Combined Mark Schemes
And Report on the Units

January 2006
OCR (Oxford, Cambridge and RSA Examinations) is a unitary awarding body, established by the University of Cambridge Local Examinations Syndicate and the RSA Examinations Board in January 1998. OCR provides a full range of GCSE, A level, GNVQ, Key Skills and other qualifications for schools and colleges in the United Kingdom, including those previously provided by MEG and OCEAC. It is also responsible for developing new syllabuses to meet national requirements and the needs of students and teachers.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners’ meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2006

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annersley
NOTTINGHAM
NG15 0DL

Telephone: 0870 870 6622
Facsimile: 0870 870 6621
E-mail: publications@ocr.org.uk
## MARK SCHEME ON THE UNITS

<table>
<thead>
<tr>
<th>Unit</th>
<th>Content</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4721</td>
<td>Core Mathematics 1</td>
<td>1</td>
</tr>
<tr>
<td>4722</td>
<td>Core Mathematics 2</td>
<td>7</td>
</tr>
<tr>
<td>4723</td>
<td>Core Mathematics 3</td>
<td>11</td>
</tr>
<tr>
<td>4724</td>
<td>Core Mathematics 4</td>
<td>15</td>
</tr>
<tr>
<td>4725</td>
<td>Further Pure Mathematics 1</td>
<td>21</td>
</tr>
<tr>
<td>4726</td>
<td>Further Pure Mathematics 2</td>
<td>27</td>
</tr>
<tr>
<td>4727</td>
<td>Further Pure Mathematics 3</td>
<td>31</td>
</tr>
<tr>
<td>4728</td>
<td>Mechanics 1</td>
<td>37</td>
</tr>
<tr>
<td>4729</td>
<td>Mechanics 2</td>
<td>43</td>
</tr>
<tr>
<td>4730</td>
<td>Mechanics 3</td>
<td>47</td>
</tr>
<tr>
<td>4732</td>
<td>Probability &amp; Statistics 1</td>
<td>51</td>
</tr>
<tr>
<td>4733</td>
<td>Probability &amp; Statistics 2</td>
<td>57</td>
</tr>
<tr>
<td>4734</td>
<td>Probability &amp; Statistics 3</td>
<td>61</td>
</tr>
<tr>
<td>4736</td>
<td>Decision Mathematics 1</td>
<td>65</td>
</tr>
<tr>
<td>4737</td>
<td>Decision Mathematics 2</td>
<td>69</td>
</tr>
</tbody>
</table>
### REPORTS FOR THE UNITS

<table>
<thead>
<tr>
<th>Unit</th>
<th>Content</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>Chief Examiner's Report for Pure Mathematics</td>
<td>76</td>
</tr>
<tr>
<td>4721</td>
<td>Core Mathematics 1</td>
<td>77</td>
</tr>
<tr>
<td>4722</td>
<td>Core Mathematics 2</td>
<td>81</td>
</tr>
<tr>
<td>4723</td>
<td>Core Mathematics 3</td>
<td>85</td>
</tr>
<tr>
<td>4724</td>
<td>Core Mathematics 4</td>
<td>88</td>
</tr>
<tr>
<td>4725</td>
<td>Further Pure Mathematics 1</td>
<td>90</td>
</tr>
<tr>
<td>4726</td>
<td>Further Pure Mathematics 2</td>
<td>92</td>
</tr>
<tr>
<td>4727</td>
<td>Further Pure Mathematics 3</td>
<td>94</td>
</tr>
<tr>
<td>4728</td>
<td>Mechanics 1</td>
<td>97</td>
</tr>
<tr>
<td>4729</td>
<td>Mechanics 2</td>
<td>99</td>
</tr>
<tr>
<td>4730</td>
<td>Mechanics 3</td>
<td>100</td>
</tr>
<tr>
<td>*</td>
<td>Chief Examiner’s Report for Probability &amp; Statistics</td>
<td>102</td>
</tr>
<tr>
<td>4732</td>
<td>Probability &amp; Statistics 1</td>
<td>103</td>
</tr>
<tr>
<td>4733</td>
<td>Probability &amp; Statistics 2</td>
<td>108</td>
</tr>
<tr>
<td>4734</td>
<td>Probability &amp; Statistics 3</td>
<td>110</td>
</tr>
<tr>
<td>*</td>
<td>Report for Decision Mathematics</td>
<td>112</td>
</tr>
<tr>
<td>4736</td>
<td>Decision Mathematics 1</td>
<td>113</td>
</tr>
<tr>
<td>4737</td>
<td>Decision Mathematics 2</td>
<td>116</td>
</tr>
<tr>
<td>*</td>
<td>Grade Thresholds</td>
<td>119</td>
</tr>
<tr>
<td>Question</td>
<td>Mark Scheme</td>
<td>January 2006</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td>--------------</td>
</tr>
</tbody>
</table>
| 1 (i)    | \( \frac{1}{x^3} = 2 \)  
\( x = 8 \) | B1 | 1 8 |
| (ii)     | \( 10^t = 1 \)  
\( t = 0 \) | B1 | 1 0 |
| (iii)    | \( (y^2)^2 = \frac{1}{81} \)  
\( y^4 = \frac{1}{81} \)  
\( y = \pm 3 \) | B1 | B1 2 3 y = 3  
B1 | B1 | B1 | y = -3 |
| 2 (i)    | \( (3x + 1)^2 - 2(2x - 3)^2 \)  
\( = (9x^2 + 6x + 1) - 2(4x^2 - 12x + 9) \)  
\( = x^2 + 30x - 17 \) | M1 | 1 9x^2 + 6x + 1 or 4x^2 - 12x + 9 soi  
A1 | A1 | 3 \( x^2 + 30x - 17 \) |
| (ii)     | \( 2x^3 + 6x^3 + 4x^3 = 12x^3 \) | B1 | 2 12 or 12 x^3 |
| 3 (i)    | \( \frac{dy}{dx} = 15x^4 - \frac{1}{2}x^{-\frac{1}{2}} \) | B1 | 15x^4  
B1 | B1 | B1 | \( kx^{-\frac{1}{2}} \)  
B1 | B1 | B1 | \( cx^4 - \frac{1}{2}x^{-\frac{1}{2}} \) only |
| (ii)     | \( \frac{d^2y}{dx^2} = 60x^3 + \frac{1}{4}x^{-\frac{3}{2}} \) | M1 | A1 | 2 Attempt to differentiate their 2 term \( \frac{dy}{dx} \) and  
get one correctly differentiated term  
\( 60x^3 + \frac{1}{4}x^{-\frac{3}{2}} \) |
| 4 (i)    | | B1 | Correct curve in one quadrant  
B1 | 2 Completely correct |
| (ii)     | | M1 | Translate (i) horizontally  
A1 | 2 Translates all of their (i) \( \begin{pmatrix} 3 \\ 0 \end{pmatrix} \)  
3 must be labelled or stated |
| (iii)    | (One-way) stretch, sf 2, parallel to the y-axis | B1 | Stretch  
B1 | (Scale) factor 2  
B1 | B1 | B1 | Parallel to y-axis o.e.  
| | | | **SR**  
| | | | Stretch B1  
<p>| | | | Sf ( \sqrt{2} ) parallel to x-axis B2 |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(i)</td>
<td>( x^2 + 3x = \left( x + \frac{3}{2} \right)^2 - \frac{9}{4} )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>( y^2 - 4y - \frac{11}{4} = (y - 2)^2 - \frac{27}{4} )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii)</td>
<td>Centre ( \left( -\frac{3}{2}, 2 \right) )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iv)</td>
<td>Radius = ( \sqrt{\frac{27}{4} + \frac{9}{4}} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(i)</td>
<td>( y = x^3 - 3x^2 + 4 )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{dy}{dx} = 3x^2 - 6x )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 3x^2 - 6x = 0 )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 3x(x - 2) = 0 )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 0 ) ( x = 2 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = 4 ) ( y = 0 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>( \frac{d^2y}{dx^2} = 6x - 6 )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 0 ) ( y'' = -6 ) ( -ve \ max )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 2 ) ( y'' = 6 ) ( +ve \ min )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii)</td>
<td>Increasing</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x &lt; 0 ) ( x &gt; 2 )</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

**Mark Scheme**

1. \( a = \frac{3}{2} \)
2. \( b = -\frac{9}{4} \) o.e.
3. \( p = -2 \)
4. \( q = -\frac{27}{4} \) o.e.
5. \( \sqrt{-their' b'^2 - their' q'^2} \) or use \( \sqrt{f^2 + g^2 - c} \)

**Notes**

- N.B. If question is restarted in this part, ft from part (iii) working only
- \( 3x^2 - 6x \) 1 term correct
- Completely correct
- \( \frac{dy}{dx} = 0 \)
- Correct method to solve quadratic
- \( x = 0, 2 \)
- \( y = 4, 0 \)
- SR one correct \((x, y)\) pair
- Correct method to find nature of stationary points (can be a sketch)
- \( x = 0 \) max
- \( x = 2 \) min
- (N.B. If no method shown but both min and max correctly stated, award all 3 marks)
- Any inequality (or inequalities) involving both their \( x \) values from part (i)
- Allow \( x \leq 0 \) \( x \geq 2 \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Mark Scheme</th>
<th>January 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7 (i)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ x = \frac{8 \pm \sqrt{64 - 44}}{2} ]</td>
<td>M1</td>
<td><strong>Correct use of formula</strong></td>
</tr>
<tr>
<td>[ = \frac{8 \pm \sqrt{20}}{2} ]</td>
<td>A1</td>
<td>[ \frac{8 \pm \sqrt{20}}{2} \text{ aef} ]</td>
</tr>
<tr>
<td>[ = 4 \pm \sqrt{5} ]</td>
<td>B1</td>
<td>[ \sqrt{20} = 2\sqrt{5} \text{ soi} ]</td>
</tr>
<tr>
<td>[ 4 \pm \sqrt{5} ]</td>
<td>A1</td>
<td><strong>4</strong></td>
</tr>
</tbody>
</table>

**Alternative method**

\[ (x - 4)^2 - 16 + 11 = 0 \]

\[ (x - 4)^2 = 5 \]

\[ x = 4 + \sqrt{5} \]

or \[ 4 - \sqrt{5} \]

**B1**

\[ +ve \text{ parabola} \]

**B1**

\[ \text{Root(s) in correct places} \]

**B1**

\[ \text{3} \]

\[ \text{Completely correct curve with roots and (0, 11) labelled or referenced} \]

| **7 (ii)** |  |  |
|  |  |  |
| \[ y = x^2 = (4 \pm \sqrt{5})^2 \]  | M1 | **y = x^2** soi |
| \[ = 16 + 5 \pm 8\sqrt{5} \]  | M1 | Attempt to square at least one answer from part (i) |
| \[ = 21 \pm 8\sqrt{5} \]  | A1 | **4** |

**Correct evaluation of** \[ (a + b \sqrt{c})^2 \text{ (a,b,c} \neq 0) \]

**A1**
<table>
<thead>
<tr>
<th>Question</th>
<th>Description</th>
<th>Marks</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 (i)</td>
<td>Attempt to eliminate $y$</td>
<td>M1 A1</td>
<td>$x^2 - 10x + 25 = 0$ AG Obtained with no wrong working seen</td>
</tr>
<tr>
<td>(ii)</td>
<td>$b^2 - 4ac = 100 - 100 = 0$</td>
<td>B1</td>
<td>Do not allow $\sqrt{(b^2 - 4ac)}$</td>
</tr>
<tr>
<td>(iii)</td>
<td>Line is a tangent to the curve</td>
<td>B1√</td>
<td>Tangent or ‘touches’ N.B. Strict ft from their discriminant</td>
</tr>
<tr>
<td>(iv)</td>
<td>$x^2 - 10x + 25 = 0$</td>
<td>M1 A1 A1</td>
<td>Correct method to solve 3 term quadratic $x = 5$ $y = 15$</td>
</tr>
<tr>
<td>(v)</td>
<td>Gradient of tangent = 5</td>
<td>B1</td>
<td>Gradient of tangent = 5</td>
</tr>
<tr>
<td></td>
<td>Gradient of normal = $-\frac{1}{5}$</td>
<td>B1√</td>
<td>Gradient of normal = $-\frac{1}{5}$</td>
</tr>
<tr>
<td></td>
<td>$y - 15 = -\frac{1}{5}(x - 5)$</td>
<td>M1</td>
<td>Correct equation of straight line, any gradient, passing through (5, 15)</td>
</tr>
<tr>
<td></td>
<td>$x + 5y = 80$</td>
<td>A1</td>
<td>$x + 5y = 80$</td>
</tr>
<tr>
<td>9</td>
<td>(i) Length AC = ( \sqrt{(8 - 5)^2 + (2 - 1)^2} )</td>
<td>M1</td>
<td>Uses ( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} )</td>
</tr>
<tr>
<td></td>
<td>= ( \sqrt{3^2 + 1^2} )</td>
<td>A1</td>
<td>( \sqrt{10} ) (± ( \sqrt{10} ) scores A0)</td>
</tr>
<tr>
<td></td>
<td>= ( \sqrt{10} )</td>
<td>A1</td>
<td>( \sqrt{(p - 5)^2 + (7 - 1)^2} )</td>
</tr>
<tr>
<td></td>
<td>Length AB = ( \sqrt{(p - 5)^2 + (7 - 1)^2} )</td>
<td>M1</td>
<td>AB = 2AC (with algebraic expression) used</td>
</tr>
</tbody>
</table>
|   | \[\begin{align*}
&= \sqrt{(p - 5)^2 + 36} \\
&= \sqrt{(p - 5)^2 + 36} = 2\sqrt{10} \\
p^2 - 10p + 25 + 36 &= 40 \\
p^2 - 10p + 21 &= 0 \\
(p - 7)(p - 3) &= 0 \\
p &= 7, 3
\end{align*}\] | A1 | Obtains 3 term quadratic = 0 suitable for solving \( (p - 5)^2 = 4 \_ \_ \) |
|   | A1 | 7 | \( p = 7 \) |
|   | A1 | | \( p = 3 \) |
|   | SR If no working seen, and one correct value found, award B2 in place of the final 4 marks in part (i) |

| (ii) | \( 7 = 3x - 14 \) | M1 | Correct method to find \( x \) |
|      | \( x = 7 \) | A1 | \( x = 7 \) |
|      | (5, 1) (7, 7) | M1 | Use \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \) |
|      | Mid-point (6, 4) | A1 | (6, 4) or correct midpoint for their AB |

<p>| Alternative method | ( y ) coordinate of midpoint = 4 M1 A1 |
| sub 4 into equation of line | M1 |
| obtains ( x = 6 ) | A1 |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Mark Scheme</th>
<th>January 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(i)</td>
<td>( a + 19d = 10 ), ( a + 49d = 70 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attempt to find ( d ) from simultaneous ( a + (n-1)d ) or equiv method</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 30d = 60 \Rightarrow d = 2 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( a + (19 \times 2) = 10 ) or ( a + (49 \times 2) = 70 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>( S = \frac{29}{2} \left( 2 \times -28 + (29 - 1) \times 2 \right) = 0 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtain ( a = -28 )</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For relevant use of ( \frac{1}{2}n(2a + (n-1)d) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For showing the given result correctly</td>
<td>AG</td>
</tr>
<tr>
<td>2</td>
<td>(i)</td>
<td>( \Delta = \frac{1}{2} \times 10 \times 7 \times \sin 80 = 34.5 \text{cm}^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For use of ( \frac{1}{2} \text{ca} \sin B ) or complete equiv.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>( b^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 80 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hence length of ( CA ) is 11.2 cm</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(iii)</td>
<td>( \sin C = \frac{10 \sin 80}{11.166...} = 0.8819... )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hence angle ( C ) is 61.9°</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For correct value 34.5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For attempted use of the correct cosine formula</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For correct value 11.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For use of the sine rule to find ( C ), or equivalent</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For correct value 61.9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(i)</td>
<td>( (1 - 2x)^2 = 1 - 24x + 264x^2 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attempt ( x^2 ) term, including attempt at binomial coeff.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtain 1 and (-24x \ldots )</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>( (1 \times 264) + (3 \times -24) = 192 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attempt coefficient of ( x^2 ) from two pairs of terms</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtain correct unsimplified expression</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtain 192</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(i)</td>
<td>perimeter = ( (15 \times 1.8) + (20 \times 1.8) + 5 + 5 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 73 \text{cm} )</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>area = ( \left( \frac{1}{2} \times 20^2 \times 1.8 \right) - \left( \frac{1}{2} \times 15^2 \times 1.8 \right) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 157.5 \text{cm}^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attempt area of sector using ( kr^2 \theta )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Find difference between attempts at two sectors</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtain 157.5 / 158</td>
<td></td>
</tr>
</tbody>
</table>
5 (i)  
\[
\frac{r}{5} = 0.96 \Rightarrow S_\infty = \frac{5}{0.04} = 125
\]
B1*  
For correct value of \( r \) used
B1  
For correct use of \( \frac{a}{1-r} \) to show given answer AG

(ii)  
\[
S_n = \frac{5(1 - 0.96^n)}{1 - 0.96} 
\]
B1  
For correct, unsimplified, \( S_n \)
Hence \( 1 - 0.96^n > 0.992 \Rightarrow 0.96^n < 0.008 \)
M1  
For linking \( S_n \) to \( 124 (> \text{or} =) \) and multiplying through by 0.04, or equiv.
A1  
For showing the given result correctly, with correct inequality throughout AG

\[
n \log 0.96 < \log 0.008
\]
M1  
For correct log statement seen or implied (ignore sign)
Hence \( \log 0.008 \approx 118.3 \log 0.96 \)
M1  
For dividing both sides by \( \log 0.96 \)
A1  
For correct (integer) value 119

6 (a) \[
\frac{2}{3} x^2 + 4x + c
\]
M1  
For \( kx^2 \)
A1  
For correct first term \( \frac{2}{3} x^2 \), or equiv
B1  
For correct second term \( 4x \)
B1  
For \(+c\)

(b)(i) \[
\int_1^4 4x^2 \, dx = \left[-4x^{-1}\right]_1^4
\]
M1  
Obtain integral of the form \( kx^l \)

\[
= 4 - \frac{4}{a}
\]
M1  
Use limits \( x = a \) and \( x = 1 \)
A1  
Obtain \( 4 - \frac{4}{a} \) or equivalent
B1V  
State 4, or legitimate conclusion from their (b)(i)

7 (i)(a) \[
\log_{10} x - \log_{10} y
\]
B1  
For the correct answer
M1  
Sum of three log terms involving 10, \( x^2, y \)
A1  
For correct term \( 2 \log_{10} x \)
A1  
For both correct terms 1 and \( \log_{10} y \)

(b) \[
1 + 2 \log_{10} x + \log_{10} y
\]
M1  
For relevant use of results from (i)
A1  
For a correct, unsimplified, equation in \( \log_{10} y \) only
A1  
For correct use of \( a = \log_{10} e \Leftrightarrow \log_{10} c = 10^a \)
A1  
For the correct value 0.215

(ii) \[
2 \log_{10} x - 2 \log_{10} y = 2 + 2 \log_{10} x + \log_{10} y
\]
M1  
For the correct answer
A1  
For correct term \( 2 \log_{10} x \)
Hence \( 3 \log_{10} y = -2 \)
M1  
For both correct terms 1 and \( \log_{10} y \)
So \( y = 10^{\frac{-2}{3}} \approx 0.215 \)
A1  
For the correct value 0.215
### Question 8

#### Part (i)

\[-2 + k + 1 + 6 = 0 \Rightarrow k = -5\]

**M1**

For attempting \(f(-1)\)

**A1**

For equating \(f(-1)\) to 0 and deducing the correct value of \(k\)

**AG**

Match coefficients and attempt \(k\)

**B2**

For correct leading term \(2x^2\)

**M1**

For attempt at complete division by \(f(x)\) by \((x + 1)\) or equiv.

**A1**

For completely correct quadratic factor

**A1**

For all three factors correct

#### Part (ii)

\[f(2) = 16 - 20 - 2 + 6 = 0\]

Hence \((x - 2)\) is a factor

**A1**

For correct identification of factor \((x - 2)\)

**M1**

For any method for the remaining factor

**A1**

For all three factors correct

\[f(x) = (x + 1)(x - 2)(2x - 3)\]

#### Part (iii)

\[\int_{-1}^{2} f(x) \, dx = \int \left( \frac{1}{2}x^4 - \frac{5}{3}x^3 - \frac{1}{2}x^2 + 6x \right) \, dx\]

\[= \left( \frac{8}{3} - \frac{40}{3} + 12 \right) - \left( \frac{1}{2} + \frac{5}{3} - \frac{1}{2} - 6 \right)\]

\[= 9\]

**A1**

For correct value 9

**M1**

For evaluation of \(F(2) - F(-1)\)

**B1**

For any two terms integrated correctly

**B1**

For all four terms integrated correctly

**B1**

For sketch of positive cubic, with three distinct, non-zero, roots

**B1**

For correct explanation that some of the area is below the axis

**1\ 2**

### Question 9

#### Part (i)

\[\sin x = 4\cos^2 x = 4(1 - \sin^2 x)\]

Hence \(4\sin^2 x + \sin x - 4 = 0\)

\[\sin x = \frac{-1 \pm \sqrt{65}}{8}\]

Hence \(\beta - \alpha = 118.02\ldots - 61.97\ldots \approx 56\)

**B1**

For correct sketch of one curve

**B1**

For correct shape and location of second curve, on same diagram

**B1**

For intercept 4 on \(y\)-axis

**B1**

For correct identification of intersections – in correct order

**M1**

For attempt to use symmetry of the graphs

**A1**

For the correct (explicit) answer for \(\beta\)

**M1**

For use of \(\tan x = \frac{\sin x}{\cos x}\)

**M1**

For use of \(\cos^2 x = 1 - \sin^2 x\)

**B1**

For showing the given equation correctly

**B1**

For correct solution of quadratic

**M1**

Attempt value for \(x\) from their solutions

**A1**

For the correct value 56

**6**

**1**
1 Obtain integral of form \( k \ln x \) \( \text{M1} \) [any non-zero constant \( k \); or equiv such as \( k \ln3x \)]
Obtain \( 3 \ln 8 – 3 \ln 2 \) \( \text{A1} \) [or exact equiv]
Attempt use of at least one relevant log property \( \text{M1} \) [would be earned by initial \( \ln x^3 \)]
Obtain \( 3 \ln 4 \) or \( \ln 8^3 – \ln 2^3 \) and hence \( \ln 64 \) \( \text{A1} \) \( \text{AG} \); with no errors

2 Attempt use of identity linking \( \sec^2 \theta \), \( \tan^2 \theta \) and 1 \( \text{M1} \) [to write eqn in terms of \( \tan \theta \)]
Obtain \( \tan^2 \theta – 4 \tan \theta + 3 = 0 \) \( \text{A1} \) [or correct unsimplified equiv]
Attempt solution of quadratic eqn to find two values of \( \tan \theta \) \( \text{M1} \) [any 3 term quadratic eqn in \( \tan \theta \)]
Obtain at least two correct answers \( \text{M1} \) [after correct solution of eqn]
Obtain all four of 45, 225, 71.6, 251.6 \( \text{A1} \) [allow greater accuracy or angles to nearest degree – and no other answers between 0 and 360]

3 (a) Attempt use of product rule \( \text{M1} \) [involving \( \ldots + \ldots \)]
Obtain \( 2x(x + 1)^6 \ldots \) \( \text{A1} \)
Obtain \( \ldots + 6x^2(x + 1)^5 \) \( \text{A1} \) [or equivs; ignore subsequent attempt at simplification]

(b) Attempt use of quotient rule \( \text{M1} \) [or, with adjustment, product rule; allow \( u/v \) confusion ]
Obtain \( \frac{(x^2 - 3)2x - (x^2 + 3)2x}{(x^2 - 3)^2} \) \( \text{A1} \) [or equiv]
Obtain \( -3 \) \( \text{A1} \) [from correct derivative only]

4 (i) State \( y \leq 2 \) \( \text{B1} \) [or equiv; allow <; allow any letter or none]

(ii) Show correct process for composition of functions \( \text{M1} \) [numerical or algebraic]
Obtain 0 and hence 2 \( \text{A1} \) [and no other value]

(iii) State a range of values with 2 as one end-point \( \text{M1} \) [continuous set, not just integers]
State \( 0 < k \leq 2 \) \( \text{A1} \) [with correct < and \( \leq \) now]

5 Obtain integral of form \( k(1 - 2x)^6 \) \( \text{M1} \) [any non-zero constant \( k \)]
Obtain correct \( -\frac{1}{12}(1 - 2x)^6 \) \( \text{A1} \) [or unsimplified equiv; allow + \( c \)]
Use limits to obtain \( -\frac{1}{12} \) \( \text{A1} \) [or exact (unsimplified) equiv]
Obtain integral of form \( ke^{2x-1} \) \( \text{M1} \) [or equiv; any non-zero constant \( k \)]
Obtain correct \( \frac{1}{2}e^{2x-1} - x \) \( \text{A1} \) [or equiv; allow + \( c \)]
Use limits to obtain \( -\frac{1}{2}e^{-1} \) \( \text{A1} \) [or exact (unsimplified) equiv]
Show correct process for finding required area \( \text{M1} \) [at any stage of solution; if process involves two definite integrals, second must be negative]
Obtain \( -\frac{1}{12} + \frac{1}{2}e^{-1} \) \( \text{A1} \) [or exact equiv; no + \( c \)]
6 (a) Either: State proportion \( \frac{460}{775} \)

Attempt calculation involving proportion
Obtain 704

Or: Use formula of form \( 275e^{kt} \) or \( 275a^t \)
Obtain \( k = 0.047 \) or \( a = 10^{\sqrt{1.6}} \)
Obtain 704

(b)(i) Attempt correct process involving logarithm
Obtain \( \ln \frac{20}{80} = -0.02t \)
Obtain 69

(ii) Differentiate to obtain \( ke^{-0.02t} \)
Obtain \( -1.6e^{-0.02t} \) (or \( 1.6e^{-0.02t} \))
Obtain 0.88

7 (i) Sketch curve showing (at least) translation in \( x \) direction
Show correct sketch with one of 2 and \( 3\pi \) indicated
… and with other one of 2 and \( 3\pi \) indicated

(ii) Draw straight line through \( O \) with positive gradient

(iii) Attempt calculations using 1.8 and 1.9
Obtain correct values and indicate change of sign

(iv) Obtain correct first iterate 1.79 or 1.78
Attempt correct process to produce at least 3 iterates
Obtain 1.82

Attempt rearrangement of \( 3\cos^{-1}(x - 1) = x \)
or of \( x = 1 + \cos(\frac{1}{x}) \)
Obtain required formula or equation respectively
8 (i) Differentiate to obtain  \[ k(5 - x^2)^{-1} \]  
Obtain correct  \(-2x(5 - x^2)^{-1}\)  
Obtain \(-4\) for value of derivative  
Attempt equation of straight line through (2, 0) with numerical value of gradient obtained from attempt at derivative  
Obtain  \(y = -4x + 8\)  

(ii) State or imply  \(h = \frac{1}{2}\)  
Attempt calculation involving attempts at \(y\) values  
Obtain  \(k(\ln5 + 4\ln4.75 + 2\ln4 + 4\ln2.75 + \ln1)\)  
Obtain 2.44  

(iii) Attempt difference of two areas  
Obtain 8 – 2.44 and hence 5.56

9 (i) State \(\sin 2\theta \cos \theta + \cos 2\theta \sin \theta\)  
Use at least one of \(\sin 2\theta = 2 \sin \theta \cos \theta\) and \(\cos 2\theta = 1 - 2\sin^2 \theta\)  
Attempt complete process to express in terms of \(\sin \theta\)  
Obtain \(3 \sin \theta - 4\sin^3 \theta\)  

(ii) State 3  
Obtain expression involving \(\sin 10\alpha\)  
Obtain 9  

(iii) Recognise cosec 2\(\beta\) as \(\frac{1}{\sin 2\beta}\)  
Attempt to express equation in terms of \(\sin 2\beta\) only  
Attempt to find non-zero value of \(\sin 2\beta\)  
Obtain at least \(\sin 2\beta = \frac{5}{\sqrt{12}}\)  
Attempt correct process to find two values of \(\beta\)  
Obtain 20.1, 69.9
1 Attempt to factorise numerator and denominator M1
num = \( xx(x - 3) \) or denom = \((x - 3)(x + 3)\) A1 Not num = \(x(x^2 - 3x)\)
Final answer = \( \frac{x^2}{x + 3} \) [Not \( \frac{xx}{x + 3} \)] A1 3 Do not ignore further cancellation.

