Reteaching Workbook

Contents Include:

113 worksheets—one for each lesson
To The Student:

This Reteaching Workbook gives you additional examples and problems for the concept exercises in each lesson. The exercises are designed to aid your study of mathematics by reinforcing important mathematical skills needed to succeed in the everyday world. The material is organized by chapter and lesson, with one skills practice worksheet for every lesson in MathMatters 3.

To the Teacher:

Answers to each worksheet are found in MathMatters 3 Chapter Resource Masters.
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RETEACHING 1-1

THE LANGUAGE OF MATHEMATICS

A set is a well-defined collection of objects called elements. The symbol \( \in \) can be used to show that an element is a member of a set. If every element of a set \( A \) is also an element of another set \( B \), then set \( A \) is a subset of set \( B \). Sets are usually described in one of these three ways:

- **Description notation** Example: \( S = \) set of even whole numbers
- **Roster notation** Example: \( \{0, 2, 4, 6, 8 \ldots \} \)
- **Set-builder notation** Example: \( S = \{x | x \) is an even whole number\} \\

**EXERCISES**

Use the following sets \( A = \{2, 4\}, B = \emptyset, C = \{2, 4, 6, 8, 10\}, \) and \( D = \{-2, -1, 0, 1, 2\} \) for Exercises 1–8. Tell if each statement is true or false.

1. \( 1 \in D \) ______
2. \( 1 \in A \) ______
3. \( -2 \in D \) ______
4. \( -3 \in D \) ______
5. \( A \subset C \) ______
6. \( B \subset D \) ______
7. \( C \subset A \) ______
8. \( B \subset A \) ______

9. Write all the possible subsets of the set \( \{x, y\} \). 

Define each set using roster notation.

10. odd numbers greater than 5

11. even negative numbers with a value less than \(-3\)

Which of the given values is a solution of each equation?

12. \( 6 - m = 4; -2, 2 \) ______________
13. \( 4n + 6 = 30; 6, 9 \) ______________
14. \( b + 7 = -8; -1, -15 \) ______________
15. \( j \div 12 = 2; 6, 24 \) ______________
RETEACHING 1-2

REAL NUMBERS

The diagram shows how each number system nests totally within the next larger one. Also given are some types of numbers each system represents.

Real numbers: \( \pi, \sqrt{3}, \sqrt{5}, 5.15115115115 \ldots \)
Rational numbers: \(-1, -0.5, -\frac{1}{4}, 0, \frac{1}{4}, 0.5, 1\)
Integers: \(\ldots -2, -1, 0, 1, 2 \ldots\)
Whole numbers: \(0, 1, 2, 3 \ldots\)
Natural numbers: \(1, 2, 3 \ldots\)

Example 1

Graph all real numbers less than 3.

Solution

The set consists of all real numbers less than 3.

\[ \begin{array}{cccccccc}
-2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\end{array} \]

Example 2

Evaluate each expression where \( x = 0.3 \).

\( a. \ - (x) \quad b. \ - (-x) \quad c. \ |x| \)
\( d. \ |x| \quad e. \ - |x| \)

Solution

\( a. \ - (x) = - (0.3) = -0.3 \)
\( b. \ - (-x) = - (-0.3) = 0.3 \)
\( c. \ |x| = |0.3| = 0.3 \)
\( d. \ |x| = |1 - 0.3| = 0.3 \)
\( e. \ - |x| = - |-0.3| = -0.3 \)

EXERCISES

Tell whether each statement is true or false.

1. \( \sqrt{2} \) is a rational number. _____
2. \(-42\) is an integer. _____
3. 0 is a natural number. _____
4. \(- \frac{3}{5}\) is an integer. _____
5. 213 is a whole number. _____
6. 0.31131113 is an irrational number. _____

Graph each set of numbers on a number line.

7. \( \left\{ \frac{1}{5}, -1 \frac{3}{8}, \sqrt{2}, 3.9 \right\} \)

8. real numbers less than or equal to \(-1\)

Evaluate each expression where \( b = -0.8 \).

9. \(-b\) _____
10. \(-(-b)\) _____
11. \(|b|\) _____
12. \(-|b|\) _____
13. \(|-b|\) _____
14. \(-|-b|\)
RETEACHING  1-3

UNION AND INTERSECTION OF SETS

This Venn diagram represents the union and intersection of two sets and their complements.

\[ A \cup B = \{2, 4, 6, 8, 10\} \]
\[ A' = \{8, 10\} \]
\[ A \cap B = \{6\} \]
\[ B' = \{2, 4\} \]

Example 1

Graph the solution set for \( x < 1 \) and \( x \geq -2 \).

Solution

Graph of \( A \) if \( A = x < 1 \)

Graph of \( B \) if \( B = x \geq -2 \)

Graph of \( A \cap B \):
\[ x < 1 \text{ and } x \geq -2 \]

SOLUTION SET

Roster notation: \{-2, and all real numbers between -2 and 1\}

Set-builder notation: \( \{x | x \text{ is a real number and } -2 \leq x < 1\} \)

Example 2

Graph the solution set for \( x \leq -2 \) or \( x > 1 \).

Solution

Graph of \( A \) if \( A = x \leq -2 \)

Graph of \( B \) if \( B = x > 1 \)

Graph of \( A \cup B \):
\[ x \leq -2 \text{ or } x > 1 \]

SOLUTION SET

Roster notation: \{-2, and all real numbers less than 2 or greater than 1\}

Set-builder notation: \( \{x | x \leq -2 \text{ or } x > 1\} \)

EXERCISES

Refer to the diagram. Find the sets named by listing the members.

1. \( A' \)  
2. \( A \cap B \)  
3. \( A \cup B \)

Graph the solution sets for each compound inequality. Then describe the solution set in two ways using roster notation and set-builder notation.

4. \( x > 2 \text{ or } x \leq -1 \)

Graph of \( A \) if \( A = x > 2 \)

Graph of \( B \) if \( B = x \leq -1 \)

Graph of \( A \cup B \):
\[ x > 2 \text{ or } x \leq -1 \]

Roster notation: ___________________  

Set-builder notation:__________________

5. \( x \geq -3 \text{ and } x < 0 \)

Graph of \( A \) if \( A = x \geq -3 \)

Graph of \( B \) if \( B = x < 0 \)

Graph of \( A \cap B \):
\[ x \geq -3 \text{ and } x < 0 \]

Roster notation: ___________________  

Set-builder notation:__________________
RETEACHING 1-4

ADDITION, SUBTRACTION, AND ESTIMATION

When you add or subtract rational numbers, you use the same rules that apply to adding or subtracting integers.

ADDITION

Rule 1: When the numbers have the same sign, add the absolute values and give the sum the same sign as the addends.

Rule 2: When the numbers have different signs, subtract the absolute values and give the sum the sign of the number with the greater absolute value.

SUBTRACTION

Rule 3: When you subtract a number, add its opposite, or its additive inverse. Be sure to use either Rule 1 or Rule 2 when adding to find the sum.

Example 1

Find each answer.

a. \(2.51 + (-0.36)\)

b. \(-\frac{7}{9} - \left(-\frac{5}{9}\right)\)

Solution

a. \(2.51 + (-0.36) = 2.15\) Use Rule 2.

b. \(-\frac{7}{9} - \left(-\frac{5}{9}\right) = \frac{-7 + 5}{9} = \frac{-2}{9}\) Use Rule 3.

Example 2

Evaluate each expression when \(x = -1.6\) and \(y = 2.1\).

a. \(x - y\)

b. \(y + x\)

Solution

a. \(-1.6 - 2.1 = -3.7\)

b. \(2.1 + (-1.6) = 2.1 - 1.6 = 0.5\)

EXERCISES

Add or subtract.

1. \(0.23 - (-0.16)\)

2. \(-3.58 + (-1.32)\)

3. \(-1.58 + 2.21 - (-3.67)\)

4. \(1\frac{3}{7} - 1\frac{5}{7}\)

5. \(-3\frac{1}{5} - (-1\frac{3}{5})\)

6. \(-1\frac{1}{10} - 2\frac{1}{5} - (-1\frac{1}{2})\)

7. \(c - d\)

8. \(d + c\)

9. \(-d - c\)

10. \(-c + d\)
RETEACHING 1-5

MULTIPLICATION AND DIVISION

When you multiply or divide two rational numbers, you use the same rules that apply to multiplying and dividing with integers.

**Rule 1:** When the numbers have the same sign, find the product or quotient of their absolute values. The product or quotient is always positive.

**Rule 2:** When the numbers have different signs, find the product or quotient of their absolute values. The product or quotient is always negative.

---

**Example 1**

Find each answer.

a. \((-6.1)(-3.2)(10)\)  
b. \(-\frac{1}{12} \div \left(-\frac{3}{8}\right)\)

**Solution**

a. \((-6.1)(-3.2)(10)\)  
   \(= (19.52)(10)\)  
   \(= 195.2\)

b. \(-\frac{1}{12} \div \left(-\frac{3}{8}\right)\)  
   \(= \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}\)

---

**Example 2**

Evaluate each expression when \(x = -0.3, y = -1.8, \) and \(z = \frac{1}{3}\).

**Solution**

a. \(xz\)  
b. \(z \div x\)  
   \((-0.3)\left(\frac{1}{3}\right) = \frac{1}{3}\left(-\frac{1.8}{0.3}\right) = \frac{1}{3}(6) = 2\)

---

**EXERCISES**

Find each answer.

1. \((-7.2)(1.8)\)  
2. \(-13.5 \div 0.5\)  
3. \(-1.25 \div -2.5\)  
4. \((-1.8)(-3.5)\)

5. \(-2\frac{1}{2} \div -1\frac{3}{4}\)  
6. \(\left(1\frac{1}{2}\right)\left(-\frac{1}{3}\right)\)  
7. \(-4\frac{1}{8} \div 2\frac{1}{4}\)  
8. \(\left(-4\frac{1}{2}\right)\left(2\frac{1}{12}\right)\)

9. \((1.84)(-0.1)(-3)\)  
10. \((-2.5)(-10)(-0.2)\)  
11. \((-0.5)(-0.5) \div 10\)

Evaluate each expression when \(x = -0.1, y = 10, \) and \(z = -\frac{1}{5}\).

12. \(xyz\)  
13. \(xz\)  
14. \(xy \div z\)  
15. \(yz \div -x\)
RETEACHING 1-6

PROBLEM SOLVING SKILLS: USE TECHNOLOGY

A spreadsheet is composed of columns and rows, whose intersections are called cells. You may enter data into each cell or you can enter a formula into the cell that will perform calculations on data in other cells.

Example

A company budget estimates gross profit (sales less cost of sales) for one of its products. Each unit sells for $15.00. Cost for one unit is $9.50. Find gross profit for sales from 1000 units to 20,000 units using 1000 unit intervals.

a. Make a spreadsheet. Show the first five lines (including headings).

b. Find the value in each cell in your spreadsheet.

Solution

a.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Units</td>
<td>Sales</td>
<td>Cost</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>A2*15</td>
<td>A2*9.5</td>
</tr>
<tr>
<td>3</td>
<td>A2 + 1000</td>
<td>A3*15</td>
<td>A3*9.5</td>
</tr>
<tr>
<td>4</td>
<td>A3 + 1000</td>
<td>A4*15</td>
<td>A4*9.5</td>
</tr>
<tr>
<td>5</td>
<td>A4 + 1000</td>
<td>A5*15</td>
<td>A5*9.5</td>
</tr>
</tbody>
</table>

b.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Units</td>
<td>Sales</td>
<td>Cost</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>15,000</td>
<td>9500</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td>30,000</td>
<td>19,000</td>
</tr>
<tr>
<td>4</td>
<td>3000</td>
<td>45,000</td>
<td>28,500</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>60,000</td>
<td>38,000</td>
</tr>
</tbody>
</table>

Note: * represents the multiplication sign.

EXERCISES

1. Find the value in each cell.

2. Complete the first three rows of this spreadsheet and find the value for each cell. Individual gifts are 60% of corporate gifts and government grants are 20% of corporate gifts. Use $5000 intervals to find total donations if corporate gifts range from $5000 to $100,000.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Corp</td>
<td>Ind</td>
<td>Govt</td>
</tr>
<tr>
<td>2</td>
<td>5000</td>
<td>A2*0.6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A2 + 5000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Corp</td>
<td>Ind</td>
<td>Govt</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Reteaching 1-7

Distributive Property and Properties of Exponents

The distributive property as well as other mathematical properties are useful in simplifying mathematical expressions.

Example 1

Simplify this expression in two ways: \(3(9 + 5)\).

Solution

Jay simplified \(3(9 + 5)\) like this.

\[
\begin{align*}
3(9 + 5) &= 3(14) \\
&= 42
\end{align*}
\]

Mel used the distributive property.

\[
\begin{align*}
3(9 + 5) &= 3(9) + 3(5) \\
&= 27 + 15 \\
&= 42
\end{align*}
\]

Example 2

Simplify.

a. \(k^2 \cdot k^3\)

b. \((p^2)^5\)

c. \((2^3 \cdot n^2)^2\)

d. \(c^5 \div c^2\)

Solution

a. \(k^2 \cdot k^3 = k^{2+3} = k^5\)

b. \((p^2)^5 = p^{2 \cdot 5} = p^{10}\)

c. \((2^3 \cdot n^2)^2 = 2^6 \cdot n^4\)

d. \(c^5 \div c^2 = c^{5-2} = c^3\)

Exercises

Use the distributive property to find each product.

1. \(0.8(10 - 0.9)\)

2. \(3\left(\frac{1}{6} - \frac{5}{12}\right)\)

3. \(4\left(\frac{3}{4}\right)\)

4. \(-0.9(10) - 0.9(4)\)

Evaluate each expression when \(x = -0.1\) and \(y = 1.5\).

5. \(x^3\)

6. \(xy^2\)

7. \((y - x)^2\)

8. \(y^2 \div -0.75\)

Simplify.

9. \((x^3)^4\)

10. \(a^4 \cdot a^2\)

11. \(b^3 \div b^2; b \neq 0\)

12. \((a^2b^3)^2\)

13. \(x^0\)

14. \(x^5 \div x^2; x \neq 0\)

15. \((7^4 \cdot a^2)^2\)

16. \(x^2 \cdot x^5\)
RETEACHING 1-8
EXONENTS AND SCIENTIFIC NOTATION
A positive number expressed in the form \( x \cdot 10^y \), where \( x \leq 10 \) and \( y \) is an integer, is expressed in **scientific notation**. Specific examples include \( 5.08 \cdot 10^5 \) and \( 7.1 \cdot 10^{-5} \). The negative exponent in the second expression indicates that \( 10^{-5} \) could also be written as \( \frac{1}{10^5} \).

**Example 1**

Simplify each expression, using the properties of exponents.

1. \( x^6 \div x^{-4} \)
2. \( x^{-5} \cdot x^{-3} \)
3. \( (x^2)^{-8} \)

**Solution**

1. \( x^6 \div x^{-4} = x^{6 - (-4)} = x^{10} \)
2. \( x^{-5} \cdot x^{-3} = x^{-5 + (-3)} = x^{-8} \)
3. \( (x^2)^{-8} = x^{2(-8)} = x^{-16} \)

**Example 2**

1. Write \( 27,350,000,000 \) in scientific notation.
2. Write \( 9.058 \cdot 10^{-3} \) in standard form.

**Solution**

1. \( 27,350,000,000 = 2.735 \cdot 10^{10} \)
2. \( 9.058 \cdot 10^{-3} = 9.058 \cdot 0.001 = 0.009058 \)

**Exercises**

Simplify each expression, using properties of exponents.

1. \( (x^4)^{-2} \)
2. \( (-x^2)^2 \)
3. \( x^{-6} \cdot x^6; x \neq 0 \)
4. \( b^7 \div b^{-8} \)
5. \( m^{12} \div m^{16} \)
6. \( c^{-2} \div c^8 \)
7. \( a^{-5} \cdot a^{-6} \)
8. \( r^8 \cdot r^{-5} \)

Write each number in scientific notation.

9. \( 42,093 \)
10. \( 729,000,000 \)
11. \( 0.0074 \)
12. \( 0.000621 \)

Write each number in standard form.

13. \( 7.3 \cdot 10^3 \)
14. \( 6.52 \cdot 10^{-3} \)
15. \( 4.21 \cdot 10^4 \)
16. \( 9.1 \cdot 10^{-4} \)
RETEACHING 2-1

PATTERNS AND ITERATIONS

You can determine the number pattern in a sequence by finding the relationship between the numbers, or terms. This relationship is the rule that describes the number sequence. A process that is repeated over and over is an iteration.

**Example 1**

Identify the pattern \(-1, 2, -4, 8 \ldots\) and find the next three terms.

**Solution**

In this pattern, each number equals \(-2\) times the number to the left.

\[
\begin{array}{cccc}
-1 & 2 & -4 & 8 \\
\cdot(-2) & \cdot(-2) & \cdot(-2) & \cdot(-2)
\end{array}
\]

The next three terms are \(-16, 32,\) and \(-64\).

**Example 2**

Complete the iteration diagram for the sequence \(2, 6, 10, 14 \ldots\) Calculate the output for 8 iterations.

**Solution**

Study the pattern of the sequence to determine the rule. Enter the initial value, the number of iterations, and the rule. Then calculate the output. The output is \(6, 10, 14, 18, 22, 26, 30, 34\).

**Exercises**

Find the next three terms in each sequence.

1. \(0, -3, -6, -9 \)
2. \(7, 14, 21, 28 \)
3. \(100, 10, 1, 0.1 \)
4. \(4, 8, 16, 32 \)
5. \(\frac{1}{3}, -\frac{2}{3}, \frac{4}{3}, -\frac{8}{3} \)
6. \(-16, 4, -1, \frac{1}{4} \)
7. \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \)
8. \(1, 8, 27, 64 \)

Complete the iteration diagram for the sequence. Calculate the output for the first six iterations.

9. \(6, 12, 18, 24 \ldots\)
10. \(-1, 5, -25, 125 \ldots\)
THE COORDINATE PLANE, RELATIONS, AND FUNCTIONS

A set of ordered pairs is a relation. The domain of a relation is the set of all possible x-coordinates. The range of a relation is the set of all possible y-coordinates. A set of ordered pairs in which each element of the domain is paired with exactly one element in the range is a function.

**Example 1**

Determine whether each relation is a function. State each domain and range.

a. \(9 \rightarrow -9\)  
   \(8 \rightarrow -8\)  
   \(7 \rightarrow -7\)

**Solution**

a. No, the element 7 in the domain is paired with two elements in the range, -9 and -7. Domain: \(\{7, 8, 9\}\)  
   Range: \(\{-9, -8, -7\}\)

b. Yes, each element of the domain is paired with only one element of the range. Domain: \(\{2, 3, 4\}\)  
   Range: \(\{1\}\)

**Example 2**

Evaluate each function.

a. \(f(x) = 4x + 5; f(2)\)
   Solution
   \(f(2) = 4(2) + 5 = 8 + 5 = 13\)

b. \(g(x) = 6x + 1; g(-4)\)
   Solution
   \(g(-4) = 6(-4) + 1 = -24 + 1 = -23\)

**Exercises**

Determine whether each relation is a function. Give the domain and range of each.

1. \(0 \rightarrow -4\)  
   Function: \(\_\_\_\_\_\_\_)  
   Domain: \(\_\_\_\_\_\_)  
   Range: \(\_\_\_\_\_\_)  
2. \(6 \rightarrow 1\)  
   Function: \(\_\_\_\_\_\_)  
   Domain: \(\_\_\_\_\_\_)  
   Range: \(\_\_\_\_\_\_)  
3. \(\{(0, 2), (1, 4), (5, 1)\}\)  
   Function: \(\_\_\_\_\_\_)  
   Domain: \(\_\_\_\_\_\_)  
   Range: \(\_\_\_\_\_\_)  
4. \(\{(2, 1), (6, 2), (1, 6)\}\)  
   Function: \(\_\_\_\_\_\_)  
   Domain: \(\_\_\_\_\_\_)  
   Range: \(\_\_\_\_\_\_)  
5. \(\{(4, 7), (7, 4), (4, 4)\}\)  
   Function: \(\_\_\_\_\_\_)  
   Domain: \(\_\_\_\_\_\_)  
   Range: \(\_\_\_\_\_\_)  

Given \(f(x) = -2x + 3\), evaluate each function.

6. \(f(0)\)  
7. \(f(8)\)  
8. \(f(-1)\)  
9. \(f(100)\)

Given \(f(x) = (3x + 4) - (2 - x)\), evaluate each function.

10. \(f(0)\)  
11. \(f(5)\)  
12. \(f\left(-\frac{1}{2}\right)\)  
13. \(f(-3)\)
LINEAR FUNCTIONS

The graph of a linear equation is a straight line.
The absolute value function is defined as
\[ g(x) = |x| = \begin{cases} \quad x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases} \]

Example 1

Graph \( y = 4x - 2 \).

Solution
Choose at least three values for \( x \) values for \( y \). Make a table to show the ordered pairs. Then plot the points and draw the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 4x - 2 )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4(-1) - 2</td>
<td>-6</td>
<td>(-1, -6)</td>
</tr>
<tr>
<td>0</td>
<td>4(0) - 2</td>
<td>-2</td>
<td>(0, -2)</td>
</tr>
<tr>
<td>1</td>
<td>4(1) - 2</td>
<td>2</td>
<td>(1, 2)</td>
</tr>
</tbody>
</table>

Example 2

Evaluate \( h(x) = |2x - 1| \) for the given value of \( x \).

a. \( h(0) \)

b. \( h(2) \)

c. \( h(-2) \)

Solution

a. \( h(0) = |2(0) - 1| = |1| = 1 \), since \(-1 < 0\)

b. \( h(2) = |2(2) - 1| = |3| = 3 \), since \(3 \geq 0\)

c. \( h(-2) = |2(-2) - 1| = |-5| = 5 \), since \(-5 < 0\)

EXERCISES

Graph each function.

1. \( y = x \)

2. \( y = 2x - 3 \)

Evaluate \( h(x) = |x + 6| \) for the graph value of \( x \).

3. \( h(0) \)

4. \( h(-2) \)

5. \( h(5) \)

6. \( h(-10) \)

Evaluate \( g(x) = |x - 5| \) for the given value of \( x \).

7. \( g(-5) \)

8. \( g(6) \)

9. \( g(-9) \)

10. \( g(15) \)
RETEACHING 2-4

SOLVE ONE-STEP EQUATIONS

For all real numbers, $a$, $b$, and $c$, if $a = b$ then:

- $a + c = b + c$ and $c + a = c + b$ ← Addition Property of Equality
- $ac = bc$ and $ca = cb$ ← Multiplication Property of Equality

Example 1

Solve each equation. Use Algeblocks to model each step if you wish.

a. $p + 7 - 15 = 9 + 15$

Solution

\[
p + 7 - 15 = 9 + 15 \quad \text{Represent equation.}
\]

\[
p - 8 = 24 \quad \text{Simplify.}
\]

\[
p - 8 + 8 = 24 + 8 \quad \text{Add 8 to each side.}
\]

\[
p = 32
\]

Example 2

Translate each sentence into an equation using $n$ to represent the unknown number. Then solve the equation for $n$.

a. One tenth of 40 is the same as the sum of $-3$ and some number.

Solution

\[
\frac{1}{10}(40) = -3 + n \quad \text{Solve.}
\]

\[
4 = -3 + n
\]

\[
7 = n
\]

Exercises

Solve each equation. Use Algeblocks to model each step if you wish.

1. $-12 + a = 32$

2. $12 - b = 16$

3. $-5 = c + 9$

4. $2d = 16$

5. $\frac{m}{9} = -5$

6. $-\frac{1}{3}f = 24$

7. $g + \frac{1}{2} = 2$

8. $\frac{h}{7} = 9 + 5$

9. $-(8)(5) = 2j$

Translate each sentence into an equation using $n$ to represent the unknown number. Then solve the equation for $n$.

10. A number subtracted from 46 is $-21$.

11. The quotient of a number divided by 11 is 0.5.
**RETEACHING 2-5**

**SOLVE MULTI-STEP EQUATIONS**

When you solve an equation requiring more than one property of equality, use the addition property before you use the multiplication property.

**Example**

Solve \(5(x - 3) = 15 + 2x\).
Use Algeblocks to model each step if you wish. Check the solution.

**Solution**

**Algebraic Notation**

\[
\begin{align*}
5(x - 3) &= 15 + 2x \\
5x - 15 &= 15 + 2x \\
-2x + 5x - 15 &= 15 + 2x - 2x \\
3x - 15 &= 15 \\
3x &= 30 \\
\frac{1}{3}(3x) &= \frac{1}{3}(30) \\
x &= 10
\end{align*}
\]

Check: \(5(x - 3) = 15 + 2x\)
\[
\begin{align*}
5(10 - 3) &= 15 + 2(10) \\
35 &= 35
\end{align*}
\]

**Explanation and Steps When Using Algeblocks**

- Represent the equation.
- Apply the distributive property.
- Add \(-2x\) to each side.
- Simplify.
- Add 15 to each side.
- Simplify.
- Multiply each side by \(\frac{1}{3}\).
- Simplify.

The solution is 10.

**EXERCISES**

Solve each equation and check the solution. Use Algeblocks to model each step if you wish.

