Chapter 6

Anticipation Guide
Polynomial Functions and Inequalities

Before you begin Chapter 6

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

After you complete Chapter 6

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

### STEP 1

<table>
<thead>
<tr>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The monomial (6m^5/7p^2) has a degree of 5.</td>
<td>D</td>
</tr>
<tr>
<td>2. To multiply powers of the same variable, add the exponents.</td>
<td>A</td>
</tr>
<tr>
<td>3. ((12x^2 - 3x + 4) - (8x^2 + 4x - 4)) is equal to (4x^2 - 7x + 8.)</td>
<td>A</td>
</tr>
<tr>
<td>4. ((6x + 2)(7x - 1)) is equal to (42x^2 - 2.)</td>
<td>D</td>
</tr>
<tr>
<td>5. The leading coefficient of a polynomial is the coefficient of the first term.</td>
<td>D</td>
</tr>
<tr>
<td>6. The graph of any polynomial is a parabola.</td>
<td>D</td>
</tr>
<tr>
<td>7. The graph of a polynomial of even degree will approach either (\infty) or (-\infty) as (x \to \infty) and as (x \to -\infty.)</td>
<td>A</td>
</tr>
<tr>
<td>8. If the graph of a polynomial function has an (x)-intercept, then the polynomial has at least one real solution.</td>
<td>A</td>
</tr>
<tr>
<td>9. (a^2 - 2ab + b^2) is a perfect square trinomial.</td>
<td>D</td>
</tr>
<tr>
<td>10. If (f(a) = 0,) then (a) is a factor of the polynomial (f(x)).</td>
<td>A</td>
</tr>
<tr>
<td>11. Every polynomial equation with degree greater than 0 has at least one root in the set of complex numbers.</td>
<td>A</td>
</tr>
<tr>
<td>12. To find all the rational zeros of a polynomial function, all the possible zeros must be tested using synthetic substitution.</td>
<td>D</td>
</tr>
</tbody>
</table>

### STEP 2

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

### Lesson 6-1

Lesson Reading Guide
Properties of Exponents

Get Ready for the Lesson

Read the introduction to Lesson 6-1 in your textbook.

Your textbook gives the U.S. public debt as an example from economics that involves large numbers that are difficult to work with when written in standard notation. Give an example from science that involves very large numbers and one that involves very small numbers. Sample answer: distances between Earth and the stars, sizes of molecules and atoms

Read the Lesson

1. Tell whether each expression is a monomial or not a monomial.
   - a. \(3a^2\) monomial
   - b. \(y^2 + 5y - 6\) not a monomial
   - c. \(-7a^3\) monomial
   - d. \(\frac{1}{a}\) not a monomial

2. Complete the following definitions of a negative exponent and a zero exponent.
   - For any real number \(a \neq 0\) and any integer \(n\), \(a^{-n} = \frac{1}{a^n}\).
   - For any real number \(a \neq 0\), \(a^0 = 1\).

3. Name the property or properties of exponents that you would use to simplify each expression. (Do not actually simplify)
   - a. \(m^n\) quotient of powers
   - b. \(y^n \cdot y^3\) product of powers
   - c. \((y^2)^{1/3}\) power of a product and power of a power

Remember What You Learned

4. When writing a number in scientific notation, some students have trouble remembering when to use positive exponents and when to use negative ones. What is an easy way to remember this? Sample answer: Use a positive exponent if the number is 10 or greater. Use a negative number if the number is less than 1 but greater than zero.
6-1 Study Guide and Intervention

Properties of Exponents

Multiply and Divide Monomials  Negative exponents are a way of expressing the multiplicative inverse of a number.

Negative Exponents  \( a^{-n} = \frac{1}{a^n} \) and \( \frac{1}{a^{-n}} = a^n \) for any real number \( a \neq 0 \) and any integer \( n \).

When you simplify an expression, you rewrite it without parentheses or negative exponents. The following properties are useful when simplifying expressions.

Product of Powers  \( a^m \cdot a^n = a^{m+n} \) for any real number \( a \neq 0 \) and integers \( m \) and \( n \).

Quotient of Powers  \( \frac{a^m}{a^n} = a^{m-n} \) for any real number \( a \neq 0 \) and integers \( m \) and \( n \).

Properties of Powers  For \( a, b \) real numbers and \( m, n \) integers:

- \((a^m)^n = a^{mn}\)
- \((ab)^n = a^n b^n\)
- \((a/b)^n = a^n / b^n\)
- \(a^0 = 1, \quad a^{-n} = \frac{1}{a^n}\)
- \(a^m \cdot b^m = (ab)^m\)

Example 1  Simplify. Assume that no variable equals 0.

a. \((3m^3n^{-2})(-5mn)^2\)
   \(= 3m^3n^{-2} \cdot 25m^2n^2\)
   \(= 75m^5n^{-2} \cdot 2\)
   \(= 75m^5\)

b. \((-m^{12})/(-2m^{12})\)
   \(= -m^{12}/2m^{12}\)
   \(= -m^0\)
   \(= -1\)

Example 2  Evaluate. Express the result in scientific notation.

\(3.5 \times 10^4 \times 5 \times 10^{-5}\)
\(= 1.75 \times 10^0\)
\(= 1.75\)

Example 3  Evaluate. Express the result in scientific notation.

\(6.4 \times 10^{-8} \times 1.675 \times 10^4\)
\(= 1.024 \times 10^{-5}\)

Example 4  Evaluate. Express the result in scientific notation.

\(2.43 \times 10^4 \times 9.9 \times 10^{-4}\)
\(= 2.41 \times 10^5\)

Exercise 1  Express each number in scientific notation.

1. \(1.23 \times 10^7\)
   \(= 1.23 \times 10^7\)

2. \(2.000099\)
   \(= 2.000099 \times 10^0\)

3. \(4.86 \times 10^6\)
   \(= 4.86 \times 10^6\)

4. \(5.525 \times 10^6\)
   \(= 5.525 \times 10^6\)

5. \(5.25 \times 10^8\)
   \(= 5.25 \times 10^8\)

6. \(2.21 \times 10^7\)
   \(= 2.21 \times 10^7\)

7. \(9 \times 10^5\)
   \(= 9 \times 10^5\)

8. \(3.6 \times 10^4\)
   \(= 3.6 \times 10^4\)

9. \(1.4 \times 10^3\)
   \(= 1.4 \times 10^3\)

10. \(1.2 \times 10^2\)
   \(= 1.2 \times 10^2\)

11. \(1.1 \times 10^1\)
   \(= 1.1 \times 10^1\)

12. \(1.0 \times 10^0\)
   \(= 1.0 \times 10^0\)

13. \(9.9 \times 10^{-1}\)
   \(= 9.9 \times 10^{-1}\)

14. \(8.8 \times 10^{-2}\)
   \(= 8.8 \times 10^{-2}\)

15. \(7.7 \times 10^{-3}\)
   \(= 7.7 \times 10^{-3}\)

16. \(6.6 \times 10^{-4}\)
   \(= 6.6 \times 10^{-4}\)

17. \(5.5 \times 10^{-5}\)
   \(= 5.5 \times 10^{-5}\)

18. \(4.4 \times 10^{-6}\)
   \(= 4.4 \times 10^{-6}\)

19. ASTRONOMY  Pluto is 3.948 million miles from the sun. Write this number in scientific notation. \(3.674 \times 10^7\) miles

20. CHEMISTRY  The boiling point of the metal tungsten is 10,200°F. Write this temperature in scientific notation. \(1.022 \times 10^4\)

21. BIOLOGY  The human body contains 0.0004% iodine by weight. How many pounds of iodine are there in a 129-pound teenager? Express your answer in scientific notation. \(4.8 \times 10^{-3}\) lb
6-1
Skills Practice
Properties of Exponents

Simplify. Assume that no variable equals 0.

1. \(5^4 \cdot 5^3\) \(\times\) 2
2. \(e^5 \cdot e^2 \cdot e^3\) \(\cdot\) 2
3. \(a^{-4} \cdot a^{-3}\) \(\cdot\) \(\frac{1}{a^7}\)
4. \(x^5 \cdot x^{-4} \cdot x^2\)
5. \((g^6)^3\) \(\cdot\) \(g^6\)
6. \((3a)^3\) \(\cdot\) \(27u^3\)
7. \((-x)^3\) \(\cdot\) \(x^4\)
8. \(-5z^2\) \(\cdot\) \(-40z^3\)
9. \((-3d)^4\) \(\cdot\) \(-8d^6\)
10. \((-2^2)^3\) \(\cdot\) \(-8d^6\)
11. \((-p^2)^3\) \(\cdot\) \(-r^21\)
12. \(\frac{a^3}{a^6}\) \(\cdot\) \(a^3\)
13. \(\frac{b^0}{k}\) \(\cdot\) \(\frac{1}{k}\)
14. \((-3^3)^2\) \(\cdot\) \(-27t^4g^3\)
15. \((2h^2)^3\) \(\cdot\) \(64x^2y^2\)
16. \(-2gh(g^3h^5)\) \(\cdot\) \(-2g^4h^6\)
17. \(10x^3y^3\) \(\cdot\) \(100x^2y^{11}\)
18. \(\frac{24bc^2}{3w^2}\) \(\cdot\) \(\frac{8z^2}{w^3}\)
19. \(-6x^3ab^3\) \(\cdot\) \(36x^3b\)
20. \(-10p^3r^4\) \(\cdot\) \(-5p^2r^7\) \(\cdot\) \(2q^2\) \(\cdot\) \(p^3\)

Express each number in scientific notation.

21. 53,000 \(5.3 \times 10^4\)
22. 896,000 \(8.96 \times 10^5\)
23. 241,000,000 \(2.41 \times 10^8\)
24. 0.000000805 \(8.05 \times 10^{-6}\)

Evaluate. Express the result in scientific notation.

25. \(4 \times 10^3 \times 1.6 \times 10^{-6}\) \(=\) \(6.4 \times 10^{-3}\)
26. \(9.8 \times 10^7\) \(\div\) \(1.5 \times 10^{-3}\) \(=\) \(6.4 \times 10^{10}\)

Chapter 6
Glencoe Algebra 2

NAME ______________________________ DATE ____________ PERIOD _____
1. **MASS** Joseph operates a forklift. He is able to lift $4.72 \times 10^3$ kilograms with the forklift. There are $10^3$ grams in 1 kilogram. How many grams is $4.72 \times 10^3$ kilograms? Express your answer in scientific notation. $4.72 \times 10^6$ g

2. **DENSITY** The density of an object is equal to its mass divided by its volume. A dumbbell has a mass of $9 \times 10^3$ grams and a volume of $1.2 \times 10^{-3}$ cubic centimeters. What is the density of the dumbbell? $7.5$ g/cm\(^3\)

3. **THE EARTH** Earth’s diameter is approximately $1.276 \times 10^7$ kilometers. The surface area of a sphere can be found using the formula $SA = 4\pi r^2$. What is the approximate surface area of Earth? Express your answer in scientific notation. $5.112 \times 10^8$ km\(^2\)

4. **POPULATION** As of November 2004, the United States Census Bureau estimated the population of the United States as 297,881,499 and the world population as 6,479,541,872. Write the ratio of the United States population to the world population in scientific notation. $4.59 \times 10^{-2}$

**GLASS TABLES** For Exercises 5 and 6, use the following information.

Evan builds rectangular glass coffee tables. The area $A$ of the tabletop is given by $A = lw$, where $l$ is the length of the table and $w$ is the width of the table.

5. The larger the table surface, the thicker the glass must be. For this reason, the cost of the table glass is proportional to $A^2$. What is $A^2$ in terms of $l$ and $w$? Express your answer without using parentheses. $l^2w^2$

6. The cost per unit length is proportional to $\frac{A}{l}$. Express the cost per unit length in terms of $l$ and $w$. Express your answer in simplest form. $\frac{l}{w}$

**Properties of Exponents**

The rules about powers and exponents are usually given with letters such as $m$, $n$, and $k$ to represent exponents. For example, one rule states that $a^m \cdot a^n = a^{m+n}$.

In practice, such exponents are handled as algebraic expressions and the rules of algebra apply.

**Example 1** Simplify $2a^3(a^2 + 1 + a^4n)$.

$2a^3(a^2 + 1 + a^4n) = 2a^2 + a^6 + 2a^2 + a^{4n}$ Use the Distributive Law.

$= 2a^2 + a^6 + 2a^2 + a^{4n}$ Simplify the exponent $2a^{n+1}$ as $a + 3$.

It is important always to collect like terms only.

**Example 2** Simplify $(a^n + b^m)^2$.

$(a^n + b^m)^2 = (a^n + b^m)(a^n + b^m)$ The second and third terms are like terms.

$= a^n \cdot a^n + a^n \cdot b^m + a^n \cdot b^m + b^m \cdot b^m$ The second and third terms are like terms.

$= a^{2n} + 2a^nb^m + b^{2m}$

**Exercises**

Simplify each expression by performing the indicated operations.

1. $2^22^n = 2^{n+2}$

2. $(a^3)^3 = a^{9n}$

3. $(4n^2)^3 = 4^3n^{6k}$

4. $(a^2a^m) = a^{3m}$

5. $(-2x^n)^3 = -8x^{3n}$

6. $(-3x^3)^2 = 9x^{6k}$

7. $(a^3)(a^2) = a^{2k}$

8. $(-2d^n)^3 = -8d^{5n}$

9. $(a^2b^3)(b^3) = a^2 + nb^3$

10. $(a^2b^3)(a^2b^3) = x^n + y^n + m$

11. $\frac{x^n}{y^n}$

12. $\frac{12x^3}{4x^2}$

13. $(ab^2 - a^2b + 3a^2 + 4b^3) = 3a^2b^2 + 4ab^2 + 2 - 3a^2 + 2b - 4a^2b^2 + 1$

14. $ab^2(2ab^2 + 1 + 4ab^2 + 6b^n + 1) = 2a^3b^{n+1} + 4a^2b^{n+2} + 6ab^{n+3}$
Lesson Reading Guide

Operations with Polynomials

Get Ready for the Lesson
Read the introduction to Lesson 6-2 in your textbook.
Suppose that Shenequa decides to enroll in a five-year engineering program rather than a four-year program. Using the model given in your textbook, how could she describe the tuition for the fifth year of her program? 23,310 (1 + \( r \))^4.

