K–12 Mathematics TEKS
Revised 2009

Texas Essential Knowledge and Skills

Printed and distributed by the Charles A. Dana Center
at The University of Texas at Austin

Texas Administrative Code (TAC), Title 19, Part II, Chapter 111,
Texas Essential Knowledge and Skills for Mathematics
(Subchapters A through D)

The 2009 revised TEKS, to be implemented beginning in 2009–2010
NEW → Includes the English Language Proficiency Standards
Mathematics Standards in the Classroom: Classroom Activities Aligned to the Texas Standards for Kindergarten-Grade 2 (1st edition, with revised TEKS; 2008)

Intended for: Grades K-2 mathematics educators

This resource provides a clarifying activity for each of the 114 student expectations in the mathematics TEKS for grades K-2, plus additional activities for some expectations. Each clarifying activity has assessment connections, which link classroom activities to the mathematics TEKS. This resource also provides 13 clarifying lessons (four each for grades K and 1; 5 for grade 2) that bring together several TEKS expectations. 257 pages.

Price (book): $40.00
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Mathematics Standards in the Classroom: Classroom Activities Aligned to the Texas Standards for Grades 3-5 (2nd edition, with revised TEKS; 2008)

Intended for: Grades 3-5 mathematics educators

This resource provides a clarifying activity for each of the 123 student expectations in the mathematics TEKS for grades 3-5, plus additional activities for some expectations. Each clarifying activity has assessment connections, which link classroom activities to the mathematics TEKS. This resource also provides six clarifying lessons (two each for grades 3, 4, and 5) that bring together several TEKS expectations. 320 pages.

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Mathematics Standards in the Classroom: Classroom Activities Aligned to the Texas Standards for Grades 6-8 (2nd edition, with revised TEKS; 2008)

Intended for: Grades 6-8 mathematics educators

This resource provides 124 clarifying activities. Each clarifying activity has assessment connections, which link classroom activities to the mathematics TEKS. This resource also provides 6 clarifying lessons (two each for grades 6, 7, and 8) that bring together several TEKS expectations in multi-day activities. 363 pages.

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Mathematics TEKS Charts (K-8 or 6-12, with revised TEKS; 2006)

The Mathematics TEKS Chart: Grades K-8 and Mathematics TEKS Chart: Grades 6-12 are full-color, poster-size wall charts. These charts are designed to conveniently display all the knowledge and skills and student expectation statements for the mathematics Texas Essential Knowledge and Skills, in a way that articulates how key themes in the TEKS develop from one grade or course to another.

Price (K-8 chart): $10.00
Price (6-12 chart): $10.00

Our products support high academic achievement among all students and help educators implement the Texas Essential Knowledge and Skills (TEKS) for improved student learning and success on the Texas Assessment of Knowledge and Skills (TAKS). For a collection of free resources to help implement the mathematics TEKS and strengthen mathematics education in Texas, see the Mathematics TEKS Toolkit at www.mathtekstoolkit.org. For links to all our products, both free and for sale, see www.utdanacenter.org/products.
Mathematics
Texas Essential Knowledge and Skills

For kindergarten through grade 12
Texas Administrative Code (TAC), Title 19, Part II
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
(Subchapters A through D)

Now including the
English Language Proficiency Standards
Texas Administrative Code (TAC), Title 19, Part II
Chapter 74, Curriculum Requirements; Subchapter A, Required Curriculum;
Rule §74.4, English Language Proficiency Standards

About the Mathematics Texas Essential Knowledge and Skills
The mathematics Texas Essential Knowledge and Skills (TEKS) were developed by the state of Texas to clarify what all students should know and be able to do in mathematics in kindergarten through grade 12. The mathematics TEKS also form the objectives and student expectations for the mathematics portion of the Texas Assessment of Knowledge and Skills (TAKS) for grades 3 through 10 and for the grade 11 exit test, as well as the End-of-Course Exams for Algebra I, Geometry, and Algebra II.

Texas school districts are required to provide instruction in the mathematics TEKS, which were originally adopted by the State Board of Education in 1997 and implemented statewide in 1998. The first revision of the mathematics TEKS was adopted by the State Board of Education in 2005 and implemented statewide in 2006–2007. This first revision of the mathematics TEKS started being assessed on the TAKS beginning in 2007–2008. The second revision of the mathematics TEKS—which is published in this booklet—was adopted by the State Board of Education in 2009 and is to be implemented statewide in 2009–2010.

About the English Language Proficiency Standards
The English Language Proficiency Standards are part of Texas state law. The Charles A. Dana Center is providing printed copies of these standards as a public service. You may access the ELPS free online at the Texas Education Agency website, Curriculum > Bilingual/ESL Education > English Language Proficiency Standards: tea.state.tx.us/curriculum/biling/elps.html.

According to the TAC administrative rule cited in (a) Introduction to the ELPS,

“(1) The English language proficiency standards … outline English language proficiency level descriptors and student expectations for English language learners (ELLs). School districts shall implement this section as an integral part of each subject in the required curriculum. The English language proficiency standards are to be published along with the Texas Essential Knowledge and Skills (TEKS) for each subject in the required curriculum.”

About the development of this resource
The Dana Center is printing these revised mathematics TEKS—along with the English Language Proficiency Standards—as a service to educators in Texas who want a bound version of the standards. In keeping with our longstanding practice, we use all funds generated through materials we publish to further our nonprofit educational mission.

This booklet is intended to help educators familiarize themselves with the latest version of the mathematics TEKS and to use these TEKS, in conjunction with the ELPS, to plan instruction and assessment.
About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center works to raise student achievement in K–16 mathematics and science, especially for historically underserved populations. We do so by providing direct service to school districts and institutions of higher education; to local, state, and national education leaders; and to nonprofits, agencies, and professional organizations concerned with strengthening American mathematics and science education.

The Center was founded in 1991 in the College of Natural Sciences at The University of Texas at Austin. Our original purpose—which continues in our work today—was to increase the diversity of students who successfully pursue careers in science, technology, engineering, and mathematics (STEM) fields.

We carry out our work by supporting high standards and building system capacity; partnering with key state and national organizations to work on emerging issues; creating and delivering professional supports for educators and education leaders; and writing and publishing education resources, including student supports.

Our staff of more than 80 researchers and education professionals has worked intensively with dozens of school systems in 20 states and with 90 percent of Texas’s more than 1,000 school districts. As one of the college’s largest research units, the Dana Center works to further the university’s mission of achieving excellence in education, research, and public service. We are committed to ensuring that the accident of where a child attends school does not limit the academic opportunities he or she can pursue.

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Resources

The TEKS for mathematics, as well as for other subject areas, can be downloaded in printable format, free of charge, via the Texas Education Agency website, www.tea.state.tx.us. The mathematics TEKS can also be downloaded free from the Mathematics TEKS Toolkit, www.mathtekstoolkit.org, a resource of the Charles A. Dana Center at The University of Texas at Austin. Bound versions of the mathematics TEKS can be ordered from the Dana Center product catalog at www.utdanacenter.org/catalog or by contacting the Dana Center at 1-866-871-9995.

The Dana Center also provides resources, including professional development, for implementing the state standards. Many resources can be found in the Mathematics TEKS Toolkit at www.mathtekstoolkit.org. See www.utdanacenter.org/catalog for our products and www.utdanacenter.org/pd to sign up for our professional development. You may also find out about professional development opportunities by calling 512-471-6190.

