, −2)</td>
</tr>
<tr>
<td>d.</td>
<td>(7, 6)</td>
<td>(3, 2)</td>
<td>(−1, 7)</td>
</tr>
<tr>
<td>e.</td>
<td>(8, 2)</td>
<td>(0, −6)</td>
<td>(−8, 4)</td>
</tr>
</tbody>
</table>

**Exercise Set 12, pp. 66–67**

1. a. The first service will charge $26 for 60 minutes of air time and $30 for 100 minutes. The second service will charge $40 for 60 minutes and $60 for 100 minutes.
   
   b. The first service will give 300 minutes of air time for $50. The second service will give 80 minutes for $50.
   
   c. 25 minutes
   
   d. The first service is the better deal if you plan to use the phone for more than 25 minutes per month.

2. a. $x = 5$  
    b. $x = −1$  
    c. $x = ±3$
   
    d. $x = 17$  
    e. $x = ±5$  
    f. $x = \frac{1}{5}$

3. a. slope: $−\frac{2}{3}$; y-intercept: 5  
    b. slope: $−\frac{3}{2}$; y-intercept: 3
   
    c. slope: 4; y-intercept: $−5$  
    d. slope: 3.4; y-intercept: 2

4. a. $AB = \sqrt{164} = 2 \sqrt{41} ≈ 12.81$ cm  
    b. $AC = 24$ m

5. 17.1 points per game

6. a. $x^7y^6$  
    b. $x^{12}y^8$  
    c. $9x^8$

7. There are many possible correct answers to these questions. We give only one example for each.
   
    a. An isosceles triangle that is not also equilateral.
8. a. A graph model with stations as vertices and edges connecting stations within 500 miles of each other will look something like this:

![Graph model diagram]

b. The graph will need three colors, so three frequencies will be needed: Stations C, B, and G can be assigned one frequency; Stations F, A, and E can be assigned a different frequency; and Station D, a third frequency.

9. $1,370.09

10. Scatterplot (i) suggests the strongest linear correlation, Scatterplot (iv) the weakest.

**Exercise Set 13, pp. 68–69**

1. a. Basic cable is $32.50 per month; each pay channel is $6.50 per month.

b. The special package would be a good deal if you wanted five or more pay channels.

2. a. \( y = -3x + 7 \)  
b. \( y = -\frac{1}{2}x + \frac{5}{2} \)  
c. \( y = 10x - 4 \)

3. The most efficient use of fencing in a rectangular shape would be a square that is 25 feet on each side. Its area would be only 625 square feet. So Sam cannot enclose a rectangular region with an area of 630 square feet. A circle with a circumference of 100 feet has a radius of \( \frac{50}{\pi} \) and an area equal to \( \pi \left( \frac{50}{\pi} \right)^2 \approx 796 \) square feet. So Sam has more than enough fence for a circular region.

4. a. This experiment can be simulated by using five random digits and letting one of them indicate a correct guess.
2.36 questions

\[
\frac{4}{50} = 0.08 = 8\%
\]

5. a. 

b. 

c. 

d. 

6. a. Has an Euler path, but not a circuit. For example, \(ACBAB\).

b. Has an Euler circuit. For example, \(ABCBCBA\).

c. Has neither an Euler circuit nor path. Eulerize by adding edges from \(B\) to \(C\) and from \(A\) to \(D\), for example. Then an Euler circuit would be \(ABCD\).

d. Has neither an Euler circuit nor path. Eulerize by adding edges from \(B\) to \(G\), \(E\) to \(F\), and \(D\) to \(I\), for example. Then it will be possible to construct an Euler circuit. For example, \(ABCGBJHJHDB\).

7. \[18 + 9 \sqrt{2} \approx 30.7\]

8. a. \[
\begin{bmatrix}
-5 & 7 & -2 \\
3 & 8 & -1
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
5 & 7 & 2 \\
3 & 8 & -1
\end{bmatrix}
\]

c. \[
\begin{bmatrix}
-3 & 8 & 1 \\
-5 & 7 & -2
\end{bmatrix}
\]

d. \[
\begin{bmatrix}
5 & 7 & 2 \\
3 & 8 & -1
\end{bmatrix}
\]

9. a. Sales were never greater than $50,000.

b. Maximum sales of $46,500 occurred in 1983.