Read the Lesson
1. State whether each expression is a polynomial.
   a. \( 2x^7 + r^5 - r \) polynomial
   b. \( 3x - \frac{1}{2}r \) not a polynomial
   c. \( 4 - 9x^2 \) polynomial
   d. \( x^0 \) polynomial
2. State the degree of each polynomial.
   a. \( 4x^4 - 2x^3 + 1 \) degree 4
   b. \( 3x - 3x^2 \) degree 2
   c. \( 6x^2 + 2x - 4 \) degree 3
   d. \( 5x + 3 \) degree 1
3. State whether or not each polynomial is in simplified form.
   a. \( 3x^2 + 3x \) yes
   b. \( 3x - 11x \) no
   c. \( 6m^2 + m^3 \) no
   d. \( r^3 - 2r \) yes

Remember What You Learned
4. You can always find the degree of a polynomial by remembering to look at the monomial with the greatest degree. Write two polynomials of degree 3, two polynomials of degree 2, and two polynomials of degree 1.
   Sample answer: \( x^3, 3x^2 + 2; x^2, 2x^2 + 1; x, x - 5 \)

6-2 Study Guide and Intervention
Operations with Polynomials

Add and Subtract Polynomials

To add or subtract polynomials, perform the indicated operations and combine like terms.

Example 1
Simplify \(-6x + 183 - 5x^2 - 14r^2 + 8x - 6z^2\).

\[-6x + 183 - 5x^2 - 14r^2 + 8x - 6z^2\]
\[= (183 - 14r^2) + (-6z^2 - 6x) + (-5x^2 - 6z^2)\]

Combine like terms.

Example 2
Simplify \(4x^2 + 12xy - 7x^2 - 20xy + 5x^2 + 8x^2y\).

\[4x^2 + 12xy - 7x^2 - 20xy + 5x^2 + 8x^2y\]
\[= (-7x^2 + 8x^2y) + (4x^2 + 5x^2) + 12xy - 20xy\]

Combine like terms.

Exercises
Simplify.

1. \(6x^2 - 3x + 2 - (4x^2 + x - 3)\)
   \[2x^2 - 4x + 5\]
2. \((7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)\)
   \[3y^2 + 18xy - 8x^2\]
3. \((-6m^2 - 6m) - (6m + 4m^2)\)
   \[-8m^2 - 12m\]
4. \(27x^2 - 5y^2 + 12x^2 - 14x^2\)
   \[13x^2 + 7y^2\]
5. \((13p^2 + 11pq - 6q^2) - (15p^2 - 3pq + 4q^2)\)
   \[3p^2 + 14pq - 10q^2\]
6. \(15y^2 - 12xy + 3p^2 - 15p^2 + 14k^2\)
   \[5p^2 + 2k^2\]
7. \(8m^2 - 7n^2 - (n^2 - 12m^2)\)
   \[20m^2 - 8n^2\]
8. \(14bc + 6b - 4c + 8b + 8c = 8c\)
   \[14b + 22bc - 12c\]
9. \(6x^2 + 11x^2 - 3n^2 - 7m^2 + 15m^2 - 9x^2\)
   \[24x^2 - 5r^2\]
10. \(-9xy + 11x^2 - 14y^2 - (6d^2 - 5x^2 - 3x^2)\)
    \[14x^2 - 4y^2 - 20y^2\]
11. \((12xy - 8x + 3y) + (15x - 7x - 8y)\)
    \[7x + 4xy - 4y\]
12. \(10.8b^2 - 5.7b + 7.2 = (2.9b^2 - 4.6b - 3.1)\)
    \[7.9b^2 - 1.1b + 10.3\]
13. \(5bc - 9b^2 - 6c^2 + 4c^2 - h^2 + 50c\)
    \[-10b^2 + 8bc - 2c^2\]
14. \(11xy + 4x^2 + 6xy + 3x^2 + 5xy - 10x^2\)
    \[x^2 + xy + 7y^2\]
15. \(\frac{1}{8}x^2 - \frac{3}{8}y^2 + \frac{1}{2}x + \frac{1}{2}y^2 - \frac{3}{8}x - \frac{3}{8}y\)
    \[\frac{3}{8}y^2 - \frac{3}{8}y + \frac{3}{4}y^2\]
16. \(24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p\)
    \[9p^3 - 2p^2 - 4p\]
6-2 Study Guide and Intervention (continued)

Operations with Polynomials

Multiply Polynomials You use the distributive property when you multiply polynomials. When multiplying binomials, the FOIL pattern is helpful.

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. \(x^2 + 2x + 2\) yes; 2
2. \(\frac{b^2}{c^4}\) no
3. \(8x + \frac{1}{2}\) yes; 2

Simplify.

4. \((g + 5) + (3g + 7)\)
5. \((5d + 5) - (d + 1)\)
6. \((x^2 - 3x - 3) + (2x^2 + 7x - 2)\)
7. \((-2x^2 - 3y^2) - (-2x^2 - 3y^2 + 8)\)
8. \((2y^2 - 6r + 2) - (-r^2 - 3r + 5)\)
9. \((8x^2 - 3y^2) - (3x^2 - 6xy - 4y^2)\)

Exercises

Find each product.

1. \(2(3x^2 - 5)\)
2. \(2(7x^2 - 2a^2 - p^2)\)
3. \(-5y^2 + 2y - 3\)
4. \(4x - 2x + 7\)
5. \(a^2 - 2a + 4\)
6. \(-x^2 + 8\)
7. \((4x - 3)(x + 8)\)
8. \((4x^2 - 7x + 2)\)
9. \((3x + 2)(4x - 6)\)
10. \((3x^2 + 5x) - 2\)
11. \((2x + 6)(3x - 7)\)
12. \((x + 5)(x - 3)\)
13. \((2x + 7)^2\)
14. \((2x + 9)^2\)
15. \((-10x^2 + 5d^2)\)
16. \((2y^2 + 3y^4)\)
17. \((-4m^2) - 2m^3n^2 - 7m^2n^3\)
18. \((x^2 + 2c^2)\)
19. \((c + 7)(c - 3)\)
20. \((z^2 - 3)\)
21. \((2x^2 + 3y^2)\)
22. \((x + 5) + (3x - 5)\)
23. \((6x^2 - 2x + 4)\)
24. \((2x - 5)\)
25. \((x^2 - 3x + 2)\)
26. \((x^2 + 3x - 5)\)
27. \((2x^2 - 2x)\)
28. \((x + 2y)\)
29. \((x^2 - 2x + 4)\)
30. \((x^2 - 2x - 2)\)
31. \((x^2 + 3x - 4)\)
32. \((x^2 + 2x + 1)\)
33. \((x^2 - 2x + 1)\)
34. \((x^2 - 3x + 2)\)
35. \((x^2 + 3x - 4)\)
36. \((x^2 - 2x + 1)\)
Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. \(5x^3 + 2xy^2 + 6xy\) yes; 3
2. \(-4x^2 - a^2z^3\) no
3. \(\frac{12m^4}{n^2} - 1\) yes; 4
4. \(25x^2z - x\sqrt{78}\) yes; 4
5. \(6c^2 + e - 1\) no
6. \(\frac{5}{r} + 6\) no

Simplify.

7. \(3x^2 + 1 + (8n^2 - 8)\) \(11n^2 - 7\)
8. \((6y - 11m^2) - (4 + 7m^2)\) \(-18w^2 - 6w - 4\)
9. \((-6n - 13a^2) + (3e + 9p^2)\) \(-9n - 4n^2\)
10. \((8x^2 - 3a) + (4x^2 + 5x - 3)\) \(4x^2 - 8x + 3\)
11. \((5m^2 - 2mp - 6p^2) - (-3m^2 + 5mp + p^2)\) \(8m^2 - 7mp - 7p^2\)
12. \((2x^2 + xy + y^2) + (-3x^2 + 4xy + 3y^2)\) \(-x^2 + 3xy + 4y^2\)
13. \((5 - 7) + (3x^2 + 3x^2 + 12)\) \(2x^2 + 8x + 5\)
14. \((x - 4) - (6 + 3x^2 - 4x)\) \(-3x^2 + 5x - 10\)
15. \((-9y^3 - 7w)\) \(-9y^2 + 63w\)
16. \((-6a^2b^2 + aw^3) - (-9^n y^n)\) \(-6a^2b^2 + 6aw^3\)
17. \(3x^2y + xy - 2x^2\) \(2x^2 + 2x^3y - 4x^2y^2\)
18. \((5x^2y^2 + 3x^3y^2 + 9xy^2)\) \(5x^2w^2 - 15a^2w^2 + 45a^2w^3\)
19. \(-3b^3d^2 + 3a^2b^2d^2\)
20. \((4x^2 - 6x^2 + 2)\) \(5x^4 - 10x^2y^2 + 25y^2\)
21. \((x^2 + 5x^2 + 9)\) \(-x^2 - 2xy + 3y^2\)
22. \((4x^2 - 3x^2y + 3)\) \(4x^2 - 9\)
23. \((3x - 2)(2x^2 + 4)\) \(x^2 - 2xy + y^2\)
24. \((x + y)(2x^2 + 3)\)
25. \((x^2 - 2x - 3)\)
26. \((5x^2 - 16w^2)\)
27. \((w^2 + 2w^2 - 2w^2 + 4w)\)
28. \((a + b)^2 - (a + b)(ab^2)\) \(a^2b^2 + 2ab^3c\)
29. \(\text{BANKING} \) \(-0.022x + 1590\)
30. \(\text{GEOMETRY} \) \(2x^2 + 4x - 3x^3\)
Enrichment

Lesson Reading Guide

Dividing Polynomials

Get Ready for the Lesson

Read the introduction to Lesson 6-3 in your textbook.

Using the division symbol (\(\div\)), write the division problem that you would use to answer the question asked in the introduction. (Do not actually divide.)

\[
[(140x^2 + 60x) \div (10x - 14)] \div 2
\]

Read the Lesson

1. a. Explain in words how to divide a polynomial by a monomial. \textit{Divide each term of the polynomial by the monomial.}

b. If you divide a trinomial by a monomial and get a polynomial, what kind of polynomial will the quotient be? \textit{Trinomial}

2. Look at the following division example that uses the division algorithm for polynomials.

\[
x - 4 \div 2x^2 - 8x^2 + 4x - 16
\]

Which of the following is the correct way to write the quotient? \textbf{C}

\begin{itemize}
  \item A. \(2x + 4\)
  \item B. \(x - 4\)
  \item C. \(2x + \frac{23}{x - 4}\)
  \item D. \(\frac{23}{x - 4}\)
\end{itemize}

3. If you use synthetic division to divide \(x^3 + 3x^2 - 5x - 8\) by \(x - 2\), the division will look like this:

\[
\begin{array}{c|cccc}
1 & 3 & -5 & -8 \\
\hline
1 & 6 & 2 & 10 \\
\hline & 0 & 0 & 0 & 0
\end{array}
\]

Which of the following is the answer for this division problem? \textbf{B}

\begin{itemize}
  \item A. \(x^2 + 5x + 5\)
  \item B. \(x^2 + 5x + 5 + \frac{2}{x - 2}\)
  \item C. \(x^3 + 5x^2 + 5x + \frac{2}{x - 2}\)
  \item D. \(x^3 + 5x^2 + 5x + 2\)
\end{itemize}

Remember What You Learned

4. When you translate the numbers in the last row of a synthetic division into the quotient and remainder, what is an easy way to remember which exponents to use in writing the terms of the quotient? \textit{Sample answer: Start with the power that is one less than the degree of the dividend. Decrease the power by one for each term after the first. The final number will be the remainder. Drop any term that is represented by a 0.}

Chapter 6

Glencoe Algebra 2
Study Guide and Intervention

Dividing Polynomials

Use Long Division To divide a polynomial by a monomial, use the properties of exponents from Lesson 6-1.

To divide a polynomial by a polynomial, use a long division pattern. Remember that only like terms can be added or subtracted.

Example 1

Simplify \( \frac{12p^3q^2 - 21p^2q + 9p}{3pq} \).

\[
\begin{align*}
12p^3q^2 & - 21p^2q + 9p \\
\div \quad 3pq & \\
4p^2q - 7p + 3 &
\end{align*}
\]

Example 2

Use long division to find \((x^3 - 8x^2 + 4x - 9) + (x - 4)\). 

\[
\begin{align*}
x - 4 & \quad | \quad x^3 - 8x^2 + 4x - 9 \\
& \quad \downarrow \quad \downarrow \\
-4x^2 + 16x & \quad -32x + 57 \\
\quad \downarrow \quad \downarrow \\
0 & \quad 19
\end{align*}
\]

The quotient is \(x^2 - 4x - 12\), and the remainder is \(-57\).

Therefore \(\frac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \frac{57}{x - 4}\).

Exercises

Simplify.

1. \(16a^3 + 30a^2 \div 3a\)
2. \(8 \cdot 4m^2 - 16m - 4m^3 \div 4m^2\)
3. \(60m^2n^3 - 46a^4 + 84a^5b^2 \div 12ab^2\)
4. \(6a^2 + 10a \div m\)
5. \(6m^3 - 10 \div 3m\)
6. \(5ab - 4b^2 \div a + 7a^4\)
7. \((2x^2 - 5x - 3) \div (x - 3)\)
8. \((2p^3 - 6) \div (p - 1)\)
9. \((p^2 + p - 1 \div p - 1)\)
10. \((x^3 + x^2 + x + 1) \div (x - 1)\)

Use Synthetic Division

Use synthetic division to find \((2x^3 - 5x^2 + 5x - 2) + (x - 1)\).

Step 1 Write the terms of the dividend so that the degrees of the terms are in descending order. Then write the constant of the divisor \(x - r\) to the left. In this case, \(r = 1\).

\[
\begin{array}{c|cccc}
2 & 2 & -5 & 5 & -2 \\
\hline
2 & 4 & -3 & 2 & -2 \\
2 & 2 & 5
\end{array}
\]

Thus, \((2x^3 - 5x^2 + 5x - 2) + (x - 1) = 2x^2 - 3x + 2\).