For more information about the Dana Center and our programs and resources, see our homepage at www.utdanacenter.org.
The Dana Center is adding the English Language Proficiency Standards to our TEKS booklets as the booklets come up for revision or reprinting. You may also access the ELPS free online at the Texas Education Agency website, Curriculum > Bilingual Education > English Language Proficiency Standards: 

ritter.tea.state.tx.us/curriculum/biling/elps.html.

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Introduction: A few of the changes in the revised Mathematics TEKS

The second revision of the mathematics TEKS—which is published in this booklet—was adopted by the State Board of Education in 2009 and is to be implemented statewide in 2009–2010. This latest revision aligned the existing mathematics TEKS to the Texas College and Career Readiness Standards (www.thecb.state.tx.us/collegereadiness/CCRS.pdf) adopted by the Texas Higher Education Coordinating Board in January 2008.

These changes are relatively minimal. Overall, the 2009 revision:

• Added 3 Student Expectations statements: 1 in middle school and 2 in high school
• Modified 3 existing Knowledge and Skills statements: 2 in middle school and 1 in high school
• Modified 4 existing student expectations: 3 in middle school and 1 in high school

There were not any changes to the elementary mathematics TEKS.

In this introduction, boldface within a TEKS statement marks the words added to the statement.

Middle School (MS) Texas Essential Knowledge and Skills: The following changes were made:

Middle School Knowledge and Skills statements: Changes

MS 7.10: In the Knowledge and Skills statement, the “physical or mathematical model” that a student can use to “describe the experimental and theoretical probability of real-life events” now includes geometric models.

MS 7.12: In the Knowledge and Skills statement, the word range is replaced by variability as a term for describing a set of data.

Middle School Student Expectations statements: Changes and additions

MS 7.4 A: The student is expected to: “generate formulas involving unit conversions within the same system (customary and metric),” in addition to the other formulas previously listed.

MS 8.1. “Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations”: An additional student expectation was added, that students are expected to: (E) “compare and order real numbers with a calculator.”

MS 8.2 D: Now the student is expected to: “use multiplication by a given constant factor (including unit rate) to represent and solve problems involving proportional relationships including conversions between measurement systems.”

MS 8.12 A: The student is expected to: “use variability (range, including interquartile range (IQR)) and select the appropriate measure of central tendency or range to describe a set of data and justify the choice for a particular situation.” The language was clarified slightly to use range as a term for variability (instead of central tendency), and the interquartile range (IQR) was added as a range students should use to describe a set of data.

High School (HS) Texas Essential Knowledge and Skills: The following changes were made:

High School Knowledge and Skills statements: Changes

Mathematical Models with Applications, M.3: “The student develops and implements a plan for collecting and analyzing data (qualitative and quantitative) in order to make decisions.” That is, this Knowledge and Skills statement now makes explicit that plans for collecting and analyzing data must include both qualitative and quantitative data.
High School Student Expectations statements: Changes and additions

Algebra II 6.C (or 2A.6.C): The student is expected to "determine a quadratic function from its roots (real and complex) or a graph."

Geometry 8. “Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to:"

“E. use area models to connect geometry to probability and statistics; and
F. use conversions between measurement systems to solve problems in real-world situations.”

To the first Knowledge and Skills statement in the congruence and the geometry of size strand, two student expectations (E and F) were added.
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§111.11. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades K-5.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.


(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Kindergarten are developing whole-number concepts and using patterns and sorting to explore number, data, and shape.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(K.1) Number, operation, and quantitative reasoning. The student uses numbers to name quantities. The student is expected to:

(A) use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects;

(B) use sets of concrete objects to represent quantities given in verbal or written form (through 20); and

(C) use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions.
(K.2) **Number, operation, and quantitative reasoning.** The student describes order of events or objects.

The student is expected to:

(A) use language such as before or after to describe relative position in a sequence of events or objects; and

(B) name the ordinal positions in a sequence such as first, second, third, etc.

(K.3) **Number, operation, and quantitative reasoning.** The student recognizes that there are quantities less than a whole.

The student is expected to:

(A) share a whole by separating it into two equal parts; and

(B) explain why a given part is half of the whole.

(K.4) **Number, operation, and quantitative reasoning.** The student models addition (joining) and subtraction (separating).

The student is expected to model and create addition and subtraction problems in real situations with concrete objects.

(K.5) **Patterns, relationships, and algebraic thinking.** The student identifies, extends, and creates patterns.

The student is expected to identify, extend, and create patterns of sounds, physical movement, and concrete objects.

(K.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to make predictions.

The student is expected to:

(A) use patterns to predict what comes next, including cause-and-effect relationships; and

(B) count by ones to 100.

(K.7) **Geometry and spatial reasoning.** The student describes the relative positions of objects.

The student is expected to:

(A) describe one object in relation to another using informal language such as over, under, above, and below; and

(B) place an object in a specified position.

(K.8) **Geometry and spatial reasoning.** The student uses attributes to determine how objects are alike and different.

The student is expected to:

(A) describe and identify an object by its attributes using informal language;

(B) compare two objects based on their attributes; and

(C) sort a variety of objects including two- and three-dimensional geometric figures according to their attributes and describe how the objects are sorted.
(K.9) **Geometry and spatial reasoning.** The student recognizes attributes of two- and three-dimensional geometric figures.

The student is expected to:

(A) describe and compare the attributes of real-life objects such as balls, boxes, cans, and cones or models of three-dimensional geometric figures;

(B) recognize shapes in real-life three-dimensional geometric figures or models of three-dimensional geometric figures; and

(C) describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).

(K.10) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions.

The student is expected to:

(A) compare and order two or three concrete objects according to length (longer/shorter than, or the same);

(B) compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same);

(C) compare two containers according to capacity (holds more, holds less, or holds the same);

(D) compare two objects according to weight/mass (heavier than, lighter than or equal to); and

(E) compare situations or objects according to relative temperature (hotter/colder than, or the same as).

(K.11) **Measurement.** The student uses time to describe, compare, and order events and situations.

The student is expected to:

(A) compare events according to duration such as more time than or less time than;

(B) sequence events (up to three); and

(C) read a calendar using days, weeks, and months.

(K.12) **Probability and statistics.** The student constructs and uses graphs of real objects or pictures to answer questions.

The student is expected to:

(A) construct graphs using real objects or pictures in order to answer questions; and

(B) use information from a graph of real objects or pictures in order to answer questions.
(K.13) **Underlying processes and mathematical tools.** The student applies Kindergarten mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:
(A) identify mathematics in everyday situations;
(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(K.14) **Underlying processes and mathematical tools.** The student communicates about Kindergarten mathematics using informal language.

The student is expected to:
(A) communicate mathematical ideas using objects, words, pictures, numbers, and technology; and
(B) relate everyday language to mathematical language and symbols.

(K.15) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.
§111.13. Mathematics, Grade 1.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 1 are building number sense through number relationships, adding and subtracting whole numbers, organizing and analyzing data, and working with two- and three-dimensional geometric figures.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(1.1) **Number, operation, and quantitative reasoning.** The student uses whole numbers to describe and compare quantities.

The student is expected to:

(A) compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models;

(B) create sets of tens and ones using concrete objects to describe, compare, and order whole numbers;

(C) identify individual coins by name and value and describe relationships among them; and

(D) read and write numbers to 99 to describe sets of concrete objects.

(1.2) **Number, operation, and quantitative reasoning.** The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects.

The student is expected to:

(A) separate a whole into two, three, or four equal parts and use appropriate language to describe the parts such as three out of four equal parts; and

(B) use appropriate language to describe part of a set such as three out of the eight crayons are red.
(1.3) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in addition and subtraction situations.