1. \(3x + 2 = 17\)
2. \(2x + 1 = 4x - 3\)
3. \(5(x + 2) = 9 + 16\)
4. \(9x - 17 = -71\)
5. \(2x + 8 = 3x - 12\)
6. \(6(x - 3) = -4 + 10\)
7. \(4(2x + 1) = 28 - 16\)
8. \(7x + 14 = 5x - 6\)
9. \(8x + 12 = 36\)
10. \(6x - 5 = -35\)
11. \(x + 9 = 3x - 15\)
12. \(4(3x - 1) = x + 40\)
13. \(4x + 5 = 49\)
14. \(3(3x - 9) = 12 + 15\)
15. \(3x - 18 = 42\)
16. \(5x - 2 = 2x + 16\)
Solve Linear Inequalities

The steps used to solve an inequality are similar to those used to solve an equation. When an inequality is multiplied by a negative number, the order of the inequality is reversed.

**Example 1**

Solve \(2x < 10\) and graph its solution on a number line.

**Solution**

Solve the inequality. Graph the inequality.

\[
\frac{1}{2} (2x) < 10 \left(\frac{1}{2}\right)
\]

\[
x < 5
\]

The solution is \(x < 5\).

**Example 2**

Graph \(y > 4x - 2\) on the coordinate plane.

**Solution**

Write the related equation.

\[
y = 4x - 2
\]

Make a table of values (ordered pairs) to graph the boundary.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

The boundary is a broken line because the boundary is not part of the solution set.

Determine the shading.

Test point: \((0, 0)\)

\[
y > 4x - 2 \rightarrow 0 > 4(0) - 2 \rightarrow 0 > -2
\]

**Exercises**

Solve each inequality and graph its solution on the number line.

1. \(3x - 2 \leq 7\)
2. \(9 < 5x - 1\)

Graph each inequality on the coordinate plane.

3. \(2x - 2y \geq -2\)
4. \(y < 2x - 3\)
RETEACHING 2-7

DATA AND MEASURES OF CENTRAL TENDENCY

Data can be organized using a tally system in a frequency table. Mean, median, and mode are measures of central tendency and can be used to analyze data.

Example

The following are a baseball team’s scores for ten games: 4, 3, 0, 3, 2, 0, 1, 1, 3, 3.

a. Construct a frequency table for the data.

b. Find the mean, median and mode for the data.

Solution

a. List each different data item. Make a tally to record the number of times each data item occurs. Total the tally marks to compute the frequency of each response.

b. Mean: divide the sum of the data by the number of data items. 

\[
\frac{0 + 0 + 1 + 1 + 2 + 3 + 3 + 3 + 3 + 4}{10} = \frac{20}{10} = 2
\]

Median: the middle value of the data. 

0, 0, 1, 1, 2, 3, 3, 3, 3, 4. Since the number of data items is even, the median is the average of the two middle numbers:

\[
\frac{2 + 3}{2} = \frac{5}{2} = 2.5
\]

Mode: the number that occurs most frequently. The mode is 3.

Exercises

Construct a frequency table for each set of data. Then find the mean, median, and mode.

1. 100, 103, 103, 100, 103, 103, 100, 104
   - Mean: ____________
   - Median: ____________
   - Mode: ____________

2. 215, 214, 212, 211, 212, 215, 215, 210
   - Mean: ____________
   - Median: ____________
   - Mode: ____________
RETEACHING 2-8
DISPLAYING DATA

Stem-and-leaf plots and histograms are two ways to display data.

Example

The test scores for the math exam were as follows: 75, 87, 68, 63, 45, 76, 75, 88, 87, 89, 93, 95, 98, 70, 72, 75, 82, 83, 66, 66.

a. Construct a stem-and-leaf plot to display the data.
b. Identify any outliers, clusters, or gaps in the data.
c. Group the data into intervals of 20. Construct a histogram to display the data.

Solution

a. The tens digit will be the stem. The units digit will be the leaf.

b. The outlier is 45 since it is the only score that is much less or much greater than the other data. The remaining data are clustered between 63 and 98. The only gap is between 45 and 63.

c. The data must be grouped into intervals of 20. Use the grouping from the stem-and-leaf plot, combining stems 4 and 5, 6 and 7, and 8 and 9.

Exercises

For each data group below, a. construct a stem-and-leaf plot to display the data, b. identify any outliers, clusters, or gaps in the data, and c. use the stem-and-leaf plot to help group the data into intervals of 20. Then construct a histogram on your own paper.

1. Ages at Recital
   17, 28, 36, 32, 65, 45, 21, 16, 29, 17, 23, 34, 42, 44, 19

   a. Ages at Recital  
   b. Outlier ________  
   c. Cluster ________  
   d. Gaps ________

2. Cost of in-line skates (in dollars)
   142, 163, 125, 126, 128, 146, 101, 131, 130, 129, 131, 125, 143, 132, 141

   a. Cost of in-line skates  
   b. Outlier ________  
   c. Cluster ________  
   d. Gaps ________
RETEACHING 2-9

PROBLEM SOLVING SKILLS: MISLEADING GRAPHS

Statistics can be useful when used properly. However, they can be misleading when the scales of graphs are manipulated, measures of central tendency are used that don’t accurately relate the data, or advertisements omit data.

Example

A high school principal used the graph at the right to illustrate the gains in mean test scores over a four-year period.

a. What is deceptive about the graph?

b. How would you change the graph so that it is not misleading?

Solution

a. The small gains in the mean scores appear more significant in the graph because of the slope of the line. This happens because: a) the break between 0 and 992 makes the first 992 points visually appear as though they are equal to 2 points, b) the dates on the horizontal scale are squeezed closely together making the line steeper and causing the increase to appear more significant.

b. You could use a vertical scale that has consistent intervals from 0–1002. The dates on the horizontal scale could be written somewhat farther apart. You might also eliminate the word ‘soar’ from the title of the graph.

EXERCISES

Solve.

1. Describe how the graph at the right is deceptive. How would you change it so that it is not misleading?

2. On another sheet of paper, make two graphs that show the increase in a high-school population for the next five years if it is estimated that there will be a 25-student increase per year. Use one graph to support claims of overcrowding and the other to support claims that the growth is controlled. Explain why each graph supports each position.
RETEACHING 3-1

POINTS, LINES, AND PLANES

In the figure at the right, points C and D determine line l. The intersection of planes A and B is line l. Points C, D and E determine plane A. Points C, D and F determine plane B. \( \overrightarrow{CE}, \overrightarrow{CD}, \) and \( \overrightarrow{DE} \) lie in plane A; and \( \overrightarrow{CF}, \overrightarrow{CD}, \) and \( \overrightarrow{DF} \) lie in plane B.

Example 1

Refer to the figure at the right.

Find the length of \( \overrightarrow{VY} \).

Solution

Using the ruler postulate, the coordinate of point V is \( -1 \). The coordinate of point Y is \( 2 \).

So the distance between points V and Y is 3, and the length of \( \overrightarrow{VY} \) is 3.

Example 2

In the figure at the right, \( AC = 92 \).

Find \( AB \).

Solution

Using the segment addition postulate, write and solve an equation.

\[
\begin{align*}
\quad d + 5 + d + 9 &= 92 \\
2d + 14 &= 92 \\
2d &= 78 \\
&= 39
\end{align*}
\]

The value of \( d \) is 39. To find \( AB \), replace \( d \) with 39 in \( d + 5 \).

\( AB = d + 5 = 39 + 5 = 44 \)

EXERCISES

Use the figure at the right for Exercises 1–3.

1. Name three points that determine plane \( W \).

2. Name three lines that lie in plane \( X \).

3. Name the intersection of planes \( W \) and \( X \).

Use the figure at the right for Exercises 4–7. Find the length of each segment.

4. \( \overline{AF} \)

5. \( \overline{GK} \)

6. \( \overline{DL} \)

7. \( \overline{AM} \)

8. In the figure at the right, \( NP = 142 \).

Find \( NO \).

9. In the figure at the right, \( RT = 105 \).

Find \( ST \).
RETEACHING 3-2

TYPES OF ANGLES

Use either the inner scale or the outer scale on a protractor to find the measure of an angle.

∠HQI and ∠IQJ are adjacent angles.

\[ m\angle HQI + m\angle IQJ = m\angle HQJ \]

Example 1

Refer to the figure below. Find \( m\angle BLC \).

Solution

Using the inner scale, \( \overrightarrow{LB} \) is paired with 150 and \( \overrightarrow{LC} \) is paired with 110.

\[ |150 - 110| = |40| = 40 \]

Using the outer scale, \( \overrightarrow{LB} \) is paired with 30 and \( \overrightarrow{LC} \) is paired with 70.

\[ |30 - 70| = |-40| = 40 \]

\( m\angle BLC = 40^\circ \)

Example 2

Refer to the figure below. Find \( m\angle GMF \).

Solution

Since \( \angle GME \) is a straight angle, \( \angle GMF \) and \( \angle FME \) are supplementary and the sum of their measures is 180°.

\[ m\angle GMF + m\angle FME = 180 \]

\[ n + 16 + n = 180 \]

\[ 2n + 16 = 180 \]

\[ 2n = 164 \]

\[ n = 82 \]

The value of \( n \) is 82. To find \( m\angle GMF \) replace \( n \) with 82 in \( n + 16 \).

\[ m\angle GMF = (n + 16)^\circ = (82 + 16)^\circ = 98^\circ \]

EXERCISES

Refer to the figure in Example 1. Find the measure of each angle.

1. \( \angle CLE \)  
2. \( \angle BLF \)  
3. \( \angle DLF \)  
4. \( \angle BLE \)  
5. \( \angle CLD \)  
6. \( \angle ELF \)  
7. \( \angle BLD \)  
8. \( \angle DLE \)

Find the measure of each angle in the figure at the right.

9. \( \angle TMV \)  
10. \( \angle TMU \)  
11. \( \angle UMV \)  
12. \( \angle SMT \)
RETEACHING 3-3
SEGMENT AND ANGLES

The midpoint of $QR$ is $M$.
$QM = MR$

Vertical angles have equal measures.

Example 1
In the figure at the right $\angle AGD$ is a right angle, and $\overrightarrow{GB}$ bisects $\angle AGC$. Find $m\angle AGB$.

Solution
Since $\angle AGD$ is a right angle, $m\angle AGD = 90^\circ$. Since $\angle AGC$ and $\angle CGD$ are complementary angles, $m\angle AGC + m\angle CGD = 90^\circ$.
It is given that $m\angle CGD = 22^\circ$.
So, $m\angle AGC + 22^\circ = 90^\circ$ and $m\angle AGC = 90^\circ - 22^\circ = 68^\circ$.
Since $\overrightarrow{GB}$ bisects $\angle AGC$, $m\angle AGB = \frac{1}{2} m\angle AGC = \frac{1}{2} (68) = 34^\circ$.

Example 2
In the figure at the right, $\overleftrightarrow{HJ}$ and $\overleftrightarrow{IK}$ intersect at point $Z$. Find $m\angle HZI$.

Solution
Since $\angle HZI$ and $\angle JZK$ are vertical angles, they are equal in measure. $m\angle HZI = m\angle JZK$
$2t + 27 = 3t$
$27 = t$
The value of $t$ is 27. To find $m\angle HZI$, replace $t$ with 27.
$m\angle HZI = (2t + 27)^\circ = [2(27) + 27]^\circ = (54 + 27)^\circ = 81^\circ$

EXERCISES

Refer to the figure at the right for Exercises 1–3.
1. Name the midpoint of $OS$. ________
2. Name the segment whose midpoint is $M$. ________
3. Name all the segments whose midpoint is point $P$.
   ________________________________

Refer to the figure at the right for Exercises 4–7. In the figure, $\overleftrightarrow{TV}$ and $\overleftrightarrow{UX}$ intersect at point $F$, and $\overrightarrow{FW}$ bisects $\angle XFV$. Find the measure of each angle.
4. $\angle XFT$ ________
5. $\angle TFU$ ________
6. $\angle XFV$ ________
7. $\angle VFW$ ________
RETEACHING 3-4
CONSTRUCTION AND LINES

If two parallel lines are cut by a transversal, then corresponding angles are equal in measure.

∠1 and ∠5 are corresponding angles, so \( m\angle 1 = m\angle 5 \).

∠2 and ∠6 are corresponding angles, so \( m\angle 2 = m\angle 6 \).

∠3 and ∠7 are corresponding angles, so \( m\angle 3 = m\angle 7 \).

∠4 and ∠8 are corresponding angles, so \( m\angle 4 = m\angle 8 \).

Example 1

In the figure at the right, \( \overrightarrow{MO} \perp \overrightarrow{AP} \). Find \( m\angle OAN \).

Solution

Since \( \overrightarrow{MO} \perp \overrightarrow{AP} \), \( \angle PAM \) is a right angle and \( m\angle PAM = 90^\circ \).

Since \( m\angle PAQ \) and \( m\angle QAM \) are complementary, \( m\angle PAQ + m\angle QAM = 90^\circ \).

It is given that \( m\angle PAQ = 26^\circ \).

So, \( m\angle QAM + 26^\circ = 90^\circ \) and \( m\angle QAM = 90^\circ - 26^\circ = 64^\circ \).

Since \( \angle QAM \) and \( \angle OAN \) are vertical angles, they are equal in measure. \( m\angle OAN = m\angle QAM = 64^\circ \)

Example 2

In the figure at the right, \( \overrightarrow{AE} \parallel \overrightarrow{BD} \). Find \( m\angle CTD \) and \( m\angle CSE \).

Solution

Since \( \angle BTF \) and \( \angle CTD \) are vertical angles, they are equal in measure.

\[
6h - 5 = 4h + 35
\]

\[
2h = 40
\]

\[
h = 20
\]

\[
m\angle CTD = (4h + 35)^\circ = [4(20) + 35]^\circ = (80 + 35)^\circ = 115^\circ
\]

\( \angle CSE \) and \( \angle CTD \) are corresponding angles, so they are equal in measure. \( m\angle CSE = m\angle CTD = 115^\circ \)

EXERCISES

In the figure at the right, \( \overrightarrow{AE} \perp \overrightarrow{FC} \). Find the measure of each angle.

1. \( \angle AYB \) _____ 2. \( \angle DYE \) _____ 3. \( \angle GYF \) _____

In the figure at the right, \( \overrightarrow{BG} \parallel \overrightarrow{CF} \) and \( \overrightarrow{AE} \perp \overrightarrow{BG} \). Find the

4. \( \angle CUE \) _____ 5. \( \angle BTS \) _____ 6. \( \angle FUS \) _____
7. \( \angle CUD \) _____ 8. \( \angle GSH \) _____ 9. \( \angle DUF \) _____
10. \( \angle EUS \) _____ 11. \( \angle EUD \) _____ 12. \( \angle HSB \) _____
RETEACHING 3-5

INDUCTIVE REASONING IN MATHEMATICS

Inductive reasoning skills can be used to make conjectures about geometric figures. Inductive reasoning is the process of observing data and making a generalization, or conjecture, based on your observations. Sometimes it is helpful to draw a picture, write a function, or make a table when making conjectures about geometric figures.

**Example**

Describe the 30th figure in the pattern.

**Solution**

Each figure consists of a number of planes that have the same line of intersection. The first figure has 2 planes; the second, 3 planes; the third, 4 planes; the fourth, 5 planes. You can see that the \( n \)th figure consists of \((n + 1)\) planes. So, the 30th figure would have \((30 + 1)\), or 31 planes with one common line of intersection.

**Exercises**

Draw the next figure in each pattern. Then describe the fifteenth figure.

1. 

The figures at the right show 2, 3, 4, and 5 parallel lines with 1 perpendicular line.

3. How many right angles are there in each figure? ________________________________

4. Find the number of right angles that would be formed by a figure containing 18 parallel lines and 1 perpendicular line. ________________________________

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CONDITIONAL STATEMENTS

Conditional statements are written in an *if-then* (hypothesis/conclusion) form.
The converse of a conditional statement is formed by interchanging the hypothesis and the conclusion.
A counterexample proves a conditional or converse is false.

**Example**

Write the converse of this statement: If \( RS \parallel TU \), then \( RS, TU, \) and \( RT \) are coplanar.
Then decide whether the statement and its converse are true or false.
If false, give a counterexample.

**Solution**

Converse: If \( RS, TU \) and \( RT \) are coplanar, then \( RS \parallel TU \).
Original statement is true, since parallel lines are coplanar by definition and for any two points in a plane, the line joining them lies in the plane (Postulate 3).
Converse is false. Counterexample: Each pair of coplanar lines could intersect to form a triangle.

**Exercises**

Write the converse of each statement. Then tell whether the given statement and its converse are true or false. If false, give a counterexample.

1. If points \( A, B, C \) and \( D \) lie on both plane \( L \) and plane \( M \), then points \( A, B, C, \) and \( D \) are collinear.

2. If \( m \angle IQJ + m \angle HQJ > 180^\circ \), then \( \angle IQJ \) and \( \angle HQJ \) are obtuse angles.

3. If three lines have one point in common, then they are coplanar.

4. If two lines are skew, then they are not coplanar.
Reteaching 3-7

Deductive Reasoning and Proofs

Most geometric proofs of theorems are based on deductive reasoning and the use of general, logical statements to arrive at a conclusion.

Example

Given: \( \angle ALB = \angle CLD \)
Prove: \( \angle ALC = \angle BLD \)

Solution

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle ALB = \angle CLD )</td>
<td>1. given</td>
</tr>
<tr>
<td>2. ( \angle ALB + \angle BLC = \angle CLD + \angle BLC )</td>
<td>2. Addition Property of Equality</td>
</tr>
<tr>
<td>3. ( \angle ALB + \angle BLC = \angle ALC ) ( \angle CLD + \angle BLC = \angle BLD )</td>
<td>3. Angle Addition Postulate</td>
</tr>
<tr>
<td>4. ( \angle ALC = \angle BLD )</td>
<td>4. Substitution Property</td>
</tr>
</tbody>
</table>

Exercises

1. Complete the proof.

Given: \( \angle 1 \) and \( \angle 2 \) are supplementary
Prove: \( s \perp t \)

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 ) and ( \angle 2 ) are supplementary</td>
<td>1. ( \angle 1 ) and ( \angle 2 ) are supplementary</td>
</tr>
<tr>
<td>2. ( m\angle 1 + m\angle 2 = 180^\circ )</td>
<td>2. definition of ( \angle 1 ) and ( \angle 2 )</td>
</tr>
<tr>
<td>3. ( m\angle 1 = m\angle 2 )</td>
<td>3. ( m\angle 1 = m\angle 2 ) Theorem</td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 1 = 180^\circ ) or ( 2m\angle 1 = 180^\circ )</td>
<td>4. Substitution Property</td>
</tr>
<tr>
<td>5. ( m\angle 1 = 90^\circ )</td>
<td>5. ( m\angle 1 = 90^\circ ) Property of Equality</td>
</tr>
<tr>
<td>6. ( \angle 1 ) is a right angle</td>
<td>6. ( \angle 1 ) is a right angle</td>
</tr>
<tr>
<td>7. ( s \perp t )</td>
<td>7. ( s \perp t )</td>
</tr>
</tbody>
</table>

2. Write a proof of the following theorem on your own paper. Use the proofs in the Example and Exercise 1 as models.

Given: \( g \perp h \)
Prove: \( \angle 1 \) and \( \angle 2 \) are complementary
RETEACHING 3-8

PROBLEM SOLVING SKILLS: USING LOGICAL REASONING

It can be useful to make a table to solve some logic problems.

Example

Four students were talking about their Friday night dates. Amy went skating. Juan and Toi each saw a movie or play, but not necessarily in that order. Toi and Jamal saw live performances. One student went to a football game.

Solution

Make a table listing each student and each activity. Put a ✔ in a box to indicate a fact is true. Put an ✗ in a box for any fact that is not true.

Since Amy went skating, place a ✔ in the box where the row Amy meets the column skating and an ✗ in the other boxes in the column and row. Juan and Toi saw a movie or play, so place an ✗ in the boxes where the rows Juan and Toi meet the columns football and skating. Thus Jamal must have seen the football game. Place a ✔ in that box. Put an ✗ in the other boxes in the row.

Toi did see a live performance, so she saw a play. Place a ✔ in the box where the row Toi meets the column play and an ✗ in the other boxes in that column and row. This leaves Juan seeing the movie. Mark the box with a ✔.

Exercise

Make a table to help solve the problem.

1. Maria, Keisha, Jo, and Leah wear jerseys numbered 5, 8, 13, and 36, but not necessarily in that order. The girl wearing 8 has a better jumpshot than Maria or Leah. Jo's jersey number is divisible by 6. The number of letters in one name is the same as her number. What number does each wear?

2. A grocery, a cafe, a boutique, and a dance studio are all on one street. Their addresses are 500 Elm, 524 Elm, 538 Elm, and 544 Elm, but not necessarily in that order. The boutique is between the cafe and the studio. The studio has the highest address. What is the address of each business?
RETEACHING 4-1

TRIANGLES AND TRIANGLE THEOREMS

Interior angles: \( \angle 1, \angle 2, \angle 3 \)
Exterior angles: \( \angle 4, \angle 5, \angle 6, \angle 7, \angle 8, \angle 9 \)
According to the triangle-sum theorem, 
\[ m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ. \]
According to the exterior angle theorem, 
\[ m\angle 9 = m\angle 1 + m\angle 2. \]

Example 1

Refer to the triangle at the right.
Find \( m\angle F \).

Solution
Use the triangle-sum theorem to write and solve an equation.
\[
m\angle F + m\angle G + m\angle H = 180 \\
4l + 2l + 3l = 180 \\
9l = 180 \\
l = 20
\]
So, the value of \( l \) is 20. From the figure, \( m\angle F = (4l)^\circ \). Substituting 20 for \( l \), \( m\angle F = (4 \cdot 20)^\circ = 80^\circ \).

Example 2

Refer to the triangle at the right.
Find \( m\angle KIJ \).

Solution
Use the exterior angle theorem to write and solve an equation.
\[
m\angle KIJ + m\angle IJK = m\angle LKJ \\
(2c + 4) + (3c - 13) = 96 \\
5c = 96 \\
c = 21
\]
So the value of \( c \) is 21. From the figure, \( m\angle KIJ = (2c + 4)^\circ \). Substituting 21 for \( c \), 
\[
\angle KIJ = (2 \cdot 21 + 4)^\circ = (42 + 4)^\circ = 46^\circ.
\]

Exercises

Find the value of \( x \) in each figure.

1.  

2.  

3.  

In the figure at the right, 
\( MO \perp KN \) and \( LO \parallel MP \). Find the measure of each angle.

4.  \( \angle LMO \)  
5.  \( \angle OLM \)  
6.  \( \angle LOM \)  
7.  \( \angle KLO \)
RETEACHING 4-2

CONGRUENT TRIANGLES

Congruent figures have the same size and the same shape. You can use the SSS (Side-Side-Side) Postulate, the SAS (Side-Angle-Side) Postulate or the ASA (Angle-Side-Angle) Postulate to show that two triangles are congruent.

Example

Given: Point $S$ is the midpoint of $UR$. Point $S$ is the midpoint of $QT; QR \cong TU$.
Prove: $\triangle QRS \cong \triangle TUS$

Solution

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Point $S$ is the midpoint of $UR$. Point $S$ is the midpoint of $QT; QR \cong TU$.</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $US \cong RS; QS \cong TS$</td>
<td>2. definition of midpoint</td>
</tr>
<tr>
<td>3. $\triangle QRS \cong \triangle TUS$</td>
<td>3. SSS postulate</td>
</tr>
</tbody>
</table>

EXERCISES

Complete the proof

Given: $BC \parallel FE; AF \parallel CD; \overline{AB} \cong \overline{DE}$
Prove: $\triangle AFE \cong \triangle DCE$

Solution

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $BC \parallel FE; AF \parallel CD; \overline{AB} \cong \overline{DE}$</td>
<td>1.</td>
</tr>
<tr>
<td>2. $\overline{AB} + \overline{BE} = \overline{DE} + \overline{BE}$</td>
<td>2. If two parallel lines are cut by a transversal, alternate interior angles are equal.</td>
</tr>
<tr>
<td>3. $\overline{AE} = \overline{BD}$, or $\overline{AE} \cong \overline{BD}$</td>
<td>3. reflexive property</td>
</tr>
<tr>
<td>4. $\overline{AB} + \overline{BE} = \overline{DE} + \overline{BE}$</td>
<td>4.</td>
</tr>
<tr>
<td>5. $\overline{AE} = \overline{BD}$, or $\overline{AE} \cong \overline{BD}$</td>
<td>5. segment addition postulate</td>
</tr>
<tr>
<td>6. $\overline{AE} = \overline{BD}$, or $\overline{AE} \cong \overline{BD}$</td>
<td>6.</td>
</tr>
<tr>
<td>7. $\triangle AFE \cong \triangle DCE$</td>
<td>7.</td>
</tr>
</tbody>
</table>

On another sheet of paper, write a two-column proof.

8. Given: $MN \parallel \overline{OP}, MN \cong OP$
   Prove: $\triangle OMP \cong \triangle MON$
RETEACHING 4-3

CONGRUENT TRIANGLES AND PROOFS

If two angles of a triangle are congruent, then the sides opposite those angles are congruent. Corresponding parts of congruent triangles are congruent (CPCTC).