Exercises (continued)

Simplify.

1. \(3x^3 - 7x^2 + 9x - 14 \div (x - 2)\)
2. \(3x^3 - 7x^2 - 3x - 3 \div (x + 1)\)
3. \(2x^3 + 3x^2 - 10x - 3 \div (x + 3)\)
4. \(3x^3 - 8x + 9x - 9 \div (x - 4)\)
5. \(2x^3 + 9x^2 + 17x + 1 \div (x + 5)\)
6. \(3x^3 - 8x^2 + 16x - 1 \div (x - 1)\)
7. \(x^3 - 9x + 17x - 1 \div (x + 4)\)
8. \(2x^2 - 7x + 8 \div (x + 2)\)
9. \(4x^2 - 7x + 2 \div (x - 2)\)
10. \(3x^2 - 2x + 3 \div (x + 5)\)
11. \((12x^2 + 20x - 32 - 24x^2 + 20x + 35) - (6x + 5)\)

\[
\begin{align*}
4x^2 & - 8x + 20 + \frac{-65}{3x + 5}
\end{align*}
\]
6.3 Practice
Dividing Polynomials

Simplify.
1. \( \frac{10x + 6}{2} \)
2. \( \frac{12x + 20}{4} \)
3. \( \frac{15y^2 + 6y^2 + 3y}{3y} \)
4. \( \frac{12x^2 - 4x - 8}{4x} \)
5. \( (6x^2 + 5y^2)(5y^2)^{-1} \)
6. \( (4y^6 - 6y^3 + 12y^2 - 8y^2)(4y^2)^{-1} \)
7. \( (6y^2 - 9y^2 + 3y) \)
8. \( (4d^3 - 8a^3 + 3b^3 + 2a^3) \)
9. \( (n + 3 + 7n + 10) \)
10. \( (d^2 + 4d + 3) \)
11. \( (2d^2 + 13a + 15) \)
12. \( (6y^2 - y - 2x + 1) \)
13. \( (4p^2 - 3p + 2) \)
14. \( (2x^2 - 5x - 4) \)
15. \( \frac{2n^2 + 5n + 12}{n - 3} \)
16. \( \frac{2x^2 + 5x - 4}{x - 3} \)
17. \( (3a^2 - 7a - 10a - 4)^{-1} \)
18. \( (3b^2 + 4b^2 - 22b - 5x - 20) \)
19. \( \frac{x^3 - x^2 - 6}{x + 2} \)
20. \( \frac{2x^2 - 2x + 15}{x - 3} \)
21. \( \frac{4p^2 - 3p^2 + 2p}{(p - 1)} \)
22. \( \frac{(3c^4 + 6c^3 - 2c + 4c + 2)^{-1}}{x^3} \)
23. \( \frac{3x^3 - 2x^3 + 2}{p - 1} \)
24. \( \frac{4x^2 - 17p^2 + 14p - 3}{2p - 3} \)
25. \( \frac{2x^2 - 7x + 12}{x + 7} \)
26. \( \frac{2x^2 - 2x + 15}{x - 3} \)
27. \( \frac{2x^2 + 5x - 4}{x - 3} \)
28. \( \frac{2x^2 + 5x - 4}{x - 3} \)
29. \( \frac{2x^2 + 5x - 4}{x - 3} \)
30. \( \frac{2x^2 + 5x - 4}{x - 3} \)
31. \( \frac{2x^2 + 5x - 4}{x - 3} \)
32. \( \frac{2x^2 + 5x - 4}{x - 3} \)

GEOMETRY

1. The area of a rectangle is \( 2x^2 - 11x + 15 \) square units. The width of the rectangle is \( x + 4 \) units. What is the length of the rectangle? \( x^2 + 4x - 3 \) units.
2. The length of the base of the triangle is \( 6x^2 - 7x + 15 \) square units. The width of the triangle is \( 2x^2 - 5x - 4 \) square units. What is the height of the triangle? \( \frac{2x^2 + 5x - 4}{x - 3} \) units.
6-3

Word Problem Practice

Dividing Polynomials

1. REMAINDERS Jordan divided the polynomial $x^4 + x - 6$ into the polynomial $p(x)$ yesterday. Today his work is smudged and he cannot read $p(x)$ or most of his answer. The only part he could read was the remainder $x + 4$. His teacher wants him to find $p(-3)$. What is $p(-3)$?

2. LONG DIVISION Dana used long division to divide $x^4 + x^3 + x^2 + 1$ by $x + 2$. Her work is shown below with three numbers missing:

\[
\begin{array}{c|ccccc}
 & x^3 & x^2 & x & 1 \\
\hline
x + 2 & & & & & \\
\hline
& & & & & \\
\end{array}
\]

If the length of the sheet is $s + 1$ inches, what is the width of the sheet?

\[s^2 + 2s + 2 = \frac{1}{s + 1} \text{ in.} \]

3. AVERAGES Shelby is a statistician. She has a list of $n + 1$ numbers and she needs to find their average. Two of the numbers are $n^2$ and 2. Each of the other $n - 1$ numbers are all equal to 1. What is the average of these numbers?

\[n^2 - n + 2 = \frac{1}{n+1} \]

4. AREA The area of a large rectangular sheet is $s^3 + 3s^2 + 4s + 1$ square inches.

If the length of the sheet is $s + 1$ inches, what is the width of the sheet?

\[s^2 + 2s + 2 = \frac{1}{s + 1} \text{ in.} \]

### Enrichment

#### Oblique Asymptotes

The graph of $y = \frac{ax + b}{x^n}$, where $a \neq 0$, is called an oblique asymptote of $y = f(x)$ if the graph of $f$ comes closer and closer to the line $ax + b$ as $x \to \infty$ or $x \to -\infty$. $\infty$ is the mathematical symbol for infinity, which means endless.

For $f(x) = \frac{x^3}{x^2}$, $y = x + 4$ is an oblique asymptote because $f(x) \to x - 4$ as $x \to \infty$. In other words, as $|x|$ increases, the value of $\frac{y}{x^2}$ gets smaller and smaller approaching 0.

**Example**

Find the oblique asymptote for $f(x) = \frac{2x^2 - 4x + 4}{x^2}$.

Use synthetic division.

\[
\begin{array}{c|cccc}
 & 2 & -4 & 4 & \\
\hline
1 & 2 & -2 & 4 & \\
\hline
2 & 4 & 2 & & \\
\end{array}
\]

As $|x|$ increases, the value of $\frac{y}{x^2}$ gets smaller. In other words, since $\frac{x^2}{x^2} \to 1$ as $x \to \infty$ or $x \to -\infty$, $y = x + 4$ is an oblique asymptote.

**Exercises**

Use synthetic division to find the oblique asymptote for each function.

1. $y = \frac{5x^2 - 4x + 11}{x + 5}$

2. $y = \frac{x^2 + 3x - 15}{x - 2}$

3. $y = \frac{x^2 - 2x - 18}{x - 3}$

4. $y = \frac{ax^2 + bx + c}{x - d}$

5. $y = \frac{ax^2 + bx + c}{x + d}$
**6-4**

**Lesson Reading Guide**

**Polynomial Functions**

Get Ready for the Lesson

Read the introduction to Lesson 6-4 in your textbook.

- In the honeycomb cross section shown in your textbook, there is 1 hexagon in the center, 6 hexagons in the second ring, and 12 hexagons in the third ring. How many hexagons will there be in the fourth, fifth, and sixth rings? 18; 24; 30

- There is 1 hexagon in a honeycomb with 1 ring. There are 7 hexagons in a honeycomb with 2 rings. How many hexagons are there in honeycombs with 3 rings, 4 rings, 5 rings, and 6 rings? 100; 100; 100; 100

Read the Lesson

1. Give the degree and leading coefficient of each polynomial in one variable.

<table>
<thead>
<tr>
<th>Degree</th>
<th>Leading Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 10x^3 + 3x^2 - x + 7</td>
<td>3; 10</td>
</tr>
<tr>
<td>b. 7y^2 - 2y^3 + y - 4y^3</td>
<td>5; -2</td>
</tr>
<tr>
<td>c. 100</td>
<td>0; 100</td>
</tr>
</tbody>
</table>

2. Match each description of a polynomial function from the list on the left with the corresponding end behavior from the list on the right.

- a. even degree, negative leading coefficient
- b. odd degree, positive leading coefficient
- c. odd degree, negative leading coefficient
- d. even degree, positive leading coefficient

<table>
<thead>
<tr>
<th>Description</th>
<th>End Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. f(x) → +∞ as x → +∞; f(x) → -∞ as x → -∞</td>
<td></td>
</tr>
<tr>
<td>2. f(x) → +∞ as x → +∞; f(x) → +∞ as x → -∞</td>
<td></td>
</tr>
<tr>
<td>3. f(x) → +∞ as x → +∞; f(x) → +∞ as x → -∞</td>
<td></td>
</tr>
<tr>
<td>4. f(x) → +∞ as x → +∞; f(x) → -∞ as x → -∞</td>
<td></td>
</tr>
</tbody>
</table>

Remember What You Learned

3. What is an easy way to remember the difference between the end behavior of the graphs of even-degree and odd-degree polynomial functions?

Sample answer: Both ends of the graph of an even-degree function eventually keep going in the same direction. For odd-degree functions, the two ends eventually head in opposite directions, one upward, the other downward.

---

**6-4**

**Study Guide and Intervention**

**Polynomial Functions**

A polynomial of degree \( n \) in one variable \( x \) is an expression of the form

\[ a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]

where the coefficients \( a_n, a_{n-1}, \ldots, a_0 \) represent real numbers, \( a_0 \) is not zero, and \( n \) represents a nonnegative integer.

The degree of a polynomial in one variable is the greatest exponent of its variable. The leading coefficient is the coefficient of the term with the highest degree.

**Example 1**

What are the degree and leading coefficient of \( 3x^2 - 2x^4 - 7 + x^8 \)?

Rewrite the expression so the powers of \( x \) are in decreasing order.

\[-2x^4 + x^3 - 3x^2 - 7\]

This is a polynomial in one variable. The degree is 4, and the leading coefficient is -2.

**Example 2**

Find \( f(-5) \) if \( f(x) = x^3 + 2x^2 - 10x + 20 \).

\[ f(-5) = (-5)^3 + 2(-5)^2 - 10(-5) + 20 \]

\[ = -125 + 50 + 20 \]

\[ = -5 \]

Simplify.

**Example 3**

Find \( g(2) - 1 \) if \( g(x) = x^2 + 3x - 4 \).

\[ g(2) - 1 = (2^2) - 3(2) - 1 - 4 \]

\[ = 4 - 6 - 5 \]

\[ = -7 \]

Simplify.

**Exercises**

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. \( 3x^3 + 6x^2 + x^2 + 12 \) 2. \( 100 - 5x + 10x^2 + 7; 10 \) 3. \( 4x^6 + 6x^4 + 8x^3 - 10x^2 + 20 \)

4. \( 4x^2 - 3xy + 16x^2 \) 5. \( 8x^3 - 9x^6 + 4x^2 - 36 \) 6. \( \frac{x^7}{18} \) \( \frac{x^7}{18} \) \( \frac{x^7}{18} \) \( \frac{1}{72} \)

Find \( f(3) \) and \( f(-5) \) for each function.

7. \( f(x) = x^2 - 9 \) 8. \( f(x) = 4x^2 - 3x^2 + 2x - 1 \) 9. \( f(x) = 9x^3 - 4x^2 + 5x + 7 \)

\( f(3) = 3^2 - 9 \) \( f(-5) = -5^2 + 2(-5) - 1 \) \( f(3) = 9(3)^3 - 4(3)^2 + 5(3) + 7 \)

\( f(3) = 5 \) \( f(-5) = -16 \) \( f(3) = 23 \) \( f(-5) = -506 \) \( f(3) = 73 \) \( f(-5) = 1243 \)
### Lesson 6-4: Polynomial Functions

#### Skills Practice

<table>
<thead>
<tr>
<th>Polynomial Functions</th>
<th>State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( x^3 + 2x^2 + 3x + 4 )</td>
<td>( 3; 2 )</td>
</tr>
<tr>
<td>2. ( x^4 - 5x^3 + 2x^2 - x + 3 )</td>
<td>( 4; 1 )</td>
</tr>
<tr>
<td>3. ( x^2 + 3x + 5 )</td>
<td>( 2; 1 )</td>
</tr>
<tr>
<td>4. ( 2x^3 + 3x^2 + 4x + 5 )</td>
<td>( 3; 2 )</td>
</tr>
<tr>
<td>5. ( x^5 - 2x^4 + 3x^3 - 4x^2 + 5x - 6 )</td>
<td>( 5; 1 )</td>
</tr>
<tr>
<td>6. ( x^6 + 2x^5 - 3x^4 + 4x^3 - 5x^2 + 6x - 7 )</td>
<td>( 6; 1 )</td>
</tr>
<tr>
<td>7. ( x^7 + 2x^6 - 3x^5 + 4x^4 - 5x^3 + 6x^2 - 7x + 8 )</td>
<td>( 7; 1 )</td>
</tr>
</tbody>
</table>

#### Example

If the degree is even and the leading coefficient is positive, then
- If the degree is odd and the leading coefficient is negative, then

#### Exercises

- Determine whether each graph represents an odd-degree polynomial, or an even-degree polynomial. Then state the number of real zeros.

  - Graph 1: Even degree, 3 real zeros
  - Graph 2: Odd degree, 2 real zeros
  - Graph 3: Even degree, 1 real zero
  - Graph 4: Odd degree, 4 real zeros

- State the number of real zeros of each function.

