STANDARD 3

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Alternate Assessment Standards for Students with Severe Disabilities

Standard 1: Mathematical Analysis
Standard 3: Number and Numeration
Standard 3: Measurement
Standard 3: Patterns and Functions
**Standard 3**
Mathematics

Students will: understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

**Mathematical Reasoning**

**Key Idea:** Students use MATHEMATICAL REASONING to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

**Performance Indicators--Students will:**

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Intermediate</th>
<th>Commencement</th>
</tr>
</thead>
<tbody>
<tr>
<td>• use models, facts, and relationships to draw conclusions about mathematics and explain their thinking</td>
<td>• apply a variety of reasoning strategies</td>
<td>• construct simple logical arguments</td>
</tr>
<tr>
<td>• use patterns and relationships to analyze mathematical situations</td>
<td>• make and evaluate conjectures and arguments using appropriate language</td>
<td>• follow and judge the validity of logical arguments</td>
</tr>
<tr>
<td>• justify their answers and solution processes</td>
<td>• make conclusions based on inductive reasoning</td>
<td>• use symbolic logic in the construction of valid arguments</td>
</tr>
<tr>
<td>• use logical reasoning to reach simple conclusions</td>
<td>• justify conclusions involving simple and compound (i.e., and/or) statements</td>
<td>• construct proofs based on deductive reasoning</td>
</tr>
</tbody>
</table>

**Four-Year Sequence Commencement**

• construct indirect proofs or proofs using mathematical induction
• investigate and compare the axiomatic structures of various geometries
**Standard 3 Mathematics**

**Students will:** understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

**Number Sense & Numeration**

**Key Idea:** Students use **NUMBER SENSE AND NUMERATION** to develop an understanding of multiple uses of numbers in the real world, use of numbers to communicate mathematically, and use of numbers in the development of mathematical ideas.

**Performance Indicators--Students will:**

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Intermediate</th>
<th>Commencement</th>
</tr>
</thead>
<tbody>
<tr>
<td>• use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances</td>
<td>• understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded and scientific notation)</td>
<td>• understand and use rational and irrational numbers</td>
</tr>
<tr>
<td>• use concrete materials to model numbers and number relationships for whole numbers and common fractions, including decimal fractions</td>
<td>• understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations</td>
<td>• recognize the order of real numbers</td>
</tr>
<tr>
<td>• relate counting to grouping and to place-value</td>
<td>• develop an understanding of numbered theory (primes, factors, and multiples)</td>
<td>• apply the properties of the real numbers to various subsets of numbers</td>
</tr>
<tr>
<td>• recognize the order of whole numbers and commonly used fractions and decimals</td>
<td>• recognize order relations for decimals, integers, and rational numbers</td>
<td></td>
</tr>
<tr>
<td>• demonstrate the concept of percent through problems related to actual situations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Four-Year Sequence Commencement**

• understand the concept of infinity
• recognize the hierarchy of the complex number system
• model the structure of the complex number system
• recognize when to use and how to apply the field properties
**Standard 3 Mathematics**

Students will: understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

**Mathematical Operations & Relationships**

Key Idea: Students use **MATHEMATICAL OPERATIONS and RELATIONSHIPS** among them to understand mathematics.

**Performance Indicators--Students will:**

<table>
<thead>
<tr>
<th><strong>Elementary</strong></th>
<th><strong>Intermediate</strong></th>
<th><strong>Commencement</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• add, subtract, multiply, and divide whole numbers</td>
<td>• add, subtract, multiply, and divide fractions, decimals, and integers</td>
<td>• use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions</td>
</tr>
<tr>
<td>• develop strategies for selecting the appropriate computational and operational method in problem-solving situations</td>
<td>• explore and use the operations dealing with roots and powers</td>
<td>• develop an understanding of and use the composition of functions and transformations</td>
</tr>
<tr>
<td>• know single digit addition, subtraction, multiplication, and division facts</td>
<td>• use grouping symbols (parentheses) to clarify the intended order of operations</td>
<td>• explore and use negative exponents on integers and algebraic expressions</td>
</tr>
<tr>
<td>• understand the commutative and associative properties</td>
<td>• apply the associative, commutative, distributive, inverse, and identity properties</td>
<td>• use field properties to justify mathematical procedures</td>
</tr>
<tr>
<td></td>
<td>• demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.)</td>
<td>• use transformations on figures and functions in the coordinate plane</td>
</tr>
<tr>
<td></td>
<td>• develop appropriate proficiency with facts and algorithms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• apply concepts of ratio and proportion to solve problems</td>
<td></td>
</tr>
</tbody>
</table>