Example 1
Refer to the triangle at the right. Find the value of $z$.

Solution
Since $EF = DF$, $\triangle DEF$ is an isosceles triangle with base $ED$. By the isosceles triangle theorem, $m \angle E = m \angle D = 25^\circ$. So $z = 25^\circ$.

Example 2
Given: $W \cong Y; VZ \cong XZ; W \perp VY$
Prove: $\angle W \cong \angle Y$

Solution

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $WZ \cong YZ; VZ \cong XZ; W \perp VY$</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $\angle VZW$ is a right angle. $\angle YZX$ is a right angle.</td>
<td>2. definition of perpendicular lines</td>
</tr>
<tr>
<td>3. $\angle VZW = 90^\circ; \angle YZX = 90^\circ$</td>
<td>3. definition of a right angle</td>
</tr>
<tr>
<td>4. $m \angle VZW = m \angle YZX$, or $\angle VZW \cong \angle YZX$</td>
<td>4. Transitive Property</td>
</tr>
<tr>
<td>5. $\triangle VZW \cong \triangle XZY$</td>
<td>5. SAS Property</td>
</tr>
<tr>
<td>6. $\angle W \cong \angle Y$</td>
<td>6. CPCTC</td>
</tr>
</tbody>
</table>

EXERCISES

Complete the proof. Given: $AB \cong BC, BE \cong BD$
Prove: $AE \cong CD$

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle ABE$ and $\angle CBD$ are vertical angles.</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $m \angle ABE = m \angle CBD$, or $\angle ABE \cong \angle CBD$</td>
<td>2. definition of vertical angles</td>
</tr>
<tr>
<td>3. $\triangle ABE \cong \triangle CBD$</td>
<td>3. SAS Property</td>
</tr>
<tr>
<td>4. $\triangle ABE \cong \triangle CBD$</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>5. $AE \cong CD$</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DRAW</th>
<th>DRAW</th>
<th>DRAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$5 \text{ in.}$</td>
<td>$a$</td>
</tr>
<tr>
<td>$3 \text{ cm}$</td>
<td>$35^\circ$</td>
<td>$65^\circ$</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>$5 \text{ in.}$</td>
<td>$9 \text{ in.}$</td>
</tr>
</tbody>
</table>
RETEACHING 4-4

ALTITUDES, MEDIANS, AND PERPENDICULAR BISECTORS

Altitudes and medians are segments connecting a vertex of a triangle to the opposite side. An altitude is perpendicular to the opposite side, while a median intersects the midpoint of the opposite side.

Example 1

Sketch all the altitudes and medians of \( \triangle ABC \).

Solution

There are three altitudes.

Example 2

Refer to the figure at the right. \( \overline{EG} \) is a perpendicular bisector.

Tell whether each statement is true or false.

a. \( \overline{EG} \) is a median of \( \triangle DEF \).

b. \( \overline{EG} \) is an altitude of \( \triangle DEF \).

Solution

a. By definition of perpendicular bisector, \( G \) is the midpoint of \( \overline{DF} \). Since \( \overline{EG} \) connects the midpoint of a line with the opposite vertex, it is a median. The statement is true.

b. By definition, perpendicular lines intersect to form right angles. Since \( \overline{EG} \) is a perpendicular segment from a line to the opposite vertex, it is an altitude. The statement is true.

EXERCISES

Trace each triangle onto a sheet of paper. Sketch all the altitudes in red and all the medians in blue.

1.  

2.  

3.  

4.  

Refer to \( \triangle HIJ \) at the right.

Tell whether each statement is true.

5. \( \angle H \cong \angle J \)  

6. \( HK = KJ \)  

7. \( \angle HKI \cong \angle JKI \)  

8. \( \triangle HIK \cong \triangle JIH \)  

9. \( IK \) is an altitude.  

10. \( IK \) is a median.  

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RETEACHING  4-5

PROBLEM SOLVING SKILLS: WRITE AN INDIRECT PROOF

An indirect proof is one in which you assume the opposite of what you want to prove. Then you show that your assumption leads to a contradiction of a theorem, postulate, or other known fact.

Example

Prove this statement: If \( \angle 1 \) is not congruent to \( \angle 2 \), then line 1 is not parallel to line \( m \).

Solution

Step 1: Assume that the conclusion is false.
Assume that line 1 is parallel to line \( m \).

Step 2: Reason logically from the assumption as follows.
By parallel lines postulate, \( \angle 1 \) is congruent to \( \angle 2 \).

Step 3: Note that the statement in Step 2 is contradictory to the given information that \( \angle 1 \) is not congruent to \( \angle 2 \).
Therefore, the assumption that line 1 is parallel to line \( m \) is false.
The given statement must be true.

EXERCISES

Complete the indirect proof by filling in the blanks.
If \( YM \) is perpendicular to \( XZ \) in this scalene triangle, then \( M \) is not the midpoint of \( XZ \).

Step 1: Assume that \( M \) is the (1) of \( XZ \).

1. __________________________

Step 2: By definition of midpoint, \( XM \equiv (2) \).
By definition of perpendicular lines, \( \angle YMX \) and \( \angle YMZ \) are (3) angles.
By definition of right angle, \( m\angle YMX = (4) \) and \( m\angle YMZ = (5) \).
By (6), \( m\angle YMX = m\angle YMZ \), or \( \angle YMX \equiv \angle YMZ \).
By the (7) property, \( \overline{YM} \equiv \overline{YM} \).
By the (8) postulate, \( \angle XYM \equiv \angle ZYM \).
By CPCTC, \( \overline{XY} \equiv (9) \).
By definition of isosceles triangle, \( \angle XMZ \) is isosceles.

Step 3: Note that the last statement (10) the given fact that the triangle is scalene. So the assumption that \( M \) is the midpoint of \( XZ \) must be (11).
The given statement must be (12).
RETEACHING 4-6

INEQUALITIES IN TRIANGLES

If two sides of a triangle are unequal in length, then the angle opposite the longer side has the greater measure. If two angles of a triangle are unequal in measure, then the side opposite the angle with the greater measure has the longer length.

Example 1

Two sides of a triangle measure \( \frac{6}{2} \) in. and \( \frac{3}{2} \) in. Find the range of lengths for the third side.

Solution

Use the variable \( l \) to represent the length in inches of the third side. By the triangle inequality theorem, these three inequalities must be true.

\[
\begin{align*}
\text{a. } & \frac{6}{2} + \frac{3}{2} > l \\
\text{b. } & \frac{6}{2} + l > \frac{3}{2} \\
\text{c. } & \frac{3}{2} + l > \frac{6}{2}
\end{align*}
\]

\[
\begin{align*}
&10 > l \\
&l > -3 \\
&l > 3
\end{align*}
\]

Inequality b is not useful since a length must be a positive number. Combining inequalities a and c results in the inequality \( 10 > l > 3 \), so the third side must be less than 10 in. and greater than 3 in.

Example 2

Refer to \( \triangle DEF \). List the sides from longest to shortest.

Solution

List the angles from the largest to smallest: \( m\angle E > m\angle D > m\angle F \). By the unequal angles theorem, opposite sides of unequal angles will be unequal in length in the same order. So, \( DF > EF > DE \).

Exercise 1

The lengths of two sides of a triangle are given. Find the range of lengths for the third side.

1. 3 mm, 5 mm
2. 1.2 in., 4.6 in.
3. 2.5 cm, 8 cm
4. In \( \triangle GHI \), \( GH = 3 \) cm, \( HI = 4 \) cm, and \( GI = 3.5 \) cm. List the angles of the triangle in order from smallest to largest. ________________
5. In \( \triangle MNO \), \( m\angle M > m\angle O \) and \( m\angle N < m\angle O \). List the sides of the triangle in order from shortest to longest. ________________

Which side is the longest side of each triangle? Which side is the shortest?

6. Longest ______ 7. Longest ______
   Shortest ______ 8. Shortest ______
**RETEACHING 4-7**

**POLYGONS AND ANGLES**

Polygons are closed plane figures formed by joining three or more coplanar segments at their endpoints. A polygon is convex when each line containing a side contains no points in the interior of the polygon. The sum of the measures of the angles of a convex polygon with \( n \) sides can be found by solving the equation \((n - 2)(180)°\).

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

**Example**

a. Find the measure of each interior angle of a regular pentagon.

b. Find the measure of each exterior angle of a regular pentagon.

**Solution**

a. A pentagon has five sides. Use the polygon-sum theorem to find the sum of the measures of the interior angles.
\[(n - 2)180° = (5 - 2)180° = (3)180° = 540°\]
Because the pentagon is regular, each interior angle is equal in measure. So, the measure of one interior angle is \(540° ÷ 5 = 108°\).

b. By the polygon exterior angle theorem, the sum of the measures of the exterior angles is 360°. Because the pentagon is regular, each exterior angle is equal in measure. So, the measure of one exterior angle is \(360° ÷ 5 = 72°\).

**Exercises**

Find the unknown angle measure or measures in each figure.

1.  
2.  
3.  
4.  

5. Find the measure of each interior angle of a regular decagon.

6. Find the measure of each exterior angle of a regular hexagon.

7. Find the sum of the measures of the interior angles of a polygon with 22 sides.
RETEACHING 4-8

SPECIAL QUADRILATERALS: PARALLELOGRAMS

Opposite sides: \( \overline{AB} \) and \( \overline{CD}, \overline{BC} \) and \( \overline{DA} \)

Opposite angles: \( \angle A \) and \( \angle C, \angle B \) and \( \angle D \)
\[ EF \parallel HG \quad m\angle 1 = m\angle 3 \]
\[ EH \parallel FG \quad m\angle 2 = m\angle 4 \]
\[ HF \text{ bisects } \overline{EG}, \text{ and } \overline{EG} \text{ bisects } \overline{HF}. \]
\[ \overline{EF} = \overline{HG} \]

Example 1

Refer to \( \square \text{IJKL} \) at the right.
Find \( m\angle I \) in \( \square \text{IJKL} \).

Solution
Since \( \angle L \) and \( \angle J \) are opposite angles, by the parallelogram-angle theorem,
\[ m\angle L = m\angle J = 55^\circ. \]

Use the polygon-sum theorem to find the sum of the measures of the interior angles.
\[ (n - 2)180^\circ = (4 - 2)180^\circ = (2)180^\circ = 360^\circ \]
Notice that \( \angle L + \angle J = 55^\circ + 55^\circ = 110^\circ. \)
It follows that
\[ m\angle I + m\angle K = 360^\circ - 110^\circ = 250^\circ. \]
Since \( \angle I \) and \( \angle K \) are opposite angles, by the parallelogram-angle theorem,
\[ m\angle I = m\angle K = 250^\circ / 2 = 125^\circ. \]

Example 2

In the figure at the right, \( \text{MNOP} \) is a square and \( PN = 9 \) cm. Find \( OZ \).

Solution
A square is also a rectangle. By the rectangle-diagonal theorem, the diagonals are equal in length. So, \( PN = OM = 9 \) cm.

By the parallelogram-diagonal theorem, the diagonals bisect each other.
So, \( OM = OZ + MZ = 2(OZ) \).
Substitute 9 for \( OM \) and solve.
\[ 2OZ = 9 \]
\[ OZ = 4.5 \]
The length of \( OZ \) is 4.5 cm.

Exercise
Find the values of \( a, b, c, \) and \( d \) in these parallelograms.

1. \( a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \)

2. \( a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \)

3. \( a = \_ \quad b = \_ \quad c = \_ \quad d = \_ \)

Quadrilateral \( \text{ABCD} \), shown at the right, is a rhombus with \( m\angle ABC = 84^\circ. \)

4. \( \angle DAB \)

5. \( \angle AED \)

6. \( \angle ADB \)

7. \( \angle DCE \)
RETEACHING 4-9

SPECIAL QUADRILATERALS: TRAPEZIODS

Bases: $\overline{AB}$ and $\overline{CD}$
Base angles: $\angle A$ and $\angle B$; $\angle C$ and $\angle D$
$\overline{AB} \parallel \overline{DC}$

Legs: $\overline{AD}$ and $\overline{BC}$

Median: $\overline{XY}$

An isosceles trapezoid has legs of equal length and base angles with equal measures.

Example

In the figure at the right, $\overline{EH} \parallel \overline{FG}$, $\overline{EH} \parallel \overline{XY}$, and $EF = HG$.

a. Find $XY$.

b. Find $m \angle G$.

Solution

Quadrilateral $EFGH$ is an isosceles trapezoid with bases $\overline{EH}$ and $\overline{FG}$.

a. Since $\overline{XY}$ is the median, apply the trapezoid-median theorem.

$XY = \frac{1}{2}(EH + FG)$

$= \frac{1}{2}(6 + 8)$

$= \frac{1}{2}(14)$

$= 7$

So, the length of $\overline{XY}$ is 7 in.

b. By the isosceles-trapezoid theorem, the base angles $\angle F$ and $\angle G$ are equal in measure. Use this fact to write and solve an equation.

$n + 36 = 2n - 12$

$n + 36 - n = 2n - 12 - n$

$36 = n - 12$

$48 = n$

So, the value of $n$ is 48. Use $m \angle G = (2n - 12)^\circ$. Substituting 48 for $n$,

$m \angle G = (2 \cdot 48 - 12)^\circ = (96 - 12)^\circ = 84^\circ$.

Exercises

A trapezoid and its median are shown. Find the value of $x$.

1. [Diagram]

2. [Diagram]

3. [Diagram]

4. [Diagram]

The given figure is a trapezoid. Find the measures of all the unknown angles.

5. [Diagram]

6. [Diagram]

7. [Diagram]

8. [Diagram]
RETEACHING 5-1

RATIOS AND UNITS OF MEASURE

A ratio is a quotient of two numbers that compares one number with another. When you write a ratio involving measurement, you must sometimes convert from one unit to another. Remember to multiply when changing a larger unit to a smaller unit and to divide when changing a smaller unit to a larger unit.

Example 1

Complete.

a. 4 pt = \( \square \) c
b. 2000 m = \( \square \) km

Solution

a. 1 pt = 2 c, so multiply by 2.
\( (2)(4) = 8 \), so 4 pt = 8 c.
b. 1 km = 1000 m, so divide by 1000.
2000 \( \div \) 1000 = 2,
so 2000 m = 2 km.

Example 2

Write the ratio of 200 mL to 1 L in lowest terms.

Solution

Write the ratio as \( \frac{200 \text{ mL}}{1 \text{ L}} \).
Then rename the measurements.
\( \frac{200 \text{ mL}}{1 \text{ L}} = \frac{200 \text{ mL}}{1000 \text{ mL}} \)
Divide to write fractions in lowest terms.
\( \frac{200 \div 200}{1 \div 1} = \frac{1}{5} \)

EXERCISES

Complete.

1. 132 in. = \( \square \) ft
2. 8 yd = \( \square \) in.
3. 5 lb = \( \square \) oz
4. 108 ft = \( \square \) yd
5. 256 cups = \( \square \) pt
6. 5 gal = \( \square \) qt
7. 3.4 L = \( \square \) mL
8. 1085 g = \( \square \) kg
9. 4.35 m = \( \square \) cm
10. 8 g = \( \square \) mg
11. 3.2 km = \( \square \) m
12. 23 mm = \( \square \) m

Write each ratio in lowest terms.

13. 3 lb to 54 oz
14. 12 ft to 5 yd
15. 8 qt to 4 gal
16. 5 g to 2 kg
17. 520 mm to 4 m
18. 3 L to 5500 mL
RETEACHING 5-2

PERIMETER, CIRCUMFERENCE, AND AREA

You can find the perimeter of a polygon by adding the lengths of its sides. You can use this formula to find the circumference of a circle: \( C = \pi d \). You can use these formulas to find the areas of the figures listed.

- Square: \( A = s^2 \)
- Rectangle: \( A = lw \)
- Parallelogram: \( A = bh \)
- Triangle: \( A = \frac{1}{2}bh \)
- Circle: \( A = \pi r^2 \)
- Trapezoid: \( A = \frac{1}{2}h (b_1 + b_2) \)

Example 1

Find the perimeter of the parallelogram.

\[ P = 2(10.1) + 2(5) = 30.2 \]

The perimeter is 30.2 m.

Example 2

Find the area of the parallelogram.

\[ A = (10.1)(4) = 40.4 \]

The area is 40.4 m².

EXERCISES

Find the perimeter or circumference of each. Then find the area.

1. \[ P = \quad A = \]
2. \[ P = \quad A = \]
3. \[ P = \quad A = \]
4. \[ C = \quad A = \]
5. \[ P = \quad A = \]
6. \[ P = \quad A = \]
7. \[ P = \quad C = \]
8. \[ A = \]
RETEACHING 5-3

PROBABILITY AND AREA

The probability of any event can be expressed as a ratio:

\[
P(\text{any event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

To find the probability that a point selected at random appears in a shaded region of a figure, you may need to select some of these formulas to help you find the area.

- **Square:** \( A = s^2 \)
- **Rectangle:** \( A = lw \)
- **Parallelogram:** \( A = bh \)
- **Triangle:** \( A = \frac{1}{2}bh \)
- **Circle:** \( A = \pi r^2 \)
- **Trapezoid:** \( A = \frac{1}{2}h (b_1 + b_2) \)

**Example 1**

Find the probability that a point selected at random is in the shaded region.

**Solution**

\[
P(\text{point in shaded circle}) = \frac{\text{area of shaded circle}}{\text{area of entire circle}}
\]

\[
P = \frac{\pi (2)(2)}{(\pi)(8)(8)} = \frac{4}{6} = \frac{1}{16}
\]

The probability that a point is in the shaded region is \( \frac{1}{16} \), or 0.0625.

**Example 2**

Suppose you lost your ring somewhere in your 1500-ft\(^2\) home. What is the probability that you will find it in the 20-ft by 12-ft den?

**Solution**

\[
P(\text{finding ring in den}) = \frac{\text{area of den}}{\text{area of entire home}}
\]

\[
P = \frac{(20)(12)}{1500} = \frac{240}{1500} = \frac{4}{25}
\]

The probability that you will find your ring in the den is \( \frac{4}{25} \), or 0.16.

**Exercises**

Find the probability that a point selected at random in each figure is in the shaded region.

1.  

2.  

3.  

4.  

5. The total area of Florida is 58,560 mi\(^2\). Of these, 4308 mi\(^2\) are inland water. If a meteor were to land somewhere in the state, what is the probability that it will splash down on the inland water?

6. The area of a yard is 420 ft\(^2\). What is the probability that any leaf landing in the yard will land in a circular wading pool that has a diameter of 14 ft? Use \( \frac{22}{7} \) for \( \pi \).
PROBLEM SOLVING SKILLS: IRREGULAR SHAPES

One strategy you can use to find the answer to a complex problem is to solve a simpler problem first. Then use these formulas to help you find the area.

Rectangle: \( A = lw \)  
Triangle: \( A = \frac{1}{2}bh \)  
Trapezoid: \( A = \frac{1}{2} h(b_1 + b_2) \)  
Circle: \( A = \pi r^2 \)

Example

Joan wants to carpet the floor of her family room. She selects carpet that costs $20 per square yard. How much money will she spend?

Solution

Step 1: Begin by finding the area in square yards.

\[ \text{Area of trapezoid: } A = \frac{1}{2} h(b_1 + b_2) \]
\[ \text{Area of rectangle: } A = lw \]
\[ A = \frac{1}{2} (4)(8 + 6) = 28 \]
\[ A = (8)(8) = 64 \]
\[ \text{The area is } 28 \text{ yd}^2. \]
\[ \text{The area is } 64 \text{ yd}^2. \]

Step 2: Then multiply the number of square yards by the cost per square yard.

\[ 20 \cdot 92 = 1840 \]

The cost of the carpeting is $1840.

Exercise

Find the area of each figure. Then tell how much money you would spend to carpet each floor if carpeting costs $15 per square yard.

1. 
2. 
3. 
4. 
5. 
6.
RETEACHING 5-5

THREE-DIMENSIONAL FIGURES AND LOCI

A prism has two bases with three or more lateral faces which are shaped like parallelograms.

A pyramid has one base with three or more lateral faces which are shaped like triangles.

Example

Name and identify the polyhedron, its vertices and each of its bases. Then identify the intersecting faces and edge at which they intersect.

Solution

The polyhedron is a hexagonal pyramid; it has one hexagonal base and six triangular faces.

Vertices: Points A, B, C, D, E, F, G

Base: BCDEFG

Intersecting faces and edges: \( \triangle ABC \) and \( BCDEFG, BC \); \( \triangle ACD \) and \( BCDEFG, CD \); \( \triangle ADE \) and \( BCDEFG, DE \); \( \triangle AEF \) and \( BCDEFG, EF \); \( \triangle AGF \) and \( BCDEFG, FG \); \( \triangle AGB \) and \( BCDEFG, GB \)

EXERCISES

Name and identify each polyhedron, each vertex and each of its bases.

1. 

2. 

3.

4. 

5. 

6.
RETEACHING 5-6

SURFACE AREA OF THREE-DIMENSIONAL FIGURES

To find the surface area (SA) of a polyhedron, add the area of each base to the sum of the areas of all the lateral sides.

Example

Find the surface area of the triangular prism.

Solution

\[ SA = \text{sum of the area of each lateral face} + 2(\text{area of base}) \]

Use the formula \( A = lw \) to find area of each lateral face.

- Face 1: \( A = (5)(12) = 60 \)
- Face 2: \( A = (5)(12) = 60 \)
- Face 3: \( A = (6)(12) = 72 \)

Use the formula \( A = \frac{1}{2}bh \) to find area of a base.

- \( A = \frac{1}{2}(4)(6) = 12 \)

Use the formula \( SA = \text{sum of the areas of the lateral faces} + 2(\text{area of base}) \).

\[ SA = 60 + 60 + 72 + (2)(12) = 192 + 24 \]
\[ = 216 \]

The surface area of the triangular prism is 216 m².

EXERCISES

Find the surface area of each figure.

1. 

2. 

3. 

4. 

5. 

6. 

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RETEACHING 5-7

VOLUME OF THREE-DIMENSIONAL FIGURES

Volume is a measure of the number of cubic units needed to fill a region of space. You can use a formula to find the volume for these figures. Remember, \( B \) represents area of the base.

- **Prism:** \( V = Bh \)
- **Cylinder:** \( V = \pi r^2 h \)
- **Cone:** \( V = \frac{1}{3} \pi r^2 h \)
- **Pyramid:** \( V = \frac{1}{3}Bh \)

**Example 1**

Find the volume. The area of the base is 110 cm\(^2\).

**Solution**

Use the formula \( V = Bh \)

\[
V = (110)(10)
\]

\[
V = 1100
\]

The volume is 1100 cm\(^3\).

**Example 2**

Find the volume of the powder left in the can.

**Solution**

Use the formula \( V = \pi r^2 h \)

\[
V = (3.14)(2)(2)(3)
\]

\[
V = 37.68
\]

**Exercises**

Find the volume of each figure.

1. Area of base: 37.8 cm\(^2\)

2.

3.

Find the volume of each container. Then find the volume of the powder left in each container.

4.

5.

6.

7. The volume of the powder poured out of the triangular prism is 120 ft\(^3\). What is the volume of the powder left inside the prism?
SLOPE OF A LINE AND SLOPE-INTERCEPT FORM

The slope of a line is measured as a ratio.

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change (change in y-coordinates)}}{\text{horizontal change (change in x-coordinates)}}
\]

The slope of a line containing point \(A(x_1, y_1)\) and point \(B(x_2, y_2)\) is \(\frac{y_2 - y_1}{x_2 - x_1}\).

**Example 1**

Find the slope of \(\overline{AB}\) containing points \(A(-4, 7)\) and \(B(4, -7)\).

**Solution**

Substituting the values into the formula, \(m = \frac{y_2 - y_1}{x_2 - x_1}\), you find \(\frac{-7 - 7}{4 - (-4)} = \frac{-14}{8} = -\frac{7}{4}\).

**Example 2**

Find the slope and the \(y\)-intercept for the line \(-2x + y = 4\).

**Solution**

Write the equation in slope-intercept form: \(y = 2x + 4\), where \(m = 2\) and \(b = 4\). The slope \((m)\) is 2 and the \(y\)-intercept \((b)\) is 4.

**Exercises**

Find the slope of the line containing the given points.

1. \(E(2, 6), F(4, 4)\)  
2. \(G(3, 1), H(5, 1)\)  
3. \(I(-1, -2), J(1, 2)\)  
4. \(K(7, -1), L(6, 2)\)  
5. \(M(8, 4), N(4, 2)\)  
6. \(O(3, 4), P(-1, -2)\)  
7. \(Q(5, 2), R(5, 8)\)  
8. \(S(9, 6), T(3, -2)\)  
9. \(U(-1, -2), V(-3, -4)\)

Find the slope and \(y\)-intercept for each line.

10. \(y = -2x\)  
11. \(3x + y = 10\)  
12. \(x = \frac{1}{3}\)  
13. \(y = x + 6\)  
14. \(4x + 2y = 12\)  
15. \(y = 3x + 1\)  
16. \(\frac{1}{3}x + \frac{1}{3}y = 1\)  
17. \(y = \frac{1}{4}x + \frac{1}{2}\)  
18. \(y = 8\)  
19. \(4x + 2y = 5\)
PARALLEL AND PERPENDICULAR LINES

Two lines are parallel when they have the same slope and different y-intercepts; \( m_1 = m_2 \).