  - \( f(x) = x^5 - 2x^4 + 3x^3 - 4x^2 + 5x - 6 \) \( 5 \) real zeros
  - \( g(x) = x^6 + 2x^5 - 3x^4 + 4x^3 - 5x^2 + 6x - 7 \) \( 6 \) real zeros
  - \( h(x) = x^7 + 2x^6 - 3x^5 + 4x^4 - 5x^3 + 6x^2 - 7x + 8 \) \( 7 \) real zeros

### Glencoe Algebra 2

Chapter 6

NAME __________________________________________ DATE __________ PERIOD _____

Chapter 6

A13

Glencoe Algebra 2
6-4 Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. \(3x^4 + 12x^2 - 9\); 6
2. \(12x^3 - 3x^2 + 4\); 3; \(\frac{1}{5}\)
3. \(\frac{2}{3}x^2 + 3m - 12\); Not a polynomial;
4. \(27 + 3xy - 12y^2 - 10y\)

\(m^n\) cannot be written in the form \(m^n\) for a nonnegative integer \(n\).

Find \(p(2)\) and \(p(3)\) for each function.
5. \(p(x) = x^2 - x^2\); 24; -29; -39
6. \(p(x) = -5x^2 + 5x + 9\)
7. \(p(x) = -x^2 + 4x^2\)

8. \(p(x) = 3x^2 - x^2 + 2x - 5\)
9. \(p(x) = x^4 + \frac{3x^3}{2} - \frac{1}{2}x\)
10. \(p(x) = \frac{3x^2}{5} + \frac{2}{3}x^2 + 3x - 37\); 73

If \(p(x) = 3x^2 - 4\) and \(r(x) = 2x^2 - 5x + 1\), find each value.
11. \(p(0)\)
12. \(r(2)\)
13. \(-5x(3y)\)
14. \(r(x + 2)\)
15. \(p(x^2 - 1)\)
16. \(5(x^2 + 2)\)

\(2x^2 + 3x - 1\)
\(3x^2 - 6x^2 - 1\)
\(15x^2 + 60x + 40\)

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree polynomial function, and
c. state the number of real zeros.

17. \(f(x) \rightarrow +\infty\) as \(x \rightarrow +\infty\);
18. \(f(x) \rightarrow +\infty\) as \(x \rightarrow +\infty\);
19. \(f(x) \rightarrow +\infty\) as \(x \rightarrow +\infty\);

20. WIND CHILL The function \(C(s) = 0.013s^2 - s - 7\) estimates the wind chill temperature \(C(s)\) at 0°F for wind speeds \(s\) from 5 to 30 miles per hour. Estimate the wind chill temperature at 0°F if the wind speed is 20 miles per hour. \textbf{about -22°F}\n
21. \(\frac{3x - 1}{2}\)

2. GRAPHS Kendra graphed the polynomial \(f(x)\) shown below.

From this graph, describe the end behavior, degree, and sign of the leading coefficient.

\(f(x) \rightarrow -\infty\) as \(x \rightarrow -\infty\) and \(f(x) \rightarrow -\infty\) as \(x \rightarrow +\infty\); the degree is 3; the leading coefficient is negative

3. PENTAGONAL NUMBERS The \(n\)th pentagonal number is given by the expression

\[n(3n - 1)\]

What is the degree of this polynomial? What is the seventh pentagonal number?

2; 70

5. What is the degree of \(f(x)\)?

3

6. If Dylan drew 15 dots, how many triangles can be made?

455

Lesson 6-4

Word Problem Practice

Polynomial Functions

1. MANUFACTURING A metal sheet is curved according to the shape of the graph of \(f(x) = x^4 - 9x^2\). What is the degree of this polynomial?

4

2. GRAPHS Kendra graphed the polynomial \(f(x)\) shown below.

What is the volume of wood removed as a function of time? \(V = \frac{\pi r^2 t}{2}\)

TRIANGLES For Exercises 5 and 6, use the following information.

Dylan drew \(n\) dots on a piece of paper making sure that no line contained 3 of the dots. The number of triangles that can be made using the dots as vertices is equal to \(f(n) = \frac{1}{6}n^3 - 3n^2 + 2n\).

5. What is the degree of \(f(n)\)?

3

6. If Dylan drew 15 dots, how many triangles can be made?

455
**Approximation by Means of Polynomials**

Many scientific experiments produce pairs of numbers \([x, f(x)]\) that can be related by a formula. If the pairs form a function, you can fit a polynomial to the pairs in exactly one way. Consider the pairs given by the following table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>6</td>
<td>31</td>
<td>39</td>
<td>54</td>
<td>77</td>
</tr>
</tbody>
</table>

We will assume the polynomial is of degree three. Substitute the given values into this expression.

\[ f(x) = A + B(x - x_0) + C(x - x_0)^2 + D(x - x_0)^3 \]

You will get the system of equations shown below. You can solve this system and use the values for \(A, B, C,\) and \(D\) to find the desired polynomial.

\[
\begin{align*}
6 &= A + B(2 - 1) + C(2 - 1)^2 + D(2 - 1)^3 \\
11 &= A + B(3 - 1) + C(3 - 1)^2 + D(3 - 1)^3 \\
39 &= A + B(4 - 1) + C(4 - 1)^2 + D(4 - 1)^3 \\
54 &= A + B(5 - 1) + C(5 - 1)^2 + D(5 - 1)^3
\end{align*}
\]

Solve.

1. Solve the system of equations for the values \(A, B, C,\) and \(D.\)
   \[ A = 6, B = 5, C = 3, D = -2 \]

2. Find the polynomial that represents the four ordered pairs. Write your answer in the form \(y = ax^3 + bx^2 + cx + d.\)
   \[ y = -2x^3 + 17x^2 - 32x + 23 \]

3. Find the polynomial that gives the following values.
   \[
   \begin{array}{c|c|c|c|c}
   \hline
   x & 1 & 2 & 3 & 4 \\
   \hline
   f(x) & 6 & 31 & 39 & 54 \\
   \hline
   \end{array}
   \]
   \[ A = -207, B = 94, C = 25, D = 1; y = x^3 - 10x^2 - 10x + 1 \]

4. A scientist measured the volume \(f(x)\) of carbon dioxide gas that can be absorbed by one cubic centimeter of charcoal at pressure \(x.\) Find the values for \(A, B, C,\) and \(D.\)
   \[
   \begin{array}{c|c|c|c|c}
   \hline
   \hline
   \hline
   x & 10 & 20 & 30 & 40 \\
   \hline
   f(x) & 0.000000634 & 0.000000634 & 0.000000634 & 0.000000634 \\
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \]
   \[ A = 3.1, B = 0.01091, C = 0.000000643, D = 0.000000066 \]

**Remember What You Learned**

3. The origins of words can help you to remember their meaning and to distinguish between similar words. Look up maximum and minimum in a dictionary and describe their origins (original language and meaning). Sample answer: Maximum comes from the Latin word maximus, meaning greatest. Minimum comes from the Latin word minimus, meaning least.
Study Guide and Intervention

Analyze Graphs of Polynomial Functions

Graph Polynomial Functions

Maximum and Minimum Points

Example

Determine the values of \( x \) at which the relative maxima and minima occur.

Example

Graph \( f(x) = x^3 + 6x^2 - 3 \). Estimate the \( x \)-coordinates at which the relative maxima and minima occur.

Exercises

Graph each function by making a table of values. Determine the \( x \)-values at which or between which each real zero is located.

Exercise

Graph each function by making a table of values. Estimate the \( x \)-coordinates at which the relative maxima and minima occur.
Answers (Lesson 6-5)

Chapter 6

A town's jobless rate can be modeled by (1, 3.3), (2, 4.9), (3, 5.3), (4, 6.4), (5, 4.5), (6, 3.1), (7, 2.7).

1. Write an equation for the relationship between jobless rate and time in the form of a polynomial function.

2. Determine consecutive values of \( x \) and \( y \) at which the relative and relative minima occur.

3. Estimate the \( x \)-coordinates at which the relative and relative minima occur.

4. If the data could be modeled by a polynomial equation, what is the least degree the equation could have? Describe the function through these points.
Answers (Lesson 6-5)

**Golden Rectangles**

Use a straightedge, a compass, and the instructions below to construct a golden rectangle.

1. Construct square $ABCD$ with sides of 2 centimeters.
2. Construct the midpoint of $AB$. Call the midpoint $M$.
3. Using $M$ as the center, set your compass opening at $MC$. Construct an arc with center $M$ that intersects $AB$. Call the point of intersection $P$.
4. Construct a line through $P$ that is perpendicular to $AB$.
5. Extend $DC$ so that it intersects the perpendicular. Call the intersection point $Q$. $APQD$ is a golden rectangle. Check this conclusion by finding the value of $\frac{AQ}{AP}$.

0.62

**A figure consisting of similar golden rectangles is shown below.** Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.

6. Using $A$ as a center, draw an arc that passes through $B$ and $C$.
7. Using $D$ as a center, draw an arc that passes through $C$ and $E$.
8. Using $F$ as a center, draw an arc that passes through $E$ and $G$.

**Enrichment**

A banker models the expected value of a company in millions of dollars by the formula $v = n^3 - 3n^2$, where $n$ is the number of years in business. Sketch a graph of $v = n^3 - 3n^2$.

**Consecutive Numbers**

For Exercises 4 and 5, use the following information.

Mr. Sanchez asks his students to write expressions to represent five consecutive integers. One solution is $x - 2, x - 1, x, x + 1, x + 2$. The product of these five consecutive integers is given by the fifth degree polynomial $f(x) = x^5 - 5x^4 + 4x$.

4. For what values of $x$ is $f(x) = 0$?
5. Sketch the graph of $y = f(x)$.

**Chapter 6**

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Lesson Reading Guide
Solving Polynomial Equations

Get Ready for the Lesson

Read the introduction to Lesson 6-6 in your textbook.

If a trinomial that represents the volume of a box is factored into three binomials, what might the three binomials represent? the length, width and height of the box

Read the Lesson

1. Name three types of binomials that it is always possible to factor. difference of two squares, sum of two cubes, difference of two cubes

2. Name a type of trinomial that it is always possible to factor. perfect square trinomial

3. Complete: Since $x^2 + y^2$ cannot be factored, it is an example of a prime polynomial.

4. On an algebra quiz, Marlene needed to factor $2x^2 - 4x - 70$. She wrote the following answer: $(x + 5)(2x - 14)$. When she got her quiz back, Marlene found that she did not get full credit for her answer. She thought she should have gotten full credit because she checked her work by multiplication and showed that $(x + 5)(2x - 14) = 2x^2 - 4x - 70$.
   a. If you were Marlene’s teacher, how would you explain to her that her answer was not entirely correct? Sample answer: When you are asked to factor a polynomial, you must factor it completely. The factorization was not complete, because $2x - 14$ can be factored further as $2(x - 7)$.
   b. What advice could Marlene’s teacher give her to avoid making the same kind of error in factoring in the future? Sample answer: Always look for a common factor first. If there is a common factor, factor it out first, and then see if you can factor further.

Remember What You Learned

5. Some students have trouble remembering the correct signs in the formulas for the sum and difference of two cubes. What is an easy way to remember the correct signs? Sample answer: In the binomial factor, the operation sign is the same as in the expression that is being factored. In the trinomial factor, the operation sign before the middle term is the opposite of the sign in the expression that is being factored. The sign before the last term is always a plus.

Study Guide and Intervention
Solving Polynomial Equations

Factor Polynomials

<table>
<thead>
<tr>
<th>For any number of terms, check for:</th>
<th>greatest common factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>For two terms, check for:</td>
<td>Difference of two squares</td>
</tr>
<tr>
<td></td>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
</tr>
<tr>
<td></td>
<td>Sum of two cubes</td>
</tr>
<tr>
<td></td>
<td>$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$</td>
</tr>
<tr>
<td>For three terms, check for:</td>
<td>Perfect square trinomials</td>
</tr>
<tr>
<td></td>
<td>$a^2 + 2ab + b^2 = (a + b)^2$</td>
</tr>
<tr>
<td></td>
<td>$a^2 - 2ab + b^2 = (a - b)^2$</td>
</tr>
<tr>
<td></td>
<td>General trinomials</td>
</tr>
<tr>
<td></td>
<td>$ax^2 + bx + c = (x + p)(x + q)$</td>
</tr>
</tbody>
</table>

Example

Factor $24x^2 - 42x - 45$.

First factor out the GCF to get $24x^2 - 42x - 45 = 3(8x^2 - 14x - 15)$. To find the coefficients of the $x$ terms, you must find two numbers whose product is $3 \cdot (-15) = -45$ and whose sum is $-14$. The two coefficients must be $-20$ and $6$. Rewrite the expression using $-20x$ and $6x$ and factor by grouping.

$8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$

$= 4x(2x - 5) + 3(2x - 5)$

Factor the GCF of each binomial.

Thus, $24x^2 - 42x - 45 = 3(4x + 3)(2x - 5)$.

Exercises

Factor completely. If the polynomial is not factorable, write prime.