The student is expected to:

(A) model and create addition and subtraction problem situations with concrete objects and write corresponding number sentences; and

(B) use concrete and pictorial models to apply basic addition and subtraction facts (up to $9 + 9 = 18$ and $18 - 9 = 9$).

(1.4) **Patterns, relationships, and algebraic thinking.** The student uses repeating patterns and additive patterns to make predictions.

The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems.

(1.5) **Patterns, relationships, and algebraic thinking.** The student recognizes patterns in numbers and operations.

The student is expected to:

(A) use patterns to skip count by twos, fives, and tens;

(B) find patterns in numbers, including odd and even;

(C) compare and order whole numbers using place value;

(D) use patterns to develop strategies to solve basic addition and basic subtraction problems; and

(E) identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as $2 + 3 = 5, 3 + 2 = 5, 5 - 2 = 3,$ and $5 - 3 = 2$.

(1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

(A) describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle);

(B) describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones;

(C) describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language; and

(D) use concrete models to combine two-dimensional geometric figures to make new geometric figures.
(1.7) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length.

The student is expected to:

(A) estimate and measure length using nonstandard units such as paper clips or sides of color tiles;
(B) compare and order two or more concrete objects according to length (from longest to shortest);
(C) describe the relationship between the size of the unit and the number of units needed to measure the length of an object;
(D) compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least);
(E) compare and order two or more containers according to capacity (from holds the most to holds the least);
(F) compare and order two or more objects according to weight/mass (from heaviest to lightest); and
(G) compare and order two or more objects according to relative temperature (from hottest to coldest).

(1.8) **Measurement.** The student understands that time can be measured. The student uses time to describe and compare situations.

The student is expected to:

(A) order three or more events according to duration; and
(B) read time to the hour and half-hour using analog and digital clocks.

(1.9) **Probability and statistics.** The student displays data in an organized form.

The student is expected to:

(A) collect and sort data; and
(B) use organized data to construct real-object graphs, picture graphs, and bar-type graphs.

(1.10) **Probability and statistics.** The student uses information from organized data.

The student is expected to:

(A) draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs; and
(B) identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.
(1.11) **Underlying processes and mathematical tools.** The student applies Grade 1 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify mathematics in everyday situations;
(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(1.12) **Underlying processes and mathematical tools.** The student communicates about Grade 1 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and
(B) relate informal language to mathematical language and symbols.

(1.13) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 2 are developing an understanding of the base-ten place value system, comparing and ordering whole numbers, applying addition and subtraction, and using measurement processes.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(2.1) **Number, operation, and quantitative reasoning.** The student understands how place value is used to represent whole numbers. The student is expected to:

(A) use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways;

(B) use place value to read, write, and describe the value of whole numbers to 999; and

(C) use place value to compare and order whole numbers to 999 and record the comparisons using numbers and symbols (<, =, >).

(2.2) **Number, operation, and quantitative reasoning.** The student describes how fractions are used to name parts of whole objects or sets of objects. The student is expected to:

(A) use concrete models to represent and name fractional parts of a whole object (with denominators of 12 or less);

(B) use concrete models to represent and name fractional parts of a set of objects (with denominators of 12 or less); and

(C) use concrete models to determine if a fractional part of a whole is closer to 0, \(\frac{1}{2}\), or 1.
(2.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts whole numbers to solve problems. The student is expected to:

(A) recall and apply basic addition and subtraction facts (to 18);

(B) model addition and subtraction of two-digit numbers with objects, pictures, words, and numbers;

(C) select addition or subtraction to solve problems using two-digit numbers, whether or not regrouping is necessary;

(D) determine the value of a collection of coins up to one dollar; and

(E) describe how the cent symbol, dollar symbol, and the decimal point are used to name the value of a collection of coins.

(2.4) **Number, operation, and quantitative reasoning.** The student models multiplication and division. The student is expected to:

(A) model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined; and

(B) model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets.

(2.5) **Patterns, relationships, and algebraic thinking.** The student uses patterns in numbers and operations. The student is expected to:

(A) find patterns in numbers such as in a 100s chart;

(B) use patterns in place value to compare and order whole numbers through 999; and

(C) use patterns and relationships to develop strategies to remember basic addition and subtraction facts. Determine patterns in related addition and subtraction number sentences (including fact families) such as 8 + 9 = 17, 9 + 8 = 17, 17 – 8 = 9, and 17 – 9 = 8.

(2.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to describe relationships and make predictions. The student is expected to:

(A) generate a list of paired numbers based on a real-life situation such as number of tricycles related to number of wheels;

(B) identify patterns in a list of related number pairs based on a real-life situation and extend the list; and

(C) identify, describe, and extend repeating and additive patterns to make predictions and solve problems.
(2.7) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

(A) describe attributes (the number of vertices, faces, edges, sides) of two- and three-dimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc.;

(B) use attributes to describe how 2 two-dimensional figures or 2 three-dimensional geometric figures are alike or different; and

(C) cut two-dimensional geometric figures apart and identify the new geometric figures formed.

(2.8) **Geometry and spatial reasoning.** The student recognizes that a line can be used to represent a set of numbers and its properties.

The student is expected to use whole numbers to locate and name points on a number line.

(2.9) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length, area, capacity, and weight/mass. The student recognizes and uses models that approximate standard units (from both SI, also known as metric, and customary systems) of length, weight/mass, capacity, and time.

The student is expected to:

(A) identify concrete models that approximate standard units of length and use them to measure length;

(B) select a non-standard unit of measure such as square tiles to determine the area of a two-dimensional surface;

(C) select a non-standard unit of measure such as a bathroom cup or a jar to determine the capacity of a given container; and

(D) select a non-standard unit of measure such as beans or marbles to determine the weight/mass of a given object.

(2.10) **Measurement.** The student uses standard tools to estimate and measure time and temperature (in degrees Fahrenheit).

The student is expected to:

(A) read a thermometer to gather data;

(B) read and write times shown on analog and digital clocks using five-minute increments; and

(C) describe activities that take approximately one second, one minute, and one hour.

(2.11) **Probability and statistics.** The student organizes data to make it useful for interpreting information.

The student is expected to:

(A) construct picture graphs and bar-type graphs;

(B) draw conclusions and answer questions based on picture graphs and bar-type graphs; and

(C) use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.
(2.12) Underlying processes and mathematical tools. The student applies Grade 2 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(2.13) Underlying processes and mathematical tools. The student communicates about Grade 2 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(2.14) Underlying processes and mathematical tools. The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.
§111.15. Mathematics, Grade 3.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 3 are multiplying and dividing whole numbers, connecting fraction symbols to fractional quantities, and standardizing language and procedures in geometry and measurement.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(3.1) **Number, operation, and quantitative reasoning.** The student uses place value to communicate about increasingly large whole numbers in verbal and written form, including money.

The student is expected to:

(A) use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999;

(B) use place value to compare and order whole numbers through 9,999; and

(C) determine the value of a collection of coins and bills.

(3.2) **Number, operation, and quantitative reasoning.** The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects.

The student is expected to:

(A) construct concrete models of fractions;

(B) compare fractional parts of whole objects or sets of objects in a problem situation using concrete models;

(C) use fraction names and symbols to describe fractional parts of whole objects or sets of objects; and

(D) construct concrete models of equivalent fractions for fractional parts of whole objects.
(3.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers.