Two lines are perpendicular if the product of their slopes is \(-1\): \( m_1 \cdot m_2 = -1 \).

Slopes of perpendicular lines are negative reciprocals of each other:
\( m_1 = -\frac{1}{m_2} \).

**Example 1**

Line \( AB \) contains points \( A (4, 0) \) and \( B (-3, 2) \). Find the slope of a line

- **a.** parallel to \( AB \).
- **b.** perpendicular to \( AB \).

**Solution**

The slope of \( \overleftrightarrow{AB} \) is \( \frac{0-2}{4-(-3)} = \frac{-2}{7} \).

**a.** Parallel lines have the same slope, so \( m = -\frac{2}{7} \).

**b.** Since slopes of perpendicular lines are negative reciprocals, the slope is \( \frac{7}{2} \).

**Example 2**

Determine whether the pair of lines is parallel, perpendicular or neither.

\( 4x + 2y = -8 \)
\( 6y = 12x + 36 \)

**Solution**

Write each in slope-intercept form.

\( 4x - 2y = -8 \rightarrow y = 2x + 4 \) \( m_1 = 2 \)
\( 6y = 12x + 36 \rightarrow y = 2x + 6 \) \( m_2 = 2 \)

Since \( m_1 = m_2 \), the lines are parallel.

**EXERCISES**

Find the slope of a line parallel to the given line and of a line perpendicular to the given line.

1. The line containing points \( G (2, 0) \) and \( H (-2, -4) \). Parallel _____ Perpendicular _____

2. The line containing points \( I (0, 5) \) and \( J (8, -4) \). Parallel _____ Perpendicular _____

3. \( 3x + 4y = 5 \) Parallel _____ Perpendicular _____

4. \( 7y = x + 7 \) Parallel _____ Perpendicular _____

Determine whether each pair of lines is parallel, perpendicular or neither.

5. The line containing points \( K (-1, 0) \) and \( L (2, 6) \).

   The line containing points \( M (4, 2) \) and \( N (3, 0) \).

6. \( -5x + 7 = 3y \) \( y = 3 \)

7. \( 3x - 6 = 2y \) \( \frac{3}{2}x - y = 5 \)
Writing Equations for Lines

An equation can be written in slope-intercept form, \( y = mx + b \), when you know the slope, \( m \), and the \( y \)-intercept, \( b \).

An equation can be written in the point-slope form, \( y - y_1 = m(x - x_1) \) when you know:

1. a point \((x_1, y_1)\) and the slope, \( m \),
2. two points \((x_1, y_1)\) and \((x_2, y_2)\), or
3. the graph with points \(A(x_1, y_1)\) and \(B(x_2, y_2)\).

**Example 1**

Write an equation of the line whose slope is 9 and contains the point \(C(−5, 2)\).

**Solution**

Given a point and the slope, write an equation in point-slope form, substituting 9 for \(m\), −5 for \(x_1\), and 2 for \(y_1\).

\[
y_2 - y_1 = m(x_2 - x_1)\\y_2 - 2 = 9(x_2 - (-5))\\y_2 - 2 = 9x_2 + 45\\y_2 = 9x_2 + 47
\]

An equation of the line is \(y = 9x + 47\).

**Example 2**

Write an equation of the line that contains points \(D(3, 7)\) and \(E(4, 1)\).

**Solution**

Given two points, use the ratio to determine slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{4 - 3} = \frac{-6}{1} = -6
\]

Then write an equation in point-slope form, substituting −6 for \(m\), 7 for \(y_1\), and 3 for \(x_1\).

\[
y_2 - y_1 = m(x_2 - x_1)\\y_2 - 7 = -6(x_2 - 3)\\y_2 - 7 = -6x_2 + 18\\y_2 = -6x_2 + 25
\]

An equation of the line is \(y = -6x + 25\).

**Exercises**

Write an equation of the line that contains the given point and slope.

1. \(m = \frac{2}{3}, V(1, 1)\) \______________
2. \(m = -2, W(-2, 0)\) \______________

3. \(m = 0, X(0, 5)\) \______________
4. \(m = -3, A(-1, -4)\) \______________

5. \(m = \frac{1}{2}, B(3, 2)\) \______________
6. \(m = \frac{1}{3}, C(9, -1)\) \______________

Write an equation of the line that contains the given points.

7. \(F(6, -6), G(-7, 7)\) \______________
8. \(H(-1, -3), I(2, 0)\) \______________
RETEACHING 6-4
SYSTEMS OF EQUATIONS
A graph can help you solve some systems of equations.
If the graphs of two lines intersect, there is one solution—the point of intersection.
If the graphs of two lines are parallel, there is no solution.
If the graphs of two lines coincide, there is an infinite number of solutions—all points on the line.

Example
Solve the system of equations by graphing. \[
\begin{align*}
y &= 3x + 4 \\
-12x + 6y &= 6
\end{align*}
\]

Solution
Graph each equation. Then read the solution from the graph.
• To graph, write each equation in the slope-intercept form and identify the \( y \)-intercept, \( b \), and the slope, \( m \).
  \[
  \begin{align*}
y &= 3x + 4 & b &= 4, \quad m = 3 \\
-12x + 6y &= 6 & 6y &= 12x + 6 & y &= 2x + 1 & b &= 1, \quad m = 2
\end{align*}
\]
• Use the \( y \)-intercept and the slope to graph each equation.
The lines intersect at the point \((-3, -5)\).
So, the solution is \((-3, -5)\).

EXERCISES
Solve each system of equations by graphing. Graph Exercises 4–6 on another sheet of paper.

1. \[
\begin{align*}
2y - 3x &= 8 \\
2y &= -2x - 2
\end{align*}
\]

2. \[
\begin{align*}
4y &= 6x - 16 \\
7x &= 14y
\end{align*}
\]

3. \[
\begin{align*}
-3x + 6y &= 12 \\
2x - 4y &= -8
\end{align*}
\]

4. \[
\begin{align*}
x + y &= -4 \\
2x + 8y &= -8
\end{align*}
\]

5. \[
\begin{align*}
x &= 4 \\
y &= -2x + 6
\end{align*}
\]

6. \[
\begin{align*}
y &= 3x + 2 \\
-3x + y &= -1
\end{align*}
\]
RETEACHING 6-5

SOLVING SYSTEMS BY SUBSTITUTION
You can use algebraic methods to find the solution to a system of equations.

Example
Solve. \[
\begin{align*}
3x + 6y &= 12 \\
3x - 6y &= 12
\end{align*}
\]
Solution
Solve the first equation for \( y \) in terms of \( x \).
\[
6y = -3x + 12 \\
y = -\frac{1}{2}x + 2
\]
Write the second equation.
\[
3x - 6y = 12
\]
Substitute \( \left(-\frac{1}{2}x + 2\right) \) for \( y \).
\[
3x - 6\left(-\frac{1}{2}x + 2\right) = 12
\]
Solve for \( x \).
\[
3x + 3x - 12 = 12 \\
6x = 24 \\
x = 4
\]
Choose one of the original equations; then substitute 4 for \( x \).
\[
3x + 6y = 12 \\
3(4) + 6y = 12 \\
12 + 6y = 12 \\
6y = 0 \\
y = 0
\]
The solution is (4, 0).
You can check the solution by substituting 4 for \( x \) and 0 for \( y \) in each equation.

EXERCISES
Solve each system of equations by the substitution method. Check the solution.

1. \[
\begin{align*}
x + y &= 4 \\
2x - y &= 5
\end{align*}
\]

2. \[
\begin{align*}
5x + y &= 0 \\
x - 2y &= 11
\end{align*}
\]

3. \[
\begin{align*}
-4x + 2y &= 8 \\
2x + 2y &= 6
\end{align*}
\]

4. \[
\begin{align*}
2x + \frac{1}{2}y &= 25 \\
-x - y &= 10
\end{align*}
\]

5. \[
\begin{align*}
x &= 4y + 16 \\
2x - y &= 53
\end{align*}
\]

6. \[
\begin{align*}
y &= 7x + 21 \\
3x - y &= 11
\end{align*}
\]

7. \[
\begin{align*}
2x + 3y &= 3 \\
x - 4y &= 7
\end{align*}
\]

8. \[
\begin{align*}
-6x + \frac{1}{2}y &= 30 \\
-x - \frac{1}{2}y &= -2
\end{align*}
\]
RETEACHING 6-6

SOLVING SYSTEMS BY ADDING, SUBTRACTING, AND MULTIPLYING

Algebraic methods of solving a system of equations include the addition/subtraction method and the multiplication and addition method.

Example

Solve.

\[
\begin{align*}
4x - 3y &= 11 \\
2x &= 4y + 3
\end{align*}
\]

Solution

Write each equation in standard form.

\[
\begin{align*}
4x - 3y &= 11 \\
2x &= 4y + 3 \rightarrow 2x - 4y = 3
\end{align*}
\]

Multiply to obtain an equivalent system of equations in which the coefficients of one of the variables are either the same or the opposite. Then add or subtract.

\[
\begin{align*}
4x - 3y &= 11 \rightarrow 4x - 3y = 11 \\
2x - 4y &= 3 \rightarrow -2(2x - 4y) = 3(-2) \rightarrow -4x + 8y = -6
\end{align*}
\]

\[5y = 5, \text{ so } y = 1\]

Choose one of the original equations, for example \(4x - 3y = 11\). Substitute 1 for \(y\).

\[
\begin{align*}
4x - 3(1) &= 11 \\
4x - 3 &= 11 \\
x &= 14 \\
x &= 3\frac{1}{2}
\end{align*}
\]

The solution is \(3\frac{1}{2}, 1\).

You can check the solution by substituting \(3\frac{1}{2}\) for \(x\) and 1 for \(y\) in each equation.

EXERCISES

Solve each system of equations. Check the solution.

1. \[
\begin{align*}
2x - y &= 4 \\
x + 2y &= 7
\end{align*}
\]

2. \[
\begin{align*}
3x - y &= 4 \\
4y &= -2x + 12
\end{align*}
\]

3. \[
\begin{align*}
x &= 3y - 6 \\
6y &= x + 3
\end{align*}
\]

4. \[
\begin{align*}
-x + 3y &= 8 \\
y &= 2x - 4
\end{align*}
\]

5. \[
\begin{align*}
6x + 6y &= 6 \\
x + y &= 1
\end{align*}
\]

6. \[
\begin{align*}
5x - y &= 4 \\
3x &= -2y + 18
\end{align*}
\]

7. \[
\begin{align*}
3y &= 2x - 9 \\
6x + 13 &= y
\end{align*}
\]

8. \[
\begin{align*}
10x + 5y &= 20 \\
x &= y + 2
\end{align*}
\]

9. The perimeter of a rectangle is 24 in. The length is twice the width. Find the dimensions.
PROBLEM SOLVING SKILLS: DETERMINANTS AND MATRICES

A system of equations can be written using the matrix equation, \( AX = B \), where \( A \) is a matrix of the coefficients, \( X \) is a matrix of variables, and \( B \) is a matrix of constants. The method of determinants can be used to solve some systems of equations.

Given: \( ax + by = e \)
\( cx + dy = f \)

Matrix Equation:
\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
e \\
f
\end{bmatrix}
\]

\[ \text{det } A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \]

Example

For the system of equations,
\[
\begin{align*}
2x + 4y &= 6 \\
3x - 2y &= 4
\end{align*}
\]
a. write the matrix equation. 

b. solve using the method of determinants.

Solution

\[
\begin{bmatrix}
2 & 4 \\
3 & -2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
6 \\
4
\end{bmatrix}
\]

\[ x = \frac{A_x}{A} = \begin{vmatrix} 4 & -2 \\ 2 & -4 \end{vmatrix} = \frac{-12 - 16}{-4 - 12} = \frac{-28}{-16} = \frac{7}{4} = 1 \frac{3}{4} \]

\[ y = \frac{A_y}{A} = \begin{vmatrix} 2 & 6 \\ 3 & 4 \end{vmatrix} = \frac{8 - 18}{-4 - 12} = \frac{-10}{-16} = \frac{5}{8} \]

The solution is \( \left(1 \frac{3}{4}, \frac{5}{8}\right)\). Check by substituting \(1 \frac{3}{4}\) for \( x \) and \(\frac{5}{8}\) for \( y \) in the original equations.

EXERCISES

For each system of equations, a. write the matrix equation and b. solve using the method of determinants.

1. \[
\begin{align*}
-3x + 4y &= 12 \\
x - 2y &= 6
\end{align*}
\]
a. b. 

2. \[
\begin{align*}
5x + y &= 10 \\
x - y &= 5
\end{align*}
\]
a. b. 

3. \[
\begin{align*}
x - y &= 16 \\
x + y &= 10
\end{align*}
\]
a. b. 

4. \[
\begin{align*}
2x - 2y &= 8 \\
-x + 3y &= 12
\end{align*}
\]
a. b. 

RETEACHING 6-8
SYSTEMS OF INEQUALITIES
You can find the solution set of a system of linear inequalities by graphing the inequalities.

Example 1
Write a system of linear inequalities for the graph at the right.

Solution

<table>
<thead>
<tr>
<th>Equation</th>
<th>Line ( l ): ( b = 3, \ m = -1 )</th>
<th>Line ( p ): ( b = -3, \ m = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shading</td>
<td>above line</td>
<td>below and including line</td>
</tr>
<tr>
<td>Symbol</td>
<td>( &gt; )</td>
<td>( \leq )</td>
</tr>
<tr>
<td>Inequality</td>
<td>( y &gt; -x + 3 )</td>
<td>( y \leq 2x - 3 )</td>
</tr>
</tbody>
</table>

The system of linear inequalities is \[
\begin{align*}
y &> -x + 3 \\
y &\leq 2x - 3
\end{align*}
\]

Example 2
Graph the solution set. \[
\begin{align*}
x - 2y &\geq 8 \\
3x + y &< 6
\end{align*}
\]

Solution
Write each inequality in slope–intercept form and make a table.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Boundary</th>
<th>Shading</th>
<th>Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x - 2y \geq 8 \rightarrow y \leq \frac{x}{2} - 4 )</td>
<td>( y = \frac{x}{2} - 4 )</td>
<td>below line</td>
<td>solid</td>
</tr>
<tr>
<td>( 3x + y &lt; 6 \rightarrow y &lt; -3x + 6 )</td>
<td>( y = -3x + 6 )</td>
<td>below line</td>
<td>dashed</td>
</tr>
</tbody>
</table>

EXERCISES

Write a system of linear inequalities for each graph.

1. [Graph]

Graph the solution set of each system of linear inequalities on another sheet of paper.

3. \[
\begin{align*}
x &> 2 \\
y &> 4
\end{align*}
\]

4. \[
\begin{align*}
x &< y + 1 \\
3 &\leq -x - y
\end{align*}
\]

5. \[
\begin{align*}
x + y &\geq 0 \\
y - x &< 4
\end{align*}
\]
**RETEACHING 6-9**

**LINEAR PROGRAMMING**

Linear programming can be used in business to solve problems involving linear inequalities.

**Example**

Graph the solution set of the system of inequalities.
Then determine the minimum value of \( P = 5x - 2y \).

**Solution**

Graph each inequality. Make a chart.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>( x + 2y \leq 4 )</th>
<th>( x \leq 2 )</th>
<th>( x \geq 0 )</th>
<th>( y \leq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope-intercept form</td>
<td>( y \leq -\frac{1}{2}x + 2 )</td>
<td>( x \leq 2 )</td>
<td>( x \geq 0 )</td>
<td>( y \leq 0 )</td>
</tr>
<tr>
<td>Boundary</td>
<td>( y = -\frac{1}{2}x + 2 )</td>
<td>( x = 2 )</td>
<td>( x = 0 )</td>
<td>( y = 0 )</td>
</tr>
<tr>
<td>Shading</td>
<td>below</td>
<td>left</td>
<td>right</td>
<td>above</td>
</tr>
<tr>
<td>Line</td>
<td>solid</td>
<td>solid</td>
<td>solid</td>
<td>solid</td>
</tr>
</tbody>
</table>

Identify vertices. Then make a table of the vertices and the value of \( P \) for each vertex.
The minimum value of \( P \) is \(-4\) when \( x = 0 \) and \( y = 2 \).

**Exercise**

Graph the solution set of each system of inequalities. Then determine the value of \( P \).

1. \[
\begin{align*}
 x - y & \leq 5 \\
 y & \geq -4 \\
 x & \geq 0 \\
 y & \leq 0 \\
\end{align*}
\]
Max \( P = 4x + 2y \)

2. \[
\begin{align*}
 -x - y & \geq -2 \\
 x & \geq 0 \\
 y & \geq 0 \\
\end{align*}
\]
Min \( P = x - 3y \)

3. \[
\begin{align*}
 -4x + 2y & \leq 8 \\
 -x - 3y & \leq 9 \\
 x & \leq 0 \\
 y & \leq 0 \\
\end{align*}
\]
Max \( P = 3x + 2y \)

4. \[
\begin{align*}
 x + 3y & \leq 6 \\
 -x - 9 & \leq 3y \\
 x & \leq 0 \\
 x & \geq -3 \\
\end{align*}
\]
Min \( P = 5x + y \)
RATIOS AND PROPORTIONS

Equivalent ratios can be named by the same fraction. A proportion is an equation stating that two ratios are equivalent. Cross-products of a proportion are equal. Use them to find the missing term in a proportion.

\[ \frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{b} = \frac{c}{d} \rightarrow ad = bc \]

**Example 1**

Solve the proportion by finding \( x \).

\[ \frac{x}{21} = \frac{5}{35} \]

**Solution**

Use cross-products to write another equation. Then solve for \( x \).

\[
35x = 5(21) \\
35x = 105 \\
\frac{1}{35}(35x) = 105 \left( \frac{1}{35} \right) \\
x = 3
\]

Find cross-products. Simplify. Multiply each side by \( \frac{1}{35} \). Simplify.

Substitute 3 for \( x \) in the original proportion. Then determine if the ratios are equivalent.

\[
\frac{3}{21} = \frac{3 \div 3}{21 \div 3} = \frac{1}{7} \\
\frac{5}{35} = \frac{5 \div 5}{35 \div 5} = \frac{1}{7}
\]

Since the ratios are equivalent, the proportion is solved for \( x = 3 \).

**Example 2**

A pattern calls for black tiles and white tiles in the ratio of 4 : 5. How many white tiles are needed if 200 black tiles are used?

**Solution**

Write a proportion. Let \( w \) = number of white tiles.

\[
\frac{4}{5} = \frac{200}{w}
\]

Use cross-products to write another equation and solve.

\[
4w = 5(200) \\
4w = 1000 \\
\frac{1}{4}(4w) = 1000 \left( \frac{1}{4} \right) \\
w = 250
\]

Find cross-products. Simplify. Multiply by \( \frac{1}{4} \). Simplify.

So 250 white tiles are needed.

**Exercises**

Solve each proportion.

1. \( \frac{a}{8} = \frac{9}{12} \)

2. \( \frac{15}{b} = \frac{6}{14} \)

3. \( \frac{6}{27} = \frac{14}{c} \)

4. \( \frac{12}{42} = \frac{d}{28} \)

5. \( \frac{16}{24} = \frac{18}{y} \)

6. \( \frac{f}{44} = \frac{16}{64} \)

Soolve.

7. A pattern calls for green tiles and blue tiles in a ratio of 3 : 4. How many green tiles are needed if 100 blue tiles are used?

8. Two entrepreneurs imported crafts for resale. The ratio of their investment was 3 : 5. What was each person's share of the $12,000 sales income?
**RETEACHING 7-2**

**SIMILAR POLYGONS**

Similar figures have the same shape. In similar polygons, corresponding angles are congruent and the measures of all corresponding sides are in proportion.

**Example 1**

Is \(ABCD \sim EFGH\)?

**Solution**

Find the missing angle measures.

\[
\begin{align*}
\angle D &= 360° - (90° + 112° + 90°) \\
&= 360° - 292° = 68°, \text{ so } \angle D \cong \angle H. \\
\angle F &= 360° - (90° + 68° + 90°) \\
&= 360° - 248° = 112°, \text{ so } \angle F \cong \angle B.
\end{align*}
\]

All corresponding angles are congruent.

Find the ratios of corresponding sides.

\[
\begin{align*}
\frac{AB}{EF} &= \frac{4}{6} = \frac{2}{3} \\
\frac{BC}{FG} &= \frac{4}{6} = \frac{2}{3} \\
\frac{CD}{GH} &= \frac{6}{9} = \frac{2}{3} \\
\frac{DA}{HE} &= \frac{6}{9} = \frac{2}{3}
\end{align*}
\]

Each pair of corresponding sides has the same ratio, so they are in proportion.

The figures are similar.

**Example 2**

Given \(JKLM \sim NOPQ\), find the length of \(NO\).

**Solution**

In similar figures, corresponding sides are in proportion. Determine the proportion between any two sides.

\[
\frac{KL}{OP} = \frac{5}{10} = \frac{1}{2}
\]

Write a ratio of corresponding sides, substituting known values. Solve by finding the cross-products.

\[
\frac{JK}{NO} = \frac{KL}{OP} \\
\frac{3}{x} = \frac{1}{2}
\]

\[6 = x\]

So, \(NO = 6\) in.

**Exercises**

Use these figures to answer Exercises 1–4. Write yes or no and explain why.

1. Is \(ABCD \sim EFGH\)?
2. Is \(ABCD \sim IJKL\)?
3. Is \(EFGH \sim MNOP\)?
4. Is \(IJKL \sim MNOP\)?

Find \(x\) in each pair of similar polygons.

5. \[
\begin{align*}
\angle A &= 75° \\
\angle D &= 55° \\
\angle B &= 75° \\
\angle C &= 12.5°
\end{align*}
\]

6. \[
\begin{align*}
\angle M &= 8° \\
\angle N &= 53° \\
\angle P &= 14° \\
\angle O &= 10°
\end{align*}
\]
RETEACHING  7-3

SCALE DRAWINGS

A scale drawing is in direct proportion to the actual object. The scale gives you the ratio of the length in the drawing to the actual size of the object.

**Example**

In this scale drawing, the ratio of the scale to the actual boat is 5 cm : 40 m.

a. Find the actual overall length of the boat.

b. The actual length of the boat at the waterline is 26.5 m. Find the scale length of the boat at the waterline.

**Solution**

The overall length of the boat in the scale drawing is 5.15 cm. The actual length of the boat at the water line is 26.5 m.

The scale is 5 cm : 40 m.

Use these facts to write and solve a proportion.

a. Let \( x \) = the actual length of the boat.

\[
\frac{5 \text{ cm}}{40 \text{ m}} = \frac{5.15 \text{ cm}}{x} \\
5x = 40(5.15) \\
x = 41.2
\]

b. Let \( y \) = scale length at waterline.

\[
\frac{5 \text{ cm}}{40 \text{ m}} = \frac{y}{26.5} \\
40y = 5(26.5) \\
y = 3.3125
\]

The actual length of the boat is 41.2 m. The scale length of the boat at the waterline is 3.3125 cm.

**EXERCISES**

Find the actual measure of each scale item.

1. scale length is 5 cm  
   scale is 1 cm : 3 m

2. scale length is 1\( \frac{1}{2} \) in.  
   scale is 1 in. : 8 in.

3. scale distance is 2.5 cm  
   scale is 10 cm : 1 km

4. scale distance is 15 in.  
   scale is 5 in. : 2 mi

Find the scale measure for each item.

5. actual length is 6 ft  
   scale is 1 in. : 15 ft

6. actual length is 45 m  
   scale is 3 cm : 10 m

7. actual distance is 75 mi  
   scale is 1 in. : 100 mi

8. actual distance is 165 km  
   scale is 15 cm : 75 km
Reteaching 7-4

Postulates for Similar Triangles

The AA (angle-angle) Similarity Postulate, the SSS (side-side-side) Similarity Postulate, or the SAS (side-angle-side) Similarity Postulate can be used to prove that two triangles are similar.

Example

Prove \( \triangle ABC \sim \triangle DEF \) using the

a. AA Similarity Postulate.

b. SSS Similarity Postulate.

c. SAS Similarity Postulate.

Solution

a. Find the measure of the missing angle in either triangle.

\[
180° - (82° + 60°) = 180° - 142° = 38°
\]

Since \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \), the triangles are similar by the AA Postulate.

b. Find the ratio of each pair of corresponding sides.

\[
\begin{align*}
\frac{AC}{DF} &= \frac{5}{7.5} = \frac{2}{3} \\
\frac{AB}{DE} &= \frac{7}{10.5} = \frac{2}{3} \\
\frac{BC}{EF} &= \frac{8}{12} = \frac{2}{3}
\end{align*}
\]

Since the ratios of all three pairs of corresponding sides are equal, the triangles are similar by the SSS Postulate.

c. Determine that one angle in one triangle is congruent to one angle in another triangle.

\( \angle A \cong \angle D \) since their measures are equal.