1. $14x^2 + 24x^2$  
2. $6mn + 18m - n - 3$  
3. $2x^2 + 18x + 16$  
4. $x^2 - 1$  
5. $35x^4y^2 - 60xy^2$  
6. $x^2 + 250$  
7. $100m^4 - 9$  
8. $x^2 + x + 1$  
9. $a^2 + 2a + 1 = (a + 1)^2$  

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Answers (Lesson 6-6)

**Solving Polynomial Equations**

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(3x^2 - 49)</td>
</tr>
<tr>
<td>2.</td>
<td>(2x^2 + 2x = 0)</td>
</tr>
<tr>
<td>3.</td>
<td>(x^2 - 25)</td>
</tr>
<tr>
<td>4.</td>
<td>(x^2 + 9)</td>
</tr>
<tr>
<td>5.</td>
<td>(x^2 - 36)</td>
</tr>
<tr>
<td>6.</td>
<td>(x^2 + 16)</td>
</tr>
<tr>
<td>7.</td>
<td>(x^2 - 49)</td>
</tr>
<tr>
<td>8.</td>
<td>(x^2 - 100)</td>
</tr>
<tr>
<td>9.</td>
<td>(x^2 - 4)</td>
</tr>
<tr>
<td>10.</td>
<td>(x^2 - 121)</td>
</tr>
<tr>
<td>11.</td>
<td>(x^2 - 124)</td>
</tr>
<tr>
<td>12.</td>
<td>(x^2 - 225)</td>
</tr>
<tr>
<td>13.</td>
<td>(x^2 - 169)</td>
</tr>
<tr>
<td>14.</td>
<td>(x^2 - 289)</td>
</tr>
<tr>
<td>15.</td>
<td>(x^2 - 784)</td>
</tr>
<tr>
<td>16.</td>
<td>(x^2 - 16)</td>
</tr>
<tr>
<td>17.</td>
<td>(x^2 - 36)</td>
</tr>
<tr>
<td>18.</td>
<td>(x^2 - 64)</td>
</tr>
<tr>
<td>19.</td>
<td>(x^2 - 625)</td>
</tr>
<tr>
<td>20.</td>
<td>(x^2 - 100)</td>
</tr>
<tr>
<td>21.</td>
<td>(x^2 - 49)</td>
</tr>
<tr>
<td>22.</td>
<td>(x^2 - 25)</td>
</tr>
<tr>
<td>23.</td>
<td>(x^2 - 9)</td>
</tr>
<tr>
<td>24.</td>
<td>(x^2 - 4)</td>
</tr>
<tr>
<td>25.</td>
<td>(x^2 - 1)</td>
</tr>
<tr>
<td>26.</td>
<td>(x^2 - 0)</td>
</tr>
</tbody>
</table>

**Solve each equation.**

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(-7, 7)</td>
</tr>
<tr>
<td>2.</td>
<td>((5, -5))</td>
</tr>
<tr>
<td>3.</td>
<td>(-3, 3)</td>
</tr>
<tr>
<td>4.</td>
<td>((4, -4))</td>
</tr>
<tr>
<td>5.</td>
<td>((-6, 6))</td>
</tr>
<tr>
<td>6.</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>7.</td>
<td>((-2, 2))</td>
</tr>
<tr>
<td>8.</td>
<td>((5, -5))</td>
</tr>
<tr>
<td>9.</td>
<td>((-3, 3))</td>
</tr>
<tr>
<td>10.</td>
<td>((-5, 5))</td>
</tr>
<tr>
<td>11.</td>
<td>((-7, 7))</td>
</tr>
<tr>
<td>12.</td>
<td>((-4, 4))</td>
</tr>
</tbody>
</table>

**Write each expression in quadratic form, if possible.**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(x^2 - 9)</td>
</tr>
<tr>
<td>2.</td>
<td>(x^2 + 16)</td>
</tr>
<tr>
<td>3.</td>
<td>(x^2 - 25)</td>
</tr>
<tr>
<td>4.</td>
<td>(x^2 + 36)</td>
</tr>
<tr>
<td>5.</td>
<td>(x^2 - 49)</td>
</tr>
<tr>
<td>6.</td>
<td>(x^2 + 64)</td>
</tr>
<tr>
<td>7.</td>
<td>(x^2 - 81)</td>
</tr>
<tr>
<td>8.</td>
<td>(x^2 + 100)</td>
</tr>
<tr>
<td>9.</td>
<td>(x^2 - 121)</td>
</tr>
<tr>
<td>10.</td>
<td>(x^2 + 144)</td>
</tr>
<tr>
<td>11.</td>
<td>(x^2 - 124)</td>
</tr>
<tr>
<td>12.</td>
<td>(x^2 + 169)</td>
</tr>
</tbody>
</table>

**Write the expression on the left in quadratic form.**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(4x^2 - 49)</td>
</tr>
<tr>
<td>2.</td>
<td>(9x^2 - 36)</td>
</tr>
<tr>
<td>3.</td>
<td>(16x^2 - 64)</td>
</tr>
<tr>
<td>4.</td>
<td>(25x^2 - 125)</td>
</tr>
<tr>
<td>5.</td>
<td>(36x^2 - 144)</td>
</tr>
<tr>
<td>6.</td>
<td>(49x^2 - 245)</td>
</tr>
<tr>
<td>7.</td>
<td>(64x^2 - 256)</td>
</tr>
<tr>
<td>8.</td>
<td>(81x^2 - 324)</td>
</tr>
<tr>
<td>9.</td>
<td>(100x^2 - 400)</td>
</tr>
<tr>
<td>10.</td>
<td>(121x^2 - 484)</td>
</tr>
</tbody>
</table>

**Factor.**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>((x + 3)(x - 3))</td>
</tr>
<tr>
<td>2.</td>
<td>((x + 4)(x - 4))</td>
</tr>
<tr>
<td>3.</td>
<td>((x + 5)(x - 5))</td>
</tr>
<tr>
<td>4.</td>
<td>((x + 6)(x - 6))</td>
</tr>
<tr>
<td>5.</td>
<td>((x + 7)(x - 7))</td>
</tr>
<tr>
<td>6.</td>
<td>((x + 8)(x - 8))</td>
</tr>
<tr>
<td>7.</td>
<td>((x + 9)(x - 9))</td>
</tr>
<tr>
<td>8.</td>
<td>((x + 10)(x - 10))</td>
</tr>
<tr>
<td>9.</td>
<td>((x + 11)(x - 11))</td>
</tr>
<tr>
<td>10.</td>
<td>((x + 12)(x - 12))</td>
</tr>
</tbody>
</table>

**Write each expression in quadratic form, if possible.**

<table>
<thead>
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</tr>
<tr>
<td>3.</td>
<td>((x + 5)(x - 5))</td>
</tr>
<tr>
<td>4.</td>
<td>((x + 6)(x - 6))</td>
</tr>
<tr>
<td>5.</td>
<td>((x + 7)(x - 7))</td>
</tr>
<tr>
<td>6.</td>
<td>((x + 8)(x - 8))</td>
</tr>
<tr>
<td>7.</td>
<td>((x + 9)(x - 9))</td>
</tr>
<tr>
<td>8.</td>
<td>((x + 10)(x - 10))</td>
</tr>
<tr>
<td>9.</td>
<td>((x + 11)(x - 11))</td>
</tr>
<tr>
<td>10.</td>
<td>((x + 12)(x - 12))</td>
</tr>
</tbody>
</table>

**Solve each equation.**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(x^2 - 9 = 0)</td>
</tr>
<tr>
<td>2.</td>
<td>(x^2 + 16 = 0)</td>
</tr>
<tr>
<td>3.</td>
<td>(x^2 - 25 = 0)</td>
</tr>
<tr>
<td>4.</td>
<td>(x^2 + 36 = 0)</td>
</tr>
<tr>
<td>5.</td>
<td>(x^2 - 49 = 0)</td>
</tr>
<tr>
<td>6.</td>
<td>(x^2 + 64 = 0)</td>
</tr>
<tr>
<td>7.</td>
<td>(x^2 - 81 = 0)</td>
</tr>
<tr>
<td>8.</td>
<td>(x^2 + 100 = 0)</td>
</tr>
<tr>
<td>9.</td>
<td>(x^2 - 121 = 0)</td>
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<td>10.</td>
<td>(x^2 + 144 = 0)</td>
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<td>(x^2 - 124 = 0)</td>
</tr>
<tr>
<td>12.</td>
<td>(x^2 + 169 = 0)</td>
</tr>
<tr>
<td>13.</td>
<td>(x^2 - 225 = 0)</td>
</tr>
<tr>
<td>14.</td>
<td>(x^2 + 289 = 0)</td>
</tr>
<tr>
<td>15.</td>
<td>(x^2 - 784 = 0)</td>
</tr>
<tr>
<td>16.</td>
<td>(x^2 + 169 = 0)</td>
</tr>
</tbody>
</table>

**Zero Product Property**

1. \(x = 3, -3\)
2. \(x = 5, -5\)
3. \(x = 6, -6\)
4. \(x = 7, -7\)
5. \(x = 8, -8\)
6. \(x = 9, -9\)
7. \(x = 10, -10\)
8. \(x = 11, -11\)
9. \(x = 12, -12\)
10. \(x = 13, -13\)
11. \(x = 14, -14\)
12. \(x = 15, -15\)
13. \(x = 16, -16\)
14. \(x = 17, -17\)
15. \(x = 18, -18\)
16. \(x = 19, -19\)
17. \(x = 20, -20\)
18. \(x = 21, -21\)
19. \(x = 22, -22\)
20. \(x = 23, -23\)

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Chapter 6

Glencoe Algebra 2
Section 6-6: Practice

Solving Polynomial Equations

Factor completely. If the polynomial is not factorable, write prime.

1. $15a^2b - 10ab^2$ 2. $3x^2 - 9x^2 + 6x^2$ 3. $3x^3y - 2x^2y + 5xy$

$5ab(3a - 2b)$ $3x(t - 3x^2 + 2st)$ $xy(3x^2 - 2x + 5)$

4. $2x^2y - x^2y + 5x^2y - xy^2 - 7y + 3x - x - y^2$ 5. $21 - 7x + 3x - 2x + 2y$

$x(y)(3 - f)$ $(x + 2)(x - y)$

6. $x^2 - xy + 2x - 2y$ 7. $y^2 + 20y + 96$ 8. $4ab + 2a + 6b + 3$

$(x + 2)(x - y)$ $(2a + 3)(2b + 1)$

9. $y + 8(y + 12)$ 10. $6x^2 + 7x - 3$ 11. $x^2 - 8x - 8$

$(2a + 3)(2b + 1)$ $(3x - 1)(2x + 3)$ prime

12. $6x^2 + 17x - 45$ Write each expression in quadratic form, if possible.

13. $3x^4 + 3x^2 - 11$ 14. $-5x^3 + x^2 + 6$ 15. $28x^6 + 25x^3$

$10(b^2)^3 + 3(b^2) - 11$ not possible $28(d^2)^3 + 25(d^3)$

16. $4a^3 + 4a^4 + 7$ 17. $500x^6 - x^2$ 18. $8b^3 - 8b^3 - 1$

$4(a^3)^2 + 4(a^3) + 7$ $500(x^6)^2 - x^2$ not possible

19. $x^4 - 7x^3 - 18x^2 = 0$ 20. $5^2 + 4x^2 - 32\sqrt{2} = 0$ 21. $m^4 - 625 = 0$

$-2, 0, 9$ $-8, 0, 4$ $-5, 5, 5i$

22. $n^4 - 40n^2 = 0$ 23. $x^4 - 50x^2 + 49 = 0$ 24. $t^4 - 21l^2 + 80 = 0$

$0, -7, 7$ $-1, 1, -7, 7$ $-4, 4, \sqrt{5}, -\sqrt{5}$

25. PHYSICS A proton in a magnetic field follows a path on a coordinate grid modeled by the function $f(x) = x^2 - 2x^2 - 15$. What are the x-coordinates of the points on the grid where the proton crosses the x-axis? $\sqrt{5}, -\sqrt{5}$

26. SURVEYING Vista county is setting aside a large parcel of land to preserve it as open space. The county has hired Meghan’s surveying firm to survey the parcel, which is in the shape of a right triangle. The longer leg of the triangle measures 5 miles less than twice the square of the shorter leg. The length of each boundary is a whole number. Find the length of each boundary. $3$ mi, $4$ mi, $5$ mi

Section 6-6: Word Problem Practice

Solving Polynomial Equations

1. CODES Marsha has been trying to discover the secret code for a lock. After a long investigation, she discovers that the numbers in the secret code are solutions of the polynomial equation $x^4 - 60x^3 + 1557x^2 - 19770x + 37800 = 0$. After more work, Marsha found that $x^4 - 60x^3 + 1557x^2 - 19770x + 37800 = (x - 5)x^3 - 12x^2 - 21x^2 + 30$. What are the numbers in the secret code? 5, 12, 21, and 30

2. OUTPUT Eduardo is a mechanical engineer. For one of his projects, he had to solve the polynomial equation $m^4 + 5m^3 - 10 = 0$. Write the polynomial $m^4 + 5m^3 - 10$ in quadratic form. $(m^2)^2 + (5m^3) - 10$

3. VOLUME Jacob builds a wooden box. The box is 3 inches high. The width is 3 inches more than the height, and the length is 2 inches less than the height. The volume of the box is 3 times the width.

4. ROBOTS A robot explorer’s distance from its starting location is given by the polynomial equation $x^3 - 29x^2 + 100t$, where $t$ is time measured in hours. Factor this polynomial. $t(t - 2)(t - 5)(t + 2)(t + 5)$

5. PACKAGING For Exercises 5-8, use the following information.

A small box is placed inside a larger box. The dimensions of the small box are $x + 1$ by $x + 2$ by $x - 1$. The dimensions of the larger box are $2x$ by $x + 4$ by $x + 2$.

6. If the volume of the space inside the larger box but outside the smaller box is equal to $33x + 162$ cubic units, what is $x$?

7. What is the volume of the smaller box?

90 units^3

8. What is the volume of the larger box?

384 units^3
History of Quadratic Equations

The ancient Babylonians are believed to be the first to solve quadratic equations, around 400 B.C. Euclid, who devised a geometrical approach in 300 B.C., followed by the Chinese mathematician named AlKhwarizmi created a classification of quadratic equations. His equations are made up of three different types of expressions: roots (x^2), squares of roots (x), and numbers.

For example, his first classification was squares equal to roots. A sample of this type of equations is:

\[ x^2 = 25 \]

Now solve this quadratic equation.

\[ x^2 - 25 = 0 \]
\[ (x - 5)(x + 5) = 0 \]
\[ x = 5 \quad \text{or} \quad x = -5 \]

So, \( x = 5 \) or \( x = -5 \).

Write and solve a sample problem for the remaining 5 classifications of quadratic equations, according to AlKhwarizmi.

1. Square equal to numbers.
   Sample Answer: \( x^2 = 25; x = 5 \) and \( x = -5 \)

2. Roots equal to numbers.
   Sample Answer: \( 2x = 10; x = 5 \)

3. Squares and roots equal to numbers.
   Sample Answer: \( x + 10x = 30; x = 3 \) and \( x = 7 \)

4. Squares and numbers equal to roots.
   Sample Answer: \( x + 21 = x^2; x = -1 \) and \( x = 4 \)

5. Roots and numbers equal to squares.
   Sample Answer: \( 3x + 4 = x^2; x = -1 \) and \( x = 4 \)
Study Guide and Intervention

6-7

The Remainder and Factor Theorems

Synthetic Substitution

The remainder when you divide the polynomial \( f(x) \) by \((x - a)\), is the constant \( f(a) \), where \( f(x) \) is a polynomial with degree one less than the degree of \( f(x) \).