The student is expected to:

(A) model addition and subtraction using pictures, words, and numbers; and  
(B) select addition or subtraction and use the operation to solve problems involving whole numbers through 999.

(3.4) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in multiplication and division situations.

The student is expected to:

(A) learn and apply multiplication facts through 12 by 12 using concrete models and objects;  
(B) solve and record multiplication problems (up to two digits times one digit); and  
(C) use models to solve division problems and use number sentences to record the solutions.

(3.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to:

(A) round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations; and  
(B) use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems.

(3.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to solve problems.

The student is expected to:

(A) identify and extend whole-number and geometric patterns to make predictions and solve problems;  
(B) identify patterns in multiplication facts using concrete objects, pictorial models, or technology; and  
(C) identify patterns in related multiplication and division sentences (fact families) such as $2 \times 3 = 6$, $3 \times 2 = 6$, $6 + 2 = 3$, $6 + 3 = 2$.

(3.7) **Patterns, relationships, and algebraic thinking.** The student uses lists, tables, and charts to express patterns and relationships.

The student is expected to:

(A) generate a table of paired numbers based on a real-life situation such as insects and legs; and  
(B) identify and describe patterns in a table of related number pairs based on a meaningful problem and extend the table.
(3.8) **Geometry and spatial reasoning.** The student uses formal geometric vocabulary. The student is expected to identify, classify, and describe two- and three-dimensional geometric figures by their attributes. The student compares two-dimensional figures, three-dimensional figures, or both by their attributes using formal geometry vocabulary.

(3.9) **Geometry and spatial reasoning.** The student recognizes congruence and symmetry. The student is expected to:

(A) identify congruent two-dimensional figures;
(B) create two-dimensional figures with lines of symmetry using concrete models and technology; and
(C) identify lines of symmetry in two-dimensional geometric figures.

(3.10) **Geometry and spatial reasoning.** The student recognizes that a line can be used to represent numbers and fractions and their properties and relationships. The student is expected to locate and name points on a number line using whole numbers and fractions, including halves and fourths.

(3.11) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses standard units to describe length, area, capacity/volume, and weight/mass. The student is expected to:

(A) use linear measurement tools to estimate and measure lengths using standard units;
(B) use standard units to find the perimeter of a shape;
(C) use concrete and pictorial models of square units to determine the area of two-dimensional surfaces;
(D) identify concrete models that approximate standard units of weight/mass and use them to measure weight/mass;
(E) identify concrete models that approximate standard units for capacity and use them to measure capacity; and
(F) use concrete models that approximate cubic units to determine the volume of a given container or other three-dimensional geometric figure.

(3.12) **Measurement.** The student reads and writes time and measures temperature in degrees Fahrenheit to solve problems. The student is expected to:

(A) use a thermometer to measure temperature; and
(B) tell and write time shown on analog and digital clocks.
(3.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

The student is expected to:

(A) collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data;

(B) interpret information from pictographs and bar graphs; and

(C) use data to describe events as more likely than, less likely than, or equally likely as.

(3.14) **Underlying processes and mathematical tools.** The student applies Grade 3 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(3.15) **Underlying processes and mathematical tools.** The student communicates about Grade 3 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(3.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to:

(A) make generalizations from patterns or sets of examples and nonexamples; and

(B) justify why an answer is reasonable and explain the solution process.
§111.16. Mathematics, Grade 4.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 4 are comparing and ordering fractions and decimals, applying multiplication and division, and developing ideas related to congruence and symmetry.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(4.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

<table>
<thead>
<tr>
<th>The student is expected to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) use place value to read, write, compare, and order whole numbers through 999,999,999; and</td>
</tr>
<tr>
<td>(B) use place value to read, write, compare, and order decimals involving tenths and hundredths, including money, using concrete objects and pictorial models.</td>
</tr>
</tbody>
</table>

(4.2) **Number, operation, and quantitative reasoning.** The student describes and compares fractional parts of whole objects or sets of objects.

<table>
<thead>
<tr>
<th>The student is expected to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) use concrete objects and pictorial models to generate equivalent fractions;</td>
</tr>
<tr>
<td>(B) model fraction quantities greater than one using concrete objects and pictorial models;</td>
</tr>
<tr>
<td>(C) compare and order fractions using concrete objects and pictorial models; and</td>
</tr>
<tr>
<td>(D) relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models.</td>
</tr>
</tbody>
</table>
(4.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers and decimals.

The student is expected to:

(A) use addition and subtraction to solve problems involving whole numbers; and

(B) add and subtract decimals to the hundredths place using concrete objects and pictorial models.

(4.4) **Number, operation, and quantitative reasoning.** The student multiplies and divides to solve meaningful problems involving whole numbers.

The student is expected to:

(A) model factors and products using arrays and area models;

(B) represent multiplication and division situations in picture, word, and number form;

(C) recall and apply multiplication facts through 12 x 12;

(D) use multiplication to solve problems (no more than two digits times two digits without technology); and

(E) use division to solve problems (no more than one-digit divisors and three-digit dividends without technology).

(4.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to:

(A) round whole numbers to the nearest ten, hundred, or thousand to approximate reasonable results in problem situations; and

(B) use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems.

(4.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns in multiplication and division.

The student is expected to:

(A) use patterns and relationships to develop strategies to remember basic multiplication and division facts (such as the patterns in related multiplication and division number sentences (fact families) such as 9 x 9 = 81 and 81 ÷ 9 = 9); and

(B) use patterns to multiply by 10 and 100.

(4.7) **Patterns, relationships, and algebraic thinking.** The student uses organizational structures to analyze and describe patterns and relationships.

The student is expected to describe the relationship between two sets of related data such as ordered pairs in a table.
(4.8) **Geometry and spatial reasoning.** The student identifies and describes attributes of geometric figures using formal geometric language.

The student is expected to:

(A) identify and describe right, acute, and obtuse angles;

(B) identify and describe parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models; and

(C) use essential attributes to define two- and three-dimensional geometric figures.

(4.9) **Geometry and spatial reasoning.** The student connects transformations to congruence and symmetry.

The student is expected to:

(A) demonstrate translations, reflections, and rotations using concrete models;

(B) use translations, reflections, and rotations to verify that two shapes are congruent; and

(C) use reflections to verify that a shape has symmetry.

(4.10) **Geometry and spatial reasoning.** The student recognizes the connection between numbers and their properties and points on a line.

The student is expected to locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths.

(4.11) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:

(A) estimate and use measurement tools to determine length (including perimeter), area, capacity and weight/mass using standard units SI (metric) and customary;

(B) perform simple conversions between different units of length, between different units of capacity, and between different units of weight within the customary measurement system;

(C) use concrete models of standard cubic units to measure volume;

(D) estimate volume in cubic units; and

(E) explain the difference between weight and mass.

(4.12) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:

(A) use a thermometer to measure temperature and changes in temperature; and

(B) use tools such as a clock with gears or a stopwatch to solve problems involving elapsed time.
(4.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

(4.14) **Underlying processes and mathematical tools.** The student applies Grade 4 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) use concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation; and

(B) interpret bar graphs.

(4.15) **Underlying processes and mathematical tools.** The student communicates about Grade 4 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(4.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to:

(A) make generalizations from patterns or sets of examples and nonexamples; and

(B) justify why an answer is reasonable and explain the solution process.
§111.17. Mathematics, Grade 5.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 5 are comparing and contrasting lengths, areas, and volumes of two- or three-dimensional geometric figures; representing and interpreting data in graphs, charts, and tables; and applying whole number operations in a variety of contexts.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(5.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

The student is expected to:

(A) use place value to read, write, compare, and order whole numbers through the 999,999,999,999; and

(B) use place value to read, write, compare, and order decimals through the thousandths place.