2 \( \frac{dy}{dx} (\sin y) = \cos y \cdot \frac{dy}{dx} \) B1
\( \frac{dy}{dx} (xy) = x \frac{dy}{dx} + y \) s.o.i. B1 [SR: If \(xy\) taken to LHS, accept
\( -x \frac{dy}{dx} + y \) as s.o.i.]
\( \cos y \cdot \frac{dy}{dx} = x \frac{dy}{dx} + y + 2x \) AEF B1
[If written as \( x \frac{dy}{dx} = \cos y \frac{dy}{dx} = x \frac{dy}{dx} + y + 2x \), accept for prev B1 but not for following marks if the \( \frac{dy}{dx} \) is used]
f\((x, y)\) \( \frac{dy}{dx} = g(x, y) \) M1 Regrouping provided \( > \) one \( \frac{dy}{dx} \) term
\( \frac{y + 2x}{\cos y - x} \) or \( \frac{y + 2x}{x - \cos y} \) or \( \frac{-2x - y}{x - \cos y} \) A1 5 ISW Answer could imply M1

3 (i) Quotient = \( 3x + ... \) B1 For correct leading term in quotient
For evidence of correct division process M1 Or for cubic
\( (x^2 - 2x + 5) \) (\( g + h \)) (+ ...
\( 3x + 4 \) A1 For correct quotient
\( -6x - 13 \) A1 4 For correct remainder ISW

(ii) \( a = 7 \) B1 √ Follow through If rem in (i) is
\( \frac{P x + Q}{x^2} \),
\( b = 20 \) B1 √ then B1 √ for \( a = 1 - P \)
\( 2 \) and B1 √ for \( b = 7 - Q \)
[SR: If B0+B0, award B1 √ for \( a = 1 + P \) AND \( b = 7 + Q \); also SR B1 for \( a = 20, b = 7 \)]

4 (i) Parts using correct split of \( u = x, \frac{dv}{dx} = \sec^2 x \) M1 1st stage result of form
\( f(x) + \int g(x) dx \)
x \( \tan x \) = \( \int \tan x dx \) A1 Correct 1\(^{st}\) stage
\( \int \tan x \frac{dx}{dx} = -\ln \cos x \) or \( \ln \sec x \) B1
\( \int \tan x + \ln \cos x + c \) or \( x \tan x - \ln \sec x + c \) A1 4

(ii) \( \tan^2 x = +1/\sec^2 x \) M1 or \( \sec^2 x = +1/\tan^2 x \)
\( \int x \sec^2 x \frac{dx}{dx} - \int x \frac{dx}{x} \) s.o.i. A1 Correct 1\(^{st}\) stage
\( x \tan x + \ln \cos x - \frac{1}{2} x^2 + c \) A1 √ 3 f.t. their answer to part (i) \(-1/2 x^2 \)
5 (i) \[
\frac{dy}{dx} = \frac{dy}{dr} \cdot \frac{dr}{dx}
\]
M1 Used, not just quoted

\[
\frac{1}{t} \quad \text{or} \quad t^{-1}
\]
A1 2 Not \(\frac{2}{2t}\) as final answer

SR: M1 for Cart conv, finding \(\frac{dy}{dx}\) & ans involv \(t + A1\) M1 is attempt only, accuracy not involved

(ii) Finding equation of tangent (using \(p\) or \(t\))

\[
p y = x + p^2
\]
A1 2 AG; \(p\) essential; at least 1 line inter working

(iii) \((25,-10) \Rightarrow p = -5\) or \(-5y = x + 25\) seen

B1 5\(y = x + 25\) seen \(\Rightarrow B0\)

Substitution of their values of \(p\) into given tgt eqn

B1 5 Producing 2 equations

Solving the 2 equations simultaneously

M1 Producing 2 equations

\((-15,-2)\) \(x = -15, y = -2\)

A1 4 Common wrong ans

\((15,8) \Rightarrow B0, M2, A0\)

6 (i) Attempt to connect \(dx, d\theta\)

\[
dx = 2 \sin \theta \cos \theta \, d\theta
\]
M1 But not \(dx = d\theta\)

A1 AEF

\[
\sqrt{1 - x} = \frac{\sin \theta}{\cos \theta}
\]
B1 Ignore any references to \(\pm\).

Reduction to \(\int 2 \sin^2 \theta \, d\theta\)

A1 4 AG WWW

(ii) \(\sin^2 \theta = k(\pm 1 + \pm \cos 2\theta)\)

M1 Attempt to change \(2 \sin^2 \theta\) into \(f(\cos 2\theta)\)

A1 Correct attempt

\[
\int \cos 2\theta \, d\theta = \frac{1}{2} \sin 2\theta
\]
B1 Seen anywhere in this part

Attempting to change limits

M1 Or Attempting to resubstitute; Accept degrees

\[
\frac{1}{2} \pi
\]
A1 5

Alternatively Parts once & use

\[
\cos^2 \theta = 1 - \sin^2 \theta
\]
(M2) Instead of the M1 A1 B1

\[
\frac{1}{2}\left(\theta - \sin \theta \cos \theta\right)
\]
(A1) Then the final M1 A1 for use of limits

7 (i) \(A = 3\)

B1 For correct value stated

\(C = 1\)

B1 For correct value stated

\(11 + 8x = A(1 + x)^2 + B(2 - x)(1 + x) + C(2 - x)\)

M1 AEF; any suitable identity

e.g. \(A - B = 0, 2A + B - C = 8, A + 2B + 2C = 11\)

A1 For any correct (f.t.) equation involving \(B\)

\(B = 3\)

A1

(ii) \((1 - \frac{x}{2})^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + ...\)

B1 s.o.i.

\((1 + x)^{-1} = 1 - x + x^2 - ...\)

B1 s.o.i.

\((1 + x)^2 = 1 - 2x + 3x^2 - ...\)

B1,B1 s.o.i.

Expansion \(= \frac{11}{2} - \frac{17}{4}x + \frac{51}{8}x^2 + ...\)

B1 5 CAO. No f.t. for wrong \(A\) and/or \(B\) and/or \(C\)
8 (i) \[\int (y-3) \, dy = \int (2-x) \, dx\] or equiv \(M1\) 
For separation & integration of both sides 
\[\frac{1}{2}y^2 - 3y = 2x - \frac{1}{2}x^2\] \(A1\) 
or \[\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2\] 
For an arbitrary const on one/both sides *B1 \{ or + M2 for equiv statement using limits\} 
Substituting \((x,y) = (5,4)\) or \((4,5)\) & finding 'c' dep*M1 \} 
\[\frac{1}{2}y^2 - 3y = -\frac{1}{2}x^2 + 2x - \frac{3}{2}\] AEF ISW \(A1\) \[5\] 
or \[\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2 + 5\] AEF

(ii) Attempt to clear fracts (if nec) & compl square \(M1\) 
\(a = 2, b = 3, k = 10\) \(A2\) \[3\] For all 3; SR: \(A1\) for 1 or 2 correct

(iii) Circle clearly indicated in a sketch \(B1\) 
Centre \((2,3)\) or their \((a,b)\) \(B1\)√ 
Radius \(\sqrt{10}\) or their \(\sqrt{k}\) \(B1\)√ \[3\] √ provided \(k > 0\)

9 (i) Using \[\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix}\] and \[\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix}\] as the relevant vectors \(M1\) i.e. correct direction vectors 
Using \(\cos \theta = \frac{a \cdot b}{|a||b|}\) AEF for any 2 vectors \(M1\) \(A1\) \[5\] Accept \(\cos \theta = \frac{a \cdot b}{|a||b|}\)
Method for scalar product of any 2 vectors \(M1\) 
Method for finding magnitude of any vector \(M1\) 
\(15°\) (15.38…), 0.268 rad \(A1\)

(ii) Produce (at least) 2 of the 3 eqns in \(t\) and \(s\) \(M1\) e.g. \(4 - 8t = -2 - 9s\), \(-6 - 2t = -2 - 5s\) 
Solve the \((x)\) and \((z)\) equations \(M1\) 
\(t = 3\) or \(s = 2\) \(A1\) for first value found 
\(s = 2\) or \(t = 3\) f.t. \(A1\)√ for second value found 
Substituting their \((t,s)\) into \((y)\) equation \(M1\) 
\(a = 1\) \(A1\) 
Substituting their \(t\) into \(l_t\) or their \((s,a)\)
into $I_2$

\[
\begin{pmatrix}
-20 \\
5 \\
-12
\end{pmatrix}
\]

M1

A1  8  Any format but not $\left( \right) + \left( \right)$
| 1. | (i) \(2 + 16i - i - 8i^2\) & \(10 + 15i\) & M1 & A1 & 2 & Attempt to multiply correctly \n & \(\frac{1}{5}(10 + 15i)\) or \(2 + 3i\) & A1 & M1 & A1 & 3 & Their part (i) or \(10 + 15i\) derived again / 5
| 2. | \(1^2 = \frac{1}{6} \times 1 \times 2 \times 3\) & B1 & \(\frac{1}{6}n(n + 1)(2n + 1) + (n + 1)^2\) & M1 & DM1 & Show result true for \(n = 1\) or 2 \n & \(\frac{1}{6}(n + 1)(n + 2\{2(n + 1) + 1\} & A1 & A1 & 5 & Specific statement of induction conclusion, with no errors seen
| 3. | (i) & M1 & A1 & A1 & 3 & Show correct expansion process, allow sign slips \n & \(2 \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1 \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\) & A1 & A1 & B1 & 1 & State that \(M\) is non-singular as \(\det M\) non-zero, \(\text{ft}\) their determinant \n & \(2 \times 5 - 1 \times 2 + 3 \times -1\) & 5 & 4 & \(u^2 + 4u + 4\) & B1 & \(u + 2\) squared and cubed correctly & M1 & A1 & A1 & 5 & Substitute these and attempt to simplify \n & \(u^3 + 6u^2 + 12u + 8\) & & & 5 & Obtain \(u^3 - 5 = 0\) or equivalent \n & \(u = \frac{3}{\sqrt{5}}\) & A1 & A1 & A1 & & Correct solution to their equation \n & \(x = 2 + \frac{3}{\sqrt{5}}\) & 5 & 5 & Obtain 2 + their answer \n & [Decimals score 0/2 of final A marks]
5. Consider the sum of three separate terms

- $8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$
  - $8\Sigma r^3 = 2n^2(n + 1)^2$
  - $6\Sigma r^2 = n(n + 1)(2n + 1)$
  - $2\Sigma r = n(n + 1)$

$$2n^3(n + 1)$$

Correct formula stated or used a.e.f.

Correct formula stated or used a.e.f.

Correct term seen

Attempt to factorise or expand and simplify

Obtain given answer correctly

<table>
<thead>
<tr>
<th>Marks</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Consider the sum of three separate terms</td>
</tr>
<tr>
<td>1</td>
<td>Correct formula stated or used a.e.f.</td>
</tr>
<tr>
<td>1</td>
<td>Correct formula stated or used a.e.f.</td>
</tr>
<tr>
<td>1</td>
<td>Correct term seen</td>
</tr>
<tr>
<td>6</td>
<td>Attempt to factorise or expand and simplify</td>
</tr>
<tr>
<td>6</td>
<td>Obtain given answer correctly</td>
</tr>
</tbody>
</table>

6. Transpose leading diagonal and negate other diagonal

- Divide by determinant

- State or imply $(AB)^{-1} = B^{-1}A^{-1}$
  - Use this result and obtain $B^{-1} = C^{-1}A$, or equivalent matrix algebra

- Matrix multn., two elements correct, for any pair
  - All elements correct if their (i)

<table>
<thead>
<tr>
<th>Marks</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Transpose leading diagonal and negate other diagonal</td>
</tr>
<tr>
<td>1</td>
<td>Divide by determinant</td>
</tr>
<tr>
<td>1</td>
<td>State or imply $(AB)^{-1} = B^{-1}A^{-1}$</td>
</tr>
<tr>
<td>1</td>
<td>Use this result and obtain $B^{-1} = C^{-1}A$, or equivalent matrix algebra</td>
</tr>
<tr>
<td>5</td>
<td>Matrix multn., two elements correct, for any pair</td>
</tr>
<tr>
<td>7</td>
<td>All elements correct if their (i)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marks</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find $A^{-1}$</td>
</tr>
<tr>
<td>1</td>
<td>Premultiply by $A^{-1}$ stated or implied</td>
</tr>
<tr>
<td>1</td>
<td>Matrix multn. Two elements correct</td>
</tr>
<tr>
<td>1</td>
<td>All elements correct</td>
</tr>
<tr>
<td>1</td>
<td>Correct $B^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marks</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find $AB$</td>
</tr>
<tr>
<td>1</td>
<td>Solve one pair of simultaneous equations</td>
</tr>
<tr>
<td>1</td>
<td>Each pair of answers</td>
</tr>
<tr>
<td>7</td>
<td>Correct $B^{-1}$</td>
</tr>
</tbody>
</table>
7.\[ (a) \text{ (i) } \sqrt{13} \]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td>1</td>
<td>Obtain correct answer, decimals OK</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>M1</td>
<td>A1</td>
<td>A1</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>-0.59</td>
<td>3</td>
<td>Obtain correct answer</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>M1</td>
<td>A1A1</td>
<td>A1</td>
<td>Correct answer written as a complex number</td>
</tr>
<tr>
<td></td>
<td>1 - 2i</td>
<td>4</td>
<td>Obtain x = 1 and y = -2</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>B1</td>
<td>B1</td>
<td>B1</td>
<td>Sketch of vertical straight line</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>2</td>
<td>Through (-0.5, 0)</td>
<td></td>
</tr>
</tbody>
</table>

8.\[ (i) \]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td>3</td>
<td>For correct vertex (2, -2)</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>3</td>
<td>For all vertices correct</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>3</td>
<td>For correct diagram</td>
</tr>
</tbody>
</table>

(ii) Either \[
\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1,B1</td>
<td>B1</td>
<td>Reflection, in x-axis</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Correct matrix</td>
<td></td>
</tr>
</tbody>
</table>

Or \[
\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}
\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1,B1</td>
<td>B1</td>
<td>Enlargement, centre O s.f.2</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Correct matrix</td>
<td></td>
</tr>
</tbody>
</table>

Or \[
\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}
\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1,B1</td>
<td>B1</td>
<td>Reflection, in the y-axis</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Correct matrix</td>
<td></td>
</tr>
</tbody>
</table>

Or \[
\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}
\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1,B1</td>
<td>B1</td>
<td>Enlargement, centre O s.f. -2</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Correct matrix</td>
<td></td>
</tr>
</tbody>
</table>

Or \[
\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}
\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1,B1</td>
<td>B1</td>
<td>Stretch, in x-direction s.f. 2</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Correct matrix</td>
<td></td>
</tr>
</tbody>
</table>

Or \[
\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}
\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1,B1</td>
<td>B1</td>
<td>Stretch, in y-direction s.f. -2</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Correct matrix</td>
<td></td>
</tr>
</tbody>
</table>
9. (i) \[
\frac{r + 2 - r}{r(r + 2)} = \frac{2}{r(r + 2)}
\]

(ii) \[
\frac{3}{2} - \frac{1}{n + 1} - \frac{1}{n + 2}
\]

(iii) (a) \[
\frac{3}{2}
\]

(b) \[
\frac{1}{n + 1} + \frac{1}{n + 2}
\]

M1 Show correct process for subtracting fractions
A1 2 Obtain given answer correctly
AG Express terms as differences using (i)
M1 Express 1st 3 (or last 3) terms so that cancelling occurs
M1 Obtain \(1 + \frac{1}{2}\)
A1 Obtain \(-\frac{1}{n + 2}, -\frac{1}{n + 1}\)
A1 Obtain correct answer in any form
B1ft 1 Obtain value from their sum to \(n\) terms

10. (i) \(\alpha + \beta + \gamma = 9\)

(ii) \(p = \frac{9 - \alpha}{2}\)

(iii) \(\alpha \beta \gamma = 29\)

(iv) \(\alpha(p^2 + q^2) = 29\)

\(q = \sqrt{\frac{29}{\alpha} - \frac{(9 - \alpha)^2}{4}}\)

(iv) Alternative method
\(2p\alpha + p^2 + q^2 = 27\)

\(q = \sqrt{27 - \frac{(9 - \alpha)^2}{4} - \alpha(9 - \alpha)}\)

B1 1 State or use other root is \(p - iq\)
B1 Substitute into (i)
M1 Obtain \(2p + \alpha = 9\)
A1 Obtain correct answer a.e.f.

A1 Substitute into (iii)
M1 Obtain unsimplified expression with no i’s
A1 Rearrange to obtain \(q\) or \(q^2\)
M1 Substitute their expression for \(p\) a.e.f.
A1 Obtain correct answer a.e.f.

M1 Substitute into \(\alpha\beta + \beta\gamma + \gamma\alpha = 27\)
A1 Obtain unsimplified expression with no i’s
M1 Rearrange to obtain \(q\) or \(q^2\)
M1 Substitute their expression for \(p\) a.e.f.
A1 Obtain correct answer a.e.f.
(i) Use standard \( \ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} \)
\[
= 3x - 9x^2/2 + 9x^3
\]
(ii) Produce ( \( 1 + x + x^2/2 \) )
\[
\text{Get } 3x - 3x^2/2 + 6x^3
\]

2 \quad Write as \( f(x) = e(x - e^x) \)
So \( f'(x) = e(1 + e^x) \)
Use \( x_{n+1} = x_n - f(x_n)/f'(x_n) \) with \( x_0 = 0.5 \)
Get \( x_1 = 0.56631, x_2 = 0.56714 \)
Get \( x_3 = 0.5671(1) \)

3 \quad Use \( A/x + (Bx + C)/(x^2 + 2) \)
Equate \( x+6 \) to \( A(x^2 + 2) + (Bx+C)x \) (or equiv.)
Use \( x = 0 \) or equiv. for \( A \) (or equate coeff, etc.)
Correctly find one of \( B, C \)
Get \( A=3, B=-3, C=1 \)

4(i)
(ii)(a)Converges to \( x=e \)
(b)Diverges (does not give either root)

5 \quad (i) Give \( x = -2 \)
Attempt to divide out
Get \( y = x + 1 \)
(ii) Write as quad. \( x^2 + x(3 - y) + (3-2y) = 0 \)
Use for real \( x, b^2 - 4ac \geq 0 \)
Produce quad. inequality in \( y \)
Attempt to solve quad. inequality
Get A.G. clearly e.g. graph

Final Draft
M1 \quad \text{Allow e.g. } 3x^2, 2! \text{ etc.}
M1 \quad \text{Attempt to simplify } (3x)^2 \text{ etc.}
A1 \quad \text{cao}

B1
M1 \quad \text{Mult. 2 reasonable attempts, each of 3 terms (non-zero)}
A1 \quad \text{From their series}

SC M1 \quad \text{Reasonable attempt at diff. and replace } x = 0 \text{ (2 correct)}
M1 \quad \text{Put their values into correct Maclaurin expansion}
A1 \quad \text{cao (Applies to either/both parts)}

B1 \quad \text{Or equivalent}
B1 \quad \text{Correct from their } f(x)
M1 \quad \text{Clear evidence of N-R on their } f, f'
A1 \quad \text{At least one to 4d.p.}
A1 \quad \text{cao to 3 d.p.}

B1
M1 \quad \text{Equate to their P.F. (e.g. if } B = 0 \text{ or } C = 0 \text{ used)}
M1 \quad \text{Include cover-up}
A1
A1

B1 \quad \text{Line from } x_1 \text{ to curve}
B1 \quad \text{Then to line}
B1 \quad \text{Clear explanation; allow use of step/staircase}

B1, B1
B1
B1

B1
M1 \quad \text{Giving } y = x+k; \text{ allow } k = 0 \text{ here}
A1 \quad \text{Must be =}

M1 \quad \text{SC Differentiate } M1
M1 \quad \text{Solve } dy/dx = 0 \text{ M1}
M1 \quad \text{Get 2 } x, y \text{ values correct } A1
M1 \quad \text{Attempt at max/min } M1
A1 \quad \text{Justify, e.g. graph, constraints on } y \ A1
6 (i) Use parts to \((-e^x \cdot x^n - \int e^x \cdot nx^{n-1} \, dx)\)
   Use limits to get \(e^1\)
   Tidy correctly to A.G.

   (ii) Use \(I_2 = 3I_2 - e^{-1}\)
       \(I_2 = 2I_1 - e^{-1}\)
       \(I_1 = I_0 = e^{-1}\)
       Work out \(I_0 = 1 - e^{-1}\) or \(I_1 = 1 - 2e^{-1}\)
       Get \(6 - 16e^{-1}\)

M1, A1

7 (i) Area under graph = \(\int \sqrt{x} \, dx\)
   > Sum of areas of rectangles from 1 to \(N+1\)
   Area of each rect. = Width x Height = \(1 \times \sqrt{x}\)

   (ii) Similarly, area under curve from 0 to \(N\)
       < sum of areas of rect. from 0 to \(N\)
       Clear explanation of A.G.

   (iii) Integrate \(x^{0.5}\) and use 2 different sets of limits M1, M1
       Get area between \(\frac{2}{3}(N+1)^{1.5} \cdot 1\) and \(\frac{2}{3}N^{1.5}\)

A1

8 (i) Max. \(r = 2\) at \(\theta = 0\) and \(\pi\)

   (ii) Solve \(r = 0\) for \(\theta\), giving \(\theta = \frac{\pi}{2}\) and \(\frac{3\pi}{2}\)

   (iii) Use correct formula with correct \(r\)
       Expand \(r\)
       Get \(\frac{A}{r} + B \cdot \cos 2\theta + C \cdot \cos 4\theta d\theta\)
       Integrate their expression correctly
       Get \(3\pi/8\)

   (iv) Express \(\cos 2\theta = \cos^2 \theta - \sin^2 \theta\) or similar
       Use \(\cos \theta = x/r\) and/or \(\sin \theta = y/r\)
       Simplify to \((x^2 + y^2)^{1.5} = 2x^2\) or similar

9 (i) Correct def' of \(\cosh x\) and \(\sinh x\)
   Expand \(2.5\) \((e^x - e^{-x})^{1.5} (e^x + e^{-x})\)
   Clearly get \(\frac{1}{2} (e^{2x} - e^{-2x})\) to A.G.