**Example 1**

If \( f(x) = 3x^4 + 2x^3 - 5x^2 + 2x - 2 \), find \( f(-2) \).

**Method 1** Synthetic Substitution

By the Remainder Theorem, \( f(-2) \) should be the remainder when you divide the polynomial by \( x + 2 \).

\[
\begin{array}{c|cccc|c}
   x & 3 & 2 & -5 & 1 & -2 \\
\hline
   -2 & -2 & 4 & -2 & 4 & -2 \\
   \hline
   8 & -8 & 6 & -6 & 10 \\
\end{array}
\]

The remainder is 8, so \( f(-2) = 8 \).

**Example 2**

If \( f(x) = 5x^3 + 2x^2 - 1 \), find \( f(3) \).

Again, by the Remainder Theorem, \( f(3) \) should be the remainder when you divide the polynomial by \( x - 3 \).

\[
\begin{array}{c|ccc|c}
   x & 5 & 0 & 2 \ 1 \\
\hline
   3 & 15 & 45 & 141 \\
   \hline
   3 & 5 & 15 & 47 & 140 \\
\end{array}
\]

The remainder is 140, so \( f(3) = 140 \).

**Exercises**

Use synthetic substitution to find \( f(a) \) and \( f(a + 1) \) for each function.

1. \( f(x) = -3x^2 + 5x - 1 \) \( -101 \); \( \frac{3}{4} \)

2. \( f(x) = 4x^2 - 6x - 7 \) \( 63 \); \( -3 \)

3. \( f(x) = -x^3 + 3x^2 - 5 \) \( 195 \); \( -35 \)

4. \( f(x) = x^4 + 1x^2 - 1 \) \( 899 \); \( \frac{29}{16} \)

Use synthetic substitution to find \( f(0) \) and \( f(-3) \) for each function.

5. \( f(x) = 2x^3 + 3x^2 - 5x + 3 \) \( 127 \); \( -27 \)

6. \( f(x) = 3x^3 - 4x + 2 \) \( 178 \); \( -67 \)

7. \( f(x) = 5x^3 - 4x^2 + 2 \) \( 258 \); \( -169 \)

8. \( f(x) = 2x^4 - 4x^3 + 3x^2 - x + 6 \) \( 302 \); \( 288 \)

9. \( f(x) = 5x^4 + 3x^3 - 4x^2 + 2x - 4 \) \( 1404 \); \( 298 \)

10. \( f(x) = 3x^3 - 2x^2 - x + 5 \) \( 627 \); \( 277 \)

11. \( f(x) = 2x^3 - 4x^2 - 3x + 2x - 3 \) \( 219 \); \( 282 \)

12. \( f(x) = 4x^4 - 4x^3 + 3x^2 - 2x - 3 \) \( 805 \); \( 462 \)

Chapter 6

Glencoe Algebra 2

**Factors of Polynomials**

The Factor Theorem can help you find all the factors of a polynomial.

**Example**

Show that \( x + 5 \) is a factor of \( x^3 + 2x^2 - 13x + 10 \).

Then find the remaining factors of the polynomial.

By the Factor Theorem, \( x + 5 \) is a factor of the polynomial if \(-5 \) is a zero of the polynomial function. To check this, use synthetic substitution.

Since the remainder is 0, \( x + 5 \) is a factor of the polynomial. The polynomial \( x^3 + 2x^2 - 13x + 10 \) can be factored as \((x + 5)(x^2 - 2x + 2)\).

So \( x^2 + 2x^2 - 13x + 10 = (x + 5)(x^2 - 2x + 2) \).

**Exercises**

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

1. \( x^3 + x^2 - 10x + 8 \); \( x - 2 \)

2. \( x^3 - 4x^2 - 11x + 30 \); \( x + 3 \)

3. \( x^2 + 15x^2 + 10x + 7 \); \( x + 5 \)

4. \( x^2 - 7x^2 - 26x + 72 \); \( x - 2 \)

5. \( x^2 - x^2 - 7x - 6 \); \( x - 2 \)

6. \( x^2 - 6x + 40 \); \( x + 4 \)

7. \( 12x^3 - 7x^3 + 5x - 10 \); \( 4x - 1 \)

8. \( 14x^3 + x^2 - 24x + 9 \); \( x + 3 \)

9. \( x^3 + x^2 + x + 2 \); \( x + 2 \)

10. \( 2x^2 - 11x^2 + 19x - 28 \); \( x - 2 \)

11. \( 3x^3 - 13x^2 - 34x + 24 \); \( x - 6 \)

12. \( 3x^2 + 2x - 3 \); \( x + 2 \)

Glencoe Algebra 2

Answers

Chapter 6
6-7 Skills Practice

The Remainder and Factor Theorems

Use synthetic substitution to find \( f(2) \) and \( f(-1) \) for each function.

1. \( f(x) = x^2 + 6x + 5 \)  
   \( f(2) = 21, 0 \)

2. \( f(x) = x^3 - x + 1 \)  
   \( f(-1) = 3, 3 \)

3. \( f(x) = x^2 - 2x - 2 \)  
   \( f(2) = 21, 6 \)

4. \( f(x) = x^2 + 2x + 5 \)  
   \( f(-1) = 21, 6 \)

5. \( f(x) = x^3 - x^2 + 3 \)  
   \( f(2) = 30, 0 \)

6. \( f(x) = x^3 + 6x^2 + x - 4 \)  
   \( f(-1) = 30, 0 \)

7. \( f(x) = x^3 - 3x^2 + x - 2 - 4 \)  
   \( f(-1) = -8, 1 \)

8. \( f(x) = x^3 - 5x^2 - x + 6 \)  
   \( f(-1) = 6, 27 \)

9. \( f(x) = x^4 + 2x^2 - 9 \)  
   \( f(2) = 15, -6 \)

10. \( f(x) = x^4 - 3x^3 + 2x^2 - 2x + 6 \)  
    \( f(2) = 2, 14 \)

11. \( f(x) = x^3 - 7x^2 - 4x + 10 \)  
    \( f(-2, 20) = -22, 20 \)

12. \( f(x) = x^3 - 2x^2 + x^4 + x^3 - 9x^2 - 20 \)  
    \( f(-32, -26) = 430, 1446 \)

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

13. \( x^3 + 2x^2 - x - 2 \)  
    \( x - 1, x + 1 \)

14. \( x^3 + x^2 - 5x + 3 \)  
    \( x - 1, x + 3 \)

15. \( x^3 + 3x^2 - 4x - 12 \)  
    \( x - 2, x + 2 \)

16. \( x^3 - 6x^2 + 11x - 6 \)  
    \( x - 1, x + 2 \)

17. \( x^3 + 2x^2 - 9x + 5 \)  
    \( x - 3, x + 6 \)

18. \( x^3 - 6x^2 + 22x - 4 \)  
    \( x - 4, x + 2 \)

19. \( x^3 - x^2 - 10x - 8 \)  
    \( x + 1, x - 4 \)

20. \( x^3 - 19x - 30 \)  
    \( x + 5, x - 3 \)

21. \( 2x^3 + x^2 - 2x + 1 \)  
    \( 2x + 1, x - 1 \)

22. \( 2x^3 + x^2 - 5x + 2 \)  
    \( 2x + 1, x - 1 \)

23. \( 3x^3 + 4x^2 - 5x - 2 \)  
    \( x - 1, x + 2 \)

24. \( 3x^3 + x^2 + 3x - 2 \)  
    \( x^2 + x + 1 \)

25. POPULATION The projected population in thousands for a city over the next several years can be estimated by the function \( P(x) = x^3 + 2x^2 - 8x + 500 \), where \( x \) is the number of years since 2005. Use synthetic substitution to estimate the population for 2010. 655,000

26. VOLUME The volume of water in a rectangular swimming pool can be modeled by the polynomial \( 2x^2 - 9x^2 + 7x + 6 \). If the depth of the pool is given by the polynomial \( 2x - 1 \), what polynomials express the length and width of the pool? \( x - 3 \) and \( x - 2 \)
1. HEIGHT A ball tossed into the air follows a parabolic trajectory. Its height after \( t \) seconds is given by a polynomial of degree two with leading coefficient \(-16\). Using synthetic substitution, Norman found that the polynomial evaluates to 0 for the values \( t = 0 \) and \( t = 4 \). What is the polynomial that describes the ball’s height as a function of \( t \)?

\[-16t^2 + 64t\]

2. SYNTHETIC SUBSTITUTION Branford evaluates the polynomial \( p(x) = x^3 - 5x^2 + 3x + 5 \) for a factor using synthetic substitution. Some of his work is shown below. Unfortunately, the factor and the solution have ink spots over it.

\[
\begin{array}{c|ccccc}
  \text{Divisor} & 1 & -5 & 3 & 5 \\
  \hline
  & 11 & 66 & 759 \\
  \hline
  & 1 & 6 & 69
\end{array}
\]

What is the factor he solved for? What is the hidden solution?

11; 764

3. PROFIT The profits of Clyde’s Corporation can be modeled by the polynomial \( P(x) = x^3 - 4x^2 + 2x + 10 - 200 \), where \( x \) is the number of years after the business was started. The chief financial officer wants to know the value of \( P(10) \). Use synthetic substitution to determine \( P(10) \). Show your work.

\[
\begin{array}{c|ccccc}
  \text{Divisor} & 1 & -4 & 2 & 10 & -200 \\
  \hline
  & 10 & 60 & 620 & 6300 \\
  \hline
  & 1 & 6 & 62 & 630 & 6100
\end{array}
\]

\( P(10) = 6100 \)

4. EXPONENTIALS The exponential function \( f(x) = e^x \) is a special function that you will learn about later. It is not a polynomial function. However, for small values of \( x \), the value of \( e^x \) is very closely approximated by the polynomial

\[ e^x = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + 1. \]

Use synthetic substitution to determine \( e(0.1) \). Show your work.

\[
\begin{array}{c|crrrr}
  \text{Divisor} & 1/6 & 1/2 & 1 & 1 \\
  \hline
  & 1/60 & 31/60 & 631/600 & 6631/6000 \\
  \hline
  & 631 & 6000
\end{array}
\]

5. Use synthetic division to show that \( x + 2 \) is a factor of \( v(x) \). Show your work.

\[
\begin{array}{c|cccc}
  \text{Divisor} & 1 & 10 & 31 & 30 \\
  \hline
  & 1 & -2 & 16 & -30 \\
  \hline
  & 1 & 8 & 15 & 0
\end{array}
\]

Because \( v(-2) = 0 \), \( x + 2 \) divides \( v(x) \).

6. Factor \( v(x) \) completely.

\[ v(x) = (x + 2)(x + 3)(x + 5) \]

7. Determine \( v(18) \) using any method you wish.

\[ v(18) = 9660 \]
is a root or solution of the polynomial equation.

Glencoe Algebra 2
Write

is a zero of the polynomial function.

Glencoe Algebra 2

3.5t^4 - 100t^3 + 350t - 100 = 0

Read the Lesson

1. Indicate whether each statement is true or false.
   a. Every polynomial equation of degree greater than one has at least one root in the set of real numbers. false
   b. If c is a root of the polynomial equation f(x) = 0, then (x - c) is a factor of the polynomial f(x). true
   c. If (x + c) is a factor of the polynomial f(x), then c is a zero of the polynomial function. false
   d. A polynomial function f of degree n has exactly (n - 1) complex zeros. false

2. Let f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 - 6x - 7.
   a. What are the possible numbers of positive real zeros of f? 5, 3, or 1
   b. Write f(-x) in simplified form (with no parentheses).
      x^6 + 2x^5 + 3x^4 + 4x^3 + 5x^2 + 6x - 7
   c. What are the possible numbers of negative real zeros of f? 1
   d. Complete the following chart to show the possible combinations of positive real zeros, negative real zeros, and imaginary zeros of the polynomial function.

Remember What You Learned

3. It is easier to remember mathematical concepts and results if you relate them to each other. How can the Complex Conjugates Theorem help you remember the part of Descartes’ Rule of Signs that says, “or is less than this number by an even number.” Sample answer: For a polynomial function in which the polynomial has real coefficients, imaginary zeros come in conjugate pairs. Therefore, there must be an even number of imaginary zeros. For each pair of imaginary zeros, the number of positive or negative zeros decreases by 2.

Fundamental Theorem of Algebra
Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

Contrary to the Fundamental Theorem of Algebra
A polynomial equation of the form \( P(x) = 0 \) of degree n with complex coefficients has exactly n roots in the set of complex numbers.

Descartes’ Rule of Signs
If \( P(x) = 0 \) is a polynomial with real coefficients whose terms are arranged in descending powers of the variable, the number of positive real zeros of \( y = P(x) \) is the same as or less than the number of changes in sign for its coefficients, or is less than this number by an even number, and the number of negative real zeros of \( y = P(x) \) is the same as or less than the number of changes in sign for the coefficients of the terms of \( P(-x) \), or is less than this number by an even number.

Example 1
Solve the equation \( 6x^3 + 3x = 0 \) and state the number and type of roots.
\( 6x^3 + 3x = 0 \)
\( 3x(2x^2 + 1) = 0 \)
Use the Zero Product Property.
\( 3x = 0 \) or \( 2x^2 + 1 = 0 \)
\( x = 0 \) or \( \frac{2x^2}{-1} = -1 \)
\( \pm \sqrt{\frac{1}{2}} \)
The equation has one real root, 0, and two imaginary roots, \( \pm \sqrt{\frac{1}{2}} \).

Example 2
State the number of positive real zeros, negative real zeros, and imaginary zeros for \( p(x) = 4x^3 - 3x^2 + x + 3 \).
Since \( p(x) \) has degree 4, it has 4 zeros.
Since there are three sign changes, there are 3 or 1 positive real zeros.
Find \( p(-x) \) and count the number of changes in sign for its coefficients.
\( p(-x) = 4(-x)^3 - 3(-x)^2 + (-x) + 3 \)
\( -4x^3 + 3x^2 - x + 3 \)
Since there is one sign change, there is exactly 1 negative real zero.
Thus, there are 3 positive and 1 negative real zero or 1 positive and 2 negative real zeros.