(5.2) **Number, operation, and quantitative reasoning.** The student uses fractions in problem-solving situations.

The student is expected to:

(A) generate a fraction equivalent to a given fraction such as 1/2 and 3/6 or 4/12 and 1/3;

(B) generate a mixed number equivalent to a given improper fraction or generate an improper fraction equivalent to a given mixed number;

(C) compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators; and

(D) use models to relate decimals to fractions that name tenths, hundredths, and thousandths.
Ch. 111, TEKS for Mathematics (Revised 2009)

(5.3) **Number, operation, and quantitative reasoning.**
The student adds, subtracts, multiplies, and divides to solve meaningful problems.

The student is expected to:

(A) use addition and subtraction to solve problems involving whole numbers and decimals;

(B) use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology);

(C) use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology), including interpreting the remainder within a given context;

(D) identify common factors of a set of whole numbers; and

(E) model situations using addition and/or subtraction involving fractions with like denominators using concrete objects, pictures, words, and numbers.

The student is expected to use strategies, including rounding and compatible numbers to estimate solutions to addition, subtraction, multiplication, and division problems.

(5.4) **Number, operation, and quantitative reasoning.**

The student estimates to determine reasonable results.

The student is expected to:

(A) describe the relationship between sets of data in graphic organizers such as lists, tables, charts, and diagrams; and

(B) identify prime and composite numbers using concrete objects, pictorial models, and patterns in factor pairs.

The student is expected to select from and use diagrams and equations such as \( y = 5 + 3 \) to represent meaningful problem situations.

(5.5) **Patterns, relationships, and algebraic thinking.**
The student makes generalizations based on observed patterns and relationships.

The student is expected to:

(A) identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures.

(B) sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid; and

(B) identify the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid.
(5.9) **Geometry and spatial reasoning.** The student recognizes the connection between ordered pairs of numbers and locations of points on a plane.

The student is expected to locate and name points on a coordinate grid using ordered pairs of whole numbers.

(5.10) **Measurement.** The student applies measurement concepts involving length (including perimeter), area, capacity/volume, and weight/mass to solve problems.

The student is expected to:
(A) perform simple conversions within the same measurement system (SI (metric) or customary);
(B) connect models for perimeter, area, and volume with their respective formulas; and
(C) select and use appropriate units and formulas to measure length, perimeter, area, and volume.

(5.11) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:
(A) solve problems involving changes in temperature; and
(B) solve problems involving elapsed time.

(5.12) **Probability and statistics.** The student describes and predicts the results of a probability experiment.

The student is expected to:
(A) use fractions to describe the results of an experiment;
(B) use experimental results to make predictions; and
(C) list all possible outcomes of a probability experiment such as tossing a coin.

(5.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

The student is expected to:
(A) use tables of related number pairs to make line graphs;
(B) describe characteristics of data presented in tables and graphs including median, mode, and range; and
(C) graph a given set of data using an appropriate graphical representation such as a picture or line graph.
(5.14) **Underlying processes and mathematical tools.** The student applies Grade 5 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(5.15) **Underlying processes and mathematical tools.** The student communicates about Grade 5 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(5.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to:

(A) make generalizations from patterns or sets of examples and nonexamples; and

(B) justify why an answer is reasonable and explain the solution process.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter B. Middle School
Text of Adopted Amendments to 19 TAC

§111.21. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 6-8.
The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

§111.22. Mathematics, Grade 6.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 6 are using ratios to describe direct proportional relationships involving number, geometry, measurement, probability, and adding and subtracting decimals and fractions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(6.1) Number, operation, and quantitative reasoning. The student represents and uses rational numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order non-negative rational numbers;

(B) generate equivalent forms of rational numbers including whole numbers, fractions, and decimals;

(C) use integers to represent real-life situations;

(D) write prime factorizations using exponents;

(E) identify factors of a positive integer, common factors, and the greatest common factor of a set of positive integers; and

(F) identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers.
(6.2) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, and divides to solve problems and justify solutions.

The student is expected to:

(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;

(B) use addition and subtraction to solve problems involving fractions and decimals;

(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates;

(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required; and

(E) use order of operations to simplify whole number expressions (without exponents) in problem solving situations.

(6.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:

(A) use ratios to describe proportional situations;

(B) represent ratios and percents with concrete models, fractions, and decimals; and

(C) use ratios to make predictions in proportional situations.

(6.4) **Patterns, relationships, and algebraic thinking.** The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.

The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area; and

(B) use tables of data to generate formulas representing relationships involving perimeter, area, volume of a rectangular prism, etc.

(6.5) **Patterns, relationships, and algebraic thinking.** The student uses letters to represent an unknown in an equation.

The student is expected to formulate equations from problem situations described by linear relationships.

(6.6) **Geometry and spatial reasoning.** The student uses geometric vocabulary to describe angles, polygons, and circles.

The student is expected to:

(A) use angle measurements to classify angles as acute, obtuse, or right;

(B) identify relationships involving angles in triangles and quadrilaterals; and

(C) describe the relationship between radius, diameter, and circumference of a circle.
(6.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to identify location in two dimensions.

The student is expected to locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers.

(6.8) **Measurement.** The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles.

The student is expected to:

(A) estimate measurements (including circumference) and evaluate reasonableness of results;

(B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter), area, time, temperature, volume, and weight;

(C) measure angles; and

(D) convert measures within the same measurement system (customary and metric) based on relationships between units.

(6.9) **Probability and statistics.** The student uses experimental and theoretical probability to make predictions.

The student is expected to:

(A) construct sample spaces using lists and tree diagrams; and

(B) find the probabilities of a simple event and its complement and describe the relationship between the two.

(6.10) **Probability and statistics.** The student uses statistical representations to analyze data.

The student is expected to:

(A) select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot;

(B) identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data;

(C) sketch circle graphs to display data; and

(D) solve problems by collecting, organizing, displaying, and interpreting data.
(6.11) **Underlying processes and mathematical tools.** The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(6.12) **Underlying processes and mathematical tools.** The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(6.13) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples; and

(B) validate his/her conclusions using mathematical properties and relationships.
§111.23. Mathematics, Grade 7.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 7 are using direct proportional relationships in number, geometry, measurement, and probability; applying addition, subtraction, multiplication, and division of decimals, fractions, and integers; and using statistical measures to describe data.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(7.1) **Number, operation, and quantitative reasoning.** The student represents and uses numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order integers and positive rational numbers;

(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and

(C) represent squares and square roots using geometric models.
(7.2) **Number, operation, and quantitative reasoning.**
The student adds, subtracts, multiplies, or divides to solve problems and justify solutions.

The student is expected to:

(A) represent multiplication and division situations involving fractions and decimals with models, including concrete objects, pictures, words, and numbers;

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;

(C) use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;

(E) simplify numerical expressions involving order of operations and exponents;

(F) select and use appropriate operations to solve problems and justify the selections; and

(G) determine the reasonableness of a solution to a problem.

(7.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:

(A) estimate and find solutions to application problems involving percent; and

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

(7.4) **Patterns, relationships, and algebraic thinking.** The student represents a relationship in numerical, geometric, verbal, and symbolic form.

The student is expected to:

(A) generate formulas involving unit conversions within the same system (customary and metric), perimeter, area, circumference, volume, and scaling;

(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and

(C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.
(7.5) **Patterns, relationships, and algebraic thinking.** The student uses equations to solve problems. 