Then determine if the adjacent sides of \( \angle A \) are in proportion to those of \( \angle D \).

\[
\frac{AC}{DF} = \frac{AB}{DE} \text{ because } \frac{5}{7.5} = \frac{7}{10.5} = \frac{2}{3}
\]

Since two pairs of corresponding sides are in proportion and the angles between those sides are congruent, the triangles are similar by the SAS Postulate.

Exercises

Determine whether each pair of triangles is similar. If so, give a reason. Write AA, SSS, or SAS.

1. Is \( \triangle GIK \sim \triangle HIJ? \) ________________

2. Is \( \triangle GIK \sim \triangle LMN? \) ________________

3. Is \( \triangle GIK \sim \triangle OPQ? \) ________________

4. Is \( \triangle LMN \sim \triangle RST? \) ________________

5. Is \( \triangle HIJ \sim \triangle RST? \) ________________

6. Is \( \triangle OPQ \sim \triangle RST? \) ________________
TRIANGLES AND PROPORTIONAL SEGMENTS

If two triangles are similar, the lengths of their altitudes or the lengths of their medians drawn to corresponding sides are in the same proportion as the sides of the triangle.

\[
\frac{\text{side}}{\text{altitude}} = \frac{\text{side}}{\text{median}}
\]

Example

Find the value of \(x\) in right triangle \(EFG\) if \(FH\) is the altitude to the hypotenuse.

Solution

Since an altitude was drawn to the hypotenuse in a right triangle, all the triangles are similar with corresponding sides in proportion.

So \(\triangle EFG \sim \triangle EHF\).

\[
\frac{20}{10} = \frac{10}{x} \\
20x = 100 \\
x = 5
\]

So the value of \(x\) in this triangle is 5.

EXERCISES

Find the value of \(x\) in each triangle. Pairs of triangles are similar. Round your answer to the nearest tenth, if necessary.

1. 
2. 
3. 
4. 
5. 
6.
Reteaching 7-6

Parallel Lines and Proportional Segments

Properties of similar triangles can be used to determine whether lines are parallel and to show proportions.

• A line parallel to one side of a triangle that intersects each of the other two sides at a point other than the vertex divides the sides proportionately.
• A segment that joins the midpoints of two sides of a triangle is parallel to the third side.

Example

Find the value of $x$ given $\overline{DE} \parallel \overline{AB}$.

Solution

Since $\overline{DE} \parallel \overline{AB}$, and $\overline{DE}$ intersects $\overline{AC}$ and $\overline{BC}$ at a point other than the vertex, $\overline{DE}$ divides the sides proportionately.

To find $x$, write and solve a proportion.

$$\frac{AD}{CD} = \frac{BE}{CE} \rightarrow \frac{5}{x} = \frac{10}{6}$$

$$30 = 10x$$

$$3 = x$$

So $x$ is 3.

Exercises

Find the value of $x$ in each figure. Round your answer to the nearest tenth, if necessary.

1. $\overline{FH} \parallel \overline{IJ}$

2. $\overline{ML} \parallel \overline{NO}$

3. $\overline{ST} \parallel \overline{RQ}$

4. $\overline{DE} \parallel \overline{BC}$

5. $\overline{JI} \parallel \overline{GH}$

6. $\overline{ON} \parallel \overline{LM}$

7. $\overline{ST} \parallel \overline{PQ}$

8. $\overline{ED} \parallel \overline{BC}$
PROBLEM SOLVING SKILLS: INDIRECT MEASUREMENT

An indirect measurement is one in which a measure is calculated using other measures. This is useful when it is difficult to make the necessary measurement. Properties of similar triangles can be used to calculate some indirect measurements.

Example

A surveyor made this sketch of measurements taken along a river. Use these measures to find the width of the river.

Solution

The vertical angles formed at the common vertex of the two triangles are congruent (by the vertical angles theorem), as are the two right angles, so the triangles are similar by the AA Postulate.

Sketch the two separate triangles to help you write and solve a proportion.

Let \( w \) = width of the river

\[
\frac{10}{50} = \frac{20}{w}
\]

\[
10w = 1000
\]

\[
w = 100
\]

So the width of the river is 100 m.

EXERCISES

Solve. Write any proportions you use to find the solution. Draw a sketch if you wish.

1. A building casts a shadow 25 ft long. A 5-ft person standing next to the building casts a 2-ft shadow. How tall is the building?

2. To measure the clearance of a highway overpass, a 6-ft tall person places a mirror 9 ft from the bridge. Then he moves 3 ft to where he can see the overpass in the mirror. What is the measure of the clearance?

3. A 200-cm clothesline pole casts a 300-cm shadow. A garbage can next to the clothesline casts a 105-cm shadow. How tall is the garbage can?

4. Find the width of the river.

5. Find the width of the lake.
RETEACHING 8-1

TRANSLATIONS AND REFLECTIONS

Two transformations that yield congruent figures include a translation or slide of a figure and a reflection of flip of a figure. When a line is reflected across the x-axis, \((x, y) \rightarrow (x, -y)\). When a line is reflected across the y-axis, \((x, y) \rightarrow (-x, y)\).

Example 1

Graph the image of \(\triangle ABC\) under a translation of 8 units up.

Solution

Step 1: To find the coordinates of the vertices of the image, add 8 to each \(y\)-coordinate of the preimage.

\[
A(-6, -4) \rightarrow A'(-6, 4) \quad B(-2, -2) \rightarrow B'(-2, 6) \quad C(-3, -6) \rightarrow C'(-3, 2)
\]

Step 2: Graph these coordinates and draw the image.

Example 2

Graph the image of \(\triangle DEF\) under a reflection across the \(y\)-axis.

Solution

Step 1: To find the coordinates of the vertices of the image, use the rule \((x, y) \rightarrow (-x, y)\).

\[
D(-5, 4) \rightarrow D'(5, 4) \quad E(-2, 5) \rightarrow E'(2, 5) \quad F(-3, 1) \rightarrow F'(3, 1)
\]

Step 2: Graph these coordinates and draw the image.

Exercise

Find the coordinates of the vertices of the image of \(\triangle DEF\) in Example 2 under a translation of:

1. 5 units right.

2. 2 units left.

Find the coordinates of the vertices of the image of \(\triangle ABC\) in Example 1 under a reflection across:

3. the \(y\)-axis.

4. the \(x\)-axis.

Graph the image of quadrilateral \(DEFG\) at the right under each transformation from the original position.

5. 6 units down

6. reflected across the \(y\)-axis
Another type of transformation that yields congruent figures is a rotation or turn. A figure is rotated or turned about a point. To draw the image of a figure after rotation, you need to know three things: the center of rotation, the amount of turn expressed in degrees or as a fractional part of the whole turn and the direction of the rotation, clockwise or counterclockwise.

Use the following rules to find the coordinates of an image.
- When a figure is rotated $180^\circ$, $(x, y) \rightarrow (-x, -y)$.
- When a figure is rotated $90^\circ$ clockwise, $(x, y) \rightarrow (y, -x)$.
- When a figure is rotated $90^\circ$ counterclockwise, $(x, y) \rightarrow (-y, x)$.

**Example**

Graph the image of $\triangle ABC$ after a rotation of $90^\circ$ clockwise about the origin.

**Solution**

Find and graph the coordinates of $A'$, $B'$ and $C'$ using the rule $(x, y) \rightarrow (y, -x)$.
- $A(-4, 5) \rightarrow A'(5, 4)$
- $B(-6, 2) \rightarrow B'(2, 6)$
- $C(-3, 1) \rightarrow C'(1, 3)$

**Exercises**

Find the coordinates of $A'$, $B'$ and $C'$ after the given rotation about the origin.

1. $A(-4, 3), B(3, 4), C(1, 2)$ rotated $90^\circ$ clockwise

2. $A(2, 1), B(4, 2), C(1, 3)$ rotated $180^\circ$

3. $A(1, -2), B(3, -2), C(4, 1)$ rotated $90^\circ$ counterclockwise

4. Graph the image of $\triangle DEF$ after a $180^\circ$ clockwise rotation about the origin.

5. Graph the image of quadrilateral $GHIJ$ after a $90^\circ$ counterclockwise rotation about the origin.
RETEACHING 8-3

DILATIONS IN THE COORDINATE PLANE

A dilation is a transformation under which a figure is enlarged or reduced to form an image that is similar to the original figure. You need to know the center of dilation and the scale factor to graph the image of a figure under dilation.

For a center of dilation at the origin and a scale factor of \( k \), use the rule \((x, y) \rightarrow (kx, ky)\) to find the coordinates of the image.

**Example**

Graph the dilation image of \( \triangle XYZ \) using a scale factor of 2 and the center of dilation at vertex \( X \).

**Solution**

The distance from \( X \) to \( Y \) is 4 units, so the distance from \( X' \) to \( Y' \) is \( 4 \cdot 2 \), or 8 units.
The distance from \( X \) to \( Z \) is 3 units, so the distance from \( X' \) to \( Z' \) is \( 3 \cdot 2 \), or 6 units.

\( X \) and \( X' \) are at the same point since \( X \) is the center of the dilation.

**Exercises**

Find the coordinates of each point under the given dilation.

1. \((-3, 2); \text{ scale factor of } 2, \text{ center of dilation at the origin} \)

2. \((8, 4); \text{ scale factor of } \frac{1}{4}, \text{ center of dilation at the origin} \)

3. Graph the dilation image of triangle \( ABC \), using a scale factor of \( \frac{1}{2} \) and the center of dilation at the origin.

4. Graph the dilation image of rectangle \( ABCD \), using a scale factor of 3 and the center of dilation at vertex \( D \).

5. Graph the dilation image of figure \( GHIJ \), using a scale factor of \( \frac{1}{2} \) and the center of dilation at vertex \( J \).
RETEACHING 8-4

MULTIPLE TRANSFORMATIONS

Sometimes two or more successive transformations can be applied to a given figure. This is called a composite of transformations.

Example

Use \( \triangle ABC \). Perform a translation 3 units right followed by a reflection over the \( y \)-axis.

Solution

Find the coordinates for the translation.
\[
A(-6, 1) \rightarrow A'(3, 1)
\]
\[
B(-4, 2) \rightarrow B'(-1, 2)
\]
\[
C(-6, -2) \rightarrow C'(-3, -2)
\]

Find the coordinates for the reflection
\[
A'(3, 1) \rightarrow A''(3, 1)
\]
\[
B'(-1, 2) \rightarrow B''(1, 2)
\]
\[
C'(-3, -2) \rightarrow C''(3, -2)
\]

Exercise

For each exercise, perform and label the composite of transformations.

1. a rotation of 180° clockwise around the origin followed by a reflection over the \( x \)-axis

2. a dilation with center at the origin and a scale factor of 2 followed by a reflection over the \( y \)-axis

3. a translation 7 units left followed by a translation 5 units up

4. a reflection over the \( y \)-axis followed by a clockwise rotation of 90° around the origin

5. a translation 6 units to the left followed by a dilation with center at vertex \( A' \) and a scale factor of \( \frac{1}{2} \)

6. a counterclockwise rotation of 90° around the origin followed by a reflection over the \( x \)-axis
RETEACHING 8-5

ADDITION AND MULTIPLICATION WITH MATRICES

A matrix is a rectangular array of elements, or numbers, arranged into rows and columns. The matrix at the right has four rows and three columns, so it is a $4 \times 3$ (read “four by three”) matrix. $A_{13}$ means the element in row 1, column 3. So $A_{13} = 7$.

Example 1

Find the sum of matrix $C$ and matrix $D$.

Solution

$C = \begin{bmatrix} 12 & 9 & 16 \\ 10 & 11 & 12 \\ 7 & 28 & 19 \end{bmatrix}$ \quad $D = \begin{bmatrix} 32 & 25 & 3 \\ 16 & 34 & 26 \\ 44 & 9 & 10 \end{bmatrix}$

$C + D = \begin{bmatrix} 12 + 32 & 9 + 25 & 16 + 3 \\ 10 + 16 & 11 + 34 & 12 + 26 \\ 7 + 44 & 28 + 9 & 19 + 10 \end{bmatrix} = \begin{bmatrix} 44 & 34 & 19 \\ 26 & 45 & 38 \\ 51 & 37 & 29 \end{bmatrix}$

Example 2

Find the product.

Solution

$7 \begin{bmatrix} 3 & 5 & 8 \\ 2 & 9 & 8 \end{bmatrix} = 7 \cdot \begin{bmatrix} 3 & 5 & 8 \\ 2 & 9 & 8 \end{bmatrix} = \begin{bmatrix} 7 \cdot 3 & 7 \cdot 5 & 7 \cdot 8 \\ 7 \cdot 2 & 7 \cdot 9 & 7 \cdot 8 \end{bmatrix} = \begin{bmatrix} 21 & 35 & 56 \\ 14 & 63 & 56 \end{bmatrix}$

EXERCISES

Identify these elements in the matrices below.

$D = \begin{bmatrix} 7 & 3 & 1 \\ 7 & 8 & 2 \end{bmatrix}$ \quad $E = \begin{bmatrix} 9 & -5 & 0 \\ -3 & 9 & 1 \end{bmatrix}$ \quad $F = \begin{bmatrix} 3 & -9 & 8 \\ 6 & 5 & -1 \end{bmatrix}$ \quad $G = \begin{bmatrix} 4 & 2 & -6 \\ -2 & 5 & 7 \end{bmatrix}$

1. $E_{23}$

2. $G_{12}$

3. $D_{21}$

4. $F_{13}$

Refer to matrices $D, E, F$ and $G$ to help you find the following.

5. $E + F$

6. $D + F$

7. $D + G$

8. $E + G$

9. $10F$

10. $4D$

11. $5G$

12. $20E$

13. $F + 2G$

14. $D + 10E$

15. $3F + D$

16. $2D + 3E$
RETEACHING 8-6

MORE OPERATIONS ON MATRICES

Matrix multiplication is done by using row-by-column multiplication. In order to multiply matrices, **the number of columns in the first matrix must be equal to the number of rows in the second matrix.**

**Example**

Let $A = \begin{bmatrix} 20 & 40 \\ 10 & 60 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & 40 & 50 & 20 \\ 20 & 30 & 60 & 10 \end{bmatrix}$. Find $AB$.

**Solution**

The first matrix has 2 rows and 2 columns so it is a $2 \times 2$ matrix.
The second matrix has 2 rows and 4 columns so it is a $2 \times 4$ matrix.

Therefore, the product will be a $2 \times 4$ matrix.

To multiply a row by a column, multiply the first element in the row by the first element in the column. Then multiply the second element in the row by the second element in the column, and so on. Finally, sum the products of the multiplications.

$AB = \begin{bmatrix} 20 \cdot 10 + 40 \cdot 20 & 20 \cdot 40 + 40 \cdot 30 & 20 \cdot 50 + 40 \cdot 60 & 20 \cdot 20 + 40 \cdot 10 \\ 10 \cdot 10 + 60 \cdot 20 & 10 \cdot 40 + 60 \cdot 30 & 10 \cdot 50 + 60 \cdot 60 & 10 \cdot 20 + 60 \cdot 10 \end{bmatrix} = \begin{bmatrix} 1000 & 2200 & 3400 & 800 \\ 1300 & 2200 & 4100 & 800 \end{bmatrix}$

So $\begin{bmatrix} 20 & 40 \\ 10 & 60 \end{bmatrix} \cdot \begin{bmatrix} 10 & 40 & 50 & 20 \\ 20 & 30 & 60 & 10 \end{bmatrix} = \begin{bmatrix} 1000 & 2200 & 3400 & 800 \\ 1300 & 2200 & 4100 & 800 \end{bmatrix}$.

**EXERCISES**

Refer to the matrices below. Find the product for each exercise, if possible. If it is not possible to multiply, write NP.

$A = \begin{bmatrix} 7 & -1 \\ 7 & 2 \end{bmatrix}$  
$B = \begin{bmatrix} -5 & 0 \\ -3 & 9 \end{bmatrix}$  
$C = \begin{bmatrix} 3 & -9 & 8 \\ 6 & 5 & -1 \end{bmatrix}$  
$D = \begin{bmatrix} 4 & 2 & -6 \\ -2 & 5 & 7 \end{bmatrix}$

1. $AB$  
2. $CD$

3. $BC$  
4. $AC$

5. $AD$  
6. $BD$

7. $BA$  
8. $CB$

9. $DA$  
10. $DC$
RETEACHING 8-7

TRANSFORMATIONS AND MATRICES

A point can be represented by an ordered pair \((x, y)\) or by the matrix \[
\begin{bmatrix}
x \\
y
\end{bmatrix}.
\]

You can also represent transformations with matrices.

\[
\begin{array}{cc}
\text{over the } x\text{-axis} & \text{over the } y\text{-axis} & \text{over the line } y = x & \text{over the line } y = -x \\
\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} & \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}
\end{array}
\]

Example

a. Represent each vertex of \(\triangle ABC\) with a \(2 \times 1\) matrix. Then combine these matrices into a single \(2 \times 3\) matrix.

b. Find the reflection image of \(\triangle ABC\) when the triangle is reflected over the line \(y = -x\).

Solution

a. You can represent the vertices as:
\[
\begin{bmatrix} 4 & 5 & 1 \\ 6 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 5 & 1 \\ 6 & 2 & 4 \end{bmatrix}
\]

b. Multiply the matrix representing a reflection over the line \(y = -x\) by the matrix representing \(\triangle ABC\).
\[
\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 5 & 1 \\ 6 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -6 & -2 & -4 \\ -4 & -5 & -1 \end{bmatrix}
\]

So the image of \(\triangle ABC\) is
\[
\begin{bmatrix} -6 & -2 & -4 \\ -4 & -5 & -1 \end{bmatrix}.
\]

Exercise

Represent each geometric figure with a matrix.

1. 

2. 

Find the reflection image of the triangle with vertices \[
\begin{bmatrix} 1 & 4 & 3 \\ -1 & -3 & -4 \end{bmatrix}.
\]

3. over the \(x\)-axis ____________________

4. over the \(y\)-axis ____________________
RETEACHING 8-8

PROBLEM SOLVING SKILLS: USE A MATRIX

You can use a matrix as a mathematical model to help you solve many different kinds of problems.

Example

A pharmacy chain sells each toothbrush for $2.50, each tube of toothpaste for $2.00 and each box of dental floss for $4.00. During the month of May, Store A sold 200 toothbrushes, 600 tubes of toothpaste and 50 boxes of dental floss; Store B sold 100 toothbrushes, 400 tubes of toothpaste and 30 boxes of dental floss; and Store C sold 50 toothbrushes, 300 tubes of toothpaste and 20 boxes of dental floss. Find the gross revenue generated by these items at each of the three stores.

Solution

Write the prices in a $1 \times 3$ matrix.

\[
\begin{bmatrix}
2.50 & 2.00 & 4.00
\end{bmatrix}
\]

Write the number of items sold in a $3 \times 3$ matrix.

\[
\begin{bmatrix}
200 & 100 & 50 \\
600 & 400 & 300 \\
50 & 30 & 20
\end{bmatrix}
\]

Then multiply. The product is a $1 \times 3$ matrix.

\[
\begin{bmatrix}
1900 & 1170 & 805
\end{bmatrix}
\]

The gross revenue generated by Store A was $1900, by Store B was $1170, and by Store C was $805.

Exercises

A grocery store chain shows prices of its greeting cards in a $1 \times 3$ matrix and number of cards sold daily at each of its three stores in a $3 \times 3$ matrix.

\[
\begin{bmatrix}
2.00 & 1.50 & 1.00
\end{bmatrix}
\]

\[
\begin{bmatrix}
10 & 15 & 12 \\
4 & 6 & 4 \\
13 & 20 & 16
\end{bmatrix}
\]

Find the gross revenue from the cards sold at

1. Store A.
2. Store B.
3. Store C.
4. Stores A and B.
5. Stores B and C.
6. Stores A, B and C.

Find the gross revenue from each type of card sold at all three stores.

7. Birthday
8. Sympathy
9. Get Well
RETEACHING 9-1

REVIEW PERCENTS AND PROBABILITY

A knowledge of probability often plays a role in decision-making where an element of chance is involved, such as making weather predictions. You can find the probability of an event using the formula below. The result, a number between 0 and 1, can be written as a fraction, decimal or percent.

\[ P(E) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} \]

**Example 1**
Suppose you flip 2 pennies. What is the probability that the flip will show 1 head and 1 tail?

**Solution**
Make a list to find the number of possible outcomes: HH, HT, TH, TT. There are 4 possible outcomes. There are 2 favorable outcomes, HT, TH.

\[ P(\text{one tail, one head}) = \frac{2}{4} \text{ or } \frac{1}{2}, \text{ or } 0.5, \text{ or } 50\% . \]

**Example 2**
Suppose you decide to roll a die 30 times. Predict how many times you will roll a number less than 3.

**Solution**
Step 1: Find the probability.
There are 6 possible outcomes. There are 2 favorable outcomes.

\[ P(\text{number less than 3}) = \frac{2}{6} \text{ or } \frac{1}{3} \]

Step 2: Multiply the total rolls by the probability:
\[ \frac{1}{3} \cdot 30 = 10 \]
You can predict you will roll a number less than 3 about 10 times in 30 rolls.

**Exercises**

Use the cards at the right. Then give the probability of drawing at random each of the following cards.

1. \( P(N) \)  
2. \( P(C) \)  
3. \( P(I) \)

Suppose you flip 3 coins. What is the probability that the flip will show:

4. 3 heads?
5. 2 tails, 1 head?

Suppose you roll a die 60 times. Predict how many times you will roll each of the following.

6. a 4  
7. a number greater than 4  
8. a number less than 4  
9. a number other than 4  
10. an odd number  
11. a number between 3 and 4
RETEACHING 9-2

PROBLEM SOLVING SKILLS: SIMULATIONS

One way to solve a complex probability problem is to model it with a simulation.

Example

1. Read the problem carefully. Suppose you are going to take a five-question true-false test. You want to determine the probability that you will answer at least 3 of the 5 questions correctly if you guess at each answer.

2. Plan what you will do. Use a coin. Let heads stand for a correct answer and tails stand for an incorrect answer. Toss the coin 5 times for each trial and record each outcome.

3. Solve by carrying out the plan.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Number of heads</th>
<th>Number of tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

4. Answer the question asked. On the trials, heads showed on 3 or more coins 2 times. Based on these results, the probability of answering 3 of the 5 questions correctly is given by

\[
P(3 \text{ or more correct}) = \frac{\text{Number of favorable outcomes}}{5} \]

5. Check to see if your answer is reasonable. Review the design of your simulation to see that you've tested what was asked for. Determine if the number of trials is sufficient.

EXERCISES

Use a simulation to solve these problems. Record your results on another sheet of paper.

1. Continue the simulation in the example for 10 additional trials. Is the probability the same as was given in the example? Explain.

2. Each question of a multiple-choice test has 4 responses. If you guess at the answers to a ten-question test, what is the probability of answering 6 questions correctly?

3. A breeder has 6 dogs. Design a simulation to determine the probability of matching 3 dogs with their correct names.
RETEACHING 9-3

COMPOUND EVENTS

A compound event is a combination of two or more single events. Two such events that cannot occur at the same time, such as drawing a spade or a diamond from a standard deck of playing cards, are called mutually exclusive events. You can use this formula to find the probability of two mutually exclusive events, A and B.

\[ P(A \text{ or } B) = P(A) + P(B) \]

To find the complement of the event, you can use the formula
\[ P(\text{not } A) = 1 - P(A). \]

**Example**

Suppose a die is rolled. What is the probability of:

a. rolling a 3 or a 6?

b. not rolling a 3 or a 6?

**Solution**

a. Because this is a mutually exclusive event, you can use the formula to find the probability.

\[ P(3 \text{ or } 6) = P(3) + P(6) \]

\[ = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \]

The probability of rolling a 3 or 6 is \( \frac{1}{3} \).

b. Use the formula to find the complement of \( P(3 \text{ or } 6) \).

\[ P(\text{not } 3 \text{ or } 6) = 1 - P(3 \text{ or } 6) \]

\[ = 1 - \frac{1}{3} = \frac{2}{3} \]

The probability of not rolling a 3 or 6 is \( \frac{2}{3} \).

**EXERCISES**

Use the cards at the right. Then give the probability of drawing at random each of the following cards.

1. \( P(R \text{ or } T) \)  
2. \( P(B \text{ or } I) \)  
3. \( P(B, I \text{ or } O) \)

Suppose you roll a die. What is the probability of:

4. rolling a 2 or a 3?  
5. not rolling a 2 or a 3?  
6. rolling a 1, 3 or 5?  
7. not rolling an odd number?  

Suppose you roll two dice. Find the probability that the sum of the numbers if:

8. 5 or 12.  
9. not 5 or 12.  
10. 3, 6, or 9.  
11. not 3, 6, or 9.
RETEACHING 9-4

INDEPENDENT AND DEPENDENT EVENTS

A compound event can be a combination of independent events or dependent events.

Independent events: ones in which the outcome of one event has no effect on the outcome of the other. If A and B are independent events, then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Dependent events: ones in which the outcome of one event depends upon the outcome of the other. If B is dependent on A, then \( P(A \text{ and } B) = P(A) \cdot P(B), \text{ given } A \).

Example 1
Suppose you roll a die twice.
What is the probability you will roll a 3, then 6?