10. \(x = 50\)

**Exercise Set 14, pp. 70–71**

1. a. \(NEXT = 1.05 \cdot NOW\) (start at 28,000) and \(NEXT = NOW + 1,500\) (start at 28,000)

b. \(s = 28,000(1.05)^t\) and \(s = 28,000 + 1,500t\)
c. Percent increase table and graph:

<table>
<thead>
<tr>
<th>Year</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28,000</td>
</tr>
<tr>
<td>1</td>
<td>29,400</td>
</tr>
<tr>
<td>2</td>
<td>30,870</td>
</tr>
<tr>
<td>3</td>
<td>32,414</td>
</tr>
<tr>
<td>4</td>
<td>34,034</td>
</tr>
<tr>
<td>5</td>
<td>35,736</td>
</tr>
<tr>
<td>6</td>
<td>37,523</td>
</tr>
<tr>
<td>7</td>
<td>39,399</td>
</tr>
<tr>
<td>8</td>
<td>41,369</td>
</tr>
<tr>
<td>9</td>
<td>43,437</td>
</tr>
<tr>
<td>10</td>
<td>45,609</td>
</tr>
</tbody>
</table>

Fixed increase table and graph:

<table>
<thead>
<tr>
<th>Year</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28,000</td>
</tr>
<tr>
<td>1</td>
<td>29,500</td>
</tr>
<tr>
<td>2</td>
<td>31,000</td>
</tr>
<tr>
<td>3</td>
<td>32,500</td>
</tr>
<tr>
<td>4</td>
<td>34,000</td>
</tr>
<tr>
<td>5</td>
<td>35,500</td>
</tr>
<tr>
<td>6</td>
<td>37,000</td>
</tr>
<tr>
<td>7</td>
<td>38,500</td>
</tr>
<tr>
<td>8</td>
<td>40,000</td>
</tr>
<tr>
<td>9</td>
<td>41,500</td>
</tr>
<tr>
<td>10</td>
<td>43,000</td>
</tr>
</tbody>
</table>

d. Neil should consider how long he will stay at the job. For the first four years, his salary will be greater with the fixed amount increase. However, if he stays at least seven years, his total earnings will be greater with the percent increase.

2. a. \((x, y) = (1, -1)\)  
   b. \((x, y) = (6, 0)\)  
   c. \((x, y) = (-4, 3)\)

3. \(y = -\frac{6}{8}x + 1\)

4. a. The regression equation is \(y = 13.3 - 1.49x\).
b. The slope of $-1.49$ says that for each increase of $\$1$ in charge, one might expect a drop of about one-and-a-half hours in hired time for baby-sitting.

c. The largest residual is for the data point $(0, 15)$. The residual is $1.7$.

d. Approximately $5.85$ hours

5. a. Possible nets for a triangular prism:

![Diagram of a triangular prism]

b. Approximately $37.9$ square inches of cardboard would be needed. The volume is approximately $7.79$ cubic inches.

6. a. $x = 6$  

b. $x = 21.5$  

c. $x = 5$ or $x = 1$  

d. $x = 5$  

e. $x = 4$

7. a. 

![Diagram of a graph]

b. $M^2 = \begin{bmatrix} 3 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

c. $M^3 = \begin{bmatrix} 2 & 3 & 3 & 4 \\ 4 & 1 & 2 & 2 \\ 3 & 1 & 2 & 2 \\ 3 & 0 & 1 & 1 \end{bmatrix}$

There are $2$ paths of length $2$ from $B$ to $D$: $BCD$ and $BAD$.

There are $4$ paths of length $3$ from $A$ to $D$: $ABCD$, $ABAD$, $ACAD$, and $ADAD$.

8. $x = -1, y = \frac{1}{3}$
9. The scales on all axes are one.

10. Approximately 89.7 inches

Exercise Set 15, pp. 72–73

1. Cheese: 10; Pepperoni: 13

2. a. slope: \(-\frac{1}{2}\); \(y\)-intercept: \(\frac{3}{2}\) b. slope: \(\frac{2}{3}\); \(y\)-intercept: \(-2\)

   c. slope: \(-\frac{1}{3}\); \(y\)-intercept: 2 d. slope: \(-4.6\); \(y\)-intercept: 7

3. \(x = 120^\circ, \, w = 42^\circ, \, y = 60^\circ, \, z = 78^\circ\)

4. a. \(y = 11 + 5x\) b. \(y = -1 - x\) c. \(y = -19 + 5x\)
   
   d. \(x = -12\) e. \(y = 15\) f. \(y = \frac{-1}{5}x - \frac{17}{5}\)

5. a. Mean \(\approx 26.3\); Median = 25. These two measures are quite close to each other, so one could argue that there is no particular preference. However, the one week in which she earned $60 probably raised the mean significantly by itself.