**Four-Year Sequence Commencement**
• use appropriate techniques, including graphing utilities, to perform basic operations on matrices
• use rational exponents on real numbers and all operations on complex numbers
• combine functions using the basic operations and the composition of two functions
### Mathematical Modeling/ Multiple Representation

**Key Idea:** Students use **MATHEMATICAL MODELING/MULTIPLE REPRESENTATION** to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

**Performance Indicators--Students will:**

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Intermediate</th>
<th>Commencement</th>
</tr>
</thead>
<tbody>
<tr>
<td>• use concrete materials to model spatial relationships</td>
<td>• visualize, represent, and transform two- and three-dimensional shapes</td>
<td>• represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs</td>
</tr>
<tr>
<td>• construct tables, charts, and graphs to display and analyze real-world data</td>
<td>• use maps and scale drawings to represent real objects or places</td>
<td>• manipulate symbolic representations to explore concepts at an abstract level</td>
</tr>
<tr>
<td>• use multiple representations (simulations, manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures</td>
<td>• use the coordinate plane to explore geometric ideas</td>
<td>• choose appropriate representations to facilitate the solving of a problem</td>
</tr>
<tr>
<td>• use variables such as height, weight, and hand size to predict changes over time</td>
<td>• represent numerical relationships in one- and two-dimensional graphs</td>
<td>• use learning technologies to make and verify geometric conjectures</td>
</tr>
<tr>
<td>• use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts</td>
<td>• use variables to represent relationships</td>
<td>• justify the procedures for basic geometric constructions</td>
</tr>
<tr>
<td></td>
<td>• use concrete materials and diagrams to describe the operation of real world processes and systems</td>
<td>• investigate transformations in the coordinate plane</td>
</tr>
<tr>
<td></td>
<td>• develop and explore models that do and do not rely on chance</td>
<td>• develop meaning for basic conic sections</td>
</tr>
</tbody>
</table>
three-dimensional transformations
• use appropriate tools to construct and verify geometric relationships
• develop procedures for basic geometric constructions
• develop and apply the concept of basic loci to compound loci

• use graphing utilities to create and explore geometric and algebraic models

• model real-world problems with systems of equations and inequalities

### Four-Year Sequence Commencement
• model vector quantities both algebraically and geometrically
• represent graphically the sum and difference of two complex numbers
• model and solve problems that involve absolute value, vectors, and matrices
• model quadratic inequalities both algebraically and graphically
• model the composition of transformations
• determine the effects of changing parameters of the graphs of functions
• use polynomial, rational, trigonometric, and exponential functions to model real-world relationships
• use algebraic relationships to analyze the conic sections
• use circular functions to study and model periodic real-world phenomena
• illustrate spatial relationships using perspective, projections, and maps
• represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations
• analyze spatial relationships using the Cartesian coordinate system in three dimensions
**Standard 3**

**Mathematics**

**Students will:** understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

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**Measurement**

**Key Idea:** Students use **MEASUREMENT** in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

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**Performance Indicators--Students will:**

<table>
<thead>
<tr>
<th>Elementary</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• understand that measurement is approximate, never exact</td>
<td>• estimate, make, and use measurements in real-world situations</td>
<td>• derive and apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts</td>
</tr>
<tr>
<td>• select appropriate standard and nonstandard measurement tools in measurement activities</td>
<td>• select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy</td>
<td>• choose the appropriate tools for measurement</td>
</tr>
<tr>
<td>• understand the attributes of area, length, capacity, weight, volume, time, temperature, and angle</td>
<td>• develop measurement skills and informally derive and apply formula in direct measurement activities</td>
<td>• use dimensional analysis</td>
</tr>
<tr>
<td>• estimate and find measures such as length, perimeter, area, and volume using both nonstandard and standard units</td>
<td>• use statistical methods and measures of central tendencies to display, describe, and compare data</td>
<td>• use statistical methods including measures of central tendency to describe and compare data</td>
</tr>
<tr>
<td>• collect and display data</td>
<td>• explore and produce graphic representations of data using calculators/computers</td>
<td>• use trigonometry as a method to measure indirectly</td>
</tr>
<tr>
<td>• use statistical methods such as graphs, tables, and charts to interpret data</td>
<td>• develop critical judgment for the reasonableness of measurement</td>
<td>• apply proportions to scale drawings, computer-assisted design blueprints, and direct variation in order to compute indirect measurements</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• relate absolute value, distance between two points, and the slope of a line to the coordinate</td>
</tr>
</tbody>
</table>
• understand error in measurement and its consequence on subsequent calculations