   (ii) Attempt to diff. and solve \(dy/dx = 0\)
       Use (ii) to get \(A \cosh x\) (B \(\sinh x + C\)) = 0
       Clearly see \(\cosh x > 0\) or similar for one
       usable factor only
       Attempt to solve \(\sinh x = -C/B\)
       Get \(x = \ln(3 + \sqrt{13}/2)\)
       Justify one answer only for \(\sinh x = -C/B\)
       Accurate test for minimum

M1 Reasonable attempt
B1

M1

M1

A1

B1

B1

M1

M1

A1

B1

B1

M1

B1

B1

B1

M1
1. Directions \([1,1,-1]\) and \([2,-3,1]\)

\[
\theta = \cos^{-1}\left[\frac{1,1,-1}{\sqrt{3}}, \frac{2,-3,1}{\sqrt{14}}\right] \\
= \cos^{-1}\left[\frac{-2}{\sqrt{42}}\right] \\
= 72.0^\circ, 72^\circ\text{ or }1.26\text{ rad}
\]

- **B1** For identifying both directions (may be implied by working)
- **M1** For using scalar product of their direction vectors
- **A1** For completely correct process for their angle
- **4** For correct answer

2. (i) Identities \(b, 6\)

- **B1** For correct identities
- **B1** For correct subgroups

(ii) \(\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}\) or \(\{8, 6, 2, 4\}\)

- **B1** For \(b \leftrightarrow 6,\ d \leftrightarrow 4\)
- **B1** For \(a, c \leftrightarrow 2, 8\) in either order

**SR** If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in \(G\) and \(H\)

3. (i) \(3y^2 \frac{dy}{dx} = \frac{dz}{dx}\)

\[
\Rightarrow \frac{dz}{dx} + 2xz = e^{-x^2}
\]

Integrating factor \(e^{\int 2x\,dx} = e^{x^2}\)

\[
\Rightarrow \frac{d}{dx}\left(xe^{x^2}\right) \text{ OR } \frac{d}{dx}\left(y^3 e^{x^2}\right) = 1
\]

\[
\Rightarrow \quad \int \frac{dz}{x e^{x^2}} \text{ OR } y^3 e^{x^2} = x (c)
\]

\[
\Rightarrow \quad y = (x + c)^{\frac{1}{2}} e^{-\frac{1}{x^2}}
\]

- **B1** For differentiating substitution
- **A1** For resulting equation in \(z\) and \(x\)
- **B1\sqrt{\text{ }}** For correct IF
- **M1** for an equation in suitable form
- **M1** For using IF correctly
- **A1** For correct integration (+c not required here)
- **A1** For correct answer \(\text{AEF}\)

(ii) As \(x \rightarrow \infty,\ y \rightarrow 0\)

- **B1** For correct statement

4. (i) \(\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})\),

\[
\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})
\]

\[
\Rightarrow \cos^2 \theta \sin^4 \theta = \frac{1}{4}\left(e^{i\theta} + e^{-i\theta}\right)^2 \left(e^{i\theta} - e^{-i\theta}\right)^4
\]

\[
= \frac{1}{4}\left(e^{2i\theta} + 2 + e^{-2i\theta}\right) \frac{1}{16}\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)
\]

- **M1** For expanding terms
- **A1** For the 2 correct expansions
- **A1** For answer obtained correctly

**SR** Allow A1 A0 for

\[
k\left(\left(\frac{e^{2i\theta} + 2 + e^{-2i\theta}}{e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}}\right), \quad k \neq \frac{1}{64}\right)
\]

- **M1** For grouping terms and using multiple angles
- **A1** For answer obtained correctly
\[ \int_0^{\pi} \cos^2 \theta \sin \theta \, d\theta = \frac{1}{32} \left[ \frac{1}{6} \sin 60 - \frac{1}{2} \sin 40 - \frac{1}{2} \sin 20 + 20 \right]_0^{\pi} = \frac{1}{32} \left[ 0 + \frac{1}{4} \sqrt{3} - \frac{1}{4} \sqrt{3} + \frac{2}{3} \pi - 0 \right] = \frac{1}{48} \pi \]

<table>
<thead>
<tr>
<th>Question</th>
<th>Mark Scheme</th>
<th>Comments</th>
</tr>
</thead>
</table>
| (i) \[ \int_0^{\pi} \cos^2 \theta \sin \theta \, d\theta = \frac{1}{32} \left[ \frac{1}{6} \sin 60 - \frac{1}{2} \sin 40 - \frac{1}{2} \sin 20 + 20 \right]_0^{\pi} = \frac{1}{32} \left[ 0 + \frac{1}{4} \sqrt{3} - \frac{1}{4} \sqrt{3} + \frac{2}{3} \pi - 0 \right] = \frac{1}{48} \pi \] | M1 | For integrating answer to (i)
| | A1 | For all terms correct
| | A1 3 | For correct answer |

5 (i) EITHER
\[ z = \sqrt{8} \cos((2k+1)\frac{\pi}{4}), \quad k = 0, 1, 2, 3 \]
\[ OR \ z = \sqrt{8} e^{(2k+1)i\frac{\pi}{4}}, \quad k = 0, 1, 2, 3 \]

(ii) \[ z = 2\sqrt{2} \left[ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \right] \]
\[ z = 2 + 2i, -2 + 2i, -2 - 2i, 2 - 2i \]
\[ (z - \alpha), (z - \beta), (z - \gamma), (z - \delta) \]

(iii) EITHER \[ (z - (2 + 2i))(z - (2 - 2i))(z - (-2 + 2i)) \]
\[ = (z^2 + 4z + 8)(z^2 - 4z + 8) \]
\[ OR \ z^4 + 64 = (z^2 + az + b)(z^2 + cz + d) \]
\[ \Rightarrow a + c = 0, b + ac + d = 0, ad + bc = 0, bd = 64 \]
\[ Obtain \ (z^2 + 4z + 8)(z^2 - 4z + 8) \]

6 (i) \[ MB = [2, 1, -2], \ OF = [4, 1, 2] \]
\[ MB \times OF = [4, -12, -2] OR k[2, -6, -1] \]

(ii) EITHER Find vector product of any two of \[ \pm[2, -1, 2], \ \pm[0, 0, 2], \ \pm[2, -1, 0] \]
and any two of \[ \pm[4, 0, 2], \ \pm[4, -1, 0], \ \pm[0, 1, 2] \]
\[ Obtain \ k[1, 2, 0] \]
\[ Obtain \ k[1, 4, -2] \]
\[ x + 2y = 2 \quad and \quad x + 4y - 2z = 0 \]
\[ OR Use \ ax + by + cz = d \quad with \]
coordinates of \[ C, M, G \quad OR \ O, E, G \quad substituted \]
\[ Obtain \ a:b:c = 1:2:0 \quad for \ CMG \]
\[ Obtain \ a:b:c = 1:4:-2 \quad for \ OEG \]
\[ x + 2y = 2 \quad and \quad x + 4y - 2z = 0 \]

<table>
<thead>
<tr>
<th>Question</th>
<th>Mark Scheme</th>
<th>Comments</th>
</tr>
</thead>
</table>
| (i) \[ \int_0^{\pi} \cos^2 \theta \sin \theta \, d\theta = \frac{1}{32} \left[ \frac{1}{6} \sin 60 - \frac{1}{2} \sin 40 - \frac{1}{2} \sin 20 + 20 \right]_0^{\pi} = \frac{1}{32} \left[ 0 + \frac{1}{4} \sqrt{3} - \frac{1}{4} \sqrt{3} + \frac{2}{3} \pi - 0 \right] = \frac{1}{48} \pi \] | M1 | For integrating answer to (i)
| | A1 | For all terms correct
| | A1 3 | For correct answer |

B1 2 For correct arguments AEF

B1 For any of \[ \pm \frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}} i \]
B1 For any one value of \[ z \]
B1 For all values of \[ z \] correct AEFcartesian

(f.t., may be implied from symmetry or factors)

(iii) EITHER \[ (z - (2 + 2i))(z - (2 - 2i))(z - (-2 + 2i)) \]
\[ = (z^2 + 4z + 8)(z^2 - 4z + 8) \]

M1 For combining factors from (ii) in pairs
M1 Use of complex conjugate pairs

A1 For correct answer

A1 For equating coefficients
M1 For solving equations

A1 For correct answer

B1 For either vector correct (allow multiples)
M1 For finding vector product of their MB and OF

B1 3 For correct vector

M1 For finding two relevant vector products

M1 For correct LHS of plane CMG
M1 For correct LHS of plane OEG
M1 For substituting a point into each equation

A1 For both equations correct AEF

A1 For use of cartesian equation of plane

A1 For correct ratio
A1 For correct ratio
M1 For substituting a point into each equation
A1 For both equations correct AEF
### (iii) EITHER  Put $x, y$ OR $z = t$ in planes

**OR** evaluate $k[1, 2, 0] \times k[1, 4, -2]$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>For solving plane equations in terms of a parameter <strong>OR</strong> for finding vector product of normals to planes from (ii)</td>
</tr>
<tr>
<td>A1</td>
<td>Obtain a correct point AEF</td>
</tr>
<tr>
<td>A1</td>
<td>Obtain correct direction AEF</td>
</tr>
</tbody>
</table>

### 7 (i) $(x^{-1}a)x^m = (x^{-1}a)x(x^{-1}a)x \ldots (x^{-1}a)x$

- $x^{-1}a \ldots a x$, associativity, $xx^{-1} = e$
- $x^{-1}a^m x = x^{-1}e x$ when $m = n$,
- not $m < n$
- $x^{-1}x = e$
- $\Rightarrow$ order $n$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
</table>
| M1      | For considering powers of $x^{-1}a$
| A1      | For using associativity and inverse properties |
| B1      | For using order of $a$ correctly |
| A1      | For using property of identity |
| A1      | For correct conclusion |

### (ii) EITHER $(x^{-1}a)xz = e$

- $\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$
- $\Rightarrow z = x^{-1}a^{-1}x$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>For attempt to solve for $z$ AEF</td>
</tr>
<tr>
<td>A1</td>
<td>For using pre- or post multiplication</td>
</tr>
<tr>
<td>A1</td>
<td>For correct answer</td>
</tr>
</tbody>
</table>

**OR**

- Use $(pq)^{-1} = q^{-1}p^{-1}$
- $OR (pq)^{-1} = r^{-1}q^{-1}p^{-1}$
- State $(x^{-1})^{-1} = x$
- Obtain $x^{-1}a^{-1}x$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>For applying inverse of a product of elements</td>
</tr>
<tr>
<td>A1</td>
<td>For stating this property</td>
</tr>
<tr>
<td>A1</td>
<td>For correct answer with no incorrect working</td>
</tr>
<tr>
<td>SR</td>
<td>Correct answer with no working scores B1 only</td>
</tr>
</tbody>
</table>

### (iii) $ax = xa \Rightarrow x = a^{-1}xa$

- $\Rightarrow x a^{-1} = a^{-1}x$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Start from commutative property for $ax$</td>
</tr>
<tr>
<td>A1</td>
<td>Obtain commutative property for $a^{-1}x$</td>
</tr>
</tbody>
</table>

### 8 (i) $m^2 + 2km + 4 = 0$

- $\Rightarrow m = -k \pm \sqrt{k^2 - 4}$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>For stating and attempting to solve auxiliary eqn</td>
</tr>
<tr>
<td>A1</td>
<td>For correct solutions, at any stage AEF</td>
</tr>
<tr>
<td>M1</td>
<td>For using $e^{lt}$ with distinct real roots of aux eqn</td>
</tr>
<tr>
<td>A1</td>
<td>For correct answer AEF</td>
</tr>
</tbody>
</table>

**OR**

- $x = e^{-lt} \left( Ae^{\sqrt{k^2 - 4}t} + Be^{-\sqrt{k^2 - 4}t} \right)$
- $x = e^{-lt} \left( A\cos \sqrt{4-k^2}t + B\sin \sqrt{4-k^2}t \right)$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>For using $e^{lt}$ with complex roots of aux eqn</td>
</tr>
<tr>
<td>A1</td>
<td>This form may not be seen explicitly but if stated as final answer earns M1 A0</td>
</tr>
<tr>
<td>A1</td>
<td>For correct answer</td>
</tr>
</tbody>
</table>

**OR**

- $x = e^{-lt} \left( C\cos \left( \sqrt{4-k^2}t + \alpha \right) \right)$

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>For using $e^{lt}$ with equal roots of aux eqn</td>
</tr>
<tr>
<td>A1</td>
<td>For correct answer. Allow $k$ for 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marking</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>For correct answer. Allow $k$ for 2</td>
</tr>
</tbody>
</table>
### (ii)(a) \( x = B'e^{-t} \sin \sqrt{3}t \)

\[
\dot{x} = B'e^{-t} \left( \sqrt{3} \cos \sqrt{3}t - \sin \sqrt{3}t \right)
\]

\( t = 0, \dot{x} = 0 \Rightarrow B' = 2\sqrt{3}, \quad x = 2\sqrt{3}e^{-t} \sin \sqrt{3}t \)

<table>
<thead>
<tr>
<th>Mark</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1 ( \checkmark )</td>
<td>For using ( t = 0, x = 0 ) correctly, f.t. from (b)</td>
</tr>
<tr>
<td>M1</td>
<td>For differentiating ( x )</td>
</tr>
<tr>
<td>A1 ( \checkmark )</td>
<td>For correct expression, f.t. from their ( x )</td>
</tr>
<tr>
<td>A1 4</td>
<td>For correct solution AEF</td>
</tr>
</tbody>
</table>

### (b) \( x \to 0 \)
\( e^{-t} \to 0 \) and \( \sin( ) \) is bounded

<table>
<thead>
<tr>
<th>Mark</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>For correct statement</td>
</tr>
<tr>
<td>B1 2</td>
<td>For both statements</td>
</tr>
</tbody>
</table>

\( x = C'e^{-t} \cos \left( \sqrt{3}t + \frac{1}{2} \pi \right) \)
### Question 1

**Part (i)**

\[0.3g - T = 0.3a\]  
\[T - 0.4g = 0.4a\]

-0.1g = 0.7a  
\[a = -1.4\]

See appendix for substituting \(a = -1.4\)

**Part (ii)**

\[0 = 2.8t - \frac{1}{2} 1.4t^2\]  
\[0 = t(2.8 - 0.7t)\]

Time taken is 4 s

OR

\[(0.3 + 0.4)a = (0.3 - 0.4)g\]

\[a = -1.4\]

\[0 = 2.8 + -1.4t\]

\[t = \frac{2.8}{1.4}\]

Time taken is 4 s

- **M1**
- **A1**

- **M1**
- **A1**

- **M1**
- **A1**

- **M1**
- **A1**

- **M2**
- **A1**

- **A1**

- **M1**

- **A1**

### Question 2

**Part (i)**

\[T \sin \alpha = 0.08 \times 1.25\]

\[= 0.1\]

\[T \cos \alpha = 0.08g\]

\[T^2 = 0.1^2 + 0.784^2\]  
\[\alpha = 7.3^\circ\]

\[T = 0.79\]

**Part (ii)**

- **M1**
- **A1**

- **M1**
- **A1**

- **M1**
- **A1**

- **A1**

- **M1**

- **A1**

- **M1**

- **A1**

- **A1**

### Question 3

**Part (i)**

\[a = 7.2 - 0.9t\]

\[T = 8\]

See also special case in appendix.

- **M1**
- **A1**

- **M1**
- **A1**

- **M1**
- **A1**

- **M1**
- **A1**

### Part (ii)

\[v(T) = 28.8\]

See also special case in appendix.

- **B1**

### Part (iii)

\[s = 3.6t^2 - 0.15t^3\]  
\[+C\]

\[s = 153.6\]  
\[+C\]

\[s\ at\ constant\ speed = 662.4\]

Displacement is 816 m

- **DM1**
- **DM1**

- **A1**

- **B1**

- **A1**

- **A1**

- **A1**

- **A1**

- **B1**

- **A1**
### 4

|   | (i) | \( F = 12\cos 15^\circ \) | M1 | Resolve horizontally (condone sin)  
Frictional component is 11.6 N | A1  
\( N + 12\sin 15^\circ = 2g \) | M1 | Accept 12\( \cos 15^\circ \)  
Normal component is 16.5 N | A1  
11.591… = \( \mu \) 16.494… | M1 | For using \( cv \ F = \mu \ cv \ N \)  
Coefficient is 0.7(0) | A1[ft]  
\( N = 2g \) | B1 | Ft \( \mu \) to 2 sf. \( \mu = 0.7027… \)  
\( F = 19.6 \times 0.7027… \) | M1 |  
20 – 13.773… = 2\( a \) | A1ft |  
Acceleration is 3.1 ms\(^{-2}\)  
\( MISSREAD \) (omits "horizontal") | A1[ft] |  
\( N = 2g - 20\sin 15 \) | B1ft |  
\( F = 0.7027 \times 14.4 \) | M1 |  
20\( \cos 15\) - 10.14 = 2\( a \) | A1ft |  
Acceleration is 4.59 ms\(^{-2}\) | A1ft | [5] | 

### 5

|   | (i) | Graph with 5 straight line segments and with \( v \) single valued. | B1 | ‘Wait’ line segment may not be distinguishable from part of the \( t \) axis. Attempt at all lines segments fully straight.  
Line segment for car stage | B1 | Mainly straight, ends on \( t \)-axis.  
Line segment for walk stage | B1 | Horizontal below \( t \)-axis. Ignore linking to axis.  
Line segment for wait stage | B1 | Can be implied by gap between walk and motor-cycle stages  
2 line segment for motor-cycle stage | B1 | Inverted V not U, mainly straight. Condone vertex below \( x \) intercept.  
(ii) | \( d = 12/8 \) | M1 | Using gradient represents accn  
Deceleration is 1.5 ms\(^{-2}\) | A1[2] | Or \( a = -1.5 \) ms\(^{-2}\)  
(iii) | \( t_{\text{walk}} = 420/0.7 \) | M1 | Using area represents displacement.  
\( t_{\text{motorcycle}} = 42 \) | B1 | Accept 600  
\( T = 8 + 600 + 250 + 42 = 900 \) | B1 | Ignore method | A1[4] |
4728 Mark Scheme     January 2006

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>(i)</td>
<td>$T_A \cos \alpha - T_B \cos \beta = W$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$T_A = T_B (= T)$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\cos \alpha &gt; \cos \beta \Rightarrow \alpha &lt; \beta$</td>
<td>A1 [3]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>For resolving 3 forces vertically, condone $W_g$, sin</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>May be implied or shown in diagram</td>
<td>AG</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii)(a)</td>
<td>$T \sin \alpha + T \sin \beta = 14$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\sin \alpha = 0.6$ and $\sin \beta = 0.8$</td>
<td>DM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension is 10 N</td>
<td>A1 [3]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Resolve 3 forces horiz accept $\cos$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii)(b)</td>
<td>$10 \cos \alpha - 10 \cos \beta = W$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 36.9^\circ$, $\beta = 53.1^\circ$</td>
<td>DM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W = 2$</td>
<td>A1 [3]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>See appendix for solution based on resolving along $RA$ and $RB$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Must use cv T, and $W$ (not $W_g$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Or $\cos \alpha = 0.8$ and $\cos \beta = 0.6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SR -1 for assuming $\alpha + \beta = 90^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ft for $T/5$ (accept 1.99)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii)</td>
<td>$R$ is below $B$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Tension is 1 N</td>
<td>B1 [2]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accept $R$ more than 0.5 m below $A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ft for $W/2$ accept $W/2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 7

#### (i)

**Initial momentum**

\[ 0.15 \times 8 + 0.5 \times 2 = 0.15 \times 8 \]

**Final momentum**

\[ 0.15 \times 8 + 0.5 \times 2 = 0.5v \]

(or \( 0.15 \times 8 = 0.5 \times (v - 2) \))

\[ v = 4.4 \]

\[ (m)g \sin \alpha = \pm (m)a \]

\[ a = (\pm) 4.9 \]

**Either** (see also part (ii))

\[ 0 = 4.4^2 - 2 \times 4.9s \]

\[ s = 1.97 \text{ or } 1.98 \text{ m} \]

**OR**

\[ v^2 = 4.4^2 - 2 \times 4.9 \times 2 \]

\[ v^2 = -0.24 \]

**OR** (see also part (ii))

\[ t = 4.4/4.9 (=0.898) \text{ with either} \]

\[ s = 4.4 \times 0.898-0.5 \times 4.9 \times 0.898 \text{ or } s = (4.4 + 0)/2 \times 0.898 \]

\[ s = 1.97 \text{ or } 1.98 \text{ m} \]

**(or loss in A’s momentum = \(0.15 \times 8\) and gain in B’s momentum = \(0.5(v - 2)\))**

**For using the principle of conservation of momentum**

Condone inclusion of \(g\) in all terms

**SR Awarded even if \(g\) in all terms**

Condone \(\cos\)

**For using \(v^2 = u^2 + 2as\) with \(v = 0\)**

Accept \(s < 2 \text{ iff } s = 4.4^2/(2 \times 4.9)\)

**For using \(v^2 = u^2 + 2as\) with \(s = 2\)**

Accept \(v^2 < 0\)

Both parts of method needed

Accept \(s < 2\)

\(cv\) for acceleration

Accept \(0.903 \leq \text{time} \leq 0.904\)

**Appropriate use of \(s = ut + \frac{1}{2}at^2\)**

Correct method for solving QE

2.171...

Or using \(s_{up}\) to find \(t_{up}\)

\(s = ut + \frac{1}{2}at^2\) with \(cv\) \(s\) in part

(i)

Not the final answer

\[ s = ut + \frac{1}{2}at^2 \text{ with } s = 0 = 1.796 \]

#### (ii)

\[ 2 = \frac{1}{2}4.9 \times t_A^2 \]

\[ t_A = 0.904 \]

**Either**

\[ 2 = (-4.4)t_A + \frac{1}{2}4.9t_B^2 \]

\[ t_B = (4.4! \div (4.4^2 +4 \times 2.45 \times 2))/4.9 \]

\[ t_B = 2.17 \]

\[ t_B - t_A = (2.17 - 0.9) = 1.27 \text{ s} \]

**OR**

\[ t_{up} = 4.4/4.9 (=0.898) \]

\[ (2 + 1.98) = 0.5 \times 4.9 \times t_{down}^2 \]

\[ t_{down} = 1.27 \]

\[ t_B - t_A = (0.9 + 1.27 - 0.9) = 1.27s \]

**OR**

\[ 0 = 4.4t - \frac{1}{2}4.9t^2 \]

(i.e. approx 1.8 s to return to start)

\[ 2 = 4.4t + 4.9t^2 \]

\[ t = 0.376 \]

\[ t_B - t_A = 1.796+0.376-0.9 = 1.27s \]
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>tanθ = ½ (θ = 18.4° at B)</td>
<td>M1</td>
<td>71.6° at C</td>
</tr>
<tr>
<td></td>
<td>3 x T sinθ = 20 x 1.5 must have two distances and no g</td>
<td>M1</td>
<td>M(A) (d=3√10)</td>
</tr>
<tr>
<td></td>
<td>T = 31.6 N</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>(i) 0 = 50 sin25° t – 9.8t²</td>
<td>M1</td>
<td>or 0=50sin25° - 9.8t &amp;2t :2x2.16</td>
</tr>
<tr>
<td></td>
<td>t = 4.31 s</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) d = 50cos25° x 4.31</td>
<td>M1</td>
<td>or u²sin(2x25°)/g</td>
</tr>
<tr>
<td></td>
<td>195 m</td>
<td>A1</td>
<td>2√ 50cos25° x their t</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>(i)a 100 J</td>
<td>B1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b 7500 Nm</td>
<td>B1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(ii) 400cosα x 25 =7500 + 100√ for = a + b</td>
<td>M1</td>
<td>sc N II gets M1A1only. This M1 for total M (a=0.08)&amp;A1for α</td>
</tr>
<tr>
<td></td>
<td>√ 400cosα x 25 =7500 + 100√ for = a + b</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>α = 40.5</td>
<td>A1</td>
<td>3 or 0.707 rads</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>(i) horiz comps in opp direct</td>
<td>B1</td>
<td>at E &amp; F</td>
</tr>
<tr>
<td></td>
<td>Right at E + Left at F</td>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(ii) 1.6x9.8x30 = 20X or 0.5x30g + 0.7x30g + 0.2x60g = 20X</td>
<td>M1</td>
<td>or 10X + 1.6gx30 = 30X M(A)</td>
</tr>
<tr>
<td></td>
<td>X = 23.5 N</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii) 1.6 ỹ = 20x0.2 + 20x0.2 +40x0.5</td>
<td>A1</td>
<td>or 1.6 ỹ =20x0.2x2+40x0.7(22.5)</td>
</tr>
<tr>
<td></td>
<td>ỹ = 17.5 cm</td>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>(i) 6m = 3mx + 2my</td>
<td>M1</td>
<td>- 3mx ok if clear on diagram</td>
</tr>
<tr>
<td></td>
<td>6 = 3x + 2y</td>
<td>A1</td>
<td>m must have been cancelled</td>
</tr>
<tr>
<td></td>
<td>e = 1 = (y-x)/2</td>
<td>M1</td>
<td>or ½,3m,2² = ½,3mx² + ½,2my²</td>
</tr>
<tr>
<td></td>
<td>x = 0.4 or 2/5</td>
<td>A1</td>
<td>sc A1A0 if x = 2, y = 0 not rejected</td>
</tr>
<tr>
<td></td>
<td>y = 2.4 or 12/5</td>
<td>A1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>(ii) 4.8m or 24m/5</td>
<td>B1</td>
<td>2m x their y or 3m(2-their x)</td>
</tr>
<tr>
<td></td>
<td>same as original dir. of A</td>
<td>B1</td>
<td>use their diagram(or dir. of B)</td>
</tr>
<tr>
<td></td>
<td>(iii) e = (2.8 - 1.0)/2.4</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.75 watch out for ± fiddles</td>
<td>A1</td>
<td>2√ (1.8/their y) with 0 ∘ e ⊕ 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Question 6

(i) \( x = 7t \)  

\[ y = -4.9t^2 \text{ or } -\frac{1}{2}gt^2 \]  

\[ y = -\frac{x^2}{10} \] (no fiddles)  

(ii) \( -20 = -\frac{x^2}{10} \)  

\[ 14.1 \text{ m} \]  

(iii)  

\[ \frac{1}{2}mv^2 = \frac{1}{2}m(7)^2 + mgx_{20} \text{ n.b. } v^2 = u^2 + 2as \]  

\[ dy/dx = -\frac{2x}{10} \] & \( \tan \theta \)  

\[ \cos \theta = \frac{7}{21} \] or \( \sin \theta = \frac{19.8}{21} \]  

\( 70.5^\circ \) to horizontal  

\[ 7.5^\circ \text{ to vertical} \]

### Question 7

(i) \( F = \frac{300}{12} \)  

\[ R = 25 \]  

(ii) \( P = 17.5 \times 12 \)  

\[ P = 210 \text{ W} \]

(iii) \( 500 = F \times 12 \)  

\[ 0.0512 \text{ ms}^{-2} \]  

(iv) \( \frac{1}{2}mv^2 = \frac{1}{2}m(19.8)^2 - 200 \times 120 \)  

\[ 14.5 \text{ ms}^{-1} \]

### Question 8

(i) \( R \cos 30^\circ = 0.1 \times 9.8 \)  

\[ R = 1.13 \text{ N} \]  

(ii) \( r = 0.8 \cos 30^\circ = 0.693 \) or \( 2\sqrt{3}/5 \)  

\[ R \cos 60^\circ = 0.1 \times 0.693 \]  

\[ \omega = 2.86 \]  

(iii) \( T = 1.96 \text{ N} \)  

(iv) \( R \cos 30^\circ = T \cos 60^\circ + 0.1 \times 9.8 \)

\[ 4.43 \text{ ms}^{-1} \]

### Question 9

(iv) \( LHS \) (or RHS)  

\[ T + 0.1 \times 9.8 \cos 60^\circ \]  

\[ 4.43 \text{ ms}^{-1} \]
1 \[\pm (5.4\cos 45^\circ - 8.7)\]  
M1 For attempting to find \(\Delta v\) in i dir’n

\[\begin{align*} 
I \cos \theta &= \pm 0.4(5.4\cos 45^\circ - 8.7) \\
I \sin \theta &= 0.4 \times 5.4 \sin 45^\circ \\
I &= \pm \sqrt{1.527^2 + 1.953^2} \\
\theta &= \tan^{-1}[1.527/(-1.953)] \\
\text{Magnitude is 2.48 kgms}^{-1} \\
\text{Direction is 142° to original dir’n.} \\
\end{align*}\]  
M1 For using Pythagoras or trig.

\[\begin{align*} 
\text{Magnitude is 2.48 kgms}^{-1} \\
\text{Direction is 142° to original dir’n.} \\
\end{align*}\]  
A1

\[\begin{align*} 
\theta &= 38.0^\circ \text{ with } \theta \text{ shown appropriately} \\
\end{align*}\]  
A1 [7] Accept \(\theta = 38.0^\circ\) with \(\theta\) shown appropriately

\begin{itemize} 
\item \[I \cos \theta = \pm 0.4(5.4\cos 45^\circ - 8.7)\]  
A1 For using \(I = m(\Delta v)\) in i direction  
(\(= \mp 1.953\))
\item \[I \sin \theta = 0.4 \times 5.4 \sin 45^\circ\]  
B1 (\(= 1.527\))
\end{itemize}

\begin{itemize} 
\item \[\theta = \tan^{-1}[1.527/(-1.953)]\]  
A1 For using Pythagoras or trig.
\end{itemize}

\begin{itemize} 
\item \[\lambda = 117.6 \text{ N}\]  
\end{itemize}

\begin{itemize} 
\item \[0.5 \times 1.6 \times \lambda \times 2 \times 3 = 1.6 \times 9.8 \times 5\]  
A2,1,0 For use of conservation of energy  
-1 each error
\end{itemize}

\[\text{v} = 5.75 \text{ ms}^{-1}\]  
4 Perp. vel. of A after impact = 4

\[ 5x0 - 2x4 = 5a + 2b \]

0.75 x 4 = b - a

Speed of B is 1 m s\(^{-1}\); direction // l.o.c and to the right

\[ v_A = \sqrt{\left(4^2 + (-2)^2\right)} \]

\[ \tan(\text{angle}) = 4/2 \]

Speed of A is 4.47 m s\(^{-1}\); direction is 63.4° to l.o.c and to the left

5 (i) For any moment equ. that includes F and all other relevant forces

\[ 1.8F = 0.9x40 + 1.4x9 \]

Magnitude is 27 N

(ii) Vertical comp. is 22 N downwards

1.2X = (40+9-27)x(3.8-1.8) + 64 x1 (1.2X = 44 + 64)

Horizontal comp. is 90 N to the left

\[ \mu = 27/90 \]

Coefficient of friction is 0.3

6 (i) For use of conservation of energy

\[ 0.5x0.3v^2 - 0.5x0.3x2^2 = 0.3x9.8x0.5\cos60 - \]

\[ 0.3x9.8x0.5\cos\theta \]

\[ v^2 = 8.9 - 9.8\cos\theta \]

(ii) For using Newton’s 2nd law radially

\[ T + 0.3x9.8\cos\theta = 0.3v^2/0.5 \]

\[ T + 2.94\cos\theta = 0.6(8.9 - 9.8\cos\theta) \]

Tension is (5.34 - 8.82\cos\theta) N

(iii) For using \( T = 0 \)

Basic value \( \theta = 52.7^\circ \)

\[ \text{Angle} = (360-52.7) - 60 \]

\[ \text{Angle turned through is 247^\circ} \]
7 (i) For using $T = \frac{\lambda}{e/L}$ once

- $T_A = 180 \times \frac{0.5}{1}$ or $T_B = 360 \times \frac{0.3}{1.2}$

- $0.3/1.2$

- $480e = 240 \text{ or } T_A = 90, T_B = 90$

For using $T_A(e) = T_B(e)$ or attempting to show $T_A = T_B$ when $BQ = 1.5$

(ii) $BQ = 1 + 0.5 = 1.5 \text{ m or } T_A = T_B$

- $T_A = 180(0.5 + x)$

- $1.2 \frac{d^2x}{dt^2} = 300(0.3-x) - 180(0.5+x)$

- $\frac{d^2x}{dt^2} = -400x$

For using Newton’s 2nd law

- Period is $2\pi/\sqrt{400} = 0.314 \text{ s}$

(iii) Max amplitude $= 1.5 - 1.2 = 0.3 \text{ m}$

- amplitude $= \frac{u}{\sqrt{400}} \text{ or } 180x0.5^2/(2x1) + 360x0.3^2/(2x1.2) + \frac{1}{2} 1.2u_{max}^2 = 180x0.8^2/(2\pi)$

For using Amp. $= \frac{u}{\omega}$ or ‘energy at equil. pos’n = energy at max. displ.’