   Without that week, the mean becomes 22.9 and the median 24.

   b. The MAD is approximately 13 which says that on a typical week, Angela’s earnings varied about $13 from the average of $26.30.

   c. The interquartile range is \(Q_3 - Q_1 = 38 - 13\) or 25. This says that in half the weeks, Angela earned between $13 and $38.

6. There are several ways to write each expression. We give results in what is something of a standard algebraic style.

   a. \(2.3^2a^2b^3 = 5.29a^2b^3\) b. \(2^4a^6 = 16a^6\) c. \(a^2bc^{-4} = \frac{a^2b}{c^4}\)

   d. \(5^22^3x^8 = 200x^8\) e. \(x^{10}y^{15}\) f. \(\frac{2y^2}{x^3}\)

7. a. 144 b. \(-9\) c. 26 d. \(-210\)
8. a. \[6x - y = 31\]
\[4x + 3y = 17\]
b. The graphs of these lines intersect at a point because the lines have unequal slopes.
c–d. Regardless of method used, the solution is \((x, y) = (5, -1)\).

9. a. \[x \leq 6\]
b. \[x \geq -32\]
c. \[x > 4 \text{ or } x < -4\]

10. a. \((x, y) = (0, 10)\)
b. \[A'B'C'D' = \begin{bmatrix} 0 & 3 & 3 & 0 \\ -4 & -4 & -10 & -10 \end{bmatrix}\]
c. \[A'B'C'D' = \begin{bmatrix} 4 & 4 & 10 & 10 \\ 0 & -3 & -3 & 0 \end{bmatrix}\]
d. 288 square units

Exercise Set 16, pp. 74–75

1. a. 0.25
   
   b. There are many possible simulations. For example, consider 10 numbers chosen randomly from 0, 1, 2, or 3. Let 0, 1, or 2 be a made free throw. Count the number of 0s, 1s, or 2s.
   
   c. Answer will depend on the results of your simulation. Find the mean of the number of free throws made in each simulation of 10 free throws.

2. a. Equations are: \(1) \ y = \frac{1}{2}x\) and \(2) \ y = -\frac{3}{2}x + 3\).  
   
   b. The intersection point is \((x, y) = (1.5, 0.75)\).

3. a. When \(t = 2\) seconds
   
   b. 68 feet
   
   c. Approximately 4.06 seconds

4. a. Every square is a special kind of rectangle in which all sides are the same length; all rectangles (including squares) have four right angles and opposite sides parallel and the same length.
b. Every rhombus is a special kind of parallelogram in which all sides are the same length; all parallelograms have opposite sides parallel and congruent and opposite angles the same size; diagonals of all parallelograms bisect each other, but in a rhombus they are also perpendicular to each other.

![Rhombus and Non-Rhombic Parallelogram](image)

c. The most common definition of trapezoid is a quadrilateral with exactly two parallel sides. A parallelogram has two pairs of parallel sides.

![Trapezoid and Parallelogram](image)

5. a. There are many possible nets for this box.

![Box Net](image)

b. The box made from the net above will require 612 square inches of cardboard.

c. Wrapping would require more paper because there would be pieces folded under and overlapping in a typical wrapping.

6. a. $\begin{bmatrix} 1 & 2 \\ 9 & 1 \end{bmatrix}$

b. $BD$ is not possible because the number of columns in $B$ is not equal to the number of rows in $D$.

c. $\begin{bmatrix} 8 & -7 & -4 \\ 2 & -10 & 21 \end{bmatrix}$

d. $\begin{bmatrix} 16 \\ 18 \end{bmatrix}$

e. $CA$ is not possible because the number of rows in $A$ is not equal to the number of columns in $C$.

f. $\begin{bmatrix} 9 & 1 \\ 5 & 14 \end{bmatrix}$

158 SOLUTIONS
7. a. \(5(2x + 3) = 10x + 15\); student did not apply distributive property correctly.
   b. \(3x - 7x = -4x\); subtraction is not commutative.
   c. \(8 - 2(3x - 4) = 16 - 6x\); the \(-2\) was not correctly distributed.
   d. \(2 + 3(x + 1) = 5 + 3x\); student apparently operated left to right, adding 2 and 3 before distributing over \((x + 1)\).