The student is expected to:

(A) use concrete and pictorial models to solve equations and use symbols to record the actions; and

(B) formulate problem situations when given a simple equation and formulate an equation when given a problem situation.

(7.6) **Geometry and spatial reasoning.** The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties.

The student is expected to:

(A) use angle measurements to classify pairs of angles as complementary or supplementary;

(B) use properties to classify triangles and quadrilaterals;

(C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and

(D) use critical attributes to define similarity.

(7.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane.

The student is expected to:

(A) locate and name points on a coordinate plane using ordered pairs of integers; and

(B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.

(7.8) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

The student is expected to:

(A) sketch three-dimensional figures when given the top, side, and front views;

(B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and

(C) use geometric concepts and properties to solve problems in fields such as art and architecture.

The student is expected to:

(A) estimate measurements and solve application problems involving length (including perimeter and circumference) and area of polygons and other shapes;

(B) connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders; and

(C) estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders.
(7.10) **Probability and statistics.** The student recognizes that a physical or mathematical model (including geometric) can be used to describe the experimental and theoretical probability of real-life events.

The student is expected to:

(A) construct sample spaces for simple or composite experiments; and

(B) find the probability of independent events.

(7.11) **Probability and statistics.** The student understands that the way a set of data is displayed influences its interpretation.

The student is expected to:

(A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and

(B) make inferences and convincing arguments based on an analysis of given or collected data.

(7.12) **Probability and statistics.** The student uses measures of central tendency and variability to describe a set of data.

The student is expected to:

(A) describe a set of data using mean, median, mode, and range; and

(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.

(7.13) **Underlying processes and mathematical tools.** The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
(7.14) **Underlying processes and mathematical tools.** The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(7.15) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples; and

(B) validate his/her conclusions using mathematical properties and relationships.
§111.24. Mathematics, Grade 8.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent both proportional and non-proportional linear relationships and using probability to describe data and make predictions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(8.1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations.

The student is expected to:

(A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;

(C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as π, \(\sqrt{2}\));

(D) express numbers in scientific notation, including negative exponents, in appropriate problem situations; and

(E) compare and order real numbers with a calculator.

(8.2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions.

The student is expected to:

(A) select appropriate operations to solve problems involving rational numbers and justify the selections;

(B) use appropriate operations to solve problems involving rational numbers in problem situations;

(C) evaluate a solution for reasonableness; and
(8.3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional or non-proportional linear relationships in problem situations and solves problems.

The student is expected to:

(A) compare and contrast proportional and non-proportional linear relationships; and

(B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.

(8.4) **Patterns, relationships, and algebraic thinking.** The student makes connections among various representations of a numerical relationship.

The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).

(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems.

The student is expected to:

(A) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and

(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

(8.6) **Geometry and spatial reasoning.** The student uses transformational geometry to develop spatial sense.

The student is expected to:

(A) generate similar figures using dilations including enlargements and reductions; and

(B) graph dilations, reflections, and translations on a coordinate plane.

(8.7) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

The student is expected to:

(A) draw three-dimensional figures from different perspectives;

(B) use geometric concepts and properties to solve problems in fields such as art and architecture;

(C) use pictures or models to demonstrate the Pythagorean Theorem; and

(D) locate and name points on a coordinate plane using ordered pairs of rational numbers.
(8.8) **Measurement.** The student uses procedures to determine measures of three-dimensional figures.

The student is expected to:

(A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models);

(B) connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and

(C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

(8.9) **Measurement.** The student uses indirect measurement to solve problems.

The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems; and

(B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.

(8.10) **Measurement.** The student describes how changes in dimensions affect linear, area, and volume measures.

The student is expected to:

(A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and

(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.

(8.11) **Probability and statistics.** The student applies concepts of theoretical and experimental probability to make predictions.

The student is expected to:

(A) find the probabilities of dependent and independent events;

(B) use theoretical probabilities and experimental results to make predictions and decisions; and

(C) select and use different models to simulate an event.
(8.12) **Probability and statistics.** The student uses statistical procedures to describe data.

The student is expected to:

(A) use variability (range, including interquartile range (IQR)) and select the appropriate measure of central tendency to describe a set of data and justify the choice for a particular situation;

(B) draw conclusions and make predictions by analyzing trends in scatterplots; and

(C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology.

(8.13) **Probability and statistics.** The student evaluates predictions and conclusions based on statistical data.

The student is expected to:

(A) evaluate methods of sampling to determine validity of an inference made from a set of data; and

(B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

(8.14) **Underlying processes and mathematical tools.** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
(8.15) **Underlying processes and mathematical tools.** The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(8.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples; and

(B) validate his/her conclusions using mathematical properties and relationships.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter C. High School
Text of Adopted Amendments to 19 TAC

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

§111.32. Algebra I (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.

(3) Function concepts. A function is a fundamental mathematical concept; it expresses a special kind of relationship between two quantities. Students use functions to determine one quantity from another, to represent and model problem situations, and to analyze and interpret relationships.

(4) Relationship between equations and functions. Equations and inequalities arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and inequalities and use a variety of methods to solve them.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(A.1) Foundations for functions.

The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(B) gather and record data and use data sets to determine functional relationships between quantities;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;
(A.2) **Foundations for functions.** The student uses the properties and attributes of functions.

The student is expected to:

(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

(A.3) **Foundations for functions.**

The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

(A.4) **Foundations for functions.**

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(C) connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$.

(A.5) **Linear functions.** The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.
(A.6) **Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in \(m\) and \(b\) on the graph of \(y = mx + b\);

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and \(y\)-intercept;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

(F) interpret and predict the effects of changing slope and \(y\)-intercept in applied situations; and

(G) relate direct variation to linear functions and solve problems involving proportional change.

(A.7) **Linear functions.** The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

(A.8) **Linear functions.** The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.
(A.9) **Quadratic and other nonlinear functions.** The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;
(B) investigate, describe, and predict the effects of changes in $a$ on the graph of $y = ax^2 + c$;
(C) investigate, describe, and predict the effects of changes in $c$ on the graph of $y = ax^2 + c$; and
(D) analyze graphs of quadratic functions and draw conclusions.

(A.10) **Quadratic and other nonlinear functions.** The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and
(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts ($x$-intercepts) of the graph of the function.

(A.11) **Quadratic and other nonlinear functions.** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;
(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and
(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.
§111.33. Algebra II (One-Half to One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(2A.1) **Foundations for functions.**
The student uses properties and attributes of functions and applies functions to problem situations.

The student is expected to:

(A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and

(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

(2A.2) **Foundations for functions.**
The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations; and

(B) use complex numbers to describe the solutions of quadratic equations.
111. Foundations for functions. The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.

111.3. The student is expected to:
   (A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;
   (B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and
   (C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

111.4. Algebra and geometry. The student connects algebraic and geometric representations of functions.

111.4.1. The student is expected to:
   (A) identify and sketch graphs of parent functions, including linear \( f(x) = x \), quadratic \( f(x) = x^2 \), exponential \( f(x) = a^x \), and logarithmic \( f(x) = \log_a x \) functions, absolute value of \( x \) \( f(x) = |x| \), square root of \( x \) \( f(x) = \sqrt{x} \), and reciprocal of \( x \) \( f(x) = 1/x \);
   (B) extend parent functions with parameters such as \( a \) in \( f(x) = a/x \) and describe the effects of the parameter changes on the graph of parent functions; and
   (C) describe and analyze the relationship between a function and its inverse.