Solution
The two events are independent.

\[
P(3, \text{ then } 6) = P(3) \cdot P(6)
= \frac{1}{6} \cdot \frac{1}{6}
= \frac{1}{36}
\]

The probability that you will roll a 3, then 6 is \( \frac{1}{36} \).

Example 2
The teacher placed Mona's name and Kay's name along with those of four other students in a bag for a random drawing.
What is the probability that Mona's name will be drawn first and not replaced and Kay's name will be drawn second?

Solution
The two events are dependent.

First draw: \( P(\text{Mona}) = \frac{1}{6} \) ← favorable outcomes

Second draw: \( P(\text{Kay}) = \frac{1}{5} \) ← favorable outcomes

\[
P(\text{Mona, then Kay}) = P(\text{Mona}) \cdot P(\text{Kay})
= \frac{1}{6} \cdot \frac{1}{5}
= \frac{1}{30}
\]

The probability that Mona's name and then Kay's name will be drawn in that order is \( \frac{1}{30} \).

EXERCISES

Suppose you roll a die twice. What is the probability you will get the following results?

1. \( P(4, \text{ then } 5) \)       2. \( P(3, \text{ then odd}) \)       3. \( P(\text{even, then odd}) \)

A sewing basket holds 3 balls of red yarn, 2 balls of blue yarn and 5 balls of green yarn.

4. Suppose you choose each ball at random and then replace it. Find each probability.
   a. \( P(\text{blue, then red}) \)       b. \( P(\text{green, then green}) \)

5. Suppose you choose each ball at random and do not replace it. Find each probability.
   a. \( P(\text{green, then red}) \)       b. \( P(\text{green, then blue}) \)
RETEACHING 9-5
PERMUTATIONS AND COMBINATIONS

A permutation is an ordered arrangement of a set of objects. In many situations, however, you may be interested only in the objects in a set, and not the order in which they can be arranged. A set in which order is ignored is a combination.

Example 1
In how many ways can 2 books be arranged on a shelf if there are 5 books to choose from?

Solution
Use this formula to find the number of permutations.
\[ nPr = \frac{n!}{(n-r)!} \]
\[ 5P_2 = \frac{5!}{(5-2)!} \]
\[ = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \]
\[ = 20 \]

Given 5 books, there are 20 ways to arrange 2 books on a shelf.

Example 2
In how many different ways can a panel of 2 judges be selected from a group of 7 candidates?

Solution
Use this formula to find the number of combinations.
\[ nCr = \frac{n!}{(n-r)!r!} \]
\[ 7C_2 = \frac{7!}{(7-2)! \cdot 2} \]
\[ = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) (2 \cdot 1)} \]
\[ = \frac{42}{2} \]
\[ = 21 \]

There are 21 ways for a panel of 2 judges to be selected from 7 candidates.

EXERCISES

Tell whether each involves a permutation or a combination. Then solve.

1. In how many ways can 3 books be arranged on a shelf if there are 6 books to choose from?

2. In how many ways can a committee of 3 be chosen from 9 students?

3. How many kinds of pizza with 3 cheeses can be made if there are 5 cheeses available?

4. A flag locker has 8 flags. How many signals can be made by hoisting 3 flags in order, one above the other?

5. On an examination, a student can answer any 2 of the 5 questions. In how many ways can she make a choice?
RETEACHING 9-6

SCATTER PLOTS AND LINES OF BEST FIT

The scatter plot and the boxplot can be used to display data.

Example 1

Use the scatter plot to predict how many cups of hot chocolate would be sold if the temperature was 45°F.

Solution

Extend the pattern. About 15 cups would be sold.

Example 2

Make a boxplot for the number of cups of hot chocolate sold in September.

Number of cups: 15, 20, 30, 12, 18, 29, 17, 25.

Solution

Step 1: Write the data from least to greatest.

Step 2: Separate the data into 4 equal groups. Find the median of all the cups. Then find the medians of the lower half and the upper half.

Step 3: Draw a box above a number line that extends from the first quartile to the third quartile. Draw a vertical line that indicates the median and extend a line from the box on either side to the highest and lowest scores.

EXERCISES

Use the scatter plot in Example 1. Predict how many cups you would sell at the next game if the:

1. temperature is 50°F
2. temperature is 0°F

3. On another sheet of paper, make a boxplot for these test scores:
   73, 98, 47, 87, 92, 75, 93, 85.
   a. What is the median?
   b. Are the data evenly distributed?
   c. What percent of the test scores are contained in the box?
   d. Write a short paragraph describing the data.

4. On another sheet of paper, make boxplot for these test scores:
   67, 79, 98, 83, 87, 56, 89, 75. What percent of the test scores are:
   a. less than 71?
   b. greater than 71?
   c. greater than 81?
RETEACHING 9-7

STANDARD DEVIATION

Measures of dispersion such as range, variance and standard deviation are often used to describe a set of data. To find the variance of a collection of numbers \( x_1, x_2, \ldots, x_n \) with a mean of \( m \), use this formula:

\[
\frac{(x_1 - m)^2 + (x_2 - m)^2 + \ldots + (x_n - m)^2}{n}
\]

To find the standard variation, find the square root of the variance. The closer the standard deviation is to zero, the closer the data are grouped around the mean. As the standard deviation becomes greater, the data become more widely dispersed.

Example

Find the variance and standard deviation for this set of test scores: 95, 88, 76, 91, 85.

Solution

Step 1: Find the mean. \[
\frac{95 + 88 + 76 + 91 + 85}{5} = 87
\]

Step 2: Find the variance. \[
\frac{(95 - 87)^2 + (88 - 87)^2 + (76 - 87)^2 + (91 - 87)^2 + (85 - 87)^2}{5}
\]

\[
= \frac{(8)^2 + (1)^2 + (-11)^2 + (4)^2 + (-2)^2}{5} = \frac{5 + 1 + 121 + 16 + 4}{5} = \frac{206}{5} = 41.2
\]

Step 3: Find the standard deviation. \( \sqrt{41.2} \approx 6.4 \) rounded to the nearest tenth

Exercises

Find the variance and standard deviation for each set of data in Exercises 1–10. Round each answer to the nearest tenth if necessary.

1. 3, 5, 4, 1, 2
2. 2, 8, 5, 9, 1
3. 4, 6, 9, 3, 8
4. 8, 2, 10, 6, 9
5. 80, 77, 82, 87, 79
6. 37, 46, 29, 21, 32
7. 78, 68, 73, 76, 85
8. 78, 78, 95, 87, 62
9. 5, 7, 9, 11
10. 4.5, 2.5, 4.5, 2.5

11. In which of Exercises 1–10 were the data more closely clustered around the mean?

12. In which of Exercises 1–10 were the data more widely dispersed?

13. LaWanda’s class took two tests. On the first test, her score was 76, the mean score was 85 and the standard deviation was 6. On the second, her score was 70, the mean score was 79 and the standard deviation was 8. On which test did she score better, relative to the scores of her classmates?
RETEACHING 10-1

IRRATIONAL NUMBERS
A nonterminating, nonrepeating decimal is an irrational number. Some radicals are irrational numbers. You can use these properties to simplify radical expressions.

\[ \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } a \geq 0 \text{ and } b > 0. \]

\[(a \sqrt{b}) (c \sqrt{d}) = (a \cdot c) (\sqrt{b} \cdot \sqrt{d})\]

Example

Simplify.

a. \[ \sqrt{80} \]

Solution

a. Rewrite the radicand as a product of two numbers, one of which is a perfect square, then simplify.

\[ \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5} \]

So \( \sqrt{80} = 4\sqrt{5} \).

b. Use the commutative and associative properties to rewrite the expression, grouping both rational factors together and both irrational factors together. Then multiply and simplify the product.

\[ (3\sqrt{5})(6\sqrt{35}) = (3 \cdot 6)(\sqrt{5} \cdot \sqrt{35}) = 18\sqrt{175} = 18\sqrt{25 \cdot 7} = 18(5\sqrt{7}) = 90\sqrt{7} \]

So \((3\sqrt{5})(6\sqrt{35}) = 90\sqrt{7}\).

c. Write the expression in fraction form, grouping integers together and radicals together. Simplify each part of the expression and rationalize the denominator.

\[ \frac{6\sqrt{17}}{3\sqrt{34}} = \frac{6 \cdot \sqrt{17}}{3 \cdot \sqrt{34}} = 2 \cdot \sqrt{\frac{17}{34}} = 2 \cdot \sqrt{\frac{1}{2}} = \frac{2 \cdot 1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2} \cdot 2}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \]

So \(6\sqrt{17} \div 3\sqrt{34} = \sqrt{2}\).

Exercise

Simplify.

1. \( \sqrt{72} \)

2. \( \sqrt{300} \)

3. \( \sqrt{242} \)

4. \( \sqrt{56} \)

5. \( \sqrt{480} \)

6. \( \sqrt{96} \)

7. \( \sqrt{368} \)

8. \( \sqrt{768} \)

9. \( (4\sqrt{3})(5\sqrt{15}) \)

10. \( (3\sqrt{2})(4\sqrt{28}) \)

11. \( (5\sqrt{8})(5\sqrt{8}) \)

12. \( (6\sqrt{21})(4\sqrt{7}) \)

13. \( (2\sqrt{5})(7\sqrt{20}) \)

14. \( \sqrt{24} \div \sqrt{18} \)
RETEACHING 10-2

THE PYTHAGOREAN THEOREM

The relationship between the two legs and the hypotenuse of a right triangle can be stated by the Pythagorean Theorem, \(a^2 + b^2 = c^2\).

Example 1

Find the unknown length. Round your answer to the nearest tenth.

Solution

Substitute the values of the leg and the hypotenuse into the Pythagorean Theorem and solve.

\[
\begin{align*}
a^2 + b^2 &= c^2 \\
a^2 + 2^2 &= 6^2 \\
a^2 + 4 &= 36 \\
a^2 &= 32 \\
\sqrt{a^2} &= \sqrt{32} \\
a &= 4\sqrt{2} \text{ or } 5.7
\end{align*}
\]

So the leg of the triangle is 5.7 cm.

Example 2

Determine if each triangle is a right triangle. Write yes or no.

Solution

Substitute the measures into the Pythagorean Theorem. The greatest measure is the hypotenuse.

\[
\begin{align*}
a^2 + 7^2 &= 8^2 \\
16 + 49 &= 64 \\
65 \neq 64 \\
\text{So \(\triangle ABC\) is a right triangle.}
\end{align*}
\]

\[
\begin{align*}
a^2 + 15^2 &= 17^2 \\
64 + 225 &= 289 \\
289 &= 289 \\
\text{So \(\triangle DEF\) is a right triangle.}
\end{align*}
\]

EXERCISES

Find the unknown length. Round your answer to the nearest tenth.

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________

Determine if each triangle is a right triangle. Write yes or no.

7. ________
8. ________
9. ________
10. \(\triangle GHI\) with 12-ft, 16-ft, 20-ft sides
11. \(\triangle HIJ\) with 5-ft, 12-ft, 13-ft sides
RETEACHING 10-3
SPECIAL RIGHT TRIANGLES

Unique formulas describe the relationship between the sides in some right triangles.

Angle measures: $30^\circ$, $60^\circ$, and $90^\circ$

$$c = 2b$$
$$a = b\sqrt{3}$$

**Example 1**

Find the missing measures. Simplify your answer.

**Solution**

Use the relationship between the sides in a 30-60-90 triangle: $DE = EF\sqrt{3}$.

$$6 = EF\sqrt{3}$$
$$\frac{1}{\sqrt{3}}(6) = EF\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)$$

Multiply each side by $\frac{1}{\sqrt{3}}$.

$$\frac{6}{\sqrt{3}} = EF$$

Rationalize the denominator.

$$2\sqrt{3} = EF$$

$$DF = 2EF = 2(2\sqrt{3}) = 4\sqrt{3}$$

So $EF$ measures $2\sqrt{3}$ cm and $DF$ measures $4\sqrt{3}$ cm.

**Example 2**

Find the missing measures. Simplify your answer.

**Solution**

Use the relationship between the sides in a 45-45-90 triangle: $HI = GH\sqrt{2}$.

$$15 = GH\sqrt{2}$$
$$\frac{15}{\sqrt{2}} = GH$$

Rationalize the denominator.

$$\frac{15\sqrt{2}}{2} = GH$$

Since $GH = GI$, each leg measures $\frac{15\sqrt{2}}{2}$ cm.

**Exercises**

Find the missing measures. Simplify your answer.

1. 
2. 
3. 

4. 
5. 
6. 

7. 
8. 
9. 

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RETEACHING 10-4
CIRCLES, ANGLES, AND ARCS

A circle contains 360 degrees.

Central angle: \( \angle ECF \)
Minor arc: \( EF \)
\( m \angle ECF = mEF \)
Inscribed angle: \( \angle EHF \)

**Example**

Find the value of \( x \) in each circle.

**Solution**

a. Since \( x \) is the measure of a central angle, it is equal to the measure of its intercepted arc. So \( x = 73^\circ \).

b. Since \( x \) is the measure of an inscribed angle, it is equal to one-half the measure of its intercepted arc. So \( x = \frac{1}{2}(156^\circ) = 78^\circ \).

c. Since \( x \) is the measure of an angle formed by the intersection of two secants inside a circle, it is equal to one-half the sum of the measures of its intercepted arcs. So \( x = \frac{1}{2}(45^\circ + 60^\circ) = \frac{1}{2}(105^\circ) = 52\frac{1}{2}^\circ \).

d. Since \( x \) is the measure of an angle formed by the intersection of two secants outside a circle, it is equal to one-half the difference of the measures of its intercepted arcs. So \( x = \frac{1}{2}(98^\circ - 26^\circ) = \frac{1}{2}(72^\circ) = 36^\circ \).

**Exercises**

Find the value of \( x \) in each circle.

1.  \( x \) 83°
2. \( x \) 108°
3. \( x \) 85°
4. \( x \) 136°
5. \( x \) 236°
6. \( x \) 48°
7. \( x \) 184°
8. \( x \) 58°
9. \( x \) 92°
RETEACHING 10-5

PROBLEM SOLVING SKILLS: CIRCLE GRAPHS

You can use a circle graph to compare data when the data are parts of a whole.

Example

Make a circle graph for this set of data.
Federal Tax Returns Processed (in thousands)

<table>
<thead>
<tr>
<th>Category</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual</td>
<td>106,994</td>
</tr>
<tr>
<td>Individual estimated tax</td>
<td>35,489</td>
</tr>
<tr>
<td>Corporate</td>
<td>3986</td>
</tr>
<tr>
<td>Other (estate, gift, etc.)</td>
<td>47,837</td>
</tr>
</tbody>
</table>

Solution

• Add all the data to find the total. $106,994 + 3986 + 35,489 + 47,837 = 194,306$
• Find what percent each category is of the total returns filed. Remember that the sum of the percents for each category is equal to 100%.

<table>
<thead>
<tr>
<th>Category</th>
<th>Total Number</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual</td>
<td>106,994</td>
<td>55%</td>
</tr>
<tr>
<td>Individual estimated tax</td>
<td>35,489</td>
<td>18%</td>
</tr>
<tr>
<td>Corporate</td>
<td>3986</td>
<td>2%</td>
</tr>
<tr>
<td>Other (estate, gift, etc.)</td>
<td>47,837</td>
<td>25%</td>
</tr>
</tbody>
</table>

• Determine the central angles by multiplying 360° by each percent calculated.

<table>
<thead>
<tr>
<th>Category</th>
<th>Percentage</th>
<th>Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual</td>
<td>55%</td>
<td>198°</td>
</tr>
<tr>
<td>Individual estimated tax</td>
<td>18%</td>
<td>65°</td>
</tr>
<tr>
<td>Corporate</td>
<td>2%</td>
<td>7°</td>
</tr>
<tr>
<td>Other (estate, gift, etc.)</td>
<td>25%</td>
<td>90°</td>
</tr>
</tbody>
</table>

• Construct the circle graph.

a. Use a compass to draw a circle.
b. Draw one radius.
c. Use a protractor to mark the proper angle degree for individual tax returns. (198°)
d. Place the protractor along the new line and mark the proper angle degree for corporate returns. (7°)
e. Repeat Step d for the individual estimated tax returns and other returns.

Exercises

Find the percent and measure of the central angle for each set of data. Make a circle graph for each set of data on your own paper. Round each answer to the nearest whole number.

1. Media Market Value (in billions)

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Publishing</td>
<td>$249.2</td>
</tr>
<tr>
<td>Cable</td>
<td>88.2</td>
</tr>
<tr>
<td>Movie/video</td>
<td>29.9</td>
</tr>
<tr>
<td>Broadcasting</td>
<td>113.1</td>
</tr>
<tr>
<td>Oxygen</td>
<td>97.5</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>15.0</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>4.5</td>
</tr>
<tr>
<td>Carbon</td>
<td>27.0</td>
</tr>
<tr>
<td>Other</td>
<td>6.0</td>
</tr>
</tbody>
</table>

2. Elements in a 150-lb person (in lb)

<table>
<thead>
<tr>
<th>Element</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen</td>
<td>97.5</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>15.0</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>4.5</td>
</tr>
<tr>
<td>Carbon</td>
<td>27.0</td>
</tr>
<tr>
<td>Other</td>
<td>6.0</td>
</tr>
</tbody>
</table>
**Reteaching 10-6**

**Circles and Segments**

There is a relationship between the parts of some line segments that intersect circles.

\[
ac = bd \quad (a + b)b = (c + d)d \quad a^2 = (b + c)b \quad AX = XB
\]

**Example**

Find the value of \(x\) in each figure.

a. \[\begin{array}{c}
\begin{array}{c}
\text{9} \\
\text{x}
\end{array}
\end{array}\]

b. \[\begin{array}{c}
\begin{array}{c}
\text{7} \\
\text{8} \quad \text{8}
\end{array}
\end{array}\]

c. \[\begin{array}{c}
\begin{array}{c}
\text{3} \\
\text{x}
\end{array}
\end{array}\]

d. \[\begin{array}{c}
\begin{array}{c}
\text{2} \\
\text{2}
\end{array}
\end{array}\]

**Solution**

a. Substitute and solve for \(x\) in the formula for two intersecting chords.
\[8(9) = 6(x)\]
\[72 = 6x\]
\[12 = x\]
So the value of \(x\) is 12

b. Substitute and solve for \(x\) in the formula for two secant segments.
\[8(7 + 8) = 5(5 + x)\]
\[8(15) = 25 + 5x\]
\[120 = 25 + 5x\]
\[95 = 5x\]
\[19 = x\]
So the value of \(x\) is 19.

c. Substitute and solve for \(x\) in the formula for a radius and a perpendicular chord.
\[3 = x\]
So the value of \(x\) is 3.

d. Substitute and solve for \(x\) in the formula for a tangent segment and a secant segment.
\[x^2 = (6 + 2)(2)\]
\[x^2 = 8(2)\]
\[x^2 = 16\]
\[x = 4\]
So the value of \(x\) is 4.

**Exercises**

1. \[\begin{array}{c}
\begin{array}{c}
\text{18} \\
\text{x} \quad \text{12} \quad \text{20}
\end{array}
\end{array}\]

2. \[\begin{array}{c}
\begin{array}{c}
\text{4} \\
\text{6}
\end{array}
\end{array}\]

3. \[\begin{array}{c}
\begin{array}{c}
\text{10} \\
\text{x} \quad \text{12}
\end{array}
\end{array}\]

4. \[\begin{array}{c}
\begin{array}{c}
\text{9} \\
\text{x}
\end{array}
\end{array}\]

5. \[\begin{array}{c}
\begin{array}{c}
\text{3} \\
\text{13}
\end{array}
\end{array}\]

6. \[\begin{array}{c}
\begin{array}{c}
\text{15} \\
\text{x} \quad \text{25}
\end{array}
\end{array}\]

7. \[\begin{array}{c}
\begin{array}{c}
\text{8} \\
\text{x} \quad \text{4}
\end{array}
\end{array}\]

8. \[\begin{array}{c}
\begin{array}{c}
\text{6} \\
\text{x} \quad \text{8}
\end{array}
\end{array}\]
RETEACHING 10-7

CONSTRUCTIONS WITH CIRCLES

A polygon is circumscribed about a circle if each of its sides is tangent to the circle.

A polygon is inscribed in a circle if each of its vertices lies on the circle.

Example 1

Inscribe the heptagon in a circle.

Solution

• Construct a perpendicular bisector of one of the sides of the heptagon.

• Construct a perpendicular bisector of another side.

• The point of intersection of these bisectors will be the center of the circle. Place the compass tip on this point.

• The distance from the center to any vertex will be the radius of the circle. Place the pencil tip on a vertex.

• Draw the circle.

Example 2

Circumscribe the nonagon about a circle.

Solution

• Construct a perpendicular bisector of one of the sides of the nonagon. Repeat for another side.

• The point of intersection of these bisectors will be the center of the circle. Place the compass tip on this point.

• Locate the point where a bisector intersects the nonagon. The distance from the center to this point will be the radius of the circle. Place the pencil tip on this point of intersection.

• Draw the circle.

EXERCISES

Draw each circle.

1. Inscribe the triangle in a circle.

2. Inscribe the pentagon in a circle.

3. Circumscribe the hexagon about a circle.

4. Circumscribe the octagon about a circle.
**RETEACHING 11-1**

**ADDING AND SUBTRACTING POLYNOMIALS**

You can simplify a polynomial when you group and then combine all of its like terms, such as:

\[ 5x^2 + 3xy + y^2 - 5xy + 2x^2 + 3xy + 3y^2 - 9x^2 + 7xy - 7xy = -2x^2 + xy + 4y^2. \]

To add two polynomials, combine their like terms and write the polynomial in standard form.

To subtract two polynomials, add the opposite of the polynomial being subtracted to the other polynomial. Then write in standard form.

### Example 1

Add \( 5x^2 + 3xy - y^2 \) and \( 2x^2 + xy + 9 \).

**Solution**

\[
\begin{align*}
5x^2 + 3xy - y^2 &= 0 \\
+ 2x^2 + xy + 0y^2 &= 9 \\
\hline
7x^2 + 4xy - y^2 + 9
\end{align*}
\]

**Example 2**

Subtract \( 4x^2 - 3x + 2 \) from \( 8x^2 - 5x - 8 \).

**Solution**

First line up like terms. Then change signs and add.

\[
\begin{align*}
8x^2 - 5x - 8 &= 8x^2 - 5x - 8 \\
-(4x^2 - 3x + 2) &= + (-4x^2 + 3x - 2) \\
\hline
4x^2 - 2x - 10
\end{align*}
\]

### Exercises

Simplify.

1. \( (3b - 6) + (4b^2 - 6b + 10) \)

2. \( (4a + b) + (2a - 3b) \)

3. \( (7m^2 + 8mn - 9) + (2m^2 - 10mn + 1) \)

4. \( (-3c^2 + 12cd - 7) + (5c^2 - 9cd + d) \)

5. \( (7a^2 - 3a + 5) - (-a^2 + 4a - 10) \)

6. \( (5b^2 + 7bc - 9c^2) - (b^2 + 9bc + 2c^2) \)

7. \( (7t^2 - 5t) - (-4t^2 + 3t - 7) \)

8. \( (7x^2 + xy - 3y^2) - (-4x^2 + 7xy + 12) \)

9. \( (8j^2 - 4j + 10) + (2j^2 - 8j + 2) \)

10. \( (-4m^2 + 2mn - n^2) - (2m^2 + 3mn - 18) \)

11. \( (8k^3 - 6k^2 + 12) - (3k^3 + 5k + 10) \)

12. \( (5ab^2 - 2ab + 4a^2b) + (-4ab + 2a^2b - 8) \)
RETEACHING 11-2
MULTIPLYING BY A MONOMIAL
When you multiply a polynomial by a monomial, the answer always has
the same number of terms as the polynomial. Remember to multiply each
term of the polynomial by the term of the monomial.

**Example 1**

Simplify \((7ab)(8c)\).

**Solution**

\[
(7ab)(8bc) = \\
(7)(a)(b)(8)(b)(c) = \\
(7)(8)(a)(b)(b)(c) = \\
56ab^2c
\]

**Example 2**

Simplify \(2x^2(x^2 + 3x - 5)\).

**Solution**

\[
2x^2(x^2 + 3x - 5) = \\
2x^2(x^2) + (2x^2)(3x) - (2x^2)(5) = \\
2x^4 + 6x^3 - 10x^2
\]

**Exercises**

Simplify.

1. \(a(abc)\)_

2. \((8xy)(9y^2z)\)_

3. \((4m^2)(8mn^2)\)_

4. \(3b(b - 8)\)_

5. \(3m^2(m - 2n)\)_

6. \(x^2(a - b)\)_

7. \(-9d(d + 6)\)_

8. \(-2a^2b(3ab^2 - 7b)\)_

9. \(4x(x^2 + 3x - 6)\)_

10. \(7n^2(8m^2n - 7mn - 6n)\)_

11. \(-8a^3(3ab^2 - 2b + b^2)\)_

12. \(-12x^2y^2(3x - 4xy + 2y)\)_

13. \(14x(4x^2 - 3x + 9)\)_

14. \(25m(-8m^2 + 6m - 4)\)_

15. \(8abc^2(a^2bc - a^2b^3 - a)\)
RETEACHING 11-3

DIVISION AND FINDING FACTORS

The process of writing \(6a^2 + 4a - 2ab\) as \(2a(3a + 2 - b)\) is called factoring. The terms \(2a\) and \(3a + 2 - b\) are the factors. To factor a polynomial, first find the greatest common factor (GCF) of its monomial terms by following these steps.