8. The dogs are vertices and edges connect dogs that cannot be walked together. Colors represent walks. Three colors are necessary as indicated in the graph model at the right. For example, Cookie and Einstein have color 1 because they can be walked together; Duke and Bruno and Flower have color 2 because they can be walked together. Alf has color 3 and will be walked by himself.

9. a. 4,400,000 cm\(^2\) b. 440 m\(^2\) c. 2,720,000,000 cm\(^3\) d. 2,720 m\(^3\)

10. The coordinates are \((1, 2.5)\).

**Exercise Set 17, pp. 76–77**

1. a. Because most of the points are above the line \(y = x\), the plot shows that performance generally improved from 1992 to 1996.
   b. For 1992 data: Range = 56; interquartile range = 16.5.
      For 1996 data: Range = 58; interquartile range = 16.5.
      These data suggest that the variation of scores in 1996 is really pretty much the same as in 1992.

2. a. Rooms cost $75 per night and meals cost $15 apiece.
   b. Assuming that all meals and rooms are charged at the same rate as the weekend getaway prices, the cost would be \(75 + 4(15) = 135\).

3. a. \(x = 144\) b. \(x = 12\) or \(x = -12\) c. \(x = 0.00694\)
   d. \(x = 130.25\) e. \(x = 3\) f. \(x = 11.413\) or \(x = -11.413\)

4. Surface area: 512 cm\(^2\)

SOLUTIONS
5. a. b. c. d. e. f.

6. a. \( y = 13 - 4x \)
d. \( y = -11 \)

b. \( y = -28 + 4x \)
e. \( x = -10 \)

c. \( y = \frac{27}{4} - \frac{1}{4}x \)

7. a. 1
d. \( 16a^{12}b^4 \)

b. 7
e. \( x^6y^2 \)
f. \( 3x^4y^3 \)

c. \( n^8 \)

8. a. If matrix \( B = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \) then the product \( AB \) will give income on each of the three days.

b. The product \( AB = \begin{bmatrix} 597 \\ 789 \\ 596 \end{bmatrix} \) tells that the show took in $597 on Friday, $789 on Saturday, and $596 on Sunday.

9. a.

<table>
<thead>
<tr>
<th>Task</th>
<th>Time</th>
<th>Prerequisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>none</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>none</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>A, B</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
<td>D, E</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>I</td>
<td>7</td>
<td>G, H</td>
</tr>
</tbody>
</table>
b. The critical path is $SBDGIF$.

c. The EFT is 19.

10. Responses will vary. Rotations can be done in any order. A rotation of $90^\circ$ followed by a rotation of $180^\circ$ will be the same as a rotation of $270^\circ$, as will a rotation of $180^\circ$ followed by a rotation of $90^\circ$. Any two translations can be combined in either order to produce the same result. A translation followed by a reflection will usually not give the same result as a reflection followed by a translation. For example, the image of $(1, 0)$ translated by magnitudes 3 and 5 and then reflected across the $y$-axis gives image point $(-4, 5)$. If the reflection is followed by the translation, the image is $(2, 5)$. The image points are not the same.

**Exercise Set 18, pp. 78–79**

1. a. The scatterplot is shown below using 1985 as $t = 0$.

![Scatterplot](image)

b. The linear regression line has equation $y = 85.3 + 26.98x$.
This leads to a prediction of 490 computers per 1,000 people in 2000 (year 15).

c. The estimate from the Almanac is greater than the estimate obtained using the regression line. They may have taken other things into consideration when making their estimate.

2. a. $x = -1$
   b. $x = 512$
   c. $x = -2$
   d. $x = 0$
   e. $x = 6$
   f. $x = 81$

3. a. The figure is a non-square rectangle because its sides are parallel to the $x$- and $y$-axes of the coordinate system, so adjacent sides are perpendicular. It is not a square because not all sides are the same length.
   b. 56 square units

4. $(1.2)^4$ is approximately 2, so four enlargements are necessary.
5. a. Predictions will vary, of course.

b. The actual change in travel time is an increase of 3.14 hours along the slower route (8.36 hours versus 11.50 hours) and an increase of only 2.12 hours on the faster route (7.08 hours versus 9.2 hours).