Maximum value of $u$ is 6

(iv) $-0.2 = 0.3 \sin 20t$

- $20t = 0.7297 + 3.142$

- Time taken is 0.194s

For relevant trig. equation

For method of obtaining relevant solution
1(i) \( \frac{2}{3} + \text{prod of 2 P's} \) or \( 1 - \frac{1}{3} \times \frac{1}{4} \)
\( = \frac{11}{12} \) or 0.917 (3 sfs)  
M1 M1 A1 3

(ii) \( \frac{1}{3} \times p \)
\( = \frac{5}{6} \) oe  
M1 M1 A1 3

\( \frac{2}{3} + \frac{1}{3} \times p = \frac{5}{6} \)

or \( \frac{1}{3}(1 - p) \)

or \( \frac{1}{3}(1 - p) = 1 - \frac{5}{6} \)

SW: \( \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \) M2 A0, unless clear this is a check

Total 6

2(i) 124.5, 4.8 B1 B1 2

(ii) mean smaller or generally smaller or means similar or hts similar oe  
More widely spread or varied oe  
Assume 2nd referred to unless clear 1st or less consistent or gter dispersion or further from mean, gter variance Not “range” greater  
Allow opposite if fit (i)

Total 6

3(i) \( \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \) or \( \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} \)
\( \times 2 \) or +  
\( \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} \)
\( = \frac{1}{5} \) AG  
M1 M1 M1 A1 4

\( \Sigma xp \)
\( = 4 \)  
M1

\( \Sigma x^2 p = (17) \) - \( \mu^2 \)  
M1

\( = 1 \)  
A1 5

\( \Sigma xp & \Sigma x^2 p, if + \) eg 4: M0 A0 (- \( \mu^2 \) poss M1)

Total 9
### 4(i)(a)

**Total area = 60 sqs**

Recognize that the total area represents 300

\[8 \times 300/60 = 40\]

- **M1**
- **M1**
- **M1**
- **A1** 4

**Attempt total area, eg 15000 or 15 cm²**

eg 1 sq = 5 or 15000 \(\div (300 \text{ or } 50)\) or 2000/50
cao

### (b)

Splitting classes

\[1.2 \times 4 \times 5 \text{ or } 0.8 \times 6 \times 5 \text{ oe}\]

- **M1**
- **M1**
- or 0.316 \(\times 5\) or 0.4 \(\times 12 \times 5\) or 24

**NB other correct eg 2 \(\times 4 \times 5 + ^{4/7} \times 2 \times 5\)**

### (ii)(a)

Box & whisker

Cum freq diag

- **B1** 1
- **B1** 1

### Total

| 9 |

### 5(i)(a)

\[\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4\]

- **M1**
- **A1** 2

**Allow index 3 or 5**

\[\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4\]

- **M1**
- **M1**
- **A1** 3

**B(5, \frac{2}{5})** stated

\[5 \times \left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 \text{ or } 0.3370 - 0.0778\]

- **M1**
- **M1**
- **A1** 3

**or \(\cdot C_\alpha \text{ or } 5 \cdot C_\beta\) \(\cdot C_\alpha \text{ or } 5 \cdot C_\beta\)** & \(a + b = 5\)

### (ii)(a)

\[\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4\]

- **M1**
- **A1f** 2

eg ft: (a) 0.0518 \(\rightarrow 0.0207\)

(a) 0.922 \(\rightarrow 0.369\)

### Total

| 10 |

### 6(ii)

\[\begin{align*}
\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 & = 0.259 \text{ (3 sfs) or } \frac{344}{625} \text{ oe} \\
\frac{2}{5} \times \frac{3}{5} & = 0.104 \text{ (3 sfs) or } \frac{344}{625} \text{ oe} \\
\end{align*}\]

- **M1**
- **M1**
- **A1** 3

**M1 either comb. 140/330: M1M1**

### (ii)

\[\begin{align*}
\left(\frac{2}{3}\right)^4 & \quad \text{or } \left(\frac{3}{5}\right)^4 \quad \text{or } \left(\frac{5}{7}\right)^4 \\
\text{or } \frac{2}{3} \times \frac{3}{5} & \quad \text{or } \frac{3}{5} \times \frac{2}{3} \\
\end{align*}\]

- **M1**
- **M1**
- **A1** 3

or \(\left(\frac{3}{5}\right)^4 \times \left(\frac{2}{3}\right)^4 \times \left(\frac{2}{3}\right)^4 \text{ or } a + b = 5\)

\[\begin{align*}
\left(\frac{3}{5}\right)^4 & \quad \text{or } \left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{3}\right)^4 \text{ or } a + b = 5\]
\end{align*}\]

**all correct**

### (iii)

\[\begin{align*}
\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 & \quad \text{or } \left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{3}\right)^4 \\
\text{or } \frac{3}{4} \times \left(\frac{1}{2}\right) & \quad \text{or } \frac{3}{4} \times \left(\frac{1}{2}\right) \\
\end{align*}\]

- **M1**
- **M1**
- **A1f** 3

**1 correct prod or “140” – any prod**

\[\begin{align*}
\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 & \quad \text{or } \left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{3}\right)^4 \\
\text{or } \frac{3}{4} \times \left(\frac{1}{2}\right) & \quad \text{or } \frac{3}{4} \times \left(\frac{1}{2}\right) \\
\end{align*}\]

- **M1**
- **A1f** 3

**ft only “140”**
### Question 7

**Part (i)**
- Binomial
  - \( n = 10, \ p = 0.9 \)
- Each seed equally likely germ
  - or \( P(\text{germ}) \) same for all seeds oe
- Seeds independent oe

Both req’d. Ignore \( q = 0.1 \)
- or seeds grown in same conditions
- Context nec’y for each B1

**Part (ii)**
- \( 0.0702 \) (3 sfs)

0.07 or 0.2639; B1
- \( \Sigma \) or \( 1-\Sigma \): 1 term extra or omit or wrong:
  - M1

**Part (iii)**
- \( 1 - "0.0702" \)
  - \( 0.9298^{20} + {20 \choose 1} \times 0.0702 \times 0.9298^{19} \)
  - \( = 0.585 \) (3 sfs)

Or 0.9298 or 0.93(0) seen
- M1 each term
- cao
  - eg ft (ii) 0.2639 → (iii) 0.0178 from correct wking: M3A0
  - \( 0.0702^{20} + {20 \choose 1} \times 0.9298 \times 0.0702^{19} \)
  - (=2.25x10^{-21}); SC M1M1
- NB ft (ii) for all M mks. But if 0.1, 0.9 used, must be clear using (ii) rounded

**Total**
- 10
8(i)(a) Ranks 1 2 3 4 5 6 7 8 9 9 8 7 6 5 4 3 2 1
3 2 1 5 4 7 8 6 9 7 8 9 5 6 3 2 4 1
\[ \Sigma d^2 = 16 \]
\[ r_s = 1 - \frac{6 \times \text{their 16}}{9 \times (9^2 - 1)} = 0.867 \text{ (3 sfs)} \]
M1 M1dep
A1 A1dep
Correct ranks Dep ranks attempted
Attempt ranks, same dir’n Correct formula with \( n = 9 \), dep M1M1
or ft (a) or ft (a)

(b) Countries with larger pops tend to have larger capital pops. oe
B1ft
Must interp & refer to context.
Not “Gd corr’n country & cap pops”
Not “Gd agree’t country & cap pops”
Not “Gd rel’nship country & cap pops”
Not “proportional”

(ii) \[ \frac{1533.76 - (337.5 \times 28.3)/9}{\sqrt{(18959.11 - 337.5^2/9)(161.65 - 28.3^2/9)}} = 0.698 \text{ (3 sfs)} \]
M1 M1
A1 A1
Correct formula = 0.47251/\(1/(6302.86 \times 72.66)\)
Not correct subst in 2 “S” formulae, any version
No wking: 0.7 M0A0; 0.70: M1A0
or nearer to 1 or nearer to 1

(iii) Increase
B1
y indep or known or given or \( x \) unknown or \( x \) dep on \( y \) oe

(iv)(a) \( x \) on \( y \)
Est country pop from cap or \( x \) from \( y \) oe
B1ind
B1ind
\( y \) indep or known or given or \( x \) unknown or \( x \) dep on \( y \) oe
or reliable because \( r \) (or \( r_s \)) high: B1
or unreliable because \( r \) (or \( r_s \)) not hi: B1
“accurate”: B0

(b) any indication different context, eg “Africa”, “remote areas” unreliable
B1dep
B1dep
Total 13
Total 72 marks
1 | (i) (a) Po(2): $1 - P(\leq 3) = 0.1429$ & M1 2 | Po(2) tables, “1 – ” used Answer, a.r.t. 0.143  
(b) Po(2/3): $e^{-2/3} \left(\frac{2}{3}\right)^2 = 0.114$ & M1 3 | Poisson formula correct, $r = 2$, any $\mu$ Answer, a.r.t. 0.114  
(ii) Foxes may congregate so not independent & B1 2 | Independent/not constant rate/singly used Any valid relevant application in context

2 | N(80/7, 400/49)  
\[ \frac{13.5 - 80}{400} = 0.725 \]  
$1 - \Phi(0.725) = 0.2343$ & B1 7 | 80/7, a.e.f (11.43)  
400/49 or 20/7 seen, a.e.f. (8.163 or 2.857)  
Standardise with $np$ & $npq$ or $\sqrt{npq}$ or $nq$, no $\sqrt{n}$  
$\sqrt{npq}$ correct Answer, a.r.t. 0.234  
[SR: Binomial, complete expression M1, 0.231  
Po(80/7) B1, complete expression M1, 0.260  
Normal approx to Poisson, B1B0 M1A0A1 M1A0]

3 | H$_0$: $p = 0.3$  
H$_1$: $p \neq 0.3$  
B(8, 0.3)  
P$(\leq 4) = 0.9420$; $P(> 4) = 0.0580$  
P$(\leq 5) = 0.9887$; $P(> 5) = 0.0113$  
Compare 0.025 or critical value 6  
Do not reject H$_0$  
Insufficient evidence that manufacturer’s claim is wrong & B1 7 | NH stated, must be this form (or $\pi$)  
AH stated, must be this form (or $\pi$) $[\mu$: B1 both]  
B(8, 0.3) stated or implied Any one of these four probabilities seen  
Either compare $P(\geq 5)$ & 0.025 / $P(\leq 4)$ & 0.975  
Or critical region $\geq 6$ with 5  
H$_0$ not rejected, can be implied, needs essentially correct method Correct conclusion in context  
[SR: Normal, Poisson: can get  
B2M1A0M0M1A1  
P$(\leq 5)$: first 4 marks. $P(= 5)$: first 3 marks only.]

4 | (i) B(80, 0.02)  
approx Po(1.6)  
$1 - P(\leq 1) = 1 - 0.5249 = 0.4751$ & M1 4 | B(80, 0.02) seen or implied, e.g. N(1.6, 1.568)  
Po$(np)$ used $1 - P(\leq 1)$ used Answer, a.r.t. 0.475  
[SR: Exact: M1 M0 M0, 0.477 A1]

(ii) $P(\leq 4) = 0.9763$, $P(\geq 5) = 0.0237$  
$P(\leq 5) = 0.9940$, $P(\geq 6) = 0.0060$  
Therefore least value is 6 & M1 3 | Evidence for correct method, e.g. answer 6  
At least one of these probabilities seen Answer 6 only  
[SR: N(1.6,1.568); $2.326 = \sqrt{r - 1.6}/\sqrt{1.568}$ M1  
r = 5 or (with cc) 6 A1  
Exact: M1 A0 A1]
5  
(i) \( \frac{0 - \mu}{\mu / 2} = -2, \)  
independent of \( \mu \)
\[ 1 - \Phi(2) = 1 - 0.9772 = 0.0228 \]
M1  
A1  
A1  
A1  
4
Standardise, allow \(-\), allow \(\mu^2/4\)  
z = 2 or \(-2\)  
z-value independent of \(\mu\) and any relevant statement  
Answer, a.r.t. 0.023

(ii) \( \Phi[(9 - 6)/3] \)
\[ \Phi(1.0) = 0.8413 \]
\[ [\Phi(1.0)]^3 = 0.59546 \]
M1  
A1  
M1  
A1  
4
Standardise and use \(\Phi[no \ \sqrt{n}]\)  
0.8413 [not 0.1587]  
Cube previous answer  
Answer, in range \([0.595, 0.596]\)

(iii) Annual increases not independent
B1  
1
Independence mentioned, in context. Allow “one year affects the next” but not “years not random”

6  
H₀: \( \mu = 32; H₁: \mu > 32 \), where \( \mu \) is population mean waist measurement  
\( \bar{x} = 32.3 \)
\[ s^2 = 52214.50/50 - \bar{x}^2 \ = 1 \]
\[ \sigma^2 = 50/49 \times s^2 \ = 50/49 \text{ or } 1.0204 \]
M1  
A1  
B1  
B1  
M1  
M1  
B1  
A1  
10
Correct formula for \(z\), can use \(s, \sigma\), aef, need \(\mu = 32\)
\( z = 2.1 \) or \(1 - \Phi(z) = 0.0179, \text{ not } -2.1 \)
Explicitly compare their 2.1 with 3.09(0) or their 0.0179 with 0.001

\( \alpha: \)  
\[ z = (32.3 - 32) \times \sqrt{49} = 2.1 \]
Compare 2.1 with 3.09 or 0.0179 with 0.001
M1  
A1  
B1

\( \beta: \)  
\[ CV = 32 + 3.09 + \sqrt{49} = 32.44 \]
Compare CV with 32.3
M1  
B1  
A1

Do not reject H₀
M1\( \Box \)
Correct conclusion, can be implied, needs essentially correct method including \(\sqrt{n}\), any reasonable \(\sigma\), but not from \(\mu = 32.3\)  
Interpreted in context

7  
(i) \( \frac{80 - c}{8/\sqrt{12}} = 2.326 \)
\[ c = 74.63 \]
M  
I  
A  
I  
B  
I  
A  
I  
4
Equate standardised variable to \(\Phi^{-1}\), allow \(-\sqrt{12}\), 8 correct  
2.326 or a.r.t 2.33 seen, signs must be correct  
Answer, a.r.t. 74.6, cwo, allow \(\leq\) or \(\geq\)

(ii)
(a) Type I error
(b) Correct
B  
I\( \Box \)
1
1
B  
I  
1
1

“Type I error” stated, needs evidence  
“Correct” stated or clearly implied  
Wrong \(c: 74 < c < 75\), B1\( \sqrt{B1} \)
\(c < 74\), both “correct”, B1. 75 < \(c\) < 80, both “Type I”, B1
Also allow if only one is answered  
\(c - \mu \sqrt{8/12} = (\pm)\Phi^{-1}\), allow no \(\sqrt{12}\) but not 80, not 0.8264

(iii) \( \frac{78.22 - \mu}{8/\sqrt{12}} = -1.555 \)
M1*d  
ep  
A1\( \Box \)  
dep*  
M1  
A1  
4
Correct including sign, \(\Box\) on their \(c\)  
Solve to find \(\mu, \ dep\), answer consistent with signs  
Answer, a.r.t. 78.2
\[
\int_0^1 x^n \, dx = \left[ \frac{x^{n+1}}{n+1} \right]_0^1 = \frac{1}{n+1} \\
k/(n+1) = 1 \text{ so } k = n + 1
\]

\[
\int_0^1 x^{n+1} \, dx = \left[ \frac{x^{n+2}}{n+2} \right]_0^1 = \frac{1}{n+2} \\
\mu = \frac{k}{n+2} = \frac{n+1}{n+2} \quad \text{AG}
\]

\[
\int_0^1 x^5 \, dx = \left[ \frac{x^6}{6} \right]_0^1 = \frac{1}{6} \\
\sigma^2 = \frac{4}{6} - \left(\frac{1}{6}\right)^2 = \frac{7}{36}
\]

\[
\text{Normal stated} \\
\text{Mean } \frac{4}{3} \quad \text{or } \frac{n+1}{n+2} \\
\text{Variance same as their (iii)/100, a.e.f., allow } \sqrt{\frac{7}{36}}
\]

\[
\text{Same distribution, translated} \\
\text{Mean 0} \\
\text{Variance } \frac{7}{36}
\]

\[
\text{Can be negative translation; or integration, must include correct method for integral} \\
(\text{Their mean}) - \frac{4}{3}, \text{ c.w.d.} \\
\text{Variance same as their (iii), or } \frac{7}{36} \text{ by integration}
\]
STATISTICS 3

1 (i) 

\[ p_S \pm z_{\text{est}} \]

Use formula, \( \sigma \) involving \( p_S \) and \( \mu \) involving \( \mu_s \).

\[ p_s = \frac{186}{400}(0.465) \]

\[ \sigma_{\text{est}} = \sqrt{\frac{0.465 \times 0.535}{400}} \]

\[ z = 1.96 \]

(0.416, 0.514)

A1

5

(ii) Councillor statement implies \( p = 0.5 \).

CI does contain 0.5 but just so councilor probably correct.

assertive

B1

Any justifiable comment Not too

2 (i) \( \sigma^2 \) unknown

B1

(ii) \( H_0: \mu = 2000 \) (or \( \geq \)), \( H_1: \mu < 2000 \)

\[ \bar{x} = 1958.2, s = 115.57 \]

B1

EITHER: Test statistic = \( \frac{1958.2 - 2000}{115.57/2} \)

\[ = -0.7234 \]

Critical value –1.638

Test statistic not in CR, accept \( H_0 \)

Accept that specification is being met

M1

OR: Critical region:

\[ \frac{\bar{x} - 2000}{115.57/2} < t \]

\[ t = -1.638 \]

\[ \bar{x} < 1905.2 \]

As above

M1

8

Conclusion in context

3 (i) Use of \( \int_{20}^{a} f(t)\,dt \)

M1

With limits and \( f(t) \) substituted

\[ \left[ -\frac{2 \cos \frac{\pi t}{60} \right]_{20}^{a} \]

A1

AG

3

Properly obtained

(ii) \( 3 \times (i) + 2 \times (1-(i)) \)

M1

Idea of expectation

A1

All correct

From equation in a, 2 or 3

Equate to 2.80 and attempt to solve

M1

a = 44.8

A1

4

Accept 45

SR: \( \frac{3}{2}(1- \text{cos}..) = 0.8 \) give max3/4
4 (i) Use Poisson distribution \( \text{M1} \) Po(5.5) or Po(55) seen

With \( \mu = 55 \)

\[ \sigma^2 = 55 \text{ M1} \]

\[ (39.5 - 55) / \sqrt{55} \text{ A1} \]

\[ -2.09 \text{ art} \ 0.982 \text{ A1} \]

(ii) \( E(X - Y) = 37 \text{ B1} \) ft \( \mu \) above

\[ \text{Var}(X - Y) = 55 + 18 \text{ M1} \]

\[ = 73 \text{ A1} \] ft \( \mu \) above

(iii) EITHER: Expectation not equal to variance OR: \( X - Y \) could be negative OR: Difference of two Poisson variables could have a negative expectation M1 Any one

So \( X - Y \) does not have a Poisson distn A1

5 (i) EITHER: Use \( \frac{1}{8}(3-1)^2 = a(3-2) \)

OR: \( a(4-2) = 1 \text{ M1} \) Continuity of \( F \)

\[ a = \frac{1}{8} \text{ A1} \] 2

(ii) \( F(1.8) = \frac{1}{4}(0.8)^2 \text{ M1} \) Appropriate use of \( F \)

\[ = 0.08 \text{ A1} \] 2

(c) \( C_{18}(8) = 1.8 \text{ A1} \) 2

(iii) \( G(y) = P(Y \leq y) = P( (X-1)^{1/2} \leq y) \text{ M1} \)

\[ = P(X \leq y^2 + 1) \text{ A1} \]

\[ = F(y^2 + 1) \text{ A1} \]

\[ G(y) = \begin{cases} 
\frac{1}{8}y^4 & (0 \leq y \leq \sqrt{2}), \\
\frac{1}{2}(y^2 - 1) & (\sqrt{2} < y \leq \sqrt{3}).
\end{cases} \text{ A1} \]

Ignore others, A1 for both ranges of \( y \) B1 5

(iv) Use \( G(y) \) to find \( C_{18}(8) \text{ M1} \)

Obtain \( \sqrt{(0.8)} \text{ A1} \)

Correct verification B1 3
6 (i) \[ s^2 = \frac{(8 \times 0.7400 + 9 \times 0.8160) / 17}{0.780} \]
\[ s = 0.780 \text{ AG} \]
Formula for pooled estimate
At least 4DP shown

(ii) Assumes braking distances have normal distributions
Use \[ x_A - x_B \pm \sigma \]
\[ t = 2.567 \]
\[ \sigma = \sqrt{\frac{0.7802(1/10 + 1/9)}{17}} \]
Must be \( t \) value
Allow 0.780
art (1.52, 3.60)

(iii) \( H_0: \mu_A - \mu_B = 2, H_1: \mu_A - \mu_B > 2 \)
Use of CV, 1.740

OR:
Critical region
\[ x_A - x_B > 2 + 1.74 \times 0.4054 \]
Not from different signs test statistic and
Insufficient evidence to accept claim


7 (i) \[ \int_2^3 x^\alpha x^{-a-1} dx = (\int_2^3 x^\alpha - a \) dx \]
Correct limits not required

(ii) \( \alpha(1-\alpha) = 1.92 \) giving 2.087 AG

(iii) Integral of 2.087x^{-3.087} from 2 to 3
\[ \int_2^{2.087} x^3 dx \]
Obtain AG

(iv) Combine last 3 cells
\[ X^2 = \frac{6.9^2}{152.9} + \frac{6.1^2}{26.9} \]
Accept one error
All correct

Use CV 5.991
ft number of sells used.
Accept that data supports Zipf's law
SR: From 6 cells: B0M1A1 (for 9.34) then B1 for 9.488, B1 Max 4/6
### 1

<table>
<thead>
<tr>
<th>Arc</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>3</td>
</tr>
<tr>
<td>FG</td>
<td>4</td>
</tr>
<tr>
<td>JL</td>
<td>5</td>
</tr>
<tr>
<td>EG</td>
<td>6</td>
</tr>
<tr>
<td>AE</td>
<td>7</td>
</tr>
<tr>
<td>BG</td>
<td>7</td>
</tr>
<tr>
<td>AB</td>
<td>8</td>
</tr>
<tr>
<td>CH</td>
<td>8</td>
</tr>
<tr>
<td>DF</td>
<td>8</td>
</tr>
<tr>
<td>GJ</td>
<td>8</td>
</tr>
<tr>
<td>HK</td>
<td>8</td>
</tr>
<tr>
<td>AC</td>
<td>9</td>
</tr>
<tr>
<td>DE</td>
<td>9</td>
</tr>
<tr>
<td>FI</td>
<td>9</td>
</tr>
<tr>
<td>GH</td>
<td>9</td>
</tr>
<tr>
<td>HL</td>
<td>9</td>
</tr>
<tr>
<td>AD</td>
<td>10</td>
</tr>
<tr>
<td>GJ</td>
<td>10</td>
</tr>
<tr>
<td>KL</td>
<td>10</td>
</tr>
<tr>
<td>GI</td>
<td>11</td>
</tr>
<tr>
<td>CG</td>
<td>12</td>
</tr>
<tr>
<td>DL</td>
<td>12</td>
</tr>
</tbody>
</table>

**Total weight = 73**

- **M1** For selecting all arcs up to \(AB\) and deleting \(AB\) in list
- **A1** For deleting \(AC, DE\) in list and selecting arcs for tree correctly, indicated in any way
- **M1** For a spanning tree drawn
- **A1** For correct (minimum) spanning tree drawn

**For total = 73**

### 2

- **M1** For temporary labels at \(B\) correct, no extras
- **M1** For temporary labels at \(E\) correct, no extras
- **A1** For permanent labels correct at \(B, C\) and \(E\) (dependent on both M marks)
- **B1** For order of labelling correct at \(C, B\) and \(E\)
- **M1** For temporary labels at \(D\) correct
- **A1** For no permanent label at \(D\)

### 3

- **B1** For a correct graph for (a)
- **B1** For a correct graph for (b)
- **B1** For a clearly correct graph for (c)
- **B1** For a clearly correct graph for (d)