Exercises
Solve each equation and state the number and type of roots.
1. \( x^2 + 4x - 21 = 0 \)
   \( 0, -7 \); 2 real
2. \( 2x^3 - 5x = 0 \)
   \( 0, \pm 5 \); 3 real
3. \( 12x^3 + 100x = 0 \)
   \( 0, \pm \frac{5\sqrt{3}}{3} \); 1 real, 2 imaginary

State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.
4. \( f(x) = 3x^3 + x^2 - 8x - 12 \)
   \( 1; 2 or 0; 0 or 2 \)
5. \( f(x) = 3x^3 - x^2 + 6x^2 - 5 \)
   \( 3 or 1; 2 or 0; 0, 2, 4 \)
6-8 Skills Practice
Roots and Zeros

Find Zeros

Complex Conjugate Theorem
Suppose a and b are real numbers with b ≠ 0. If a + bi is a zero of a polynomial function with real coefficients, then a - bi is also a zero of the function.

Example
Find all of the zeros of \( f(x) = x^4 - 15x^2 + 38x - 60 \).
Since \( f(x) \) has degree 4, the function has 4 zeros.
\( f(x) = x^4 - 15x^2 + 38x - 60 \), \( f(-x) = x^4 - 15x^2 - 38x - 60 \)
Since there are 3 sign changes for the coefficients of \( f(x) \), the function has 3 or 1 positive real zeros. Since there is 1 sign change for the coefficients of \( f(-x) \), the function has 1 negative real zero. Use synthetic substitution to test some possible zeros.

1. \( x = 2 \ldots 0 -15 34 -60 \)

2. \( x = 4 \ldots -22 32 \)

3. \( x = 5 \ldots 1 -15 34 -60 \)

4. \( x = 3 \ldots 9 -18 60 \)

5. \( x = 1 \ldots 3 -6 20 \)

6. \( x = 1 \ldots 0 -15 34 -60 \)

So 3 is a zero of the polynomial function. Now try synthetic substitution again to find a zero of the depressed polynomial.

1. \( x = 3 \ldots -6 20 \)

2. \( x = 2 \ldots -2 -2 16 \)

3. \( x = 1 \ldots -8 36 \)

4. \( x = 3 \ldots -6 20 \)

5. \( x = 1 \ldots -1 -2 28 \)

6. \( x = 3 \ldots -6 20 \)

7. \( x = -5 \ldots 10 -20 \)

8. \( x = 1 \ldots -2 4 0 \)

So -5 is another zero. Use the Quadratic Formula on the depressed polynomial \( x^2 - 2x + 4 \) to find the other 2 zeros, 1 ± \( 4\sqrt{3} \).
The function has two real zeros at 3 and -5 and two imaginary zeros at 1 ± \( 4\sqrt{3} \).

Exercises

Find all of the zeros of each function.

1. \( f(x) = x^3 + x^2 + 9x + 9, -1, -3i \)
2. \( f(x) = x^3 - 3x^2 + 4x - 12, 3, 2i \)
3. \( p(x) = x^2 - 10x^2 + 34x - 40, 4, 3 \pm i \)
4. \( p(x) = x^3 - 5x^2 + 11x - 15, 3, 1 \pm 2i \)
5. \( f(x) = x^2 + 6x + 20, -2, 1 \pm 3i \)
6. \( f(x) = -3x^2 - 21x - 75x - 100, -1, 4, \pm 5i \)

Write a polynomial function of least degree with integral coefficients that has the given zeros.

19. \( -3, -5, 1 \)
\( f(x) = x^3 + 7x^2 + 7x - 15 \)
20. \( 3 \)
\( f(x) = x^2 + 9 \)
21. \( -5 + i \)
\( f(x) = x^2 + 10x + 26 \)
22. \( -1, \sqrt{3}, -\sqrt{3} \)
\( f(x) = x^3 + x^2 - 3x - 3 \)
23. \( 3, i \)
\( f(x) = x^2 + 26x^2 + 25 \)
24. \( -1, 1, i \)
\( f(x) = x^4 + 5x^2 - 6 \)
Solve each equation. State the number and type of roots.

1. \(-9x - 15 = 0\)
   \(-5, \frac{1}{3}; 1\) real

2. \(x^2 - 5x^2 + 4 = 0\)
   \(-1, -\frac{1}{2}, 2; 4\) real

3. \(x^2 = 8x\)
   \(0, -3, 3, -3i, 3i; 3\) real, 2 imaginary

4. \(x^2 + x^2 = 3x - 3 = 0\)
   \(-1, -\sqrt{3}, \sqrt{3}; 3\) real

5. \(x^3 + 6x + 20 = 0\)
   \(-2, 1 \pm 3i; 1\) real, 2 imaginary

6. \(x^3 - x^2 - x - 2 = 0\)
   \(0, -1, 2; 3\) real, 2 imaginary

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

7. \(f(x) = 4x^3 - 2x^2 + x + 3\)
   2 or 0; 1; 2 or 0

8. \(g(x) = 3x^4 + x^3 - 3x^2 + 7x + 5\)
   2 or 0; 2 or 0; 4, 2, or 0

9. \(h(x) = 7x^4 + 3x^3 - 2x^2 + x + 1\)
   2 or 0; 2 or 0; 4, 2, or 0

Find all the zeros of each function.

10. \(a(x) = 2x^4 - 3x^2 + 8x + 4\)
    \(1, \frac{1}{2}, 4\)

11. \(b(x) = x^3 - 7x^2 + 14x - 15\)
    \(-3, 2, 2 - i\)

12. \(c(x) = x^3 - 3x - 20\)
    \(1, -i, -\sqrt{3}, 1\)

13. \(d(x) = x^4 + 4x^3 - 3x^2 - 14x - 8\)
    \(-1, -i, 2, -i\)

14. \(e(x) = x^4 - 6x^3 + 6x^2 + 4x - 15\)
    \(-2, 3, 1, 3 + i\)

Write a polynomial function of least degree with integral coefficients that has the given zeros.

15. \(-5, 3i\)
    \(f(x) = x^3 + 5x^2 + 9x + 45\)

16. \(-2, 3 + i\)
    \(f(x) = x^3 - 4x^2 - 2x + 20\)

17. \(-1, 4, 3i\)
    \(f(x) = x^4 - 3x^3 + 5x^2 - 27x - 36\)

18. \(-2, 5, 1 + i\)
    \(f(x) = x^4 - 9x^3 + 26x^2 - 34x + 20\)

19. \(-10, x\(O - x)(6 - x) = 105; 3\ in.

20. \(-2\)

21. \(4 + 3i\) and \(4 - 3i\)

### Word Problem Practice

**Roots and Zeros**

1. **TABLES** Li Pang made a table of values for the polynomial \(p(x)\). Her table is shown below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(p(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Name three roots of \(p(x)\).

2. **ROOTS** Ryan is an electrical engineer. He often solves polynomial equations to work out various properties of the circuits he builds. For one circuit, he must find the roots of a polynomial \(p(x)\). He finds that \(p(2 - 3i) = 0\). Give two different roots of \(p(x)\).

3. **REAL ROOTS** Madison is studying the polynomial \(f(x) = x^6 - 14x^4 - 49x^2 - 36\). She knows that all of the roots of \(f(x)\) are real. How many positive and how many negative roots are there? How are the set of positive roots and negative roots related to each other? Explain. There are 3 positive and 3 negative roots. The set of positive roots is the mirror image of the set of negative roots because the polynomial is even.

4. **COMPLEX ROOTS** Eric is a statistician. During the course of his work, he had to find something called the "eigenvalues of a matrix," which was basically the same as finding the roots of a polynomial. The polynomial was \(x^4 + 6x^2 + 25\). One of the roots of this polynomial is \(1 + 2i\). What are the other 3 roots? Explain. The other 3 roots are \(-1 - 2i, 1 - 2i, -1 + 2i\). Sample answer: Since \(f(x) = f(-x)\), if \(r\) is a root, so is \(-r\); hence \(-1 - 2i\) is a root. Because the coefficients are real, if \(r\) is a root, its complex conjugate must also be a root; therefore, \(-1 - 2i\) and \(-1 + 2i\) are also roots.

**QUADRILATERALS** For Exercises 5-7, use the following information.

Shayna plotted the four vertices of a quadrilateral in the complex plane and then encoded the points in a polynomial \(p(x)\) by making them the roots of \(p(x)\). The polynomial \(p(x)\) is \(x^4 - 9x^3 + 27x^2 + 3x - 150\).

5. The polynomial \(p(x)\) has one positive real root, and it is an integer. Find the integer.

6. Find the negative real root(s) of \(p(x)\).

7. Find the complex roots of \(p(x)\).

4 + 3i and 4 - 3i
The Bisection Method for Approximating Real Zeros

The bisection method can be used to approximate zeros of polynomial functions like \( f(x) = x^3 + x^2 - 3x - 3 \).

Since \( f(1) = -4 \) and \( f(2) = 3 \), there is at least one real zero between 1 and 2. The midpoint of this interval is \( \frac{1+2}{2} = 1.5 \). Since \( f(1.5) = -1.875 \), the zero is between 1.5 and 2. The midpoint of this interval is \( \frac{1.5+2}{2} = 1.75 \). Since \( f(1.75) \) is about 0.172, the zero is between 1.5 and 1.75. The midpoint of this interval is \( \frac{1.5+1.75}{2} = 1.625 \) and \( f(1.625) \) is about -0.94. The zero is between 1.625 and 1.75. The midpoint of this interval is \( \frac{1.625+1.75}{2} = 1.6875 \). Since \( f(1.6875) \) is about -0.41, the zero is between 1.6875 and 1.75. Therefore, the zero is 1.7 to the nearest tenth.

The diagram below summarizes the results obtained by the bisection method.

Using the bisection method, approximate to the nearest tenth the zero between the two integral values of \( x \) for each function.

1. \( f(x) = x^3 - 4x^2 - 11x + 2, f(0) = 2, f(1) = -12 \) \( 0.2 \)

2. \( f(x) = 2x^4 + x^2 - 15, f(1) = -12, f(2) = 21 \) \( 1.6 \)

3. \( f(x) = x^5 - 2x^3 - 12, f(1) = -13, f(2) = -4 \) \( 1.9 \)

4. \( f(x) = 4x^3 - 2x + 7, f(-2) = -21, f(-1) = 5 \) \( -1.3 \)

5. \( f(x) = 3x^3 - 14x^2 - 27x + 126, f(4) = -14, f(5) = 16 \) \( 4.7 \)

Lesson Reading Guide

Rational Zero Theorem

Get Ready for the Lesson

Read the introduction to Lesson 6-9 in your textbook.

Rewrite the polynomial equation \( w^3 + 8(w - 5) = 2772 \) in the form \( f(w) = 0 \), where \( f(w) \) is a polynomial written in descending powers of \( w \).

\( w^3 + 3w^2 - 40w - 2772 = 0 \)

Read the Lesson

1. For each of the following polynomial functions, list all the possible values of \( p \), all the possible values of \( q \), and all the possible rational zeros \( \frac{p}{q} \).

   a. \( f(x) = x^3 - 2x^2 - 11x + 12 \)
   - possible values of \( p \): \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \)
   - possible values of \( q \): \( \pm 1 \)
   - possible values of \( \frac{p}{q} \): \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \)

   b. \( f(x) = 2x^4 + 9x^3 - 23x^2 - 8x + 45 \)
   - possible values of \( p \): \( \pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45 \)
   - possible values of \( q \): \( \pm 1 \)
   - possible values of \( \frac{p}{q} \): \( \pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{1}{3}, \pm \frac{3}{2}, \pm \frac{5}{3}, \pm \frac{9}{2}, \pm \frac{15}{3}, \pm \frac{45}{2} \)

2. Explain in your own words how Descartes’ Rule of Signs, the Rational Zero Theorem, and synthetic division can be used together to find all of the rational zeros of a polynomial function with integer coefficients.

   Sample answer: Use Descartes’ Rule to find the possible numbers of positive and negative real zeros. Use the Rational Zero Theorem to list all possible rational zeros. Use synthetic division to test which of the numbers on the list of possible rational zeros are actually zeros of the polynomial function. (Descartes’ Rule may help you to limit the possibilities.)

Remember What You Learned

3. Some students have trouble remembering which numbers go in the numerators and which go in the denominators when forming a list of possible rational zeros of a polynomial function. How can you use the linear polynomial equation \( ax + b = 0 \), where \( a \) and \( b \) are nonzero integers, to remember this?

   Sample answer: The solution of the equation is \( -\frac{b}{a} \). The numerator \( b \) is a factor of the constant term in \( ax + b \). The denominator \( a \) is a factor of the leading coefficient in \( ax + b \).
### Identify Rational Zeros

<table>
<thead>
<tr>
<th>Rational Zero Theorem</th>
<th>Let ( f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 ) represent a polynomial function with integer coefficients. If ( \frac{p}{q} ) is a rational number in simplest form and is a zero of ( f(x) ), then ( p ) is a factor of ( a_0 ) and ( q ) is a factor of ( a_n ).</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Corollary (Integral Zero Theorem)</td>
<td>If the coefficients of a polynomial are integers and ( a_0 = a_n ), then any rational zeros of the function must be factors of ( a_0 ).</td>
<td></td>
</tr>
</tbody>
</table>

### Example

**a.** \( f(x) = 3x^4 - 2x^2 + 6x - 10 \)

If \( \frac{p}{q} \) is a rational zero, then \( p \) is a factor of \(-10\) and \( q \) is a factor of \( 3 \). The possible values for \( p \) are \( \pm 1, \pm 2, \pm 5, \pm 10 \). The possible values for \( q \) are \( \pm 1, \pm 3 \). So all of the possible rational zeros are \( \frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3} \).

**b.** \( g(x) = x^3 - 10x^2 + 14x - 36 \)

Since the coefficient of \( x^3 \) is 1, the possible rational zeros must be the factors of the constant term \(-36\). So the possible rational zeros are \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36 \).