**Step 1:** First find the greatest possible number that will divide each coefficient evenly.

**Step 2:** Then find each variable that is included in all the monomial terms. Then find its paired factor.

Then find its paired factor.

**Example 1**

Find the greatest common factor (GCF) of \(4x^2 + 8xy^2\). Then find its paired factor.

**Solution**

4 will divide evenly into both 4 and 8. The variable \(x\) is the only variable factor of every term. So the GCF of \(4x^2 + 8xy^2\) is \(4x\).

To find the paired factor, divide each term by the GCF, \(4x\).

\[
\frac{4x^2}{4x} + \frac{8xy^2}{4x} = x + 2y^2
\]

The paired factors are \(4x\) and \(x + 2y^2\), so \(4x^2 + 8xy^2 = 4x(x + 2y^2)\).

**Example 2**

Find the greatest common factor (GCF) of \(10a^2 + 5ab\). Then find its paired factor.

**Solution**

5 will divide evenly into both \(10a^2\) and \(5ab\). The variable \(a\) is the only variable factor of every term. So the GCF of \(10a^2 + 5ab\) is \(5a\).

To find the paired factor, divide each term by the GCF, \(5a\).

\[
\frac{10a^2}{5a} + \frac{5ab}{5a} = 2a + b
\]

The paired factors are \(5a\) and \(2a + b\), so \(10a^2 + 5ab = 5a(2a + b)\).

**Exercises**

Find the GCF for each polynomial. Then find its paired factor.

1. \(9x + 12\)
2. \(a^2 + 4a\)
3. \(7a^3 - 14a^2\)
4. \(15y^4 + 12y^2z\)
5. \(8x^5 - 5x^4 + 2x^3\)
6. \(8x^{10} - 24x^5 + 6x^4\)
7. \(27y^5 - 9y^3\)
8. \(a^3b^2 - 3a^4b\)
9. \(9y^4 - 3y^3 + y^2\)
10. \(15k^3 + 5k + 10\)
11. \(12x^4 + 6x^2 - 3x\)
12. \(2f^3 - 18f^2 + 8f\)
13. \(16m^2n + 4mn - 8mn^2\)
14. \(12a^2b^2 + 8ab^2 + 16b^2\)
15. \(-9y^2z - 12y^4z^3 + 15y^3z^4\)
RETEACHING 11-4
MULTIPLYING TWO BINOMIALS
To multiply a binomial by a binomial, multiply each term of one binomial by each term of the other and then combine like terms.

Example
Find \((a + 2b)(3a - 4b)\).

Solution
You can use the distributive property to help you multiply binomials. Be sure to combine like terms.

\[
(a + 2b)(3a - 4b) = a(3a - 4b) + 2b(3a - 4b) = 3a^2 - 4ab + 6ab - 8b^2 = 3a^2 + 2ab - 8b^2
\]

EXERCISES
Find each product. Use the method with which you feel most comfortable.

1. \((x - 3)(x - 8)\)
2. \((a + 3)(a + 2)\)
3. \((m - 9)(m - 5)\)
4. \((w + 2)(w + 9)\)
5. \((n + 7)(n - 8)\)
6. \((p - 6)(p + 9)\)
7. \((2t - 4)(t + 3)\)
8. \((5s - 8)(2s - 4)\)
9. \((3b + 4)(2b + 7)\)
10. \((4 - x)(2 + x)\)
11. \((a + b)(a + b)\)
12. \((4m - 2n)(3m + 6n)\)
13. \((3a - 2b)(3a + 2b)\)
14. \((5c - 8d)(3c + 4d)\)
15. \((12x + 4y)(10x - 7y)\)
16. \((2x + y)(2x + y)\)
17. \((2x - y)(2x - y)\)
18. \((2x + y)(2x - y)\)
RETEACHING 11-5
FINDING BINOMIAL FACTORS IN A POLYNOMIAL

In developing a strategy to find binomial factors, it is important to follow this strategy.

Step 1: First look for a common monomial factor.
Step 2: Within the polynomial, make pairs of terms that share monomial factors.
Step 3: Extract the monomial factors in each pair.
Step 4: When the binomials in each pair are identical, the monomials create a second binomial factor.

Example 1
Find factors for $4ac + 12bc - 2ad - 6bd$.

Solution

$$4ac + 12bc - 2ad - 6bd = 2(2ac + 6bc - ad - 3bd) = 2[(2ac + 6bc) - (ad + 3bd)] = 2[2c(a + 3b) - d(a + 3b)] = 2(2c - d)(a + 3b)$$

Example 2
Find factors for $ax + bx + cx + ay + by + cy$.

Solution

There is no common monomial factor.

Go to Step 2.

$ax + bx + cx + ay + by + cy = (ax + bx + cx) + (ay + by + cy) = x(a + b + c) + y(a + b + c)$

Step 1

Step 2

Step 3

Step 4

EXERCISES

Find factors for each polynomial.

1. $10ac - 4ad + 15bc - 6bd$

2. $ac - ad + 2bc - 2bd$

3. $eg + 3eh - fg - 3fh$

4. $12wy - 6wz + 8xy - 4xz$

5. $24pr - 40ps - 12qr + 20qs$

6. $3ac - 3ad + 2bc - 2bd$

7. $36wy - 8wz - 9xy + 2xz$

8. $9ac + 3ad + 27bc + 9bd$

9. $8qs + 16qt - 2rs - 4rt$

10. $2ax + 2ay + 2az + 3bx + 3by + 3bz$

11. $6ax + 4ay + 8az - 3bx - 2by - 4bz$

12. $18mr - 4ms - 6mt + 36nr - 8ns - 12nt$
RETEACHING 11-6

SPECIAL FACTORING PATTERNS

You can use these patterns to help you factor a perfect-square trinomial or the difference of two squares.

- Perfect square trinomial, addition: \(a^2 + 2ab + b^2 = (a + b)^2\)
- Perfect square trinomial, subtraction: \(a^2 - 2ab + b^2 = (a - b)^2\)
- Difference of two squares: \(a^2 - b^2 = (a + b)(a - b)\)

Example 1

Find binomial factors for \(s^2 - 10s + 25\).

Solution

The first and last terms are perfect squares: \(s^2 = s \cdot s\) and \(25 = 5 \cdot 5\).
The middle term is twice the square roots of the first and last terms: \(2 \cdot 5 \cdot s\), or \(10s\).
The sign in each binomial must be negative because the middle term of the trinomial is negative and the last term is positive. So \(s^2 - 10s + 25 = (s - 5)(s - 5)\), or \((s - 5)^2\).
Check by multiplying: \((s - 5)(s - 5) = s^2 - 10s + 25\).

Example 2

Find binomial factors for \(x^2 - 9\).

Solution

The first and last terms are perfect squares: \(x^2 = x \cdot x\) and \(9 = 3 \cdot 3\).
Write the sum and difference of the square root of each term: \((x + 3)(x - 3)\).
Check by multiplying: \((x + 3)(x - 3) = x^2 - 9\).

EXERCISES

If possible, find binomial factors for each trinomial. Be sure to factor common monomial factors first. At least one does not have factors.

1. \(r^2 - 16r + 64\)  
2. \(t^2 - 81\)  
3. \(w^2 + 12w + 36\)

4. \(c^2 + 16c + 64\)  
5. \(a^2 - 16\)  
6. \(x^2 + 8x + 16\)

7. \(9y^2 - 49\)  
8. \(25n^2 - 30n + 16\)  
9. \(4b^2 - 40b + 100\)

10. \(9z^2 + 12z + 4\)  
11. \(25a^2 - 60a + 36\)  
12. \(4x^2 - 64\)
RETEACHING 11-7

FACTORIZING TRINOMIALS

When you factor trinomials, it is important to keep these points in mind.

- The first term in the trinomial is the product of the first terms of the binomials.
- The last term in the trinomial is the product of the second terms of the binomials.
- The coefficient of the middle term of the trinomial is the sum of the second terms of the binomials.
- If the sign of the last term of the trinomial is negative, the signs in each binomial factor are different; if the sign is positive, the signs in each binomial factor are the same—either positive or negative, depending on the sign of the trinomial’s middle term.

Example

Find binomial factors for \(a^2 - 3a - 10\).

Solution

To find the first terms, factor \(a^2\): \((a\quad)(a\quad)\)
To find the last terms, factor 10: \((a\ 5)(a\ 2)\)
Determine the signs: \((a - 5)(a + 2)\)

Since the sign of the last term in the polynomial is negative, the signs in each binomial factor must be different. Notice that \(-5a + 2a = -3a\), the middle term of the trinomial.

Check your work by multiplying: \((a - 5)(a + 2) = a^2 - 3a - 10\)
The answer checks. ✔

EXERCISES

Find binomial factors for each trinomial.

1. \(a^2 + 5a + 6\)
2. \(x^2 - 9x - 10\)
3. \(m^2 + 6m - 16\)
4. \(t^2 - 2t - 35\)
5. \(s^2 + 11s + 24\)
6. \(k^2 + k - 20\)
7. \(r^2 - r - 72\)
8. \(x^2 - 11x + 28\)
9. \(y^2 + 8y + 12\)
10. \(b^2 + 10b + 24\)
11. \(d^2 + 9d + 20\)
12. \(p^2 - 4p - 21\)
13. \(k^2 + 2k - 120\)
14. \(r^2 - r - 90\)
15. \(z^2 + 5z - 150\)
RETEACHING 11-8

PROBLEM SOLVING SKILLS: THE GENERAL CASE

It is often helpful to use algebra to help you create a general case when looking for patterns in mathematics.

Example

Explore the pattern when a single monomial factor can be extracted before factoring a perfect-square trinomial involving addition. Work forward from $m(2x + 3)^2$ as the specific example.

Solution

Multiply the binomial factors using the FOIL method.

<table>
<thead>
<tr>
<th>General case</th>
<th>Specific case</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $m(ax + by)^2$</td>
<td>$2(2x + 3)^2$</td>
</tr>
<tr>
<td>b. $m(a^2x^2 + abxy + abxy + b^2y^2)$</td>
<td>$2(4x^2 + 6x + 6x + 9)$</td>
</tr>
<tr>
<td>c. $m(a^2x^2 + 2abxy + b^2y^2)$</td>
<td>$2(4x^2 + 12x + 9)$</td>
</tr>
<tr>
<td>d. $ma^2x^2 + 2mabxy + mb^2y^2$</td>
<td>$8x^2 + 24x + 18$</td>
</tr>
</tbody>
</table>

Study the pattern beginning with the polynomials in c and d.

There is a common factor—$m$ in the general case and 2 in the specific case. It is important always to extract monomial factors before looking for binomial factors. The product of the F and L terms (4 and 9) in the trinomial is identical to the product of the O and I coefficients (6 and 6). The coefficients of O and I (6 and 6) add to give the second term (12) of the trinomial. Both the F and L terms are perfect squares.

EXERCISES

Explore the pattern involved when a single monomial factor can be extracted before factoring a trinomial involving the difference of two squares.

Use $m(ax + by)(ax - by)$ for the general case and $5(2x + 4)(2x - 4)$ as the specific example.
RETEACHING 11-9
MORE ON FACTORING TRINOMIALS

The FOIL method you used in multiplying binomials in Lesson 5 can help you factor trinomials that contain two binomial factors. When the F-coefficient is *not* 1, try different pairs of factors until you find the pair that gives the correct middle term. Always check by multiplying.

**Example**

Find binomial factors for $6x^2 + 11x + 3$.

**Solution**

<table>
<thead>
<tr>
<th>F</th>
<th>O + I</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6x^2$</td>
<td>$+11x$</td>
<td>$+3$</td>
</tr>
</tbody>
</table>

The F-coefficient is 6; the L-term is 3. The sum of O + I is 11.

1 • 6 Possible factors
2 • 3

The signs in the binomial factors are positive because both O and L are positive.

Try 1 and 6 for F, 1 and 3 for L: $(x + 3)(6x + 1) = 6x^2 + 19x + 3$ Doesn't check!

$(x + 1)(6x + 3) = 6x^2 + 9x + 3$ Doesn't check!

Try 2 and 3 for F, 1 and 3 for L: $(3x + 3)(2x + 1) = 6x^2 + 9x + 3$ Doesn't check!

$(2x + 3)(3x + 1) = 6x^2 + 11x + 3$ It checks! ✔

**EXERCISES**

Use + or − to complete each binomial factor.

1. $8x^2 - 38x + 24 = (4x ___ 3)(2x ___ 8)$
2. $15x^2 + 26x + 8 = (5x ___ 2)(3x ___ 4)$
3. $18x^2 + 16x - 2 = (9x ___ 1)(2x ___ 2)$
4. $42x^2 - 13x - 40 = (7x ___ 8)(6x ___ 5)$
5. $12x^2 + 12x - 9 = (6x ___ 9)(2x ___ 1)$
6. $28x^2 - 5x - 3 = (7x ___ 3)(4x ___ 1)$

Find binomial factors for each trinomial. Check by multiplying.

7. $4x^2 + 8x + 3$
8. $6x^2 - 11x + 4$
9. $8x^2 - 8x - 6$

10. $9x^2 - 6x - 8$
11. $15x^2 + 14x + 3$
12. $8x^2 - 2x - 3$

13. $9x^2 - 18x + 8$
14. $15x^2 + x - 2$
15. $6x^2 - 5x - 50$

16. $6x^2 - 17x - 14$
17. $6x^2 + 21x - 12$
18. $10x^2 - 15x - 25$
RETEACHING 12-1

GRAPHING PARABOLAS

A quadratic equation in \( x \) contains an \( x^2 \) term and involves no term with a higher power of \( x \). When the domain of a quadratic function is the set of real numbers, the graph is a parabola. The vertex of a parabola is the minimum or maximum point of a parabola depending upon whether it opens upward or downward.

Example

Graph \( y = 2x^2 - 4 \) for the domain of real numbers. Then locate the vertex of the parabola.

Solution

Make a table of five ordered pairs by selecting \( x \)-values and solving the equation to find \( y \)-values. Then graph the points and draw a smooth curve. To find the vertex, look for a \( y \)-value that has only one \( x \)-value. The vertex is (0, -4). The axis of symmetry is the line \( x = 0 \).

EXERCISES

Graph each function for the domain of real numbers. Then give the coordinates of the vertex for each graph.

1. \( y = x^2 + 3 \)

2. \( y = -x^2 + 2 \)

3. \( y = 2x^2 \)

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex  

Vertex
RETEACHING 12-2

THE GENERAL QUADRATIC FUNCTION

The general quadratic function may be written as $f(x) = ax^2 + bx + c$.
The general form of the quadratic function is defined by the standard quadratic equation $y = ax^2 + bx + c$, where $a$, $b$ and $c$ are real numbers and $a \neq 0$. You can use $x = -\frac{b}{2a}$ to find the $x$-value of the vertex and identify the axis of symmetry.

Example

Find the vertex and axis of symmetry for $y = x^2 - 2x + 1$. Then graph each equation.

Solution

Find the coordinates of the vertex.

$x = \frac{-b}{2a}$

$y = x^2 - 2x + 1$  Substitute 1 into the equation

$x = \frac{-2}{2(1)}$  Substitute $-\frac{b}{a}$ for $b$ and 1 for $a$.

$x = \frac{2}{2}$, or 1

The vertex is (1, 0). The axis of symmetry is $x = 1$. Make a table of ordered pairs, graph the points and draw a smooth curve.

EXERCISES

Find the vertex and identify the axis of symmetry. Then graph each function.

1. $y = x^2 + 2x + 3$

2. $y = -2x^2 + 3$

3. $y = x^2 + 2x - 3$

Vertex  ________________  Vertex  ________________  Vertex  ________________
Axis of symmetry _______  Axis of symmetry _______  Axis of symmetry _______
RETEACHING 12-3

FACTOR AND GRAPH

You can either graph the quadratic function or find the \( x \)-intercepts (solutions) of the quadratic equation by factoring. There can be zero, one or two possible solutions.

Example 1

Use a graphing calculator to determine the number of solutions for \( y = x^2 - 1 \). If the equation has one or two solutions, find the exact solution(s) by factoring.

Solution

Graph the equation on a graphing calculator. The graph of the equation crosses the \( x \)-axis in two places. So there are two solutions.

To solve by factoring, let \( y = 0 \).

\[
\begin{align*}
x^2 - 1 &= 0 \\
(x + 1)(x - 1) &= 0 \\
x &= -1, x = 1
\end{align*}
\]

The solutions for \( y = x^2 - 1 \) are \( x = -1 \) and \( x = 1 \).

Example 2

Use a graphing calculator to determine the number of solutions for \( x^2 + 6x = y - 9 \). If the equation has one or two solutions, find the exact solution(s) by factoring.

Solution

Graph the equation on a graphing calculator. The graph of the equation crosses the \( x \)-axis in one place. So there is one solution.

Write the equation in standard form: \( y = x^2 + 6x + 9 \)

To solve by factoring, let \( y = 0 \).

\[
\begin{align*}
x^2 + 6x + 9 &= 0 \\
(x + 3)(x + 3) &= 0 \\
x &= -3
\end{align*}
\]

The solution for \( y = x^2 + 6x + 9 \) is \( x = -3 \).

EXERCISES

Use a graphing calculator to determine the number of solutions for each equation. For equations with one or two solutions, find the exact solutions by factoring.

1. \( y = x^2 - 81 \) 
2. \( y = x^2 - 5x - 6 \)
3. \( y = x^2 - 2x + 1 \) 
4. \( y = x^2 + 6x - 27 \)
5. \( y = x^2 + 2x - 15 \) 
6. \( y = x^2 - 2x - 35 \)
7. \( x^2 = x + 56 + y \) 
8. \( y = x^2 - x - 6 \)
9. \( 2x^2 = y - x \) 
10. \( y + x = x^2 + 1 \)
11. \( y + 9x = 3x^2 \) 
12. \( x^2 + 8x = 9 + y \)
13. \( y = \frac{1}{2}x^2 + 5x \) 
14. \( y = \frac{1}{4}x^2 - 4 \)
RETEACHING 12-4

COMPLETING THE SQUARE

Making a perfect square for an expression in the form $x^2 + bx$ is called completing the square. You can use this process to help you solve quadratic equations.

Remember, $c = \left(\frac{b}{2}\right)^2$, so $x^2 + bx + c = x^2 + bx + \left(\frac{b}{2}\right)^2$.

Example 1

Complete the square for $x^2 + 6x$.

Solution

\[x^2 + 6x + \left(\frac{b}{2}\right)^2\] Add $\left(\frac{b}{2}\right)^2$ to complete the square.

\[x^2 + 6x + \left(\frac{6}{2}\right)^2\] Find $\left(\frac{b}{2}\right)^2 \cdot \left(\frac{6}{2}\right)^2 = 3^2 = 9$

\[x^2 + 6x + 9\]
The answer is $x^2 + 6x + 9$.

Example 2

Solve by completing the square $x^2 + 6x + 5 = 0$

Solution

\[x^2 + 6x = -5\] Add $-5$ to each side.

\[x^2 + 6x + \left(\frac{6}{2}\right)^2 = -5 + \left(\frac{6}{2}\right)^2\] Add $\left(\frac{b}{2}\right)^2$ to each side.

\[x^2 + 6x + 9 = -5 + 9\]

\[(x + 3)^2 = 4\] Factor then simplify.

\[x + 3 = \sqrt{4}\]
\[x + 3 = \pm 2\]
\[x = -3 - 2, \text{ so } x = -5\]
\[x = -3 + 2, \text{ so } x = -1\]

EXERCISES

Complete the square.

1. $x^2 + 8x$  
2. $x^2 - 4x$
3. $x^2 + 10x$  
4. $x^2 + 2x$
5. $x^2 - 12x$  
6. $x^2 - 14x$

Solve by completing the square.

7. $x^2 + 6x - 7 = 0$  
8. $x^2 - 4x - 5 = 0$
9. $x^2 - 16x - 17 = 0$  
10. $x^2 + 10x + 9 = 0$
11. $x^2 - 12x + 11 = 0$  
12. $x^2 + 4x - 45 = 0$
13. $x^2 + 10x + 16 = 0$  
14. $x^2 + 12x + 11 = 0$
15. $x^2 + 24x + 44 = 0$  
16. $x^2 - 8x - 33 = 0$
RETEACHING 12-5

THE QUADRATIC FORMULA

The quadratic formula can be used to solve all types of quadratic equations in the form \( ax^2 + bx + c = 0, a \neq 0 \), and is stated in these terms:

If \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

**Example 1**

Use the quadratic formula to solve \( 2x^2 - 3x - 2 = 0 \).

**Solution**

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}
\]

\[
x = \frac{3 \pm \sqrt{9 + 16}}{4}
\]

\[
x = \frac{3 \pm \sqrt{25}}{4}
\]

\[
x = \frac{3 \pm 5}{4}
\]

\[
x = \frac{3 + 5}{4}, \text{ or } 2 \quad \text{and} \quad x = \frac{3 - 5}{4}, \text{ or } -\frac{1}{2}
\]

There are two solutions: 2 and \(-\frac{1}{2}\).

**Example 2**

Use the quadratic formula to solve \( x^2 - 4x - 10 = 0 \).

**Solution**

\[
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}
\]

\[
x = \frac{4 \pm \sqrt{16 + 40}}{2}
\]

\[
x = \frac{4 \pm \sqrt{56}}{2}
\]

\[
x = \frac{4 \pm \sqrt{4(14)}}{2} = \frac{4 \pm 2\sqrt{14}}{2} = 2 \pm \sqrt{14}
\]

There are two solutions: \( x = 2 + \sqrt{14} \) and \( x = 2 - \sqrt{14} \).

**EXERCISES**

Use the quadratic formula to solve each equation. Remember, some equations may have only one solution.

1. \( x^2 - 7x + 10 = 0 \)
2. \( 6x^2 - 4x - 10 = 0 \)
3. \( 10x^2 - 6x - 4 = 0 \)
4. \( x^2 + 20x + 100 = 0 \)
5. \( x^2 - 4x - 12 = 0 \)
6. \( 2x^2 + 10x + 12 = 0 \)
7. \( x^2 + 10x - 12 = 0 \)
8. \( x^2 - 6x - 2 = 0 \)
9. \( x^2 - 14x - 12 = 0 \)
10. \( x^2 + 4x - 1 = 0 \)
11. \( 2x^2 + 2x - 24 = 0 \)
12. \( x^2 - 14x + 25 = 0 \)
13. \( x^2 + 30x + 225 = 0 \)
14. \( 2x^2 + 8x + 4 = 0 \)
RETEACHING 12-6
DISTANCE FORMULA

The distance, \( d \), between any two points \((x_1, y_1)\) and \((x_2, y_2)\) on the coordinate plane is given by the distance formula.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

**Example 1**
Use the distance formula to find the distance between point \(A(3, -1)\) and point \(B(2, -2)\).

**Solution**
\[
d = \sqrt{(2 - 3)^2 + [-2 - (-1)]^2}
\]
\[
= \sqrt{(-1)^2 + (1)^2}
\]
\[
= \sqrt{1 + 1}
\]
\[
= \sqrt{2} \approx 1.4
\]

The distance between points \(A\) and \(B\) is approximately 1.4 units.

**Example 2**
Use the midpoint formula to find the midpoint of the line segment with endpoints \(C(-2, 1)\) and \(D(6, -3)\).

**Solution**
\[
(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
\[
= \left(\frac{-2 + 6}{2}, \frac{1 - 3}{2}\right)
\]
\[
= \left(2, \frac{-2}{2}\right)
\]
\[
= (2, -1)
\]

The midpoint of \(CD\) is \((2, -1)\).

**Exercises**

Use the distance formula to find the distance between each pair of points. If necessary, round your answer to the nearest tenth.

1. \((0, 3), (-2, 0)\)  
2. \((-3, 2), (-4, 7)\)  
3. \((-4, 2), (-2, -3)\)

4. \((-2, -3), (-1, -4)\)  
5. \((2, 7), (-3, -5)\)  
6. \((2, 7), (3, 5)\)

7. \((-3, 4), (5, 2)\)  
8. \((4, 5), (-3, -1)\)  
9. \((4, 3), (3, -6)\)

Use the midpoint formula to find the midpoint between each pair of points.

10. \((4, 5), (2, 3)\)  
11. \((3, 5), (7, 1)\)  
12. \((-4, 8), (2, -4)\)

13. \((-5, 3), (1, 5)\)  
14. \((6, 5), (-4, -3)\)  
15. \((-1, -2), (-3, -2)\)

16. \((0, -7), (8, -1)\)  
17. \((-9, -8), (-3, 6)\)  
16. \((-10, 5), (6, -3)\)
RETEACHING 12-7

PROBLEM SOLVING SKILL: USE GRAPHS TO WRITE EQUATIONS

If you are given the graph of a quadratic equation or three or more points on the graph, you can find the equation.