(ii) \(\begin{align*}
2n & \quad \text{if } n \text{ is even} \\
2n + 1 & \quad \text{if } n \text{ is odd}
\end{align*}\)

- **M1** (2) For treating the cases \(n\) odd and \(n\) even separately
- **A1** For both rules correct
### Question 4

#### (i)

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>s</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>-3</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>60</td>
</tr>
</tbody>
</table>

For overall structure correct, including two slack variable columns
For a correct initial tableau, with no extra constraints added

(2)

#### (ii)

Pivot on 2 in x column

\[
\begin{align*}
nr_1 &= r_1 + 5npr \\
nr_2 &= r_2 + 2 \\
nr_3 &= r_3 - 6npr \\
1 &\quad 0 \quad -3.5 \quad 13 \quad 2.5 \quad 0 \quad 25 \\
0 &\quad 1 \quad -1.5 \quad 2 \quad 0.5 \quad 0 \quad 5 \\
0 &\quad 0 \quad 14 \quad -8 \quad -3 \quad 1 \quad 30 \\
\end{align*}
\]

For the correct pivot choice for their tableau
For dealing with the pivot row correctly (formula or numerical)
For dealing with the other rows correctly (formulae or numerical)
For a correct tableau (not ft)

\[
\begin{align*}
x &= 5, \quad y = 0, \quad z = 0 \\
P &= 25 \\
\end{align*}
\]

For reading off x, y and z from their tableau
For reading off P from their tableau

### Question 5

#### (i)

\[
\begin{align*}
x &= \text{number of lengths swum using breaststroke} \\
y &= \text{number of lengths swum using backstroke} \\
z &= \text{number of lengths swum using butterfly} \\
\end{align*}
\]

Maximise \(2x + y + 5z\)

#### (ii)

\[
\begin{align*}
x + y + z &\geq 8 \\
2x + 0.5y + z &\leq 10 \\
x &\geq 2, \quad y \geq 2, \quad z \geq 2 \\
\end{align*}
\]

For defining variables as ‘number of lengths swum’ using each stroke, or equivalent
For a correct expression using their variables
For a correct expression using their variables
For a correct expression using their variables
For correct expressions using their variables

#### (iii)

\[
\begin{align*}
(2, 4), (2, 8), (3.3, 2.7) \\
2\times2 + 8 = 12 \\
2\times3.33 + 2.67 = 9.33 \\
\end{align*}
\]

For plotting the sloping lines correctly
For completely correct shading
For two correct vertices from their graph
For all three vertices correct to at least 1 dp
For calculating P at vertices or using a valid line of constant profit or writing down their max point

#### (iv)

Swim 2 lengths using breaststroke, 8 lengths using backstroke and 2 lengths using butterfly

Total = 22 style marks

For interpreting their solution in the context of the original problem (at least for x and y)
For calculating the number of marks for their solution
6 (i) \[ A-B-D-E-G-F-C-A \]
42 minutes
\[ A-B-D-C-F-G-E-A \]
46 minutes
Upper bound = 42 minutes

For \[ A-B-D-E-G-F-C \], with or without closing tour
For 42

For \[ A-B-D-C-F-G-E \], with or without closing tour
For 46
For the smaller of their two times

(ii) Odd nodes: \( B \ D \ E \ F \)

\[ BD = 5 \quad BE = 6 \quad BF = 16 \]
\[ EF = 10 \quad DF = 14 \quad DE = 7 \]
120 minutes
Travel \( BD \), \( EG \) and \( FG \) twice (accept \( BD \), \( EGF \))
3 times

Total weight of tree = 31 minutes
Two least weight arcs from \( G \) have weight 5+5 =10 minutes
Lower bound = 31 + 10 = 41 minutes

For a diagram or listing showing a tree connecting the vertices \( A, B, C, D, E \) and \( F \), but not \( G \)
For a diagram showing this tree (vertices need to be labelled, but arc weights are not needed)
For stating or using \( GE \), \( GF \) or 5+5 or 10
For 41 or 10 + their 31 calculated
For identifying or using \( B D E F \)

(iii) Odd nodes: \( B \ D \ E \ F \)

\[ BD = 5 \quad BE = 6 \quad BF = 16 \]
\[ EF = 10 \quad DF = 14 \quad DE = 7 \]
120 minutes
Travel \( BD \), \( EG \) and \( FG \) twice (accept \( BD \), \( EGF \))
3 times

Total weight of tree = 31 minutes
Two least weight arcs from \( G \) have weight 5+5 =10 minutes
Lower bound = 31 + 10 = 41 minutes

For a diagram or listing showing a tree connecting the vertices \( A, B, C, D, E \) and \( F \), but not \( G \)
For a diagram showing this tree (vertices need to be labelled, but arc weights are not needed)
For stating or using \( GE \), \( GF \) or 5+5 or 10
For 41 or 10 + their 31 calculated
For identifying or using \( B D E F \)

Total = 6500 seconds or 108 minutes 20 seconds

7 (i) Original list: 34 42 27 31 12 48 24 37
1st pass: 34 27 31 12 48 24 37 42
2nd pass: 27 31 12 34 24 37 42 48
3rd pass: 27 12 31 24 34 37 42 48
4th pass: 12 27 24 31 34 37 42 48
5th pass: 12 24 27 31 34 37 42 48
6th pass: 12 24 27 31 34 37 42 48

Swaps = 5+5+2+2+1 =15
Comparisons = 7+6+5+4+3+2 = 27

Original list: 95 74 61 87 71 82 53 57
1st pass: 74 95 61 87 71 82 53 57
2nd pass: 61 74 95 87 71 82 53 57
3rd pass: 61 74 87 95 71 82 53 57
4th pass: 61 74 87 95 71 82 53 57
5th pass: 61 74 87 95 71 82 53 57
6th pass: 53 61 71 74 82 87 95 57
7th pass: 53 57 61 71 74 82 87 95

Swaps = 1+2+1+3+2+6+6 = 21
Comparisons = 1+2+2+4+3+6+7 = 25

Each script is looked at once
so the time taken is roughly proportional to the number of scripts

(iv) Splitting 100 scripts takes 50 seconds
so splitting 500 scripts takes about 250 seconds
Sorting 50 scripts takes 250 seconds = 0.1 \times 50^2
Sorting 250 scripts takes about 0.1 \times 250^2
= 6250 seconds
Total = 6500 seconds or 108 minutes 20 seconds

Two least weight arcs from \( G \) have weight 5+5 =10 minutes
Lower bound = 31 + 10 = 41 minutes

For a diagram or listing showing a tree connecting the vertices \( A, B, C, D, E \) and \( F \), but not \( G \)
For a diagram showing this tree (vertices need to be labelled, but arc weights are not needed)
For stating or using \( GE \), \( GF \) or 5+5 or 10
For 41 or 10 + their 31 calculated
For identifying or using \( B D E F \)

For result of first pass correct (underlined entries may be omitted)
For second and third passes correct, must be using bubble sort
For fourth and fifth passes correct, must be using bubble sort
For sixth pass correct, from correct method
For seventh pass correct, from correct method
For ‘each script is looked at once’, or equivalent
For ‘proportional’, or equivalent

M1
A1
B1 (6)

For result of first pass correct (underlined entries may be omitted)
For second and third passes correct, must be using bubble sort
For fourth and fifth passes correct, must be using bubble sort
For sixth pass correct, from correct method
For seventh pass correct, from correct method
For ‘each script is looked at once’, or equivalent
For ‘proportional’, or equivalent

M1
A1
B1 (6)

For ‘each script is looked at once’, or equivalent
For ‘proportional’, or equivalent

M1
A1
B1 (6)

For result of first pass correct (underlined entries may be omitted)
For second and third passes correct, must be using bubble sort
For fourth and fifth passes correct, must be using bubble sort
For sixth pass correct, from correct method
For seventh pass correct, from correct method
For ‘each script is looked at once’, or equivalent
For ‘proportional’, or equivalent

M1
A1
B1 (6)

nb decreasing or numbers misread ⇒ M only

Underlined entries may be omitted

nb decreasing or numbers misread ⇒ M only

Underlined entries may be omitted

nb decreasing or numbers misread ⇒ M only

Underlined entries may be omitted

For 6500 or equivalent

M1
A1
B1 (6)

For 6500 or equivalent

M1
A1
B1 (6)
1 (i) $A_1$ or $A_2$ or both joined to $G$, $P$ and $R$
$B_1$ or $B_2$ or both joined to $G$, $T$, $W$ and $Y$
and $C_1$ or $C_2$ or both joined to $P$, $W$ and $Y$

(ii) $A_1$ paired with $G$ and $A_2$ with $P$, or vice versa; $B_1$ paired with $T$ and $B_2$ with $W$, or vice versa; one of $C_1$, $C_2$ paired with $Y$ and the other left unpaired

(iii) $R - A_2 - P - C_2$ or in reverse
Accept $R - A - P - C$ or in reverse
Amanda gets green and red
Ben gets turquoise and white
Carrie gets yellow and pink

(iv) Amanda gets pink and red
Ben gets green and turquoise
Carrie gets white and yellow
### 2

<table>
<thead>
<tr>
<th>Stage</th>
<th>State</th>
<th>Action</th>
<th>Cost</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>6+3 = 9</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4+4 = 8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>6+3 = 9</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>7+4 = 11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>5+8 = 13</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3+9 = 12</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Shortest route: (3; 0) – (2; 1) – (1; 0) – (0; 0)
Length of shortest route = 12 km

**M1**

For completing cost column for stage 2

**A1**

For completing cost column for stage 3

**M1**

For completing minimum column for stage 2

**A1**

For completing minimum column for stage 3

**B1**

For this route (not in reverse)

**B1**

For 12

---

### 3

(i) $6+3+2+4 = 15$ litres per second

(ii) $6+2+3+6+2+2 = 21$ litres per second

Do not use arc DG as it is crossed twice

or $X = \{S,A,B,C,E,F\}$  $Y = \{D,G,H,I,T\}$

(iii) SC cannot be full since the most that can leave it is $2+4 = 6$ litres per second

AD cannot be full since the most that can enter it is $2+3 = 5$ litres per second

The most that can flow in SB is $2+3 = 5$ litres per second

**M1**

For a feasible flow

(may imply vertex labels)

**A1**

For a feasible flow of 14 litres per second

(directions must be shown for the A mark)

**B1**

For 15

**M1**

For showing how 21 (given) was worked out

**A1**

For explaining why arc DG is not used

**B1**

For 6 and ‘out’ or equivalent

**B1**

For 5 and ‘in’ or equivalent

**B1**

For SB = 5

**B1**

For 14

**M1**

For a feasible flow

**A1**

For explicitly stating that this cut = 14

**B1**

For explaining how maximum flow = minimum cut shows that 14 is the maximum here

(At least referring to ‘this flow’ and ‘this cut’, not just stating ‘max flow = min cut)
<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>£260</th>
<th>B1</th>
<th>M1</th>
<th>A1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>£260</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Reduce rows

- 0 60 30 10
- 0 90 90 60
- 0 80 70 40
- 10 10 20 0

Reduce columns

- 0 50 10 10
- 0 80 70 60
- 0 70 50 40
- 10 0 0 0

---

Cross through 0’s using as few lines as possible

- 0 50 10 10
- 0 80 70 60
- 0 70 50 40
- 10 0 0 0

Augment by 10

- 0 40 0 0
- 0 70 60 50
- 0 60 40 30
- 20 0 0 0

---

Cross through 0’s using as few lines as possible

- 0 40 0 0
- 0 70 60 50
- 0 60 40 30
- 20 0 0 0

Augment by 30

- 30 40 0 0
- 0 40 30 20
- 0 30 10 0
- 50 0 0 0

---

Allocation

- \( A = Y \)
- \( B = W \)
- \( C = Z \)
- \( D = X \)

Cost = £180

---

(iii) Hungarian algorithm finds the minimum cost complete matching

- B1 For ‘minimum cost’ or equivalent

- M1 For correct answer with units

- M1 Or scaled throughout by 10

- M1 For correct method for reducing rows

- M1 For correct method for reducing columns

- A1 For a correct reduced cost matrix

- M1 For correct crossing out for their reduced cost matrix. Likely to be shown on reduced cost matrix.

- M1 For correct augmenting from their reduced cost matrix and their crossing out

- A1 For a correct solution after first augmenting

- M1 For correct crossing out for their augmented matrix. Likely to be shown on matrix.

- M1 For correctly augmenting their matrix in one step by an amount greater than 10 (or greater than 1 if scaling has been used)

- A1 For correct final matrix from completely correct method

- B1 For this allocation

- B1 For £180
The dummy activity after C is needed because G follows C only but F follows both B and C. The dummy activity after H is needed because both H and I directly join the same pair of vertices (events).

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
<th>Immediate predecessors</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>B,C</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
<td>C</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
<td>D</td>
</tr>
<tr>
<td>I</td>
<td>4</td>
<td>D</td>
</tr>
<tr>
<td>J</td>
<td>3</td>
<td>E,F</td>
</tr>
</tbody>
</table>

Minimum completion time = 11 days
Critical activities: A, D, I

1 day

Number of workers required = 4

Delay activity E by 2 days, so the start changes from 5 to 7 (it happens on day 8). This causes activities H and J to shift. J shifts by 1 day and H by 2 days.

For explaining the relationships between B, C, F, G
For identifying H and I
For A, B, C having no predecessors
For D, E, F, G correct
For H, I, J correct
For carrying out forward pass
For all early event times correct
For carrying out backwards pass
For all late event times correct
For 11 with units
For A, D, I only
For the first four days correct
For a resource histogram with no 'hanging' cells
For days five onwards correct
For 4 or follow through their histogram if possible
For stating how much to move E
For saying that H and J (only) will need to shift
For describing how much H and J move
| 6 | (i) | In column Y: -3 < 5 so A does not dominate B  
In column X: -3 < 2 (or column Z: 1 < 4) so B does not dominate A | B1 | For -3 < 5 or equivalent  
B1 | For -3 < 2 or 1 < 4 or equivalent |
| (ii) | The worst outcomes for Maria are:  
X lose 4, Y lose 5, Z lose 4 | M1 | For finding column maxima  
A1 | For rejecting 5 as being bigger than 4, or using a word like 'lose' or '-4, -5, -4' |
| (iii) | If Lucy plays B she could win as much as 5 | B1 | For '5 is the most she can win' or equivalent  
B1 | For 'add 3 throughout matrix' or equivalent |
| (iv) | Need to add 3 throughout matrix to make values non-negative, this removes the 3 again | M1 | For showing where one of the expressions came from, or for referring to 'when Maria plays each of her strategies' or equivalent in a non-specific way  
A1 | For specifically linking the expressions to Maria choosing strategy X, strategy Y and strategy Z in that order |
| (v) | Having added 3 throughout, the expected number of points win by Lucy when Maria chooses strategy X is $5p_1 + 0p_2 + 7p_3$, and similarly the second expression is the expected number of points won by Lucy when Maria chooses strategy Y and the third expression is the expected number of points won by Lucy if Maria chooses strategy Z | M1 | For reference to 'number of points won by Lucy' or equivalent  
A1 | For reference to 'the worst outcome' or equivalent |
| (vi) | The number of points that Lucy can expect to win cannot be less than the worst of the three expressions, so it is less than or equal to each of them. | M1 | For reference to 'number of points won by Lucy' or equivalent  
A1 | For reference to 'the worst outcome' or equivalent |
| (vii) | \[2(p_1) + 4(p_3) = 2p_1 + 4(1-p_1) = 4-2p_1 \text{ (given)}\]  
\[4p_1 - 3(1-p_1) = 7p_1 - 3 \text{ (given)}\] | B1 | For $2p + 4(1-p)$  
B1 | For $7p - 3$ |
| (viii) | $4 - 2p_1 = 7p_1 - 3 \Rightarrow p_1 = \frac{7}{9}$  
$p_1 = \frac{7}{9} \Rightarrow 2 \frac{4}{7}, p_1 = 0 \Rightarrow \min(4, -3) = -3,$  
$p_1 = 1 \Rightarrow \min(2, 4) = 2$  
Maximin is when $p_1 = \frac{7}{9} \Rightarrow$ choose randomly between A and C so that A is chosen with probability $\frac{7}{9}$ | B1 | For solving $4 - 2p_1 = \text{their expression to get a probability}$  
M1 | For evaluating 4-2$p_1$ at their $p_1$ and the values -3 and 2  
A1 | For reference to maximin, or equivalent, leading to selection of $p = \frac{7}{9}$, or in context |
Report on the Units
January 2006
General Comments

All seven Core and Further Pure modules were available at this session and each unit proved to be a suitable test for candidates. The preparation of many candidates had clearly been very thorough and much excellent work was seen. The best candidates showed not only competence and insight in dealing with the various topics, but clarity and accuracy in their communication of mathematical ideas. In particular, good candidates were always precise in their use of algebraic notation.

The List of Formulae booklet which is available to candidates contains formulae which are potentially of use in all the units except Core Mathematics 1. It was evident, however, that many candidates did not make effective use of the booklet and so sometimes used formulae which were wrong or presented solutions which were longer than they would have been if a relevant result from the booklet had been used or adapted. It is expected that candidates will be thoroughly familiar with the booklet, knowing which results are there as well as those which are not.

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.
General Comments

The majority of candidates were able to make a good attempt at this paper, with a significant proportion achieving 60 marks or more. Most candidates attempted all questions and many scored well on the longer, more structured questions towards the end of the paper.

As in previous sessions, candidates demonstrated excellent understanding of differentiation but still made errors in calculations involving fractions and/or negative numbers.

Some candidates had failed to learn formulae correctly, particularly the formula for solving a quadratic equation, but also those needed to find the mid-point of a line and the radius of a circle.

Candidates appeared to have enough time to finish the paper and there was some evidence of checking of answers.

Comments on Individual Questions

1  This question was generally well answered.

   In part (i), most candidates stated $x = 8$ although $x − 6$ was also seen quite often.

   Part (ii) was also answered well, $t = 0.1$ being the most common wrong answer. A few candidates used logarithms but they usually failed to evaluate their final expression, not realising that $\log 1 = 0$ (or that $\log 10 = 1$).

   In part (iii), even the very brightest candidates failed to appreciate that $y = −3$ was a solution. Many of the candidates who scored 71 marks overall on the paper lost the single mark here.

2  In part (i), almost all candidates scored the first 2 marks for correctly squaring one of the bracketed expressions. Only the very weakest omitted the middle term when squaring. However, even able candidates were careless with signs, thus losing the final mark. It was also disappointing to see $2(2x − 3)^2$ become $(4x − 6)^2$ in a significant minority of scripts.

   Part (ii) was often done by expanding the expression fully, although good candidates saved themselves time by identifying the required $x^3$ terms. As in part (i), a mark was too often lost due to a lack of care with signs.

3  Both parts of this question were very well answered with a large majority of candidates scoring all 5 marks. Almost all candidates realised the need to differentiate in each part and were able to rewrite $\sqrt{x}$ correctly in index form. It was pleasing to see confident handling of negative fractional indices in many scripts.
Once again, far too many candidates lost marks through poor graph sketching. Some candidates produced a casual pen drawing without using a ruler to draw axes; others laboriously plotted points on graph paper. Centres need to emphasise the importance of sketching graphs, as many candidates seem to lack an understanding of this technique.

In part (i), although the majority of candidates knew the general shape of the curve, marks were frequently lost by the lack of quality in drawing curves approaching asymptotes. Other than this, the common error was to draw the curve \( y = \frac{1}{x} \).

In part (ii), most candidates knew that they needed to translate their curve parallel to an axis. Of those who chose the correct direction, a significant proportion lost a mark by omitting to indicate how far the curve had moved.

Part (iii) was poorly answered, although there was much variation by centre. Most candidates knew that it was a stretch but were let down by a lack of precision in their descriptions; for example, ‘in the \( y \)-axis’ instead of ‘parallel to the \( y \)-axis’ was common, as was the word ‘squash’.

This question was confidently tackled by most candidates with many scoring full marks, although not all perceived the link between the first two parts and the final two parts of the question and restarted in part (iii).

In parts (i) and (ii), completing the square was well done although, as in last year’s papers, there was evidence of some candidates’ inability to manipulate fractions. For example, in part (i), \( \left( \frac{1}{2} \right)^2 \) was simplified to \( \frac{9}{2} \) or \( \frac{9}{4} \). In part (ii), \(-4 - \frac{11}{4}\) often became \( -\frac{15}{4} \) or, in a few cases, \( 4 \frac{11}{4} \).

Part (iii) was generally answered well although \((-3, 4)\) was a common wrong answer, as was \( \left( \frac{3}{2}, -2 \right) \).

Part (iv) proved more challenging with many candidates including the \( -\frac{11}{4} \) term twice in their calculation of the radius, while others thought that the radius was simply \( \sqrt{\frac{11}{4}} \). Once again, there were errors in arithmetic involving negative signs.

Most candidates did the first two parts of this question extremely well, gaining full marks.

Part (i) was answered well by most, with almost every candidate differentiating correctly. However, it was disappointing to see so many errors in the solution of the two-term quadratic equation. Some candidates divided by \( x \), omitting the solution \( x = 0 \). Others made slips in factorising. Those who used the quadratic formula generally fared well, but some were not able to use it correctly with the constant term being zero. A very small number of candidates factorised the original equation instead of differentiating, giving the coordinates of intersections with the \( x \)-axis instead of the stationary points.

For part (ii), the vast majority of candidates used the second derivative, although alternative methods such as a sketch of the graph or consideration of gradients either side of the \( x \) values found in (i) were also seen. These candidates usually scored well.

As in other questions, candidates often did not understand the link between the final part of the question and previous working. Part (iii) was often omitted completely. Some candidates sketched the curve correctly but then gave a wrong, or wrapped, inequality. Others checked a few integer values for \( x \) and then guessed at the solution. Only the strongest candidates scored both marks here.
7 This question discriminated well between candidates. Many candidates gained full or almost full marks while there were some who were unable to make any progress at all.

In part (i), most candidates were able to recall the quadratic formula correctly and obtained \( \frac{8 \pm \sqrt{20}}{2} \). After this, many made errors in simplifying this fraction, the most frequently seen wrong answers being \( 4 \pm \sqrt{10} \) and \( 4 \pm 2\sqrt{10} \). A minority of candidates completed the square in this part, usually successfully.

In part (ii), most candidates recognised that the graph was a parabola although other shapes were seen and there were quite a few parabolas with a maximum rather than a minimum point. Some graphs showed negative roots which were inconsistent with the (often correct) roots identified in part (i). It was common for one mark to be lost because the intercept \((0, 11)\) was not shown on an otherwise correct curve. As in Q4, a few candidates worked out the coordinates of points and then plotted the curve on graph paper.

Few candidates appeared to notice that part (iii) was linked to part (i) although some realised that it involved a disguised quadratic. Only the very best candidates managed to progress beyond the initial statement \( y = x^2 \) to earn more than the first mark. Having calculated the roots of the equation in (i) again, wrong working included trying to find the square roots of \( 4 \pm \sqrt{5} \) or, less commonly, \( (4 \pm \sqrt{5})^2 = 16 \pm 5 \). A few candidates rearranged the equation in part (iii) to make \( y^{\frac{1}{2}} \) the subject and then squared both sides. This was often successful. However, it was clear that many candidates had no idea at all of how to tackle this part of the question. Many started by ‘squaring’ to get either \( y^2 - 8y + 121 = 0 \) or \( y^2 - 64y + 121 = 0 \) and then attempted to solve.

8 There were many completely correct solutions to this question and even the less able candidates scored well.

Parts (i) and (ii) were done correctly by most candidates. Only a tiny number used the square root in finding the discriminant.

Part (iii) was done less well, with many candidates simply stating ‘one real root’ or referring to the line and/or the curve touching the \( x \)-axis. Very many stated that the line crossed the curve at one point and thus did not earn the mark.

Although many candidates did not link part (iv) to earlier work in part (i), most were able to gain full marks nevertheless.

As in previous papers, part (v) on the equation of a line was done well, although a few good candidates lost the final mark by not giving the equation with integer coefficients, as required by the question. Again, some candidates were let down by careless errors involving negative number manipulation. A significant minority differentiated to get the gradient expression \( 2x - 5 \) and then stated that the gradient was \( \frac{5}{2} \).
Candidates used a wide variety of methods to tackle part (i) of this question. Although some candidates found the algebraic approach too demanding, most made some progress, often using a geometric approach involving use of Pythagoras’ theorem or, in a few cases, vectors. These candidates often found at least one correct value for \( p \), although their thinking was not always easy to follow. Many other candidates started well, obtaining correct expressions for \( AC \) and \( AB \), but were unable to solve the resulting equation. In most cases, this was because they had equated their expression for \( AB \) to \( \sqrt{20} \) leading to a quadratic equation with no real roots. Other common errors included using \( AC = 2AB \) or squaring one expression but not the other. More worryingly, however, were those candidates who showed little understanding of square roots by stating, for example, that

\[
\sqrt{36 + (p - 5)^2} = 6 + p - 5.
\]

In part (ii), most candidates could find the mid-point of the line joining two points correctly but many made no use of the line \( y = 3x - 14 \). Some failed to justify their choice of \( p \) value from part (i) and these candidates scored only 2 marks from 4.
4722: Core Mathematics 2

General Comments

This paper was accessible to the vast majority of candidates, and overall the standard was very good. There were a number of straightforward questions where candidates who had mastered routine concepts could demonstrate their knowledge; other questions had aspects that challenged even the most able candidates.

As on previous papers, only the most able candidates could manipulate logarithms accurately, though candidates are becoming more proficient in using logarithms to solve exponential equations. A number of candidates also demonstrated poor algebraic skills, particularly order of operations, use of indices and manipulating indices.

This paper contained several parts where candidates were asked to prove a given statement. It is very important in this type of question that sufficient detail is provided to convince the examiner that the principle has been fully understood. Candidates must also ensure that any intermediate values in their working are accurate enough to justify the final answer to the specified degree of accuracy. A significant number failed to gain full marks due to using a rounded value from a previous part of the question.

Whilst some scripts contained clear and explicit methods, on others the presentation was poor making it difficult to follow methods used and decipher answers given. This is especially true if a candidate starts a second attempt at a question. Some candidates squashed all their work on to a few pages of the answer booklet, making it difficult for examiners to follow. Whilst numerical methods were usually easy to follow, the same could not be said of questions requiring an explanation. This was particularly noticeable in Q8(iii), where a number of candidates gave vague and confusing explanations that did not involve the correct mathematical terminology. When asked for a sketch graph, candidates must appreciate that there is no need for graph paper, and this can even sometimes be counter-productive. A simple sketch on the lined paper is all that is required, but they must still ensure that the intention is clear. Sloppiness when sketching a curve could imply a lack of understanding, such as overlapping parts on a \( y = \tan x \) curve when the behaviour should be asymptotic.

Most candidates seemed familiar with the necessary formulae and could quote them accurately, but there were a worrying number of slips on formulae that are actually given in the booklet supplied to candidates. It is important that candidates have access to the formulae booklet throughout the course, and that they become fully conversant with its contents.