111.5. Algebra and geometry. The student knows the relationship between the geometric and algebraic descriptions of conic sections.

111.5.1. The student is expected to:
   (A) describe a conic section as the intersection of a plane and a cone;
   (B) sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph;
   (C) identify symmetries from graphs of conic sections;
   (D) identify the conic section from a given equation; and
   (E) use the method of completing the square.

111.6. Quadratic and square root functions. The student understands that quadratic functions can be represented in different ways and translates among their various representations.

111.6.1. The student is expected to:
   (A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;
   (B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and
   (C) determine a quadratic function from its roots (real and complex) or a graph.
(2A.7) **Quadratic and square root functions.** The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.

The student is expected to:

(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions; and

(B) use the parent function to investigate, describe, and predict the effects of changes in $a$, $h$, and $k$ on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations.

(2A.8) **Quadratic and square root functions.** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems;

(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula;

(C) compare and translate between algebraic and graphical solutions of quadratic equations; and

(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

(2A.9) **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;

(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;

(C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;

(D) determine solutions of square root equations using graphs, tables, and algebraic methods;

(E) determine solutions of square root inequalities using graphs and tables;

(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and

(G) connect inverses of square root functions with quadratic functions.
(2A.10) **Rational functions.** The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

- **(A)** use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior;
- **(B)** analyze various representations of rational functions with respect to problem situations;
- **(C)** determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;
- **(D)** determine the solutions of rational equations using graphs, tables, and algebraic methods;
- **(E)** determine solutions of rational inequalities using graphs and tables;
- **(F)** analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and
- **(G)** use functions to model and make predictions in problem situations involving direct and inverse variation.

(2A.11) **Exponential and logarithmic functions.** The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

- **(A)** develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses;
- **(B)** use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior;
- **(C)** determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities;
- **(D)** determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods;
- **(E)** determine solutions of exponential and logarithmic inequalities using graphs and tables; and
- **(F)** analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.
§111.34. Geometry (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

(b) Knowledge and skills.

(G.1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system.

The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and

(C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.
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(G.2) **Geometric structure.** The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements. The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;

(B) construct and justify statements about geometric figures and their properties;

(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;

(D) use inductive reasoning to formulate a conjecture; and

(E) use deductive reasoning to prove a statement.

(G.4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(G.5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:

(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties;

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

(C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and

(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.
(G.6) **Dimensionality and the geometry of location.** The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.

The student is expected to:

(A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;

(B) use nets to represent and construct three-dimensional geometric figures; and

(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.

(G.7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and

(C) derive and use formulas involving length, slope, and midpoint.

(G.8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.

The student is expected to:

(A) find areas of regular polygons, circles, and composite figures;

(B) find areas of sectors and arc lengths of circles using proportional reasoning;

(C) derive, extend, and use the Pythagorean Theorem;

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations;

(E) use area models to connect geometry to probability and statistics; and

(F) use conversions between measurement systems to solve problems in real-world situations.
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(G.9) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures.

The student is expected to:

(A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;

(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;

(C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and

(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

(G.10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems.

The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and

(B) justify and apply triangle congruence relationships.

(G.11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems.

The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;

(B) use ratios to solve problems involving similar figures;

(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.
§111.35. Mathematics, Precalculus

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students use functions, equations, and limits as useful tools for expressing generalizations and as means for analyzing and understanding a broad variety of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, trigonometry, and calculus and to model physical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model functions and equations and solve real-life problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to:

(A) describe parent functions symbolically and graphically, including \( f(x) = x^n \), \( f(x) = \ln x \), \( f(x) = \log_a x \), \( f(x) = 1/x \), \( f(x) = e^x \), \( f(x) = |x| \), \( f(x) = a^x \), \( f(x) = \sin x \), \( f(x) = \arcsin x \), etc.;

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.
The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems.

The student is expected to:

(A) apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, and compositions with absolute value functions, including $|f(x)|$, and $f(|x|)$, to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

The student uses functions and their properties, tools and technology, to model and solve meaningful problems.

The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

(D) use properties of functions to analyze and solve problems and make predictions; and

(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems.

The student is expected to:

(A) represent patterns using arithmetic and geometric sequences and series;

(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and

(D) apply sequences and series to solve problems including sums and binomial expansion.
The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations.

The student is expected to:

(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;

(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;

(C) convert between parametric and rectangular forms of functions and equations to graph them; and

(D) use parametric functions to simulate problems involving motion.

The student uses vectors to model physical situations.

The student is expected to:

(A) use the concept of vectors to model situations defined by magnitude and direction; and

(B) analyze and solve vector problems generated by real-life situations.
§111.36. Mathematical Models with Applications (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.

(b) Introduction.

(1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(M.1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems.

The student is expected to:

(A) compare and analyze various methods for solving a real-life problem;

(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(M.2) The student uses graphical and numerical techniques to study patterns and analyze data.

The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data;

(B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences;

(C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and

(D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.
§111.36. Mathematics, Mathematical Models with Applications

(M.3) The student develops and implements a plan for collecting and analyzing data (qualitative and quantitative) in order to make decisions.

The student is expected to:
(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;
(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and
(C) determine the appropriateness of a model for making predictions from a given set of data.

(M.4) The student uses probability models to describe everyday situations involving chance.

The student is expected to:
(A) compare theoretical and empirical probability; and
(B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.

(M.5) The student uses functional relationships to solve problems related to personal income.

The student is expected to:
(A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions;
(B) solve problems involving personal taxes; and
(C) analyze data to make decisions about banking.

(M.6) The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit.

The student is expected to:
(A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option;
(B) use amortization models to investigate home financing and compare buying and renting a home; and
(C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.

(M.7) The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.

The student is expected to:
(A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options;
(B) analyze and compare coverage options and rates in insurance; and
(C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.
(M.8) The student uses algebraic and geometric models to describe situations and solve problems.

The student is expected to:

(A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology;

(B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and

(C) use direct and inverse variation to describe physical laws such as Hook’s, Newton’s, and Boyle’s laws.

(M.9) The student uses algebraic and geometric models to represent patterns and structures.

The student is expected to:

(A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and

(B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter D. Other High School Mathematics Courses

Statutory Authority: The provisions of this Subchapter D issued under the Texas Education Code, §28.002, unless otherwise noted.

§111.51. Implementation of Texas Essential Knowledge and Skills for Mathematics, Other High School Mathematics Courses.

The provisions of this subchapter shall be implemented by school districts beginning September 1, 1998, and at that time shall supersede §75.63(o), (q)-(u), and (cc) of this title (relating to Mathematics).

Source: The provisions of this §111.51 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.52. Independent Study in Mathematics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of Independent Study in Mathematics. Required prerequisites: Algebra II, Geometry. Students may repeat this course with different course content for a second credit.

(b) Content requirements. Students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics, such as theory of equations, number theory, non-Euclidean geometry, advanced survey of mathematics, or history of mathematics. The requirements for each course must be approved by the local district before the course begins.

(c) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student’s mentor.