6. a. slope: \(\frac{5}{2}\), y-intercept: \(-\frac{7}{2}\)
   b. slope: \(-\frac{2}{3}\), y-intercept: 4
   c. slope: 2.5, y-intercept: \(-11\)
   d. slope: \(-2.1\), y-intercept: 4

7. Surface area: 204 cm\(^2\); Volume: 120 cm\(^3\)

8. a. Has only rotation symmetry about the center of the image through angles of 60°, 120°, and 360°.
   b. Has only line reflection symmetry across a vertical line through the center of the image.
   c. There are five lines of symmetry corresponding to lines through the centers of two opposite spokes. There are five more lines of symmetry located halfway between any two spokes and going through the center. The image has rotation symmetry of 36°, 72°, 108°, 144°, 180°, 216°, 252°, 288°, and 324° about the center of the wheel.
   d. This figure looks very much like one with bilateral or line reflection symmetry. However, if you look closely at the way the snakes cross the staff, reflection will not give a perfect match of the two sides.

9. a. The graph does not have an Euler circuit because two vertices \((B\) and \(E)\) have an odd number of adjacent edges. It does have an Euler path because there are exactly two odd vertices.
   b. One Euler path is \(BCDEABE\).
   c. There are four paths of length 3 from \(A\) to \(B\): \(ABAB, AEAB, ABEB, ABCB\).

10. Under Option One, Jesse will pay a total of $1,440 over the three years of the loan from her parents. Option Two is not described completely. If one assumes that she lets the value of the outstanding loan increase at an annual compounding rate of 14% with no repayment until the end, then she will have to repay about $1,777.85 at the end. If one assumes the more normal practice of making equal monthly payments so that the loan balance is $0 at the end of three years, the calculation requires
\[ NEXT = NOW + \left( \frac{0.14}{12} \right) NOW \] 

- payment (starting at 1,200). This is equivalent to 

\[ NEXT = 1.011666 \times NOW \] 

- payment. Equal monthly payments of about $41 will pay this off to $0 in 36 months. If one assumes the unusual simple interest, the interest on $1,200 at 14% for three years will be $504, so she would have to repay $1,704 at the end of that time period. In any case, borrowing the money from her parents is the best choice.

**Exercise Set 19, pp. 80–81**

1. a. The law of large numbers suggests that as the accuracy of estimates for a probability by simulation improves as sample size increases. Thus, one would find it more likely to find numbers of boys as low as 40% (2 of 5) in the smaller hospital than in the larger hospital (8 of 20).

b. Simulations could involve flipping a fair coin 20 times (counting the number of heads as the number of boys) for the large hospital, or 5 times for the small hospital.

c. Results of an actual simulation will vary, but they should support the general principle stated in Part a.

2. a. \((x, y) = (1, 1)\) 

b. \((x, y) = \left(\frac{3}{4}, \frac{37}{4}\right)\) 

c. \((x, y) = (1.5, -1.5)\)

3. Radius of the base: 7 cm; Surface area: \(182\pi\) cm\(^2\)

4. 

<table>
<thead>
<tr>
<th></th>
<th>Sm</th>
<th>Med</th>
<th>Lg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep Dish</td>
<td>492</td>
<td>489</td>
<td>318</td>
</tr>
<tr>
<td>Thin Crust</td>
<td>306</td>
<td>360</td>
<td>303</td>
</tr>
</tbody>
</table>

5. a. Daily use: \(C = 2n\); Season pass: \(C = 20 + 0.5n\)

b. The total cost will be the same for the two plans only when \(n = 13.3\), so they are never exactly equal for any single number of days.

c. If you expect to use the pool more than 13 times in a summer, the season pass will save you money. Otherwise, pay each time you swim.