Comments on Individual Questions

1 (i) This question was generally well done, with candidates recognising the need to formulate and solve two simultaneous equations. These equations were generally correct, though some candidates used \( a + nd \), and others confused \( n \) and \( u_n \). A number of candidates employed a more informal method to find \( d \) \( (60 \div 30) \), but then sometimes struggled to find \( a \).

(ii) Most candidates could quote a correct formula for the sum, and then substitute in their values from (i). Candidates with incorrect values for \( a \) and/or \( d \) did not seem perturbed that their sum was not equal to zero, and did not reconsider previous answers. Those candidates who used the formula \( \frac{1}{2}n(a + l) \) sometimes lost marks by not showing explicitly how they arrived at their value for \( u_{29} \).

2 (i) Most candidates made a good attempt at the area of the triangle, though some assumed that it was either a right-angled or an isosceles triangle. Some candidates did not seem familiar with the formula and first found the perpendicular height of the triangle.
(ii) Despite the appearance of the cosine rule in the formula book, a number could not quote it accurately, though most appreciated that this was the required method. A few candidates struggled to evaluate the formula correctly, with $149 - 140 \cos 80$ becoming $9 \cos 80$.

(iii) Most candidates realised the need to use the sine rule and attempted to do so correctly, though some attempted angle $A$, despite calling it angle $C$. However, many candidates failed to gain the final mark due to not using a sufficiently accurate value from part (ii). Whilst intermediate steps may be written down to 3 significant figures, candidates must appreciate the need to use the full value in subsequent calculations.

3 (i) Most candidates could make a good attempt at the binomial expansion, but algebraic insecurity led to a number of errors. Most candidates could obtain the first two terms of 1 and $24x$, though the latter was not always of the correct sign. In most solutions the third term was a product of the correct binomial coefficient and an attempt at squaring $-2x$ but a lack of brackets led to errors, with $132x^2$ being the most common mistake. A few candidates found the first three terms in descending not ascending powers.

(ii) This part of the question was generally done well, and two of the three marks were available for correct working using incorrect values from part (i). A number of candidates identified the relevant terms and produced concise solutions, whilst others carried out a full expansion. Some candidates did not appreciate that the answer was derived from two terms, and there were a number of sign errors.

4 (i) This question was generally very well done, with candidates being able to quote the correct formula and use it accurately to produce efficient and concise solutions. A few candidates found only one arc length, or found the difference in the perimeters of the two sectors. Some candidates attempted to use degrees instead, mostly with little success, which was also the case for those attempting to find fractions of the circumference. A minority of candidates continue to think that $1.8$ radians means an angle of $1.8\pi$.

(ii) This was also very well done, with the majority of candidates attempting to find the difference of two areas. Most could quote and use the correct formula, though some omitted the $\frac{1}{2}$ and just used $r^2 \theta$, and others confused the area of a sector with the area of a segment.

5 (i) This question was very well done, with most candidates gaining both of the marks available.

(ii) Whilst many candidates could attempt an expression for the sum of $n$ terms, and relate it to 124 using an inequality, there were very few who could prove the given inequality convincingly. $5(1 - 0.96^n)$ often became $5 - 4.8^n$, and many could not cope with manipulating the inequality sign. A surprising number did not even attempt to prove the given inequality but went straight into solving it. Candidates are becoming increasingly proficient at using logarithms to solve equations and many could obtain the value of 118.3 but then struggled to make the correct final conclusion. Few appreciated that $\log 0.96$ is negative, necessitating a change in the sign when it is used to divide, leading to $n < 118.3$. Only the most astute candidates realised that the smallest integer value of $n$ was required, with 118 and 118.3 being the most common answers. Even those who recognised that 119 was the required final value often concluded with an inequality rather than a statement of $n = 119$. 

85
6 (a) This question was very well done, with many candidates gaining all the marks available. Most could obtain the correct index in the first term, though a few lost a mark by having a coefficient of $3/2$ not $2/3$. Others omitted the constant of integration, or still had an integral sign remaining in their final answer.

(b)(i) Once again, the integration was very well done, with most candidates obtaining an integral of the form $kx^{-1}$. Many then also managed the correct coefficient, though in some cases $-4x^{-1}$ subsequently became $-(4x)^{-1}$. Virtually all candidates then appreciated the need to use the limits $a$ and $1$, though an insecurity with indices then often led to $4(1)^{-1}$ becoming $1/4$. A large number of candidates did not then stop at the required answer, but equated the answer to zero or introduced inequalities, possibly because they had been told that $a > 1$ and they felt that this had to be used in some way.

(ii) This part of the question caused more problems, with some candidates being unable to attempt it at all, and others leaving their final answer still containing $\infty$. Only the most astute appreciated the link between this and the previous part, and most started the entire question again.

7 (i)(a) Virtually all candidates gained this mark, but many then struggled to gain further credit in this question.

(b) Most candidates could split the given expression into two terms, but failed to take the further, crucial step of separating the 10 and the $x^2$. The most common error by far was for $\log (10x^2)$ to become $2\log (10x)$, an approach that gained no credit.

(ii) Most candidates recognised the link with previous work and could gain a mark for using their two previous results in the given equation, but there were slips with both signs and the factor of 2. Many then struggled to make further progress, as previous errors meant that their equation did not simplify to one in $\log y$ only. Even those who were using two correct expressions often made a sign error leading to $3\log y = 2$. Whilst a number of candidates could then make further progress with the question, many did not appreciate how to obtain a solution for $y$. $e$ was sometimes used as the inverse of $\log_{10}$ and others viewed $\log y$ as a product and divided by just the log part. Logarithms continue to be an area of uncertainty for many candidates. Whilst some misapplied correct logarithm rules, others appeared to be completely confused by the entire topic.

8 (i) This question was generally well answered. Most candidates attempted $f(-1)$, though some lost a mark through not explicitly equating this to zero. Some candidates used more cumbersome methods, such as division or matching coefficients, but these had varying degrees of success. There was a pleasing degree of success in finding the quadratic factor and a variety of methods were used. Whilst many then continued and correctly found the full factorisation, a number could not progress beyond the quadratic factor.

(ii) Most candidates could correctly integrate the given expression and then attempt $F(2) - F(-1)$, but a significant number then struggled to evaluate the expression correctly. Sign errors were abundant.
(iii) The standard of the sketch graphs was very varied, both in quality and content. Whilst an accurate drawing on graph paper was not required, it was important that candidates took enough care to ensure that their intention was clear. Many candidates attempted to sketch a cubic curve, but these were not always a positive cubic, with three distinct non-zero roots, as required. Examiners were disappointed at the quality of the explanations, with many candidates unable to use clear, unambiguous words. Some felt that the area under the $x$-axis was ignored and others described how to find the actual area, without explaining why the answer to (ii) did not give the correct area of the region.

9 (i) There were many good sketches seen, though a significant number offered accurate drawings on graph paper. Whilst the general shape of both graphs was usually correct, some had incorrect periods. $y = \tan x$ did not always pass through the origin and often only had a range of $-1 < y < 1$, despite $y = 4\cos x$ being correctly drawn. A number of candidates did not identify the intercept on the $y$-axis.

(ii) Assuming that the graphs had two points of intersection, most candidates attempted to identify $\alpha$ and $\beta$, though a significant number had them the wrong way around on the graph. This was probably due to assuming that $\alpha < \beta$ applied to the $y$-coordinates and not the $x$-coordinates. Only the most able candidates could express $\beta$ in terms of $\alpha$ explicitly, though a few more could deduce some kind of relationship.

(iii) Proving the given equation was generally very well done, with most candidates being familiar with the two identities and able to apply them accurately. Actually solving the equation caused more problems. Most recognised it as a quadratic in $\sin x$, but many then attempted to factorise it. Those who attempted to use the formula could usually do so correctly, but there were a number of slips with both the discriminant and the denominator. Of the candidates who got this far, many assumed that the roots were the actual values for $\alpha$ and $\beta$, with only the most able candidates appreciating that these were the solutions for $\sin x$. A number could then find $62^\circ$ correctly, but couldn’t find the second angle, either because they had an incorrect relationship between $\alpha$ and $\beta$, or because they expected the second angle to come from the second root of the quadratic. Despite all of this, a pleasing number of correct solutions were seen.
4723: Core Mathematics 3

General Comments

This paper enabled candidates to display their mathematical ability. Several of the questions were accessible to all but a few whereas there were a few requests which challenged the most able of candidates. There were not too many candidates with very low marks and, pleasingly, a good number of candidates recorded full marks. Time was not a problem for candidates.

One aspect of algebraic technique which was not always handled competently was the use of brackets. When the brackets were already present in the question, such as in Q3(a), there were no problems. However, when candidates had to incorporate brackets into their expressions, problems sometimes arose, with necessary brackets being either absent or incorrectly located. Questions where problems were noted included Qs 3(b), 5 and 9. Sometimes a candidate rescued the situation by proceeding as if the missing brackets were present but, on other occasions, the candidate was led astray by the absence of brackets. For example, in Q9(i), it was not uncommon for candidates to write \(2\sin \theta \cdot 1 - \sin^2 \theta + 1 - 2\sin^2 \theta \cdot \sin \theta\) and to follow this with the incorrect \(2\sin \theta - \sin^2 \theta + 1 - 2\sin^5 \theta\).

Questions in which it is required to confirm a given result obviously encourage candidates to check their solution when they do not reach the result. There are other situations when candidates would benefit by being aware of the nature of the answer to be expected. In Q5, awareness of integration should have led candidates to expect that one definite integral would be positive and the other negative. In Q8, consideration of the diagram should have led to the expectation that the gradient of \(PQ\) would be negative and that the area of \(A\) would be less than the area of the triangle \(OPQ\). In the majority of cases where candidates obtained an unreasonable answer, there was little evidence that they had checked their working to find and correct an error.

Comments on Individual Questions

1. This question was answered well and most candidates demonstrated the appropriate use of logarithm properties to obtain the given answer convincingly. Some included a step such as \(\ln 512 - \ln 8 = \frac{\ln 512}{\ln 8}\) in their solution; full credit was not available for solutions containing such an error. A number of candidates correctly reached \(3\ln 8 - 3\ln 2\) but then resorted to their calculators and the use of decimal approximations; the exact result cannot then be claimed legitimately.

2. Many candidates used the appropriate identity to form an equation quadratic in \(\tan \theta\). There were some slips with the identity or when simplifying or solving the equation but it was common for the four correct angles to be obtained without difficulty. The first step of some candidates was to rewrite the given equation in terms of \(\sin \theta\) and \(\cos \theta\). Such attempts seldom made any progress (although there are at least two approaches – each much more difficult than the expected method of solution – which can be used successfully).

3. This was a straightforward question for many candidates and they obtained all six marks without difficulty. The product rule is required in part (a); this was not always realised and solutions such as \(12x(x + 1)^5\) were not uncommon. A few candidates used the binomial theorem to rewrite the given expression as the sum of seven separate terms before attempting differentiation. No simplification of the derivative was requested in the question but many candidates did show confidence with algebraic skills to express their correct derivative as \(2x(4x + 1)(x + 1)^5\).

A statement of the quotient rule is given in the List of Formulae but there was a significant number of candidates who applied the rule incorrectly, confusing numerator and denominator or making sign
errors. Some candidates omitted to substitute the value 1 and others made errors in their simplification of a correct derivative and thus ended with an incorrect value.

4 Many candidates had no difficulty in writing down the correct answer for the range in part (i). But many other responses did raise doubts about candidates’ understanding of range. The answer $f(x) \geq 2$ appeared often, contradicting the clear evidence provided by the given graph; presumably this was the result of an unconsidered step whereby $x \geq 0$ for the domain became $f(x) \geq f(0)$ for the range.

The vast majority of candidates answered part (ii) correctly. However, correct responses to part (iii) were rare. Some candidates considered the graph of $y = f(x)$ and readily deduced the correct answer but, in the majority of cases, the approach involved squaring both sides to obtain $4 - 4\sqrt{x} + x = k^2$ and no sensible progress was made.

5 Most candidates earned some credit for this question but correct solutions needed a combination of accurate integration, an understanding of how to deal with the part of the area below the $x$-axis and the retention of exact values. Many excellent solutions were seen but it was disappointing that many candidates struggled with one or more of these aspects of the question. Some candidates adopted the approach of integrating $(1 - 2x)^5 - (e^{2x-1} - 1)$ but sign errors were sometimes a problem. Other candidates evaluated the two areas separately but did not always deal appropriately with the negative definite integral. The integration steps were often incorrect with results such as $\frac{1}{6}(1 - 2x)^6$ and $2e^{2x-1} - x$ among the more common errors. A few candidates tried the use of $\int x \, dy$ but found the integrals too demanding and a few thought that $\int x^2 \, dx$ was required.

6 There were two common correct approaches to part (a). Some candidates realised that exponential increase meant that the quantity $X$ was increasing by a factor 1.6 over time intervals of length 10 and obtained the correct answer without difficulty. Other candidates attempted to set up a formula for $X$ in terms of $t$. This approach was less successful as there were problems in using an appropriate formula or in retaining sufficient accuracy in the solution to reach an acceptable value of $X$. A common incorrect approach led to the answer 605, the result of assuming that the values of $X$ followed an arithmetic progression as $t$ increased by equal steps.

The two requests in part (b) were answered very well with the majority of candidates answering both correctly. Some faulty work with logarithms was seen in part (i) and some candidates substituted $t = 30$ in part (ii) before attempting differentiation, but most candidates displayed confidence in dealing with this topic.

7 In part (i), candidates generally recognised that the curve to be drawn was the result of a stretch and translation of the given curve. Most had the details of these transformations correct although, in some cases, the stretch was parallel to the $x$-axis or parallel to the $y$-axis but with a scale factor $\frac{1}{2}$ or the translation was in the negative $x$ direction. The response to part (ii) was disappointing with many candidates not recognising $y = x$ as the straight line to be drawn. Some drew no line and common incorrect responses included a line parallel to the $y$-axis, a line parallel to the $x$-axis and a line joining the points $(2, 0)$ and $(0, 3\pi)$.

Many attempts at parts (iii) and (iv) were marred by the use of angles measured in degrees instead of in radians. In part (iii), most candidates indicated that they were expecting to see a change in sign but those working in degrees and those substituting into $3\cos^{-1}(x - 1)$ looked in vain. For those candidates working with radians, the iteration in part (iv) was handled well with the individual iterates shown and the correct value of $\alpha$ found. The final part of the question was not answered well, with many candidates referring vaguely to one equation being the inverse of the other or carrying out a numerical calculation involving their value of $\alpha$. A clear mathematical explanation,
showing a step-by-step rearrangement of $x = 1 + \cos(\frac{1}{2}x)$ to $3\cos^{-1}(x - 1) = x$, or vice-versa, was not noted often.

8 Correct differentiation of $\ln(5 - x^2)$ was a problem for many candidates and wrong derivatives such as $\frac{1}{5 - x^2}$ and $\frac{2x}{5 - x^2}$ were frequently noted. Most candidates knew how to find the equation of the tangent though a few formed an equation in which the gradient remained an expression involving $x$.

Part (ii) was answered well with most candidates adopting a systematic process to establish the correct value. There were some errors with the value of $h$, some slips with the evaluation of the relevant $y$ values and some instances of an incorrect application of Simpson’s rule in which the factors 4 and 2 were associated with the wrong $y$ values.

Many answered part (iii) correctly but it was surprising how often this apparently simple request was answered incorrectly. Obviously there were problems if the answers to part (i) or part (ii) or both were wrong although both marks in part (iii) were available for correct work following plausible earlier answers. However, some candidates seemed unsure about how their results from part (i) and (ii) related to the diagram. Some claimed the coordinates of $Q$ as $(0, \ln 5)$ and others found only the area of the triangle.

9 The vast majority of candidates earned the first mark in part (i) by expressing $\sin 3\theta$ as $\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$. Most were aware of the identities then needed to complete the solution and many accurate and clear solutions were seen. Other attempts were not so assured; dealing with $\cos 2\theta$ was a problem for some.

Progress with parts (ii) and (iii) depended on an appreciation that the identity from part (i) was needed but that appreciation was absent in many cases and no progress was made. In part (ii), the two required values followed readily once the expression had been rewritten as $3\sin(10\alpha)$. Part (iii) proved to be a challenging final request but a number of candidates showed excellent technique to express the equation in terms of $\sin 2\beta$ and to find the two values of $\beta$. Many other candidates did not use the identity from part (i) and their solutions tended to meander without much purpose and often contained errors such as $\frac{\sin 6\beta}{\sin 2\beta} = \sin 3\beta$. 
Report on the units taken in January 2006

4724: Core Mathematics 4

General Comments

An important indicator of a good candidate is the presentation of the answers; an able mathematician has clarity of thought and a consequent awareness of giving clear, concise presentation. Candidates who followed this dictum produced reliable work of a high standard; others, whose work was untidy and unclear, often made mistakes and their mathematics was generally of poor quality. However, the vast majority of the work was excellent.

Comments on Individual Questions

1. This question based on the syllabus item “Simplify rational expressions, including factorising and cancelling” gave most candidates a good start; very few simplified too far by cancelling the correct answer to an inappropriate one. A few candidates attempted long division with just a few making satisfactory progress.

2. The techniques of implicit differentiation were well known; a few careless errors crept into the solution e.g. the mis-reading of ‘y’ for ‘sin y’, the derivative of sin y being-cos y, and the transference of ‘xy’ to the left hand side of the equation with a derivative of −x \( \frac{dy}{dx} \) + y instead of \(-\left(x \frac{dy}{dx} + y\right)\).

3. The methods of long division and the use of an identity were both prevalent in this question and the first part was generally done well. The answers to the second part could be easily deduced from the remainder in the first part but the majority effectively started again.

4. The first stage of the integration by parts was rarely effected incorrectly so it was surprising that, in quite a few cases, the next stage integral of \( \tan x \) became \( \sec^2 x \) (particularly as the result is quoted in the List of Formulae). Needless to say, the writing of \( \int \tan x \, dv \) was much more common than \( \int \tan x \, dv \).

In the second part, a connection between \( \tan^2 x \) and \( \sec^2 x \) was generally used (though, as this is not in the List of Formulae, sign errors were not rare) either directly or by another application of integration by parts.

5. In the first part, the direct production of \( \frac{dy}{dx} \) from \( \frac{dy}{dt} \) was very common and few mistakes occurred other than the simplification of \( \frac{2}{2t} \) to \( t \); however, the obvious lack of confidence with parameters shown by a number of candidates led them to convert parametric to cartesian, so missing the simpler work involved with the parameters.

The second part often had confusion between \( t \) and \( p \) although, needless to say, the given answer was always present on the final line.

In the final part it was heartening to see that most candidates realised that \( p \) was −5 and not 5; most obtained the result from solving \( p^2 = 25, \ 2p = -10 \) simultaneously though some, unusually, substituted \( (25, -10) \) into the equation of the tangent. On the other hand, it was surprising how many worked out the equations of the tangents again even though the format had been given in part (ii).
6 In part (i) it was rare to see \( \text{d}x \) being just replaced by \( \text{d}\theta \) but a little surprising to see \( \sin^2 \theta \) being changed into a \( \cos 2\theta \) variant before differentiation took place (generally satisfactorily) – perhaps candidates had already looked at part (ii). The reduction to \( 2 \sin^2 \theta \) was not always totally convincing and candidates must realise that, when an answer is given, very close inspection will be made of the working. In part (ii), most changed the limits to 0 and \( \frac{\pi}{2} \) but some used 0 and 90; a very few tried re-substituting but this device was rather unsuitable and rarely produced the correct answer.

7 Again, it was heartening to see the most usual identity in correct form on nearly every occasion and part (i) usually only failed because of carelessness or writing \( (1 + x^2) \) instead of \( (1 + x)^2 \). In part (ii), the expansion of \( (2-x)^{-1} \) caused most trouble, often being changed to \( 2 \left( 1 - \frac{x}{2} \right)^{-1} \) or \( 2 \left( 1 - x \right)^{-1} \). The expansions of \( B(1+x)^{-1} \) and \( C(1+x)^{-2} \) were frequently spoilt by the candidate trying to do too much all at once: e.g. the unsimplified expansions of \( (1 + x)^{-1} \) and \( (1 + x)^{-2} \) would be simplified in the head and multiplied by \( B \) or \( C \) simultaneously instead of performing the two operations in separate stages.

8 The technique of separation of variables in part (i) was generally understood but many candidates failed to show the arbitrary constant, thus making nonsense of the substitution of \((5,4)\). However, they were still able to (and generally did) complete the squares and produce some reasonable format in part (ii). In part (iii), it would have been helpful if candidates had stated that their locus, of sometimes indeterminate shape, was intended to be a circle.

9 The vast majority chose the correct vectors to determine the angle between the lines. In part (ii), it was good to see a systematic approach in the solution of the simultaneous equations.
Report on the units taken in January 2006

4725: Further Pure Mathematics 1

General comments

In general, most candidates worked sequentially through the paper and few seemed to be under time pressure. Completely correct solutions to all questions were seen and no question proved to be inaccessible. A good number of candidates scored high marks on this paper.

The level of algebraic skill shown was pleasingly high, but errors at any early stage in, for instance, Qs 4 and 5 led to a considerable proportion of the marks being lost.

Qs 2, 6, 9 and 10 proved to be the most demanding but candidates found the remaining questions, in general, more straightforward.

Comments on Individual Questions

1 Most candidates answered part (i) correctly, the most common error being to make
\[2 - 8i^2 = 2 - 8(-1) = 10.
\]
In (ii) the majority of candidates saw how to use the conjugate, but the error,
\[(2 - i)(2 + i) = 3,
\]was quite common.

2 As in June 2005, the presentation of the Induction proof was not of the standard required by the examiners. A demonstration that the result is true when \(n = 1\) was often very vague. The assumption that the result is true for some particular value leading to the truth that the result is true for the next value was sometimes similarly vague and the mark for the explanation of the Induction conclusion was not earned in a high proportion of scripts.

3 The method of evaluating the determinant was known by the majority of candidates, with sign errors being the most frequent loss of marks.

Part (ii) showed that the definition of singularity was not clearly understood. Many candidates thought that a non-zero determinant meant a singular matrix, or that a positive determinant meant a non-singular matrix, while others gave information about the inverse matrix, with no mention of non-singularity.

4 Algebraic errors in the expansion of \((u + 2)^3\) and the simplification of the equation in \(u\), caused many candidates to lose marks. A surprisingly large number of candidates found the correct value for \(u\), but failed to state the corresponding value for \(x\). Very few candidates gave decimal answers, which was pleasing as the exact value was requested.

5 This question was answered correctly by most candidates, with few making algebraic errors or failing to show sufficient working to justify the given answer.

6 The inverse matrix in part (i) was generally correct, with omission of the determinant being the most common error.

Quite a large proportion of candidates either used no matrix algebra or incorrect pre- or post-multiplication in trying to find either \(B^{-1}\) or \(B\). Very few candidates used the result \((AB)^{-1} = B^{-1}A^{-1}\). Candidates who used a general matrix for \(B\) and then solved simultaneous equations to find the elements invariably made an error in their solution.

7 In part (a) most knew how to find the modulus and argument of a complex number, the most frequent error being to give the argument of \(w\) instead of \(w^e\).
In part (b) most candidates found the real and imaginary parts of $u$, but a significant number did not give their answer as a complex number.

A considerable number of candidates thought the locus in part (c) was a circle. A rather high proportion of candidates who sketched a vertical straight line, made no indication of scale, which was penalised.

The transformation geometry was generally well understood, with many candidates scoring full marks. As in Q7, omission of scales on diagrams, when there was no evidence of coordinates being found elsewhere was penalised. There was confusion in some descriptions between a stretch in one direction and an enlargement.

It was pleasing to see that a good proportion of candidates wrote down sufficient terms to see that four elements from the differences remained rather than just assuming that the first and last elements were the ones required. Most found the sum to infinity correctly, but many did not see that part (iii)(b) required the difference between the sum to infinity and the answer to part (ii).

This question proved to be quite demanding, and many candidates scored only the mark for each of parts (i) and (iii). A significant number did not appreciate that the complex roots were a conjugate pair and produced a considerable amount of work that made no progress. Those who used the conjugate roots were generally able to find correct answers in one form or another for both parts (ii) and (iv), while some candidates used the fact that $\Sigma \alpha \beta = 27$ to find $q$ in terms of $\alpha$, some finding the simplified value $\sqrt[3]{\left|\alpha - 3\right|}$. 

94
Report on the units taken in January 2006

4726: Further Pure Mathematics 2

General Comments

Most candidates found the examination accessible, answering the questions in the order set. Candidates appeared to be well-prepared for the range of questions, and many candidates produced sound answers. There were many excellent scripts showing a variety of methods.

The main area of concern was in questions in which either the answer to be proved was given or where an explanation was called for. Candidates should appreciate that in these cases a full and detailed answer is expected. The lack of such precision was particularly noticeable in Q7 where the use of such words as “obviously” or “clearly” led to a loss of marks for many candidates.

A few candidates appeared to have problems with time. This was apparently due to overlong methods, where instructions in questions were not read carefully enough or were ignored. Examples include “write down” in Q1 and “state” in Q4.

Comments on Individual Questions

1. The majority of candidates successfully “wrote down” the expansion for \( \ln(1+3x) \), with few errors. Some marks were lost by the lack of brackets around \( (3x) \) or by using factorials in the denominator. In part (ii), candidates tended to expand the brackets beyond the \( x^2 \) that was required. Candidates who opted for expansion by repeated differentiation were often successful but wasted some time, particularly if it was applied to part (ii). Candidates should be aware of the standard expansions given to them in the List of Formulae booklet.

2. Most candidates showed a good awareness of Newton-Raphson, ensuring that an \( f(x) \) of the form \( \pm( e^{-x} - x ) \) was given as an initial step. Other interesting choices, such as \( xe^x - 1 \), were also used. The use of calculators was usually good enough to give the correct answer, although some candidates did not give sufficient decimal places in their early estimates to justify the final answer correct to 3 decimal places. Candidates who used methods other than Newton-Raphson were given no credit.

3. This proved to be a straightforward question in which very few errors were seen.

4. (i) The use of the insert caused few problems to candidates, although a surprising number of centres failed to attach it to the script concerned. The three lines given on the insert were not always used for the explanation to accompany the diagram, and although candidates generally went from \( x_1 \) to the curve and then to \( y = x \), they did not always continue in a convincing manner. Those who mentioned “step” or “staircase” were given credit. The best solutions showed \( x_2, x_3 \ldots \) and explained the process.

(ii) Even candidates who showed an understanding in part (i) believed that the iteration converged to \( \beta \) in (a). In (b), any form of words was allowed which indicated that the iteration diverged, although there were some very convoluted ways of putting it.