Example

The graph of a quadratic equation contains the three points (−2, 0), (1, −3), and (0, −4). Find the equation of the parabola.

Solution

Substitute each ordered pair into the general form of the quadratic equation, \( y = ax^2 + bx + c \), to create a system of three equations.

For (−2, 0) \( 0 = 4a - 2b + c \)

For (1, −3) \( -3 = a + b + c \)

For (0, −4) \( -4 = c \)

Substitute \( c = -4 \) in the other two equations.

\( 0 = 4a - 2b - 4 \)

\( 4 = 4a - 2b \)

\( 4 = 4a - 2b \)

\( 6a = 6 \)

\( a = 1 \)

To find \( b \), substitute \( a = 1 \) and \( c = -4 \) into \(-3 = a + b + c\) and solve.

\(-3 = 1 + b - 4\)

\( 0 = b \)

Since \( a = 1, b = 0 \) and \( c = -4 \), the equation is \( y = x^2 - 4 \).

Exercises

1. The graph of an equation contains the three points (0, −3), (3, 12) and (−2, −3).

Find the equation of the parabola.  ________________________________

2. The graph of an equation contains the three points (0, 4), (−3, −2) and (1, 10).

Find the equation of the parabola.  ________________________________

3. The graph of an equation contains the three points (0, −6), (2, 0) and (−1, −6).

Find the equation of the parabola.  ________________________________
Reteaching 13-1

The Standard Equation of a Circle

The standard equation of a circle is \((x - h)^2 + (y - k)^2 = r^2\), where \(r \neq 0\). The point \((x, y)\) lies on the circumference of the circle, and point \((h, k)\) is the center. If \(h = 0\) and \(k = 0\), the standard equation reduces to \(x^2 + y^2 = r^2\).

Example 1

Write an equation for each circle with the given radius and center.

a. radius 3 units and center \((0, 0)\)

Solution

a. Since the center is at the origin, the equation is of the form \(x^2 + y^2 = r^2\).

\[
x^2 + y^2 = r^2 \\
x^2 + y^2 = 3^2 \\
x^2 + y^2 = 9
\]

Example 2

Find the radius and the center for each circle.

a. \(x^2 + y^2 = 25\)

Solution

a. Since the equation is of the form \(x^2 + y^2 = r^2\), the center is at the origin.

\[
x^2 + y^2 = r^2 \\
x^2 + y^2 = 25 \\
r^2 = 25, \text{ so } r = 5
\]

The radius is 5; the center \((0, 0)\).

Exercises

Write an equation for each circle.

1. radius 5, center \((3, -4)\)

2. radius 3, center \((-5, 0)\)

Find the radius and center for each circle.

3. \(x^2 + y^2 = 144\)

4. \((x - 3)^2 + (y + 6)^2 = 21\)

5. \((x + 4)^2 + (y + 5)^2 = 34\)

6. \((x - 2)^2 + (y - 4)^2 = 18\)
MORE ON PARABOLAS

A parabola consists of all those points that are equidistant from a fixed point, called the focus, and a fixed line called the directrix. The equation of the form \( x^2 = 4ay \) represents a parabola with these properties:

- The vertex is \((0, 0)\).
- The focus is \((0, a)\).
- The directrix is the line \( y = -a \).
- If \( a > 0 \), the parabola opens upward. If \( a < 0 \), the parabola opens downward.

**Example 1**

Find the focus and directrix of the equation \( x^2 = 2y \).

**Solution**

\( x^2 = 4ay \) and \( x^2 = 2y \) both describe a parabola with the properties described above. So \( 4ay = 2y \) and \( a = \frac{1}{2} \).

The focus is \( (0, \frac{1}{2}) \).

The directrix is \( y = -\frac{1}{2} \).

**Example 2**

Find the simple equation for a parabola that has its vertex, at the origin and focus at \((0, -4)\).

**Solution**

\[ x^2 = 4ay \]

Substitute \(-4\) for \( a \).

\[ x^2 = 4(-4)y \]

Simplify.

\[ x^2 = -16y \]

Write the equation.

**EXERCISES**

Find the focus and directrix of each equation.

1. \( x^2 = -12y \)
2. \( x^2 = 5y \)
3. \( x^2 = -24y \)
4. \( x^2 = 16y \)
5. \( x^2 = -13y \)
6. \( x^2 - 8y = 0 \)

Find the simple equation for each parabola with vertex located at the origin.

7. Focus \((0, -7)\)
8. Focus \((0, 8)\)
9. Focus \((0, 2)\)
10. Focus \((0, -9)\)
11. Focus \((0, -5)\)
12. Focus \( \left(0, -\frac{1}{3}\right) \)
RETEACHING 13-3

PROBLEM SOLVING SKILLS: VISUAL THINKING

Often one needs to visualize what an object will look like when viewed from another direction.

Example

Imagine that the three-dimensional figure is cut as shown and that the two resulting pieces are separated. What shape would be formed at the cross section?

Solution

The oval shape that is formed is called an ellipse.

EXERCISES

Imagine that a three-dimensional figure is cut as shown and that the two resulting pieces are separated. What shape would be formed at the cross section?

1.  

2.  

3.  

4.  

5.  

6.  

7.  

8.  

9.  

10.  

11.  

12.  

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RETEACHING 13-4
ELLIPSES AND HYPERBOLAS

The standard form for the equation of an ellipse with its center at the origin (0, 0) and foci $F_1$ and $F_2$ on the x-axis is $rac{x^2}{a^2} + rac{y^2}{b^2} = 1$. The standard form for the equation for a hyperbola that is symmetric around the origin (0, 0) and has foci $F_1$ and $F_2$ on the x-axis is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

**Example 1**
Find the equation of the ellipse with foci (3, 0) and (−3, 0) and x-intercepts (−4, 0) and (4, 0).

**Solution**
Substitute known values into the equation and simplify.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

\[
\frac{x^2}{16} + \frac{y^2}{7} = 1
\]

Multiply by 112.

7$x^2 + 16y^2 = 112$ Write the equation.

**Example 2**
Find the equation of a hyperbola with center (0, 0) in which $a = 5$, $b = 2$, and the foci are on the x-axis.

**Solution**
Substitute known values into the equation and simplify.

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

\[
\frac{x^2}{25} - \frac{y^2}{4} = 1
\]

Multiply by 100.

$4x^2 - 25y^2 = 100$ Write the equation.

**Exercises**

Find the equation of each ellipse.

1. Foci (2, 0) and (−2, 0) and x-intercepts (−4, 0) and (4, 0) _________________________
2. Foci (4, 0) and (−4, 0) and x-intercepts (5, 0) and (−5, 0) _________________________
3. Foci (−5, 0) and (5, 0) and x-intercepts (−6, 0) and (6, 0) _________________________
4. Foci (−2, 0) and (2, 0) and x-intercepts (7, 0) and (−7, 0) _________________________
5. Foci (3, 0) and (−3, 0) and x-intercepts (5, 0) and (−5, 0) _________________________
6. Foci (−6, 0) and (6, 0) and x-intercepts (−7, 0) and (7, 0) _________________________

Find the equation of each hyperbola with its center at the origin (0, 0) and foci on the x-axis.

7. $a = \pm 4$ and $b = \pm 2$ _________________________
8. $a = \pm 3$ and $b = \pm 4$ _________________________
9. $a = \pm 6$ and $b = \pm 4$ _________________________
10. $a = \pm 5$ and $b = \pm 8$ _________________________
11. $a = \pm 9$ and $b = \pm 7$ _________________________
12. $a = \pm 7$ and $b = \pm 5$ _________________________
Reteaching 13-5

Direct Variation

Direct variation can be represented by an equation in the form $y = kx$ where $k$ is a nonzero constant and $x \neq 0$. The constant $k$ is called the constant of variation.

Example 1

Write the question for a direct variation when one pair of values is $x = 32$ and $y = 14$.

Solution

Substitute in the equation and solve for $k$.

$$y = kx$$

$$14 = k(32)$$

$$\frac{14}{32} = k$$

$$0.4375 = k$$

The equation is $y = 0.4375x$.

Example 2

The weight of a metal rod varies directly with its length. A rod 4 cm long weighs 12 g. How much does a rod 20 cm long weigh?

Solution

First find $k$. Then solve.

$$y = kx$$

$$12 = k(4)$$

$$3 = k$$

$$y = 3x$$

$$y = 3(20)$$

$$y = 60$$

A rod that is 20 cm long weighs 60 g.

Exercises

Write the equation for the direct variation for each pair of values.

1. $x = 40; y = 20$

2. $x = 25; y = 12$

3. $x = 15; y = 3$

4. $x = 30; y = 12$

5. $x = 25; y = 20$

6. $x = 16; y = 10$

Find each answer.

7. The weight of a metal rod varies directly with its length. A rod 5 cm long weighs 12 g. How much does a rod 40 cm long weigh?

8. The length of a shadow at a given time is directly proportional to the height of an object. If an 8-ft tree casts a 6-ft shadow, how high is a building that casts a 90-ft shadow at the same time?

9. A freight train can travel 260 mi in 5 hours. How far can it travel in 8 hours?

10. The weight of earth varies directly with the weight on the moon. An astronaut weighs 210 lbs on Earth and 35 lb on the moon. How much would a 180-lb astronaut weigh on the moon?
RETEACHING 13-6

INVERSE VARIATION

Inverse variation can be represented by an equation in the form \( y = \frac{k}{x} \)
where \( k \) is a nonzero constant and \( x \neq 0 \). When the value of one variable
increases, the value of the other variable decreases.

**Example 1**

Write the equation in which \( y \) varies inversely with \( x \) if one pair of values is
\( y = 64 \) and \( x = 0.6 \).

**Solution**

Substitute in the equation and solve for \( k \).

\[
y = \frac{k}{x}
\]

\[
64 = \frac{k}{0.6}
\]

\[
64 \times 0.6 = k
\]

\[
38.4 = k
\]

The equation is \( y = \frac{38.4}{x} \).

**Example 2**

The resistance in an electrical circuit with constant voltage is inversely proportional to the current.
If a light bulb has a resistance of 60 ohms when a current of 2 amperes flows through it, what is the resistance in another bulb when a current of 1.25 amperes flows through it?

**Solution**

First find \( k \). Then solve.

\[
y = \frac{k}{x}
\]

\[
y = \frac{k}{2}
\]

\[
60 = \frac{k}{2}
\]

\[
120 = k
\]

\[
y = \frac{120}{1.25}
\]

\[
y = 96
\]

The bulb has a resistance of 96 ohms.

**Exercises**

Write the equation in which \( y \) varies inversely with \( x \) for each pair of values.

1. \( x = 0.3; y = 40 \)  
2. \( x = 2; y = 36 \)

3. \( x = 4; y = 35 \)  
4. \( x = 0.9; y = 100 \)

5. \( x = 0.6; y = 75 \)  
6. \( x = 3; y = 74 \)

Find each answer.

7. The resistance in an electrical circuit with constant voltage is inversely proportional to the current. If a light bulb has a resistance of 55 ohms when a current of 2 amperes flows through it, what is the resistance in another bulb when a current of 1.375 amperes flows through it? 

8. The time that it takes to drive a given distance varies inversely with the rate of speed. If Rosita drives 50 mi/h for 4 hours, how long would it take her to drive the same distance at 60 mi/h?
RETEACHING 13-7
QUADRATIC INEQUALITIES

You can use quadratic equations like these to help you graph quadractic inequalities.

Circle: \( x^2 + y^2 = r^2 \) where \( r \neq 0 \) and center is at the origin (0, 0)

Parabola: \( x^2 = 4ay \) with center at the origin and focus is on the y-axis

Ellipse: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) with center at the origin and foci on the x-axis

Hyperbola: \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) with foci on the x-axis and symmetric around the origin

Example

Solve this system of inequalities by graphing: \( x^2 + y^2 \geq 25 \) and \( y < -x^2 \).

Solution

Graph the circle with a radius of 5. The circle is the solution set, so draw the circle with a solid line. Point (0, 0) and the region that contains it are not in the solution set, so the area outside the circle is in the solution set.

Graph parabola \( y < -x^2 \). The parabola is not in the solution set, so draw the parabola with a dashed line. Point (0, 0) is in the solution set so the area inside the parabola is in the solution set.

The cross-hatched section shows the intersection of the two equations.

EXERCISES

Graph each system of inequalities.

1. \( y > x^2 \) and \( 16x^2 + 9y^2 \leq 144 \)
2. \( 9x^2 - 4y^2 > 36 \) and \( x^2 + y^2 \leq 9 \)
3. \( x^2 + y^2 < 16 \) and \( 25x^2 + 4y^2 \leq 100 \)
RETEACHING 13-8

EXPONENTIAL FUNCTIONS

An exponential function has the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

<table>
<thead>
<tr>
<th>Properties of an Exponential Function</th>
<th>1. The function is continuous and one-to-one.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2. The domain is the set of all real numbers.</td>
</tr>
<tr>
<td></td>
<td>3. The x-axis is the asymptote of the graph.</td>
</tr>
<tr>
<td></td>
<td>4. The range is the set of all positive numbers if $a &gt; 0$ and all negative numbers if $a &lt; 0$.</td>
</tr>
<tr>
<td></td>
<td>5. The graph contains the point $(0, a)$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exponential Growth and Decay</th>
<th>If $a &gt; 0$ and $b &gt; 1$, the function $y = ab^x$ represents exponential growth.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>If $a &gt; 0$ and $0 &lt; b &lt; 1$, the function $y = ab^x$ represents exponential decay.</td>
</tr>
</tbody>
</table>

**Example 1**

Sketch the graph of $y = 0.1(4)^x$. Then state the function's domain and range.

Make a table of values. Connect the points to form a smooth curve.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.025</td>
<td>0.1</td>
<td>0.4</td>
<td>1.6</td>
<td>6.4</td>
</tr>
</tbody>
</table>

The domain of the function is all real numbers, while the range is the set of all positive real numbers.

**Example 2**

Determine whether each function represents exponential growth or decay.

a. $y = 0.5(2)^x$
   exponential growth, since the base, 2, is greater than 1

b. $y = -2.8(2)^x$
   neither, since $-2.8$, the value of $a$ is less than 0

c. $y = 1.1(0.5)^x$
   exponential decay, since the base, 0.5, is between 0 and 1

**Exercises**

Sketch the graph of each function. Then state the function's domain and range.

1. $y = 3(2)^x$
2. $y = -2(\frac{1}{4})^x$
3. $y = 0.25(5)^x$

Determine whether each function represents exponential growth or decay.

4. $y = 0.3(1.2)^x$
5. $y = -5(\frac{4}{5})^x$
6. $y = 3(10)^{-x}$
**LOGARITHMIC FUNCTIONS**

**Definition of Logarithm with Base b**

Let $b$ and $x$ be positive numbers, $b \neq 1$. The logarithm of $x$ with base $b$ is denoted $\log_b x$ and is defined as the exponent $y$ that makes the equation $b^y = x$ true.

The inverse of the exponential function $y = b^x$ is the **logarithmic function** $x = \log_b y$.

This function is usually written as $y = \log_b x$.

**Properties of Logarithmic Functions**

1. The function is continuous and one-to-one.
2. The domain is the set of all positive real numbers.
3. The $y$-axis is an asymptote of the graph.
4. The range is the set of all real numbers.
5. The graph contains the point $(1, 0)$.

**Example 1**

Write an exponential equation equivalent to $\log_3 243 = 5$.

$3^5 = 243$

**Example 2**

Write a logarithmic equation equivalent to $6^{-3} = \frac{1}{216}$.

$\log_6 \frac{1}{216} = -3$

**Example 3**

Evaluate $\log_8 16$.

$8^1 = 16$, so $\log_8 16 = \frac{4}{3}$.

**EXERCISES**

Write each equation in logarithmic form.

1. $2^7 = 128$
2. $3^{-4} = \frac{1}{81}$
3. $\left(\frac{1}{7}\right)^3 = \frac{1}{343}$

Write each equation in exponential form.

4. $\log_{15} 225 = 2$
5. $\log_3 \frac{1}{27} = -3$
6. $\log_4 32 = \frac{5}{2}$

Evaluate each expression.

7. $\log_4 64$
8. $\log_2 64$
9. $\log_{100} 100,000$

10. $\log_5 625$
11. $\log_{27} 81$
12. $\log_{25} 5$

13. $\log_2 \frac{1}{128}$
14. $\log_{10} 0.00001$
15. $\log_4 \frac{1}{32}$

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RETEACHING 14-1

BASIC TRIGONOMETRIC RATIOS

For a given angle in a right angle, the ratio of the lengths of two sides are always the same.

sine \( \sin \theta = \frac{\text{length of side opposite } \angle A}{\text{hypotenuse}} \)

cosine \( \cos \theta = \frac{\text{length of side adjacent to } \angle A}{\text{hypotenuse}} \)

tangent \( \tan \theta = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A} \)

Example

Given: \( \triangle ABC \)

a. Find \( \sin A \).  
   b. Find \( \cos A \).  
   c. Find \( \tan A \).

Solution

a. \( \sin A = \frac{\text{length of side opposite } \angle A}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{6}{10} = \frac{3}{5} \)

b. \( \cos A = \frac{\text{length of side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{8}{10} = \frac{4}{5} \)

c. \( \tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A} = \frac{BC}{AC} = \frac{6}{8} = \frac{3}{4} \)

Exercise

Use the figures to find each ratio. Express answers in simplest terms.

1. \( \tan D \)  
   \( \sin E \)  
   \( \cos E \)  
   \( \cos D \)

2. \( \tan G \)  
   \( \sin I \)  
   \( \sin G \)  
   \( \cos I \)

3. \( \cos L \)  
   \( \sin J \)  
   \( \sin L \)  
   \( \tan J \)

4. \( \tan O \)  
   \( \sin O \)  
   \( \tan N \)  
   \( \cos N \)
RETEACHING 14-2
SOLVING RIGHT TRIANGLES
A right triangle is solved by finding the measures of all its angles and sides. You can find the missing parts of a right triangle by using trigonometric ratios (sine, cosine, tangent) along with the Pythagorean Theorem and the sum of the angles property.

\[
\begin{align*}
\text{sine} &= \frac{\text{length of opposite side}}{\text{length of hypotenuse}} \\
\text{cosine} &= \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} \\
\text{tangent} &= \frac{\text{length of opposite side}}{\text{length of adjacent side}}
\end{align*}
\]

Example

Find the following in \(\triangle ABC\).

a. \(m\angle B\)

b. \(m\angle C\)

c. \(AC\) to the nearest tenth

Solution

a. Determine which trigonometric ratio relates the length of the known side (\(AB\) and \(CB\)) and the unknown acute angle (\(\angle B\)). The ratio is the cosine.

\[
\cos B = \frac{\text{length of side adjacent to } \angle B}{\text{length of hypotenuse}} = \frac{AB}{BC} = \frac{6}{17} \approx 0.3529
\]

Use your calculator or the Table of Trigonometric Ratios to find the measure of the angle with a cosine value of about 0.3529. The \(m\angle B \approx 69^\circ\).

b. \(m\angle C = 180^\circ - (90^\circ + 69^\circ) = 21^\circ\). So the measure of \(\angle C\) is about 21°.

c. Use the Pythagorean Theorem to find \(AC\):

\[
(AC)^2 + (AB)^2 = (CB)^2.
\]

\[
(AC)^2 + 6^2 = 17^2
\]

\[
(AC)^2 + 36 = 289
\]

\[
(AC)^2 = 253
\]

\[AC \approx 15.9\]

EXERCISES

For each triangle, find the measures of line segments to the nearest tenth and angles to the nearest degree.

1. \(DF \quad m\angle D \quad m\angle E\)

2. \(m\angle H \quad GI \quad GH\)

3. \(JK \quad JL \quad m\angle K\)

4. \(m\angle O \quad m\angle N \quad ON\)
RETEACHING 14-3

GRAPHING THE SINE FUNCTION

The acute angle formed by the intersections of the terminal side and the x-axis is the **reference angle**. You can use reference angles to find trigonometric ratios for obtuse angles.

The relationships of the sides in these triangles can help you find trigonometric ratios.

- **30°-60°-90°**: Length of side opposite 30° angle: \( x \)
  - Length of side opposite 60° angle: \( x \sqrt{3} \)
  - Length of hypotenuse: \( 2x \)

- **45°-45°-90°**: Length of each leg: \( x \)
  - Length of hypotenuse: \( x \sqrt{2} \)

**Example**

Find \( \sin 855° \).

**Solution**

- To form an angle of 855°, the initial side must complete two 360° rotations, then continue an additional 135°.
- The measure of the reference angle is the difference between the additional rotation and the measure of a straight angle. The reference angle measures \( 180° - 135° = 45° \), so a 45°-45°-90° triangle is formed.
- The leg adjacent to the 45° angle measures \( -1 \) relative to the x-axis. The leg opposite the angle measures \( +1 \) relative to the y-axis. The terminal side, or hypotenuse, is always positive.
- Find the length of the terminal side of the 45°-45°-90° triangle:
  \[ c = s \sqrt{2} = 1 \sqrt{2} \text{ or } \sqrt{2} \]
- To find \( \sin 855° \), find the sine of the reference angle.
  \[ \sin 855° = \sin 45° = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{+1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \]

**EXERCISES**

Find each ratio by drawing a reference angle.

1. \( \sin 390° \)
2. \( \sin 120° \)
3. \( \sin 405° \)
4. \( \sin 945° \)
5. \( \sin 510° \)
6. \( \sin 600° \)
7. \( \sin 1035° \)
8. \( \sin 840° \)
9. \( \sin 420° \)
RETEACHING 14-4

EXPERIMENTING WITH THE SINE FUNCTION

A periodic function repeats the same pattern over and over.
The length of one complete copy is the **period**.
The period of a graph \( y = \sin nx \) is \( \frac{360^\circ}{n} \).
The **amplitude** of a periodic function is a measure of the height and depth of a curve.
The amplitude of the graph \( y = n \sin x \) is \( |n| \).
The **position** of the graph of a periodic function is the relationship between the graph and the standard graph of the function.
The position of \( y = \sin x \pm n \) is the graph of \( y = \sin x \) raised or lowered \( n \) units.

**Example**

State the period and the amplitude for the graph of the equation \( y = 6 \sin 5x + 4 \). Then describe its position.

**Solution**

Use the rules to state the period, amplitude, and position of each graph.
Find the period by substituting 5 for \( n \) in \( y = \sin nx \). The period is \( \frac{360^\circ}{n} \).
Since \( \frac{360^\circ}{5} = 72^\circ \), the period is 72°.
Find the amplitude by substituting 6 for \( n \) in \( y = n \sin x \). The amplitude is 6 units.
Find the position of the graph by substituting \(+ 4\) for \( n \) in \( y = \sin x \pm n \).
The position of the graph is raised 4 units above its normal position.

**Exercises**

State the period and the amplitude for the graph of each equation. Describe its position.

1. \( y = 3 \sin 5x - 2 \)
   - Period ______________
   - Amplitude ______________
   - Position ______________

2. \( y = 5 \sin 3x + 1 \)
   - Period ______________
   - Amplitude ______________
   - Position ______________

3. \( y = 3 \sin 4x - 3 \)
   - Period ______________
   - Amplitude ______________
   - Position ______________

4. \( y = 4 \sin 1.5x + 5 \)
   - Period ______________
   - Amplitude ______________
   - Position ______________
PROBLEM SOLVING SKILLS: CHOOSE A STRATEGY

Once you understand a problem, you must then make a plan to help you solve the problem. Part of your plan will involve deciding upon the strategy or strategies you will use. Some of the strategies you have used in this book include the following:

- Choose a Method and an Operation
- Using Logical Reasoning
- Solve a Simpler Problem
- Use Indirect Measurement
- Using Statistics
- Making a Circle Graph
- Solve a Quadratic Equation
- Using a Spreadsheet
- Find a Pattern
- Using Linear Programming
- Use a Matrix
- Study the General Case
- Choose a Formula
- Writing and Solve an Equation
- Reading and Interpreting Graphs
- Guess and Check
- Draw a Diagram
- Work Backwards
- Make an Organized List
- Make a Model

EXERCISES

Solve. Tell what strategy or strategies you used. Round each answer to the nearest tenth if necessary.

1. Jason joins 2 triangular quilt pieces to form a square with sides of 4 in. What is the length of the side of each triangle that will be joined together to make the square?

2. The difference in Carrie’s age and Carter’s age is 5. The sum of their ages is 31. Carrie is the oldest. Find the age of each person.

3. Huang, José, and Sam ran a 100-m dash. José did not come in second; Huang did not come in third. José’s time was 2 seconds faster than that of the oldest boy in the race. In what order did the boys finish the race?

4. A cable car ascends from the Skier’s Lodge—altitude of 2315 m—to the top of the mountain—altitude 3459 m. The length of the cable is 2168 m. What is the distance from the lodge to the point directly under the summit of the mountain?

5. The sun is shining on a tree 40 ft tall. The sun’s rays form a 35° angle with the ground. How long is the shadow cast by this tree?

6. Suppose there are 25 students in your class. If each student shakes hands with every other student once and only once, how many handshakes will there be?

7. Tiffany rode her bicycle 8 mi south from her home to visit her grandmother and then 6 mi west to go to the library. From this point, there is a direct route home. How many miles is it from her home to the library?