6. a. Perimeter: \(\sqrt{45} + \sqrt{40} + 5 \approx 18.03\) units

b. Area: 15 square units

7. a. \(m \leq 6\) 

b. \(y \geq 9\) 

c. \(r < -5\) 

d. \(c > 2\)
8. a. \( x = \frac{5}{3} \) or \( x = -\frac{5}{3} \)  
   b. \( x = -5 \) or \( x = 3 \)  
   c. \( x = -0.5 \) or \( x = -6 \) 

9. a. \( y = -1 - \frac{1}{3}x \)  
   b. \( y = -11 + 3x \)  
   c. \( y = \frac{19}{3} - \frac{1}{3}x \) 

10. a. Same size and shape; 90° counterclockwise rotation 
    b. Neither same size nor same shape; shear 
    c. Same shape; size transformation with magnitude \( \frac{1}{4} \) 
    d. Same size and shape; 180° rotation 

**Exercise Set 20, pp. 82–83** 

1. a. Option 1: \( C = 180 + 5n \); Option 2: \( 75 + 8n \) 
   b. Option 1: \( C = 380 \); Option 2: \( C = 395 \) 
   c. 35 guests 
   d. If you plan on more than 35 guests, choose Option 1; otherwise use Option 2. 

2. a. \( x = -29 \)  
   b. \( x = -2 \)  
   c. \( x = 4 \) 
   d. \( x = \frac{3}{7} \)  
   e. \( x = \sqrt{11} \) or \( x = -\sqrt{11} \)  
   f. \( x = 3.5 \) or \( x = -1 \) 

3. Plot 1 seems to have a correlation coefficient of 0.9; Plot 2 has a correlation coefficient of 0.5; Plot 3 has a correlation coefficient of \(-0.8\). 

4. a. No, the best model of the given condition is \( y = a(0.8^x) \) and \( 0.8^{0.5} \neq 0.90 \) (although it is very close). The drug decays at a decreasing rate, so it decays more quickly in the first half-hour than it does in the second. 
   b. About three hours, since \( 0.8^3 = 0.512 \) 
   c. After approximately 7 hours 

5. a. 9  
   b. \( 4\sqrt{2} \)  
   c. \( 10\sqrt{2} \) 
   d. \( 5\sqrt{6} \)  
   e. \( 30\sqrt{7} \) 

6. The perimeter of the base will double, the surface area will be four times as large, and the volume will be eight times the original volume. 

7. 6 weeks
8. The earliest finish time will be 19 since two critical paths require this time. The earliest start time for C is 7, for D is 4, for E is 12, and for G is 16. The latest start times for A and B are 0, for C is 7, for D is 4, for E is 12, and for G is 16.

9. The quadrilateral is a trapezoid with \( AB \) parallel to \( CD \). The slope of \( AB \) is \( \frac{-2}{3} \), the slope of \( BC \) is 4, the slope of \( CD \) is \( \frac{-2}{3} \), and the slope of \( DA \) is \( \frac{1}{2} \). The lengths are \( AB = \sqrt{13}, BC = \sqrt{17}, CD = \sqrt{52}, DA = \sqrt{20} \).

10. a. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]

b. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]

c. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]

d. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]

e. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]

f. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]

g. 

\[ y \\
\begin{array}{c}
\text{\( x \)}
\end{array} 
\]
Solutions to Practice Sets for Standardized Tests

<table>
<thead>
<tr>
<th>Practice Set 1, pp. 86–88</th>
<th>Practice Set 2, pp. 89–92</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (b)</td>
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<td>2. (b)</td>
<td>2. (b)</td>
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<tr>
<td>3. (a)</td>
<td>3. (c)</td>
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<tr>
<td>4. (b)</td>
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<tr>
<td>5. (c)</td>
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<tr>
<td>6. (d)</td>
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</tr>
<tr>
<td>7. (c)</td>
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<tr>
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<tr>
<td>9. (a)</td>
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<th>Practice Set 3, pp. 93–96</th>
<th>Practice Set 4, pp. 97–100</th>
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<tbody>
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<td>1. (c)</td>
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<tr>
<td>Practice Set 5, pp. 101–104</td>
<td>Practice Set 6, pp. 105–108</td>
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<th>Practice Set 7, pp. 109–112</th>
<th>Practice Set 8, pp. 113–116</th>
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<td>10. (c)</td>
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</table>
Practice Set 9, pp. 117–120

1. (c)  
2. (a)  
3. (b)  
4. (e)  
5. (c)  
6. (e)  
7. (b)  
8. (d)  
9. (c)  
10. (d)

Practice Set 10, pp. 121–124

1. (e)  
2. (b)  
3. (d)  
4. (d)  
5. (a)  
6. (c)  
7. (d)  
8. (d)  
9. (a)  
10. (c)