5. (i) A variety of methods was seen for the oblique asymptote, including dividing out and equating coefficients. Marks were awarded for the correct equation even with minor algebraic errors.
(ii) Methods were firmly in two camps. The first involved the consideration of the quadratic $x^2 + x(3 - y) + (3 - 2y) = 0$ and the condition for real values of $x$. This was an example of an answer being given, and so the final solution of the inequality for $y$ should have been justified and not merely copied down. It was apparent that some candidates were not sure why they were considering $b^2 - 4ac$, but marks were awarded for a reasonable attempt at the set method.

The second method involved differentiation, either of the original $y$ or of the $y$ developed in dividing out in part (i). Many candidates found the turning points and stopped. Better solutions involving testing for maximum and minimum and sketching, possibly including obvious points and/or approaches to asymptotes, were rare.

(i) Again, this question included a given answer. Candidates who showed that they were attempting to use the limits in the first part of the integral were not penalised if they merely wrote down $-e^{-1}$. A surprising number of candidates did not use the limits at all to produce this term.

(ii) Errors in this part were mainly algebraic, with some confusion as to the meaning, and then the evaluation, of $I_0$ or $I_1$.

Candidates in general did not “explain” at all well. Statements such as “Rectangles = $\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots$” were given no credit. The best solutions explained the left-hand side of the inequality in terms of the total area or summation of the areas of rectangles (up to $N$) compared to the area under the graph (from 1 to $N + 1$), together with an indication of the area of a single rectangle as $1 \times \sqrt{x}$ for the correct values of $x$. Others noted that translating the original rectangles one unit to the left helped in explaining part (ii). Candidates should monitor the number of marks awarded for each part of each question. In part (iii), it was surprising to see the inequalities the wrong way round and/or the integrals not attempted.

(i) It was sufficient to “state” $r = 2$ for $\theta = 0, \pi$ without any working. Answers outside the given values were ignored, but extra answers within the given values were penalised.

(ii) Most candidates solved $r = 0$, sometimes producing too many “solutions”. A minority were less successful employing differentiation.

(iii) It was pleasing that the majority of candidates could deal with the integral involved, with a lot of correct solutions. There were a few candidates who produced a wrong formula for $\cos^2 2\theta$, but, for minor errors, four out of five marks were possible.

(iv) Candidates were less successful in this part, wrongly using $x = r \cos 2\theta$ or merely replacing $r$ with $\sqrt{x^2 + y^2}$ and $\theta$ with $\tan^{-1}(y/x)$. Others attempted a final equation with $y$ as the subject. As long as these candidates showed a simplified form at some stage of their working, they were penalised by time constraints and not by losing marks.

(i) Only minor errors were ever seen in this part.

(ii) It may be that candidates were getting to the end of the paper, but again there was evidence of a lack of care in how many read the question. For full marks, it was necessary to show that there was “just one” turning-point. Many candidates showed that there was at least one turning-point but did not justify it as the only one. Most candidates used the hint in part (i) to solve $2 \sinh 2x - 6 \cosh x = 0$, whilst those who opted for an immediate substitution for $\sinh x$ and $\cosh x$ in terms of $e^x$ and $e^{-x}$ had more problems. Most used the second differential test for the type of turning-point, but accurate numeric evidence was required for both marks.
4727: Further Pure Mathematics 3

General Comments

The entry for this session was small because the majority of centres use the whole of Year 13 to prepare their candidates for this unit. There was some evidence that some candidates had not fully covered all the topics in the specification. As all four sections of the specification are always tested in each paper, it is desirable for candidates to have had as much experience as possible before taking the examination. The paper appeared to be of the correct length, and there was no indication that candidates had either run out of time or had much time to spare.

Comments on Individual Questions

1 This was a straightforward question which required the use of the scalar product to find the angle, and candidates’ answers were almost entirely correct.

2 (i) Candidates had no difficulty in stating the identity element in each group, and the proper subgroups were usually correct.

(ii) Although the request to show corresponding elements had not been set recently, most candidates knew what was expected and answered confidently. A few gave only the orders of the elements, without identifying the correspondence between them. As the corresponding elements can easily be deduced from the orders, some credit was given for this.

3 A majority of candidates scored well on this question. Among others there was some uncertainty about differentiating and using the substitution, and little more credit could then be obtained. In part (ii) the fact that \( y \) approaches 0 as \( x \to \infty \) was not given as often as expected.

4 (i) This part required careful attention to detail, but it was done quite well. Not all candidates used the suggested method of expanding \( (e^{i\theta} + e^{-i\theta})^2 \) and \( (e^{i\theta} - e^{-i\theta})^4 \) and then grouping terms, but all other valid methods could gain full credit.

(ii) This was an easy application of the result of the first part and the correct answer was usually found.

5 Although most candidates gained some marks on this question, only the best were able to complete all three parts. The majority of answers to part (i) were correct, and a fair number managed to find the equivalent cartesian values to give the linear factors of \( z^4 + 64 \). But the concept of combining complex conjugate pairs of factors to give the quadratic factors was seen in only a few scripts.

6 This standard type of vector question is usually done confidently, but in this session it was evident that some candidates lacked the experience required, or even had not completed the work for the topic. Among those who knew what to do, part (i) was found straightforward. In part (ii) some answers found the two-parameter form of the equation of the planes without attempting to find the cartesian form. Vector problems often have several different possible methods, and it requires practice to know which is the best to use in a particular case. When the cartesian form of the equation of a plane is requested, changing from the parametric form without using a vector product usually takes longer than more direct methods. Answers to part (iii) by candidates who had progressed that far were good.
This question was the most demanding on the paper, and only a small number of good answers were seen. Very careful reasoning was required in the proof of part (i). To gain full credit here it was necessary to note that \( a^n = e \), as given, but that \( a^m \neq e \) when \( m < n \). In contrast, parts (ii) and (iii) were easier, and some fair answers were given.

(i) This question should have been found reasonably straightforward, but in fact candidates had more trouble than expected with distinguishing between the three cases. The auxiliary equation was usually solved correctly, and correct answers to part (a) were given. However there seemed to be problems with writing \( \sqrt{k^2 - 4} \) as \( i\sqrt{4 - k^2} \) in order to obtain the trigonometrical part of the solution in part (b). Part (c) was usually correct, with a few slips in writing the variables as \( x \) and \( t \).

(ii) Those candidates who had coped with the trigonometrical solutions in part (i)(b) were able to make fair attempts at this part.

(iii) Although it was easy to state that \( x \to 0 \), it was only the best candidates who gave a fully correct explanation, referring not only to \( e^{-t} \) approaching 0, but also to the fact that the sine function is bounded.
Report on the units taken in January 2006

Mechanics

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.
General Comments

The quality of the candidates’ work was high, and very few poor scripts were seen. Indeed the commonest outcome for all questions, save one, was the award of full marks. Nevertheless, some candidates would have found their ability better rewarded by:

- reading questions more carefully, either to assimilate all the information given, or appreciate better the results being required in their answers (Qs 1(ii), 3(iii) and 4(iv)),
- selecting swifter methods of solution, and not recalculating earlier results needlessly (Qs 1(ii) and 7(ii)),
- having greater facility with problems involving two dimensions (Qs 2, 4(iv), 6).

Specific references to these matters are made in the comments on individual questions below.

Comments on Individual Questions

1 (i) This question was convincingly answered by candidates who clearly used their $a$ in the direction specified in the question. Examiners are entitled to withhold marks from scripts if a candidate has used a letter with a defined meaning in an idiosyncratic way.

(ii) The most common error in this question was to calculate only the time taken for particle $P$ to come to rest. The most common way to answer part (ii) was to start by finding the time taken to come to rest, then calculate the distance $P$ descends in this time, and then find the time required for the particle to ascend to its initial position. Solutions based on substituting $s = 0$ into $s = ut + \frac{1}{2}at^2$ were extremely rare.

2 (i) This part of the question was well answered, though a few candidates included $g$ in their term for $ma$.

(ii) The second part of the question was frequently completed successfully, though some candidates who obtained the correct equation by resolving vertically were unable to solve their two correct simultaneous equations. More often candidates resolved parallel to the string or incorrectly included the acceleration.

3 (i) This was very well attempted and almost all candidates scored full marks.

(ii) Nearly all candidates successfully completed this.

(iii) Most candidates started this part of the question with the correct integration of $vdt$, though a few then deleted their correct expression and used the equation $s = \frac{(u^2 + v^2)}{2}t$ instead, perhaps not appreciating that it is a constant acceleration formula.

Though very many candidates completed the question correctly, a large minority simply substituted 31 as the value for $t$, not understanding how there were two stages in the motion of the motorcycle.

4 (i) Nearly all solutions were based on the correct trigonometric ratio, and this part of the question was well answered, though some candidates calculated the weight of the block.

(iii) Again almost all candidates were successful with the calculation.

(iii) Nearly all candidates employed the correct method for finding a coefficient of friction. However, there were frequently problems associated with the correct degree of accuracy. Candidates who used values from parts (i) and (ii) rounded to three significant figures gave wrong answers, and it was very common for the initial zero in the answer treated as a
significant figure, so 0.70 (and 0.7) were often seen.

(iv) Though correct solutions to the entire problem were often seen, very many candidates noted only the increase in the force from 12 N to 20 N, and overlooked the change in its direction. It was also common for the effect of the change(s) on the normal component of the contact force to be overlooked, and such candidates continued to use the frictional force found in part (i). Even candidates who did recalculate the frictional force might use an excessively approximate value for the coefficient of friction, so obtaining an inaccurate value for the acceleration.

5  
(i) Only a small minority of candidates interpreted "sketch" in an artistic sense, but it was common to see "straight lines" drawn without the aid of a straight edge. Many candidates were unaware that the horizontal "walk" section of the diagram should have been below the $t$-axis, and not linked to it by any line segments, either vertical or inclined.

(ii) This part of the question was well answered. Although nearly all candidates could find the time spent walking, the time spent on the motor-cycle was seen less often.

6  
Candidates found this the most taxing question on the paper. It was the question with the most uniform spread of marks.

(i) Although candidates could set up an equation for the vertical equilibrium of the system, it was unusual for the behaviour of the cosine function to be correctly used in the solution.

(ii)(a) Candidates who attempted this part of the question were likely to spend too much effort in calculating angles, which in turn rendered an exact value for $T$ approximate.

(ii)(b) Although the solution for $W$ requires the same essential work as (i), it was common to see progress made in one part of the question but not the other. Some scripts were marred at this stage by multiplying together $W$ and $g$.

(iii) The most frequently seen answers were that the ring would be at $B$, consistent with an elastic string, and that the tension would be zero, the string being slack. The most common non-zero value for tension was the weight of the ring.

7  
(i) Very many scripts included a correct calculation for the velocity of $B$ after the collision. The next part of this question was frequently completed correctly, the most usual difficulty being finding the correct value for the acceleration of $B$ on the inclined plane.

(ii) Again, many instances were seen of candidates correctly analysing the motion of the particles on the plane, leading to the award of full marks. In most cases, candidates found the time for $B$ to ascend and descend separately. There were few instances of a candidate using $s = ut + at^2/2$, to obtain the time in a single calculation, as observed in the comments on Q1. Many scripts showed the recalculation of the acceleration of $A$ and $B$ for each stage of the motion.
4729: Mechanics 2

General Comments

The majority of candidates showed reasonable understanding of all topics. Unusually, the questions which caused most difficulty were the ones for which diagrams were given (1, 4 and 8). Few candidates appeared to run out of time and many scored very highly. Only a small percentage of the candiature was totally unprepared. As general hints, candidates must be spatially aware and be familiar with geometry, trigonometry and specifically the components of forces.

Comments on Individual questions

1. Many candidates showed either a lack of thought or understanding in answering the first question of the paper. Resolving forces to find the tension was not an appropriate method as the force at the hinge was not known and was simply ignored by the majority of candidates. However, there were many perfect solutions gained through taking moments about the hinge at $A$.

2. This question was generally well answered. Only a few candidates used an incorrect method in part (i) and found half the required time from $O$ to $A$. Part (ii) was well answered.

3. This question was also well answered. Most candidates followed the hint and used work/energy in solving part (ii).

4. This question was found challenging. Very few candidates were successful in drawing the correct directions for the horizontal components of forces at $E$ and $F$. Part (ii) was not well answered. Very few candidates demonstrated understanding in their methods. Part (iii) was found to be more straightforward but there was some confusion about the distances required for taking moments about the axis $AD$.

5. This question was found to be straightforward. However, a significant number of candidates complicated their calculations in part (i) by using the conservation of kinetic energy rather than the fact that $e = 1$. A large number of candidates failed to state the direction of the impulse in part (ii). In this part, the $m$ was often missing from the answer $4.8m$. Part (iii) was well answered with few sign fiddles.

6. Part (i) was well answered with very few fiddles seen to gain the given answer. In part (ii), most candidates used the given formula with $y$ set to the value -20. Part (iii) was well answered.

7. Most candidates understood this question well and answered it accurately. Surprisingly, a fairly common fault was in part (iii) where many lost one mark out of five by quoting their answer as 0.05 thinking that this was to 3 sig. fig. accuracy. Balancing the work/energy equation in part (iv) was often found tricky. A hint for tackling this is to first consider the situation without any resistance.

8. This question was found to be the most difficult one of the examination. The main difficulty was in identifying forces, components and resolving in appropriate directions. In part (i) many either confused sin/cos or incorrectly derived the reaction to be $mg\cos30^\circ$. The one mark in part (iii) was gained by the majority of candidates but part (iv) was answered poorly. Many assumed that the reaction force from part (i) remained the same for the new set up.
4730: Mechanics 3

General Comments

Candidates were very well prepared for examination and this is reflected in the marks obtained. Only a small number of candidates scored fewer than half of the total marks available.

Some candidates, a significant minority, were content to use a ‘recipe’ method in Qs 6(i) and 7(ii). This approach worked better in Q7(ii) than in Q6(i). Candidates should be aware of the limitations of a particular recipe, and the recipe should not be regarded as an alternative to an understanding of the underlying mechanics principle.

Comments on Individual Questions

1  This was generally well attempted although there was a number of sources of error including; omission of mass (usually accidentally after a correct statement in symbols), obtaining $\theta = 38.0^\circ$ without making clear how this relates to the required direction (indication of $\theta$ in a diagram would have been sufficient), incorrect triangle (representing impulse as the sum of momenta instead of the difference), obtaining the value of $83.0^\circ$ for an angle in the correct triangle instead of the corresponding obtuse angle value of $97.0^\circ$.

2  This question was very well attempted, although some candidates inappropriately sought to use $a = v \frac{dv}{dx}$ instead of $a = \frac{dv}{dt}$ in one or both parts. It was disappointing to see errors of transposition such as transferring 0.5 on the left hand side of the equation of motion, to the right hand side, as a multiplier instead of a divisor, and similarly basic errors in solving $2 = 1 + \frac{k}{6}$. Obtaining $t = 2$ from $t^2(t^2 + 1) = 20$ by inspection was deemed acceptable, but intending candidates should be aware that this is the case here only because the uniqueness of $t = 2$ as a viable answer is immediately apparent ($X[X + 1]$ is clearly greater than 20 for $X > 4$ and less than 20 for $0 < X < 4$).

3  (i) This was very well attempted and almost all candidates scored full marks.

   (ii) This was also very well attempted, most candidates equating the energy when the particle is 4.5m below $O$ with that when the particle is in its initial position, rather than when it is at $O$. In this case there are however two terms representing elastic potential energy in the resulting equation, one of which was often omitted.

4  This question was very well attempted and errors in method were rare. However many errors were made by weaker candidates in constructing and solving the relevant equations.

5  (i) This was well attempted and most candidates took moments of the forces on $BC$, about $B$, to obtain the given answer correctly. However some candidates took moments of the forces on the whole system, about $A$. Candidates who did this correctly could not make progress because of the presence of a term representing a horizontal force of unknown magnitude, and candidates who omitted the moment of the horizontal force at $C$ could not obtain the given answer.

   (ii) This was fairly well attempted, although many candidates made errors of sign, and other errors arising from unnecessarily long methods (e.g. taking moments rather than resolving forces to find the vertical component of the force exerted on $BC$ at $B$).

   (iii) This was well attempted although some candidates had the values of $F$ and $R$ reversed on substitution.
Almost all candidates applied the principle of conservation of energy and many did so successfully. However some candidates made errors in dealing with the GPE, particularly those who chose a datum level other than the horizontal through $O$. A significant minority used the formula $v^2 = 2ga(1 - \cos \theta)$, failing to recognise that this applies only in cases equivalent to that for which the particle moves on the outer surface of a smooth circular cylinder with initial conditions $v = 0$ and $\theta = 0$.

This part of the question was very well attempted. However some candidates used the wrong sign for the term representing the component of the weight, and some candidates had $\sin \theta$ in this term instead of $\cos \theta$.

Almost all candidates used $T = 0$ appropriately but relatively few reached the correct answer for the angle turned through.

Most candidates scored all four marks available although there was a considerable variation in the level of mathematical elegance used in obtaining the marks.

Most candidates recognised the need to use Newton’s second law although many of the attempts to obtain $\ddot{x} = -400x$ were flawed. Almost all candidates realised the need to obtain this equation because the formula $T = 2\pi / \omega$ was well known, so that (with $T = 0.314$ given) the equation is effectively a given answer. A significant minority of candidates used the formula $\omega = \sqrt{\frac{1}{m} \left( \frac{\lambda_1}{L_1} + \frac{\lambda_2}{L_2} \right)}$ in lieu of Newton’s second law. This is acceptable here because the question informs the candidate that the motion is simple harmonic, but intending candidates should be aware that Newton’s second law cannot be avoided in all (or even most) questions of this type.

As in part (ii) there was a tendency to work back from the given answer, the formula $u = A\omega$ (or more generally $v^2 = \omega^2 (A^2 - x^2)$) being well known. This approach produced a lot of confusion between $u$ as the maximum value of $v$, and the maximum possible value of $u$ for which the equation $\ddot{x} = -400x$ can apply continuously. Thus $A = 0.3$ was rarely satisfactorily justified, or even distinguished as the greatest possible amplitude.

Many candidates equated $0.3\sin 20t$ with $1.3$, and among those who used $-0.2$ or $+0.2$ relatively few were able to obtain the correct answer.
Probability and Statistics

Chief Examiner’s Report

As before, the general standard of work on the Probability and Statistics part of the examination is pleasingly high. There are many candidates who can answer relatively taxing questions. However, verbal questions continue to reveal that candidates’ understanding is not always on a par with their ability to obtain correct numerical answers. It is also clear that, while some centres have clearly taken note of the remarks in these Reports, others have not, with the same mistakes and misconceptions continuing to appear from one year to the next.

Examiners report that many candidates do not complete the grid on the front of the first side of their answer booklet. This grid is used to display the marks for each individual question on the whole paper; the question-numbers of questions answered in subsequent answer booklets or loose sheets should be included on the front of the first booklet (and not on the subsequent booklet or loose sheets). Question numbers should not be subdivided; question numbers such as “3(i), 3(ii)” etc should not be given.

Centres are particularly asked to note the following points which have been agreed by the Examiners responsible for these papers in order to encourage good practice.

- Answers given to an excessive number of significant figures (such as “probability = 0.11853315”), which have not in the past lost marks, may in future be penalised.
- Hypothesis tests are likely no longer to include the explicit instruction “stating your hypotheses clearly”; any answer to a question involving hypothesis tests should include a statement of hypotheses unless they are already given in the question.
- Likewise, questions that involve critical regions (particularly for significance tests using discrete distributions) may not ask explicitly for the relevant probabilities to be quoted from tables, but candidates should always write down the values of these probabilities.
- Conclusions to hypothesis tests should be stated in terms that acknowledge the uncertainty involved. Thus “the mean height is 1.8” is too assertive and may not gain full credit; a statement such as “there is insufficient evidence that the mean height is not 1.8” is much to be preferred.

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.
4732: Probability and Statistics 1

General Comments

Most candidates showed a good understanding of a fair proportion of the mathematics in this paper. There were some very good scripts.

A few candidates (fewer than usual) ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. Some rounded answers to varying degrees of accuracy, apparently at random. In some cases, marks were lost through premature rounding of intermediate answers.

In certain calculations a calculator can give the answer immediately, without the need to show working (e.g. Q8(ii)). However, candidates need to be aware that they risk losing all the available marks if the answer is incorrect but no working is shown. It is wise to show some working or at least to double-check the calculation.

In questions requiring written answers, candidates commonly failed to gain the marks because they gave only statistical responses, without interpretation and/or reference to the relevant context.

Only a few candidates seemed to run out of time.

Most candidates failed to fill in the question numbers on the front page of their answer booklet, despite the clear instruction to do so.

In order to understand more thoroughly the kind of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

Use of the formula booklet MF1 was generally better than in June 2005. Nevertheless there were still a few candidates who made no use of the formula booklet, but used formulae from memory. More often than not, these formulae were incorrect. Others tried to use the given formulae, but clearly did not understand how to use them properly. Some candidates used the less convenient versions of formulae from the formula booklet, e.g. \( \sum (x - \bar{x})^2 \). The volume of arithmetic involved in these versions led to errors in most cases. Even the more convenient versions were sometimes misinterpreted, e.g. \( \sum x^2 - \frac{\sum x^2}{n} \).

Candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1. They need to understand which formulae are the simplest to use and also how to use them.

A few candidates used the binomial tables as if the probabilities were individual, rather than cumulative.
Comments on Individual Questions

1 (i) This part was successfully answered by most candidates, although a few just multiplied the two given probabilities. A tree diagram is possibly helpful here. However, some candidates drew extra branches, showing Jenny passing on her first attempt and then either passing or failing on a fictitious second attempt. These candidates sometimes included working from these incorrect branches in their calculation, achieving the correct answer by a rather suspect route.

(ii) Many candidates found $\frac{5}{6} - \frac{2}{3}$ but went no further. Some found the correct answer of $\frac{1}{2}$, but then went on to find $P(\text{John fails on his first attempt and passes on his second attempt}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$. A few simply combined the two given probabilities in some fashion. A few arrived at the correct answer by luck from $\frac{1}{6} + \frac{1}{3}$. These scored no marks.

2 (i) The effect of a linear transformation on $\mu$ and $\sigma^2$ were understood by most candidates. However, many candidates attempted to use methods appropriate for combining groups, thus wasting much time. These were sometimes successful for the mean but never for the variance. A common incorrect calculation for the mean was $24.5/50 + 100 = 100.49$. Some candidates gave 104.8 for the variance.

(ii) Interpretation of the greater standard deviation was sometimes weak. Some candidates thought that a greater standard deviation meant that the heights were greater. Many stated that the “range” of heights was greater. This did not score the mark. Answers such as “Greater spread” or “Less consistency” were accepted. It was also pleasing to see the occasional answer such as “The difference from the mean is greater on average”. A few candidates confused the two groups of heights.

(iii) Many candidates found the unweighted mean of the two means.

3 (i) The use of a geometric or binomial distribution was common. Many candidates arrived at the correct answer by the incorrect method: $\frac{3}{5} \times \frac{1}{2} \times \frac{2}{3}$. This scored no marks. Some candidates found $P(\text{BPP or PBP or PPB})$. A few resorted to using the values given in the table and $\Sigma p = 1$.

(ii) This was one of the better answered parts in the paper. Some errors were seen, e.g. $(\Sigma xp)/4$, $\Sigma xp^2$, $\Sigma (xp)^2$. Some candidates who obtained an incorrect answer for $E(X)$ or $E(X^2)$ then arrived at a negative value for $\text{Var}(X)$, but this appeared to give them no concern. Some omitted to subtract $(E(X))^2$.

4 (i)(a) Many candidates used the standard approach to histograms:

Frequency density = Frequency / Class width.

However, very few of these candidates actually arrived at a value for the frequency density represented by the height of one square. More commonly this approach led to confusion and/or incorrect answers. Many candidates did not know what to do with their numbers such as 15 000 and 300. Some combined the total frequency with a bogus “total class width” of 6000 – 1000 = 5000. A far more fruitful approach was to find the frequency represented by the area of one square. Candidates taking this approach generally answered this part correctly, taking only one or two lines of working to do so.
(i)(b) There was a similar contrast between the clumsy and the elegant methods in this part. A further difficulty was that some candidates interpreted the phrase “calculate an estimate” to mean that an approximate method was adequate. These candidates estimated the equivalent number of whole squares which would represent the frequency for £2700 to £3200. Some estimates were close enough to gain partial credit but most were not. Some candidates ignored the difference in height between the two blocks. The most efficient method was to calculate exactly the equivalent number of whole squares.

(ii) There appeared to be a common misunderstanding of the properties and advantages of different statistical diagrams. Many candidates gave a cumulative frequency graph as the answer to (a) rather than (b). Some gave a pie chart or frequency polygon in (b).

This question required candidates to distinguish between situations giving rise to a geometric distribution, a binomial distribution and neither of these. Some candidates failed to do this. It was also common for correct working to be followed by incorrect answers.

(i)(a) This part was generally well answered, although a few candidates included a binomial coefficient.

(i)(b) Many candidates appeared to be unfamiliar with questions on the Geometric distribution involving “less than”. Just \( (\frac{1}{4})^5 \) was common. Some tried to use a binomial method. Some of those who understood the question correctly, used the long method, adding terms. A few of these omitted a term or added an extra one. Only a disappointingly small number of candidates used the most efficient method, and even some of these used a power of 3 or 5 instead of 4.

(ii)(a) Many candidates omitted the binomial coefficient, probably because they were still thinking in terms of a Geometric distribution. Some candidates recognised the binomial situation, but surprisingly, most used the formula rather than the binomial tables.

(ii)(b) This unusual question was well answered by a good number of candidates, although some multiplied their answer to part (i)(b), instead of part (ii)(a), by \( \frac{2}{5} \). On the other hand, despite the clear lead-in given by part (ii)(a), the use of \( B(6, \frac{2}{5}) \) was common.

This was the least successfully answered question. A few used an incorrect formula, \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \). Some candidates used permutations. Incorrect combinations such as \( \binom{11}{7} \) were common.

(i) A very large number of candidates found the correct two combinations, but added them.

(ii) \( \binom{7}{4} \) was often seen but used incorrectly. As in part (i), many candidates added instead of multiplying. Candidates who attempted direct probability methods often used \( \frac{1}{4} \) or \( \frac{1}{7} \) or \( \frac{6}{7} \), each of which is incorrect.

(iii) Many candidates calculated the number of ways of not choosing A1 by \( \binom{4}{3} \) instead of \( \binom{3}{2} \). Others used \( \binom{5}{4} \) instead of \( \binom{6}{3} \) for the number of ways of choosing B1. Others considered (A1 and not B1) but not (not A1) .
Almost all candidates correctly gave B(10, 0.9). For the assumptions, very many candidates just listed parrot-fashion the conditions for the use of the binomial. Because no context was referred to, these scored no marks. Most candidates seemed confused between “conditions” and “assumptions”. In this case, two of the conditions (Repeated trials and Only two outcomes to each trial) are not assumptions that have to be made, but are inherent in the situation as described. The other two conditions are not inherent and have to be assumed to be true if a Binomial distribution is to be used. (The probability of each seed germinating is the same, and Germination of each seed is independent of the others). Some candidates gave the first of these in the form “Each seed must be planted in the same conditions”, which was accepted as a good answer in context.