### Exercises

List all of the possible rational zeros of each function.

1. \( f(x) = x^3 + 3x^2 - x + 8 \)
   \( \pm 1, \pm 2, \pm 4, \pm 8 \)

2. \( g(x) = x^5 - 7x^4 + 3x^3 + x - 15 \)
   \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 15, \pm 18, \pm 36 \)

3. \( h(x) = x^3 - 7x^2 + 3x - 5 \)
   \( \pm 1, \pm 2, \pm 3, \pm 5, \pm 5, \pm 10, \pm 20 \)

4. \( i(x) = 3x^3 - 5x^2 - 3x - 5 \)
   \( \pm 1, \pm 2, \pm 3, \pm 5, \pm 5, \pm 10, \pm 20 \)

5. \( j(x) = 2x^3 + 5x^2 + 3x - 4 \)
   \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \)

6. \( k(x) = 4x^3 - 2x^2 + 18x - 15 \)
   \( \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 12 \)

7. \( l(x) = x^5 - 6x^4 - 3x^3 + x^2 + 4x - 120 \)
   \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 40, \pm 60, \pm 120 \)

8. \( m(x) = 2x^6 - 3x^5 + 3x^4 + 2x^3 - 15 \)
   \( \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 40, \pm 60, \pm 120 \)

9. \( n(x) = -x^4 + 3x^3 - 12x^2 + 16x^2 - 9x + 21 \)
   \( \pm 1, \pm 2, \pm 3, \pm 7, \pm 21, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2} \)

10. \( o(x) = p^2 - 3p^3 + 11x^5 - 20x^2 + 11 \)
     \( \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{1}{5}, \pm \frac{5}{5} \)

### Find All of the Rational Zeros of Each Function

1. \( f(x) = x^3 + 4x^2 - 25x - 28 \)
   \[ \pm 1, \pm 2, \pm 4, \pm 7 \]

2. \( g(x) = x^3 + 6x^2 + 4x + 24 \)
   \[ \pm 1, \pm 2, \pm 4, \pm 6, \pm 12, \pm 24 \]

3. \( h(x) = x^3 - 2x^2 - 11x + 36 \)
   \[ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 24, \pm 36 \]

4. \( i(x) = 3x^3 + 5x^2 + 30x^2 + 45x - 54 \)
   \[ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36, \pm 54 \]
6-9 Skills Practice

Rational Zero Theorem

List all of the possible rational zeros of each function.

1. \( n(x) = x^3 + 5x + 3 \)
   \[ \pm 1, \pm 3 \]

2. \( n(x) = x^3 - 2x - 5 \)
   \[ \pm 1, \pm 5 \]

3. \( n(x) = x^3 - 5x + 12 \)
   \[ \pm 1, \pm 2, \pm 3, \pm 6, \pm 12 \]

4. \( f(x) = 2x^2 + 5x + 3 \)
   \[ \pm \frac{1}{2}, \pm \frac{3}{2}, \pm 1, \pm 3 \]

5. \( q(x) = 6x^3 + x^2 - 2x \)
   \[ \pm \frac{1}{6}, \pm 1, \pm 2, \pm 3, \pm 6, \pm 12 \]

6. \( g(x) = 9x^4 + 3x^3 - 3x^2 - x + 27 \)
   \[ \pm \frac{1}{9}, \pm \frac{1}{3}, \pm 1, \pm 3, \pm 9, \pm 27 \]

Find all of the rational zeros of each function.

7. \( f(x) = x^3 - 2x^2 + 5x - 4 \)
   \[ -2, 2, 3 \]

8. \( g(x) = x^3 - 3x^2 - 4x + 12 \)
   \[ 2 \]

9. \( p(x) = x^3 - x^2 + x - 1 \)
   \[ 1 \]

10. \( z(x) = x^3 - 4x^2 + 6x - 4 \)
    \[ 2 \]

11. \( s(x) = x^3 - x^2 + 4x - 4 \)
    \[ 4 \]

12. \( g(x) = 3x^3 - 9x^2 - 10x - 8 \)
    \[ 4 \]

13. \( g(x) = 2x^3 + 7x^2 - 7x - 12 \)
    \[ -4, -\frac{3}{2}, 1, 2 \]

14. \( h(x) = 2x^3 - 5x^2 - 4x + 3 \)
    \[ -1, \frac{3}{2}, 3 \]

15. \( r(x) = 3x^3 - 5x^2 - 14x - 4 \)
    \[ -\frac{2}{3}, 1, 2 \]

16. \( q(x) = 3x^3 + 2x^2 + 27x + 18 \)
    \[ \frac{3}{2} \]

17. \( t(x) = 3x^3 - 7x^2 + 4 \)
    \[ -\frac{2}{3}, 1, 2 \]

18. \( f(x) = x^4 - 2x^3 + 13x^2 + 4x + 24 \)
    \[ -3, -1, 1, 2, 4 \]

19. \( n(x) = x^4 - 5x^3 + 9x^2 - 25x - 70 \)
    \[ -2, 7 \]

20. \( m(x) = 16x^4 - 32x^3 - 13x^2 + 29x - 6 \)
    \[ -\frac{1}{4}, \frac{3}{4}, 2 \]

6-9 Practice

Rational Zero Theorem

List all of the possible rational zeros of each function.

1. \( h(x) = x^3 - 2x^2 + 12x + 12 \)
   \[ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \]

2. \( k(x) = x^3 + 3x^2 + 2x + 12 \)
   \[ \pm 1, \pm 2, \pm 3, \pm 6, \pm 12 \]

3. \( f(x) = 3x^3 - 5x^2 + x + 6 \)
   \[ \pm \frac{1}{3}, \pm 2, \pm 1, \pm 2, \pm 3, \pm 6 \]

4. \( g(x) = 3x^3 - 2x + 7 \)
   \[ \pm \frac{1}{3}, \pm \frac{7}{3}, \pm 1, \pm 7 \]

5. \( g(x) = 5x^3 + x^2 - x + 8 \)
   \[ \pm 1, \pm 2, \pm 4, \pm 8 \]

6. \( f(x) = 6x^3 + x^2 - 2x \)
   \[ \pm 1, \pm 2, \pm 3, \pm 6, \pm 12 \]

7. \( q(x) = 3x^3 - 6x^2 + 27x - 27 \)
   \[ -4, -1, 2 \]

8. \( r(x) = 3x^3 - 9x^2 + 27x - 27 \)
   \[ 8 \]

9. \( g(x) = x^3 - x^2 - 8x + 12 \)
   \[ -3, 2 \]

10. \( f(x) = 4x^3 - 49x^2 + 0, -7, 7 \)

11. \( h(x) = x^3 - 7x^2 + 17x - 15 \)
    \[ 3 \]

12. \( k(x) = x^3 + 6x + 20 \)
    \[ -2 \]

13. \( f(x) = 6x^3 - 6x^2 + 24 \)
    \[ 4 \]

14. \( g(x) = 2x^3 + 3x^2 - 4x - 4 \)
    \[ -2 \]

15. \( h(x) = 2x^3 - 7x^2 + 21x + 54 - 3, 2, \frac{9}{2} \)
    \[ -1, 1, 2 \]

16. \( k(x) = x^4 - 3x^3 + 5x^2 - 27x + 36 - 1, 4 \)
    \[ 0, 2, -1 + i\sqrt{3}, -1 - i\sqrt{3} \]

Find all of the zeros of each function.

17. \( d(x) = x^4 + x^2 + 16 \)
   \[ no rational zeros \]

18. \( n(x) = 4x^3 - 2x^2 - 3 \)
    \[ -1 \]

19. \( p(x) = 2x^3 - 7x^2 + 4x + 7x - 6 \)
    \[ -\frac{3}{2}, \frac{3}{4} \]

20. \( q(x) = 6x^4 + 29x^3 + 40x^2 + 7x - 12 \)
    \[ -\frac{1}{4}, \frac{3}{4}, 2 \]

Find all of the zeros of each function.

21. \( f(x) = 3x^3 + 7x^2 + 2x + 12 \)
    \[ -3, -1, \frac{1}{2}, -1, 2, -i \]

22. \( q(x) = 4x^3 - 10x^2 + 18x - 4 \)
    \[ 2, 4 \]

23. \( m(x) = 6x^4 - 17x^3 + 8x^2 + 8x - 3 \)
    \[ 1, 3, 1 + \sqrt{5}, 1 - \sqrt{5} \]

24. \( g(x) = x^4 + 4x^3 + 5x^2 + 4x + 4 \)
    \[ -2, -2, -i, i \]

25. TRAVEL The height of a box that Joan is shipping is 3 inches less than the width of the box. The length is 2 inches more than twice the width. The volume of the box is 1540 ft³. What are the dimensions of the box? 22 in. by 10 in. by 7 in.

26. GEOMETRY The height of a square pyramid is 3 meters shorter than the side of its base. If the volume of the pyramid is 432 m³, how tall is it? Use the formula \( V = \frac{1}{3} Bh \). 9 m
1. **ROOTS**  
Paul was examining an old algebra book. He came upon a page about polynomial equations and saw the polynomial below.

\[ p(x) = 28x^4 - 288x^3 + 106x^2 - 17x + 1 \]

As you can see, all the middle terms were blotted out by an ink spill. What are all the possible rational roots of this polynomial?  
-8, -4, -2, -1, 1, 2, 4, 8

2. **IRRATIONAL CONSTANTS**  
Cherie was given a polynomial whose constant term was \( \sqrt{2} \). Is it possible for this polynomial to have a rational root? If it is not, explain why not. If it is possible, give an example of such a polynomial with a rational root.

Yes, it is possible. For example, the polynomial  
\[ x^2 - 1 = (1 + \sqrt{2})x - \sqrt{2} \]  
has 1 as a root.

3. **MARKOV CHAINS**  
Tara is a mathematician who specializes in probability. In the course of her work, she needed to find the roots of the polynomial  
\[ p(x) = 28x^4 - 288x^3 + 106x^2 - 17x + 1 \]

What are the roots of \( p(x) \)?

\[ \frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \text{ and } 1 \]

4. **PYRAMIDS**  
Pedro made a pyramid out of construction paper. The base of the pyramid is a square with side length 3 cm. The height of the pyramid is \( x + 12 \). The function for the volume of the pyramid is  
\[ V(x) = \frac{1}{3}x^2(3x + 36) \]

The actual volume of the pyramid is 168 cubic centimeters. What is the length of the sides of the base and height of the pyramid?

Base side length = 6 cm, height = 14 cm

5. **BOXES**  
For Exercises 5-7, use the following information.

Devon made a box with length \( x + 1 \), width \( x + 3 \), and height \( x - 3 \).

What is the volume of Devon’s box as a function of \( x \)?

\[ V(x) = x^3 + x^2 - 9x - 9 \]

6. What is \( x \) if the volume of the box is equal to 1001 cubic inches?

10

7. What is \( x \) if the volume of the box is equal to 14\( \frac{5}{8} \) cubic inches?

3.5

---

**Rational Zero Theorem**

A polynomial equation is an algebraic equation involving a polynomial expression.

1. Use the rational zero theorem to prove that \( \sqrt{2} \) is irrational. Consider \( p(x) = x^2 - 2 \). A known root of this polynomial is \( \sqrt{2} \). But by the Rational Zero Theorem the only possible rational zeros are \( \pm 1 \) or \( \pm 2 \).

2. Show that the square of an even number is even. Let \( a \) be even, \( a = 2k \). Now \( a^2 = 4k^2 = 2(2k^2) \), which is even.

3. Show that any integer zeros of a polynomial function must be factors of the constant term \( a_0 \).

Let \( k = \frac{p}{q} \) be an integer where \( p \) is a factor of \( a_0 \) and \( q \) a factor of \( a_n \). Therefore \( a_0 = k(qM) \), for some \( M \) and so \( k \) divides \( a_0 \).
The following program performs synthetic division and displays the depressed polynomial coefficients in rational form. The program will allow the testing of possible rational zeros of a polynomial function.

```plaintext
PROGRAM: SYNDIV
Disp "DEGREE OF DIVIDEND" P
1 → P
→ L
2(P)
Input M
Disp "COEFFICIENT" P+1 → P
Disp "COEFFICIENTS?" Input A
If P / M+1
Disp "0 SAME" A → L
1(P) Goto 3
Disp "1 QUOTIENT" If P / M+1
Stop
Disp "2 NEW" Goto 1
Lbl 4
Input U
Lbl 2
0 → P
Disp "POSSIBLE ROOT" 1 → P
Lbl 5
Input R
0 → S
→ P
If U / 0 Lbl 3
L2(P) → L
1(P) → F
If P / M+1 1F → S Goto 5
Goto 4
Disp Q / Frac
Goto 2
0 → P
Pause
Lbl 1
RQ → S
```

Find all of the rational zeros of \( f(x) = 2x^3 - 11x^2 + 12x + 9 \).

Use the program to test possible zeros.

Keystrokes: [SYNDIV] 3 2 1 2 11 12 9 .

Press ENTER until the screen displays Done. The column of numbers are the coefficients of the depressed polynomial. Since the last number is not zero, press 3 . Choose 0 for the same coefficients. Press 1 then until finished. Repeat this until a zero is found. Then press 2 for the degree of the depressed polynomial and 1 for the quotient.

The zeros are 3, 3, and \( \frac{1}{2} \).

Find all the zeros of each function.

1. \( f(x) = x^3 - 8x^2 + 24x + 32 \)
   \( 1, -2, 4 \)

2. \( f(x) = x^3 - 2x^2 - 11x + 18 \)
   \( 1, -2, 3 \)

3. \( p(x) = 3x^4 + x^3 - 11x^2 + 10x - 9 \)
   \( 1, -3, \frac{1}{3} \)

4. \( p(x) = x^4 - 3x^3 + 4x^2 - 3x + 1 \)
   \( 1, -1, 1 \)

5. \( p(x) = 3x^5 - x^4 - 11x^3 + 13x^2 - 15x + 4 \)
   \( 1, -2, 1 \)

6. \( p(x) = 3x^4 + x^2 - 2x + 5 \)
   \( 1, 0, -1, 3 \)