• use geometric relationships in relevant measurement problems involving geometric concepts

**Four-Year Sequence Commencement**

• derive and apply formulas relating angle measure and arc degree measure in a circle
• prove and apply theorems related to lengths of segments in a circle
• define the trigonometric functions in terms of the unit circle
• relate trigonometric relationships to the area of a triangle and to the general solutions of triangles
• apply the normal curve and its properties to familiar contexts
• design a statistical experiment to study a problem and communicate the outcomes, including dispersion
• use statistical methods, including scatter plots and lines of best fit, to make predictions
• apply the conceptual foundation of limits, infinite sequences and series, the area under a curve, rate of change, inverse variation, and the slope of a tangent line to authentic problems in mathematics and other disciplines
• determine optimization points on a graph
• use derivatives to find maximum, minimum, and inflection points of a function
### Standard 3
**Mathematics**

**Students will:** understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

### Ideas of Uncertainty

**Key Idea:** Students use IDEAS of UNCERTAINTY to illustrate that mathematics involves more than exactness when dealing with everyday situations.

### Performance Indicators--Students will:

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<tbody>
<tr>
<td>• make estimates to compare to actual results of both formal and informal measurement</td>
<td>• use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology</td>
<td>• judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics</td>
</tr>
<tr>
<td>• make estimates to compare to actual results of computations</td>
<td>• use estimation to solve problems for which exact answers are inappropriate</td>
<td>• judge the reasonableness of a graph produced by a calculator or computer</td>
</tr>
<tr>
<td>• recognize situations where only an estimate is required</td>
<td>• estimate the probability of events</td>
<td>• use experimental or theoretical probability to represent and solve problems involving uncertainty</td>
</tr>
<tr>
<td>• develop a wide variety of estimation skills and strategies</td>
<td>• use simulation techniques to estimate probabilities</td>
<td>• use the concept of random variable in computing probabilities</td>
</tr>
<tr>
<td>• determine the reasonableness of results</td>
<td>• determine probabilities of independent and mutually exclusive events</td>
<td>• determine probabilities using permutations and combinations</td>
</tr>
<tr>
<td>• predict experimental probabilities</td>
<td>• make predictions using unbiased random samples</td>
<td></td>
</tr>
</tbody>
</table>
• interpret probabilities in real-world situations
• use a Bernoulli experiment to determine probabilities for experiments with exactly two outcomes
• use curve fitting to predict from data
• apply the concept of random variable to generate and interpret probability distributions
• create and interpret applications of discrete and continuous probability distributions
• make predictions based on interpolations and extrapolations from data
• obtain confidence intervals and test hypotheses using appropriate statistical methods
• approximate the roots of polynomial equations
Standard 3
Mathematics

Students will: understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Patterns and Functions