This very easy binomial probability can be read directly from the tables, but many candidates failed to realise this. Some found P(X = 8) or 1 – P(X = 8) or 1 – P(X ≤ 8). Some attempted a very long (but correct) method, adding eight different probabilities using the formula. Some used the tables, but found P(X < 8).

Some candidates failed to recognise a binomial distribution and attempted methods using, e.g., \(19/20\). Others used B(20, 0.9) or B(200, 0.9). Some did not use their answer from part (ii), but started from scratch, usually incorrectly. A few used the correct distribution, but found only one term instead of two. Others used the wrong tail, reading the question as “fewer than 8”, as in part (ii). Some found the correct terms but evaluated them incorrectly. Another method which displayed some misunderstanding was 0.9298^20 + 0.9298^19. The number of correct solutions to this part was small.

This was answered correctly by most candidates. A few found the differences of the raw data pairs instead of their ranks. Some made arithmetical errors. Others used an incorrect formula, either \(\frac{6\Sigma d^2}{n(n^2-1)}\) or \(\frac{1-6\Sigma d^2}{n(n^2-1)}\). A few found \(r\) instead of \(r_s\).

Most candidates quoted, parrot-fashion, answers such as “Good correlation between the populations of these countries and their capital cities” or “A strong relationship between the populations of these countries and their capital cities” or “The ranks of country populations and capital city populations are similar”. Answers such as these did not score the mark. To score the mark, answers had to interpret the value of \(r_s\) in context, e.g. “Counties with large populations tend to have capital cities with large populations.” Another common answer which showed partial understanding was “As the population of the country increases, so does the population of the capital city.” A common wrong answer was that the populations of countries and capitals were “in proportion”.

This was well answered, except for the few candidates who did not use the formula booklet or who misinterpreted the given formulae or who chose the less convenient versions for \(S_{xx}\) etc.

This part was also generally well answered. Some candidates referred to “skew”, which is irrelevant. Others thought that because the values of \(y\) for France and the UK were large, their removal would decrease \(r\).

Most candidates chose the correct regression line and gave a correct reason. Some described \(y\) as the “controlled” variable, which is incorrect, but shows some understanding. A few chose \(y\) on \(x\), and gave as a reason that an \(x\) value was to be estimated from a \(y\) value.
(iv)(b) Many candidates recognised the inappropriateness of applying the regression line in such a different context. Some ignored the context and stated that because $r$ is fairly high, and because the estimate would be an interpolation, it would be fairly reliable. These gained partial credit. Some candidates referred neither to context nor to $r$ but only to interpolation. These scored no marks. The use of the word “accurate” rather than “reliable” resulted in the loss of a mark for many candidates.
4733 Probability and Statistics 2

General Comments

This paper was found generally accessible and it produced a good range of marks. There were many very
good candidates who produced high quality scripts; however, it seemed that some would have benefited from
more preparation, especially in terms of detail. As usual there were some seriously under-prepared
candidates.

The usual problems arose with this specification. Mistakes that have been repeatedly identified in the past
include wrong or omitted continuity corrections (and the converse, the insertion of continuity corrections
when they are wrong); the similar omission, or wrong inclusion, of a factor of \( \sqrt{n} \) in the standard deviation;
incorrect use of tables, particularly with errors such as \( P(X \geq 5) = 1 - P(X \leq 5) \); and poor understanding of
verbal questions. As usual, numerical calculations were better done than verbal interpretations, and the
impression persists that many candidates do not really understand what they are doing, even when they get
correct numerical answers.

As was mentioned in the report on the Summer 2005 examination, in future it is expected that questions will
not explicitly include instructions to “state hypotheses” or “show all relevant probabilities”; it will be
assumed that these are an essential part of full solutions. Candidates may also in future be penalised for
giving too many significant figures in their answers if this is clearly inappropriate.

Rather too many candidates think that \( “P(X < 9)” \) means \( “P(X \leq 8)” \), even for a continuous distribution.

Comments on Individual questions.

1. Most could get the two calculations correct, though use of Po(0.7) tables for the distribution Po \((\lambda)\)
is not acceptable. For part (ii), in order to score both marks it was necessary to give a reason why
one of the standard conditions for the validity of a Poisson distribution might not hold, and to state
that reason. Thus answers such as “Foxes may go around in groups so they are not independent” or
“some parts of the region might be more favourable to foxes than others, so the average rate would
not be constant” would score full marks. This was a comparatively rare example of a context in
which “foxes might not occur singly” would have been an appropriate reason, though as explained in
previous reports neither “singly” nor “occurring at random” is generally a safe reason to give.

2. Most correctly used a normal approximation. Some used \( nq \) instead of \( npq \) as the variance, but most
good candidates scored at least 5 out of 7 here. The continuity correction was often wrong, with
several instances of candidates saying that \( P(X > 13) = P(X \geq 12) = P(X > 11.5) \).

3. As usual the significance test using a binomial distribution was poorly done. Many could not state
the hypotheses correctly. (It is good practice to state the meaning of the letter used in the hypothesis
test – e.g. “where \( p \) is the population proportion of chocolates with hard centres” – but very few
candidates do this.) As usual many seemed unable to do an exact binomial test and attempted to use
a normal approximation. In this particular question there was a further problem in that many did not
seem to realise that they had to test for the upper tail (because 5 is more than the expected value of
3); solutions such as “\( P(X \leq 5) = 0.9887 > 0.975 \) so do not reject \( H_0 \)” were incorrect.

4. Here the Poisson approximation was essential; the normal approximation is very poor and scored
few marks. In part (ii), it was very disappointing that, despite the clear instruction, many candidates
failed to write down any of the necessary probabilities such as “\( P(X \leq 4) = 0.9763, P(X \leq 5) =
0.9940 \)”. Without at least one of these probabilities, full marks could not be scored, and if neither
was shown and the final answer was not 6, no credit at all could be given.
This question was poorly answered. In part (i), errors included inclusion of a spurious continuity correction, failure to square-root $\frac{1}{4}\mu^2$, failure to simplify $\mu/(\frac{1}{2}\mu)$ to 2, and failure to obtain the final numerical answer as well as to state that it was independent of $\mu$.

In the second part, many attempted to use the mean of three years (with a variance of 9/3), rather than to find the probability for one year and cube it.

The final part produced many impressive (and often long) responses, but the answer “because the years are not chosen randomly” was unacceptable and would probably have been so even with a different wording of the question. As has been stated before, clear thinking on this issue is not encouraged by concentrating on the word “random”.

Many scored full, or nearly full marks on this question with apparent ease, with omission of the 50/49 factor the commonest error. But many others betrayed serious confusion; it is essential to use the hypothesis value of $\mu$, here 32, as $\mu$ in the normal distribution calculation. Those who wrote “N(32.3, ...)” and found $P(\bar{X} < 32)$ scored few marks, as did those who omitted the $\sqrt{n}$ factor. Most of the marks for a question of this sort are given for method, not calculation. Again, hypotheses were poorly stated; in the absence of a verbal definition, only the symbol $\mu$ should be used, and those who used $\bar{X}$ (or $\bar{w}$) were on completely the wrong track.

The main problems in parts (i) and (iii) here were omission of the $\sqrt{12}$ factor, and getting the wrong sign. Quite a few candidates rounded off their answers to the nearest integer, which is incorrect. A surprising number failed to realise that the answer to part (ii)(b) was that the test produced the correct result; the perhaps automatic response of “Type II error” was common. In part (iii), using 80 rather than the value of $c$ was a common error.

This question produced a very wide range of responses. At one extreme, a good many dealt very confidently with the first four parts and showed excellent algebraic and calculus skills; some of those who did not realise that the answers to part (v) could simply be written down obtained the correct values by integration, either by expanding $(y + \frac{4}{3})^3$ or by using parts. At the other extreme, it was disappointing to find many candidates (presumably including some taking Further Maths) who thought that $x^nx^n = x^{2n}$ or that the value of $kx^{n+1}$ when $x = 1$ was $kn^{n+1}$.

Most realised that the distribution in part (iv) was normal.

It should be noted that, in part (v), use of the integrals $\int (y + \frac{4}{3})^3 \, dx$ and $\int (y + \frac{4}{3})^5 \, dx$, although they give apparently correct answers, are wrong as these find the expectation and variance of $(Y + \frac{4}{3})$, rather than $Y$. 

112
4734: Probability and Statistics 3

General Comments

The general performance of candidates was similar to those of the January 2005 paper. Fewer, however, scored very high marks. This was, perhaps, mainly due to the unusual hypotheses in Qs 2 and 6. Some candidates believe that a test statistic should always be positive, and this can lead to an incorrect conclusion in a one-tailed test. \( z \) values should be obtained from \( z = \frac{\bar{x} - \mu}{\sigma_{x}} \), as recommended in previous reports.

Comments on Individual Questions

1  (i)  This proved to be a question in which most of the candidates could score well and the required interval was usually obtained correctly.

(ii)  The comment was not expected to be too assertive with statements such as “the councillor may be right”.

2  (i)  Many referred only to the small sample size whereas the \( t \)-test requires an unknown population variance.

(ii)  The hypothesis “At least 2000” would not be rejected if \( \mu \geq 2000 \) were correct, and rejected if \( \mu < 2000 \) were correct. The hypotheses should then be \( H_{0}: \mu = 2000, H_{1}: \mu < 2000 \). It is also acceptable to have \( H_{0}: \mu \geq 2000 \). Despite this not always being known, candidates did usually score well.

3  This was the least well-answered question on the paper, in both parts although, with the given answer, part (i) was found to be easier.

4  Most candidates could pick up some marks but it was generally poorly answered. It was often realised that a normal approximation to a Poisson distribution was required, but the required continuity correction was not always used. The principles for finding the mean and variance in part (ii) were usually known but their relevance in part (iii) was not always appreciated.

5  The finding of cdfs, and their relevance to finding pdfs of related variables, is a topic that more candidates are becoming familiar with. It was pleasing to find so many excellent solutions.

6  Candidates seemed very familiar with the independent sample test, but it was not always applied correctly. The required formula was usually known for finding the unbiased estimate of variance, but since the answer was given, a value to more than 3 decimal places was required before rounding in order to obtain full credit.

   In part (ii) some candidates used a \( z \)-value in their test despite the hint in part (i).

   The hypotheses in part (iii) were often incorrect and the further assumption asked for was often shortened to “normal distributions” without any context. It should always be made clear which distributions are involved.
Report on the units taken in January 2006

7 (i) Most knew which integral was required to obtain $E(X)$ but there were some problems with the limits.

(ii) Candidates were expected to obtain the value of $\alpha$ from a relevant equation, but some verified the given value, which was not acceptable.

(iii) This was the most difficult part, but the given answer meant that those unsuccessful here could still obtain full credit in the final part.

(iv) Most candidates could obtain several marks here, the main error was to give 3 for the number of degrees of freedom, instead of 2 (from 4 cells).
Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.
Report on the units taken in January 2006

4736: Decision Mathematics 1

General Comments

A good spread of marks was achieved with only a few very poor scripts. Even the weaker candidates could usually score some marks on every question. Most candidates were able to complete the examination in the time allowed, although some spent far too long on writing out every swap and comparison in the bubble and shuttle sorts, giving too many iterations of the Simplex algorithm or making multiple attempts at the linear programming question.

In general, the quality of the presentation of the candidates’ answers was good. Some candidates were unable to read their own working and created extra work for themselves as well as forfeiting accuracy marks.

Comments on Individual Questions

1 This was a straightforward start for most candidates. Some candidates did not use the ordered list of arc weights to show the order in which they had selected or rejected the arcs, but most were able to construct an appropriate minimum spanning tree. A few candidates had used Prim’s algorithm instead of Kruskal’s.

2 Many very good answers. The weaker candidates had extra temporary labels (there is no need to record a temporary label if it is larger than the current temporary labels at a vertex) or gave permanent labels to vertex $D$, or both. Some candidates left both $D$ and $E$ without permanent labels and so also lost the mark for the order of assigning the permanent labels.

3 (i) Most candidates were able to follow the algorithm accurately; a few drew out every step and some seemed to extrapolate from the first two diagrams to give incorrect answers for parts (c) and (d). A common mistake was to only have six arcs in the diagram for $n = 3$.

(ii) The better candidates realised that the odd and even values of $n$ behaved differently and were able to give a rule for the number of arcs in these two cases. The weak candidates assumed that the formula would be $\frac{1}{2}n(n+1)$, despite the fact that this did not fit their values.

4 (i) Most candidates were able to set up an initial Simplex tableau. Some candidates omitted the column corresponding to $P$ and a few had sign errors in the objective row. When slack variables have been added, the problem becomes:

$$\text{maximise } P$$

subject to

$$\begin{align*}
5x + 4y + 3z & = 0 \\
2x - 3y + 4z + s & = 10 \\
6x + 5y + 4z + t & = 60
\end{align*}$$

and $x, y, z, s, t \geq 0$

This is represented in matrix form using the tableau:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>-5</th>
<th>4</th>
<th>3</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>-3</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>
(ii) Some good answers, candidates who showed the operations they were using were able to gain method marks even when they had made arithmetic slips. Most candidates could find the appropriate pivot element (the 2 in the second row) and were able to carry out pivot operations of the form \( \text{current row} = \text{current row} + \text{multiple of pivot row} \) to get a basis column with a 1 replacing the pivot value and a zero in the other rows.

5 (i) Very few candidates were able to identify the variables correctly. Many just said ‘breaststroke’, ‘backstroke’ and ‘butterfly’, instead of ‘\( x = \) number of lengths of breaststroke’, and so on. The identification of the objective function often appeared in amongst the constraints, even when the appropriate combination of the variables was selected. Several candidates claimed that the objective was \( x + y + z \) (which represents the total distance) instead of maximising the style marks.

(ii) Many candidates just wrote down every combination of \( x, y, z \) and numbers that they could see despite the question having set out quite clearly where the constraints were to come from.

(iii) Most candidates were able to draw the boundary lines for the constraints given, but some assumed that the feasible region would be the quadrilateral with the origin as one vertex. Quite a few of the graphs were not sufficiently accurate for the candidates to be able to read off the vertices with any confidence. The question asked candidates to write down the coordinates of the feasible region but some chose to ignore the non-integer valued point completely.

(iv) Some candidates were able to interpret their solution from part (iii) in the context of the original problem to describe the number of lengths of each stroke that Findley should swim and a few gave the total number of points that he could expect as being 22 points.

6 (i) Most candidates were able to write down the order of visiting the vertices for at least one of the tours obtained by using the nearest neighbour method although some then chose the longer route as the better upper bound. If a diagram is used, the direction of travel around the tour should be indicated.

(ii) Part of Prim’s algorithm is to show the order in which the vertices are included and to state the weight of the minimum spanning tree. Several candidates showed the two shortest arcs from \( G \) on their tree and consequently lost the credit for the minimum spanning tree. Some candidates calculated the length of the spanning tree but did not use it to find a lower bound for the journey time.

(iii) Most candidates gained marks on this question but several who correctly identified \( BD \) and \( EF \) as the routes to be repeated did not state that the arcs that needed to be travelled twice were \( BD, EG \) and \( GF \). Several candidates claimed that \( D \) was travelled through six times, instead of three times.

7 (i) Many candidates gave incoherent answers in which they just showed pages of swaps with no clear indication of where the passes ended. It was only necessary to show the results at the end of each pass. A large number of candidates made errors in copying their own figures from one row or column to the next and many did not seem to know the bubble sort algorithm. Even those who did perform the algorithm correctly often went on to carry out an unnecessary seventh pass or stopped at the end of the fifth pass. The algorithm can be stopped early only when no swaps have been made in an entire pass.
(ii) Shuttle sort was somewhat more successfully carried out than bubble sort, but even so many candidates were not able to count the total number of comparisons and the total number of swaps accurately. Some candidates sorted into decreasing order, despite the question clearly asking for a sorted list in increasing order.

(iii) Only a few candidates were able to explain that each script needed to be looked at once and so the time taken is, roughly, proportional to the number of scripts. Some candidates seemed to have the right sort of idea but were not able to convincingly explain their thoughts, several just missed the point completely.

(iv) Quite a few candidates were able to calculate the total time as being \(250 + 6250 = 6500\) seconds, some candidates did not appreciate that the sorting of the two piles took place simultaneously. The majority of the candidates just scaled the total time for splitting and sorting the 100 scripts by a factor of \(5^2\).
4737: Decision Mathematics 2

General Comments

A good spread of marks was achieved and most candidates were able to complete the examination in the time allowed.

In general, the quality of the presentation of the candidates’ answers was good. A few candidates submitted messy work that was squeezed into a tiny space; this invariably led to a loss of accuracy and a consequent loss of marks.

Comments on Individual Questions

1  
(i) Most candidates were able to draw the correct bipartite graph, a few did not realise that both $A1$ and $A2$ needed to be matched to $G$, $P$ and $R$.
(ii) Nearly all the candidates completed the second diagram to show the incomplete matching. A few then superimposed their alternating paths and almost obscured the matching in the process.
(iii) Most candidates gave a valid alternating path, but several did not give a path consisting of just three arcs (the shortest alternating path), indeed some candidates seemed determined to make the longest alternating path they could.
(iv) Almost all the candidates were able to find the matching, even those who had lost their way in constructing the alternating path in part (iii).

2  
Most candidates scored full marks on this question, including reading off the route. A few candidates gave the route backwards, but most started at $(3; 0)$, and a few omitted either the start or the finish vertex. A few candidates made numerical errors but were usually saved by having the network given in the question.

3  
(i) Most candidates were able to calculate the capacity of the cut with $X = \{S, A, B, C\}$, $Y = \{D, E, F, G, H, I, T\}$ although some found the capacity of the cut $\alpha$ instead.
(ii) Several candidates realised that they did not need to include the arc $DG$ in their calculation, but their explanations of why were often confused. The best answers referred to the fact that $D$ and $G$ are both on the sink side of the cut.
(iii) Many good answers, although many candidates did not give the maximum flow in the arc $SB$. 
Report on the units taken in January 2006

4  (iv) Most candidates found a flow of 14 litres per second, although they did not always show their flow clearly and often omitted the direction arrows. A few candidates were concerned that the diagram in the question paper did not show flow directions, and some did not appreciate that the fluid could flow in either direction. Although some candidates correctly found the cut of 14 litres per second, rather more just used the flows instead of the capacities and so thought that every cut had capacity 14.

Only a small minority of candidates were able to state that since they had found a flow of 14 litres per second and a cut of 14 litres per second this must be the maximum flow and the minimum cut. One or two candidates went as far as explaining that each cut causes a restriction to the flow so every flow is less than or equal to every cut and hence maximum flow \( \leq \) every cut and thus having found a flow equal to a cut it must be the maximum flow (and the minimum cut).

The majority of the candidates either assumed that the cut they had found was the minimum cut with no reasoning given or they just said maximum flow equals minimum cut but did not refer to the flow and the cut that they had found.

(i) Most candidates calculated the value 260, but several omitted the units.

(ii) Many correct or very nearly correct solutions. Some candidates stopped before they had reached an augmented reduced cost matrix with a zero cost complete matching. A few candidates did not augment efficiently and went up in four single steps instead of just using two augmentations and some candidates only reduced the rows before beginning to augment.

(iii) Most candidates were able to gain the mark but only the best gave a completely correct description by using words such as ‘minimum cost allocation’.

5  (i) Few candidates were able to explain the necessity for the dummies, often just giving very general descriptions of what dummy activities are. The dummy between \( C \) and \( F \) is needed because \( F \) follows both \( B \) and \( C \) whereas \( G \) follows just \( C \), the dummy after \( H \) is needed because otherwise activities \( H \) and \( I \) would share a common start point and a common end point. This would mean that when the algorithm is run (mechanically, rather than using ‘common sense’) there would be ambiguity over which value to use.

(ii) Most candidates understood what was required to complete the precedence table. A few listed the activities that followed the activities instead of those that they followed from.

(iii) Most candidates were able to make a good attempt at the forward and backward passes although quite a few ignored the dummy and gave a late event time of 6 instead of 5 after activity \( C \) and rather more did not give an early or a late event label times for the activities instead of the events (vertices) between them. The minimum project completion time needed units of days.

(iv) Many correct answers, but also several candidates who thought that \( C \) would need to increase by 2 days as a consequence of earlier errors.

(v) The resource histogram was not done well; a large number of candidates drew a graph with ‘holes’ in it or ‘overhanging blocks’ instead of a histogram. A resource histogram should look like a histogram in statistics, except that the vertical axis shows ‘number of workers’. It is not necessary to label the activities \( A, B, C \), etc. on the histogram.
(vi) Many candidates tried to move activity $E$ to after activity $J$ without realising that this contravened the original precedences. Several candidates realised that activity $H$ needed to be delayed but fewer realised that they also needed to move $J$. It was sometimes difficult to interpret where an activity was being moved to, in particular there was confusion between the reading on the scale and which day it referred to (e.g. day 1 is from time 0 and time 1). Some candidates gave a schedule listing the order of the activities carried out by each worker.

(i) Most candidates identified the appropriate values to show that $A$ does not dominate $B$ (-3 < 5 in column $Y$) and $B$ does not dominate $A$ (-3 < 2 in column $X$ or 1 < 4 in column $Z$). Some candidates just listed inequality signs without explaining how this showed the dominance.

(ii) A few candidates unnecessarily redrew the pay-off matrix from Maria’s perspective. Some candidates gave column minima of giving column maxima and then explaining why column $Y$ does not give the minimax (play-safe) strategy.

(iii) Most candidates realised that the row minima were all equal and hence each is a maximin and there is no preference between $A$, $B$ and $C$ in terms of play-safe (cautious play), however if Lucy plays row $B$ then she could win as many as 5 points (the maximum possible) if Maria plays column $Y$ (rather than playing safe).

(iv) Most candidates realised that 3 has to be subtracted from $m$ because previously 3 has been added throughout the matrix. Some candidates thought that $M$ stood for the matrix or just said that 3 was the most Lucy could lose.

(v) To justify the expressions given, candidates needed to write out the matrix with 3 added throughout and then explain how column $X$ led to an expected gain of $5p_1 + 0p_2 + 7p_3$ and similarly for column $Y$ and column $Z$. Some candidates seemed to think that $A$, $B$ and $C$ had ‘become’ $p_1$, $p_2$ and $p_3$ and some candidates used rows instead of columns.

(vi) Candidates’ explanations were generally poor. The expressions in part (v) give the expected gain for Lucy for each of Maria’s choices and for each combination of the probabilities $p_1$, $p_2$ and $p_3$. $m$ represents Lucy’s minimum expected gain for each combination of the probabilities, which is equal to (at least) one of the expressions from part (v) and is lower than the others.

Even if Maria does her worst, Lucy can choose the probabilities to maximise her minimum expected gain, and she may even do better than this in the long run if Maria does not play optimally.

Many candidates discussed the probabilities needing to sum to 1 or talked vaguely about ‘it’ needing to be like this so that the Simplex method would work.

(vii) Most candidates were able to recover to show where $4 - 2p_1$ came from and to find the other expression as $7p_1 - 3$. Some candidates used the augmented matrix and so got the expressions $7 - 2p_1$ and $7p_1$ without realising that they needed to then subtract 3.

(viii) Most candidates were able to solve their equations from part (vii) to give a probability $p_1$ and some put this back into one or both equations to find the expected number of points won by Lucy in this case. Some candidates also calculated the number of points won when $p_1 = 0$ and when $p_1 = 1$ for at least one of the two expressions. Only a few candidates realised that the smaller number of points for each value of $p_1$ corresponded to the vertices of the lower boundary and thus selected the value of $p_1$ to maximise the minimum expected number of points won.
## Unit Threshold Marks

<table>
<thead>
<tr>
<th>Unit</th>
<th>Maximum Mark</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>4721</td>
<td>Raw</td>
<td>72</td>
<td>57</td>
<td>49</td>
<td>42</td>
<td>35</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4722</td>
<td>Raw</td>
<td>72</td>
<td>57</td>
<td>49</td>
<td>41</td>
<td>34</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4723</td>
<td>Raw</td>
<td>72</td>
<td>55</td>
<td>48</td>
<td>41</td>
<td>34</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4724</td>
<td>Raw</td>
<td>72</td>
<td>63</td>
<td>55</td>
<td>47</td>
<td>39</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4725</td>
<td>Raw</td>
<td>72</td>
<td>59</td>
<td>51</td>
<td>43</td>
<td>36</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4726</td>
<td>Raw</td>
<td>72</td>
<td>59</td>
<td>51</td>
<td>43</td>
<td>36</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4727</td>
<td>Raw</td>
<td>72</td>
<td>58</td>
<td>50</td>
<td>43</td>
<td>36</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4728</td>
<td>Raw</td>
<td>72</td>
<td>55</td>
<td>48</td>
<td>41</td>
<td>35</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4729</td>
<td>Raw</td>
<td>72</td>
<td>58</td>
<td>50</td>
<td>43</td>
<td>36</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4730</td>
<td>Raw</td>
<td>72</td>
<td>57</td>
<td>50</td>
<td>43</td>
<td>36</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4732</td>
<td>Raw</td>
<td>72</td>
<td>56</td>
<td>49</td>
<td>42</td>
<td>35</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4733</td>
<td>Raw</td>
<td>72</td>
<td>58</td>
<td>50</td>
<td>43</td>
<td>36</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4734</td>
<td>Raw</td>
<td>72</td>
<td>54</td>
<td>47</td>
<td>40</td>
<td>33</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4736</td>
<td>Raw</td>
<td>72</td>
<td>56</td>
<td>49</td>
<td>42</td>
<td>35</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>4737</td>
<td>Raw</td>
<td>72</td>
<td>55</td>
<td>48</td>
<td>41</td>
<td>34</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>UMS</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>
Report on the units taken in January 2006

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

<table>
<thead>
<tr>
<th>Maximum Mark</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>3890</td>
<td>300</td>
<td>240</td>
<td>210</td>
<td>180</td>
<td>150</td>
<td>120</td>
</tr>
<tr>
<td>3892</td>
<td>300</td>
<td>240</td>
<td>210</td>
<td>180</td>
<td>150</td>
<td>120</td>
</tr>
<tr>
<td>7890</td>
<td>600</td>
<td>480</td>
<td>420</td>
<td>360</td>
<td>300</td>
<td>240</td>
</tr>
</tbody>
</table>

The cumulative percentage of candidates awarded each grade was as follows:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>U</th>
<th>Total Number of Candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>3890</td>
<td>14.9</td>
<td>31.4</td>
<td>52.6</td>
<td>72.5</td>
<td>90.3</td>
<td>100</td>
</tr>
<tr>
<td>3892</td>
<td>50.0</td>
<td>50.0</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>7890</td>
<td>21.9</td>
<td>65.6</td>
<td>84.4</td>
<td>87.5</td>
<td>93.8</td>
<td>100</td>
</tr>
</tbody>
</table>

For a description of how UMS marks are calculated see; www.ocr.org.uk/OCR/WebSite/docroot/understand/ums.jsp

Statistics are correct at the time of publication