**Key Idea:** Students use PATTERNS and FUNCTIONS to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

**Performance Indicators--Students will:**

<table>
<thead>
<tr>
<th>Elementary</th>
<th>Intermediate</th>
<th>Commencement</th>
</tr>
</thead>
<tbody>
<tr>
<td>• recognize, describe, extend, and create a wide variety of patterns</td>
<td>• recognize, describe, and generalize a wide variety of patterns and functions</td>
<td>• use function vocabulary and notation</td>
</tr>
<tr>
<td>• represent and describe mathematical relationships</td>
<td>• describe and represent patterns and functional relationships using tables, charts and graphs, algebraic expressions, rules, and verbal descriptions</td>
<td>• represent and analyze functions using verbal descriptions, tables, equations, and graphs</td>
</tr>
<tr>
<td>• explore and express relationships using variables and open sentences</td>
<td>• develop methods to solve basic linear and quadratic equations</td>
<td>• translate among the verbal descriptions, tables, equations and graphic forms of functions</td>
</tr>
<tr>
<td>• solve for an unknown using manipulative materials</td>
<td>• develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another</td>
<td>• analyze the effect of parametric changes on the graphs of functions</td>
</tr>
<tr>
<td>• use a variety of manipulative materials and technologies to explore patterns</td>
<td>• verify results of substituting variables</td>
<td>• apply linear, exponential, and quadratic functions in the solution of problems</td>
</tr>
<tr>
<td>• interpret graphs</td>
<td>• apply the concept of similarity in relevant situations</td>
<td>• apply and interpret transformations to functions</td>
</tr>
<tr>
<td>• explore and develop relationships among two- and three-dimensional geometric shapes</td>
<td>• use properties of polygons to classify them</td>
<td>• model real-world situations with the appropriate function</td>
</tr>
<tr>
<td>• discover patterns in nature, art, music, and literature</td>
<td></td>
<td>• apply axiomatic structure to algebra and geometry</td>
</tr>
</tbody>
</table>
• explore relationships involving points, lines, angles, and planes

• develop and apply the Pythagorean principle in the solution of problems

• explore and develop basic concepts of right triangle trigonometry

• use patterns and functions to represent and solve problems

Four-Year Sequence Commencement

• solve equations with complex roots using a variety of algebraic and graphical methods with appropriate tools
• understand and apply the relationship between rectangular form and polar form of a complex number
• evaluate and form the composition of functions
• use the definition of a derivative to examine properties of a function
• solve equations involving fractions, absolute values, and radicals
• use basic transformations to demonstrate similarity and congruence of figures
• identify and differentiate between direct and indirect isometrics
• analyze inverse functions using transformations
• apply ideas of symmetries in sketching and analyzing graphs of functions
• use the normal curve to answer questions about data
• develop methods to solve trigonometric equations and verify trigonometric functions
• describe patterns produced by processes of geometric change, formally connecting iteration, approximations, limits, and fractals
• extend pattern and compute the nth term in numerical and geometric sequences
• use the limiting process to analyze infinite sequences and series
• use algebraic and geometric iteration to explore patterns and solve problems
• solve optimization problems
• use linear programming and difference equations in the solution of problems
STANDARD 1
Analysis, Inquiry and Design

Mathematical Analysis

Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers and develop solutions.

Key Idea: Symbolic representation is used to communicate mathematically.

ALTERNATE ASSESSMENT

Performance Indicators--Students:

• use mathematics and symbolism to communicate in mathematics

• compare and describe quantities

• demonstrate knowledge of mathematical relationships

• relate mathematics to their immediate environment
**STANDARD 3**

**Mathematics**

**Number Sense & Numeration**

**Students will:** understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

**Key Idea:** Students use number sense and numeration to communicate mathematically and use numbers in the development of concrete mathematical ideas.

**ALTERNATE ASSESSMENT**

**Performance Indicators--Students:**

- use single digit whole numbers to identify locations, quantify groups of objects, and measure distances

- use concrete materials to model numbers and number relationships for whole numbers and simple fractions

- relate counting to grouping using manipulatives

- recognize the order of whole numbers up to 12 and commonly used simple fractions

- recognize coins and dollars and their value
STANDARD 3  
Mathematics

**Measurement**  
*Students will:* understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

**Key Idea:** Students use measurement in real-world situations.

**ALTERNATE ASSESSMENT**

**Performance Indicators--Students:**
- use appropriate standard and nonstandard measurement tools in measurement activities
- understand the simple attributes of length, weight, volume, time, and temperature
- measure the length or volume of an object
- collect and display simple data
Patterns and Functions

Students will: understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Key Idea: Students use mathematical patterns in a real-world situation.

ALTERNATE ASSESSMENT

Performance Indicators--Students:
• recognize and duplicate simple patterns

• use a variety of manipulative materials and technologies to explore patterns

• recognize simple patterns in nature, art, music, and literature