Source: The provisions of this §111.52 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.53. Advanced Placement (AP) Statistics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for Advanced Placement (AP) Statistics are prescribed in the College Board Publication Advanced Placement Course Description: Statistics, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.53 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.54. Advanced Placement (AP) Calculus AB (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus AB are prescribed in the College Board Publication Advanced Placement Course Description Mathematics: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.54 adopted to be effective September 1, 1998, 22 TexReg 7623.
§111.55. Advanced Placement (AP) Calculus BC (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus BC are prescribed in the College Board Publication Advanced Placement Course Description: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.55 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.56. IB Mathematical Studies Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Studies Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Studies Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.56 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.57. IB Mathematical Methods Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Methods Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Methods Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.57 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.58. IB Mathematics Higher Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Higher Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematical Studies Subsidiary Level or IB Mathematical Methods Subsidiary Level.

(b) Content requirements. Content requirements for IB Mathematics Higher Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.58 adopted to be effective September 1, 1998, 22 TexReg 7623.
§111.59. IB Advanced Mathematics Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Advanced Mathematics Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematics Higher Level.

(b) Content requirements. Content requirements for IB Advanced Mathematics Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.59 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.60. Concurrent Enrollment in College Courses.

(a) General requirements. Students shall be awarded one-half credit for each semester of successful completion of a college course in which the student is concurrently enrolled while in high school.

(b) Content requirements. In order for students to receive state graduation credit for concurrent enrollment courses, content requirements must meet or exceed the essential knowledge and skills in a given course.

Source: The provisions of this §111.60 adopted to be effective September 1, 1998, 22 TexReg 7623.
§74.4. English Language Proficiency Standards.

Texas Administrative Code (TAC), Title 19, Part II, Chapter 74, Curriculum Requirements; Subchapter A, Required Curriculum; Rule §74.4, English Language Proficiency Standards.

The English Language Proficiency Standards are part of Texas state law (Texas Administrative Code, Title 19, Education; part 2, Texas Education Agency; Chapter 74, Curriculum Requirements; Subchapter A, Required Curriculum; Rule §74.4, English Language Proficiency Standards).

You may access the ELPS free online at the Texas Education Agency website, Curriculum > Bilingual Education > English Language Proficiency Standards: ritter.tea.state.tx.us/curriculum/biling/elps.html.

According to the TAC administrative rule cited in (a) Introduction to the ELPS,

“(1) The English language proficiency standards … outline English language proficiency level descriptors and student expectations for English language learners (ELLs). School districts shall implement this section as an integral part of each subject in the required curriculum. The English language proficiency standards are to be published along with the Texas Essential Knowledge and Skills (TEKS) for each subject in the required curriculum.”
§74.4. English Language Proficiency Standards.

(a) Introduction.

(1) The English language proficiency standards in this section outline English language proficiency level descriptors and student expectations for English language learners (ELLs). School districts shall implement this section as an integral part of each subject in the required curriculum. The English language proficiency standards are to be published along with the Texas Essential Knowledge and Skills (TEKS) for each subject in the required curriculum.

(2) In order for ELLs to be successful, they must acquire both social and academic language proficiency in English. Social language proficiency in English consists of the English needed for daily social interactions. Academic language proficiency consists of the English needed to think critically, understand and learn new concepts, process complex academic material, and interact and communicate in English academic settings.

(3) Classroom instruction that effectively integrates second language acquisition with quality content area instruction ensures that ELLs acquire social and academic language proficiency in English, learn the knowledge and skills in the TEKS, and reach their full academic potential.

(4) Effective instruction in second language acquisition involves giving ELLs opportunities to listen, speak, read, and write at their current levels of English development while gradually increasing the linguistic complexity of the English they read and hear, and are expected to speak and write.

(5) The cross-curricular second language acquisition skills in subsection (c) of this section apply to ELLs in Kindergarten–Grade 12.

(6) The English language proficiency levels of beginning, intermediate, advanced, and advanced high are not grade-specific. ELLs may exhibit different proficiency levels within the language domains of listening, speaking, reading, and writing. The proficiency level descriptors outlined in subsection (d) of this section show the progression of second language acquisition from one proficiency level to the next and serve as a road map to help content area teachers instruct ELLs commensurate with students’ linguistic needs.

(b) School district responsibilities. In fulfilling the requirements of this section, school districts shall:

(1) identify the student’s English language proficiency levels in the domains of listening, speaking, reading, and writing in accordance with the proficiency level descriptors for the beginning, intermediate, advanced, and advanced high levels delineated in subsection (d) of this section;

(2) provide instruction in the knowledge and skills of the foundation and enrichment curriculum in a manner that is linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s levels of English language proficiency to ensure that the student learns the knowledge and skills in the required curriculum;

(3) provide content-based instruction including the cross-curricular second language acquisition essential knowledge and skills in subsection (c) of this section in a manner that is linguistically accommodated to help the student acquire English language proficiency; and

(4) provide intensive and ongoing foundational second language acquisition instruction to ELLs in Grade 3 or higher who are at the beginning or intermediate level of English language proficiency in listening, speaking, reading, and/or writing as determined by the state’s English language proficiency assessment system. These ELLs require focused, targeted, and systematic second language acquisition instruction to provide them with the foundation of English language vocabulary, grammar, syntax, and English mechanics necessary to support content-based instruction and accelerated learning of English.

(c) Cross-curricular second language acquisition essential knowledge and skills.

(1) Cross-curricular second language acquisition/learning strategies. The ELL uses language learning strategies to develop an awareness of his or her own learning processes in all content areas. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. The student is expected to:
(A) use prior knowledge and experiences to understand meanings in English;

(B) monitor oral and written language production and employ self-corrective techniques or other resources;

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary;

(D) speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known);

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment;

(F) use accessible language and learn new and essential language in the process;

(G) demonstrate an increasing ability to distinguish between formal and informal English and an increasing knowledge of when to use each one commensurate with grade-level learning expectations; and

(H) develop and expand repertoire of learning strategies such as reasoning inductively or deductively, looking for patterns in language, and analyzing sayings and expressions commensurate with grade-level learning expectations.

(2) Cross-curricular second language acquisition/listening. The ELL listens to a variety of speakers including teachers, peers, and electronic media to gain an increasing level of comprehension of newly acquired language in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in listening. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. The student is expected to:

(A) distinguish sounds and intonation patterns of English with increasing ease;

(B) recognize elements of the English sound system in newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters;

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions;

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed;

(E) use visual, contextual, and linguistic support to enhance and confirm understanding of increasingly complex and elaborated spoken language;

(F) listen to and derive meaning from a variety of media such as audio tape, video, DVD, and CD ROM to build and reinforce concept and language attainment;

(G) understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to unfamiliar;

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations; and

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.
(3) Cross-curricular second language acquisition/speaking. The ELL speaks in a variety of modes for a variety of purposes with an awareness of different language registers (formal/informal) using vocabulary with increasing fluency and accuracy in language arts and all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in speaking. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. The student is expected to:

(A) practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible;

(B) expand and internalize initial English vocabulary by learning and using high-frequency English words necessary for identifying and describing people, places, and objects, by retelling simple stories and basic information represented or supported by pictures, and by learning and using routine language needed for classroom communication;

(C) speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired;

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency;

(E) share information in cooperative learning interactions;

(F) ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments;

(G) express opinions, ideas, and feelings ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics;

(H) narrate, describe, and explain with increasing specificity and detail as more English is acquired;

(I) adapt spoken language appropriately for formal and informal purposes; and

(J) respond orally to information presented in a wide variety of print, electronic, audio, and visual media to build and reinforce concept and language attainment.

(4) Cross-curricular second language acquisition/reading. The ELL reads a variety of texts for a variety of purposes with an increasing level of comprehension in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. For Kindergarten and Grade 1, certain of these student expectations apply to text read aloud for students not yet at the stage of decoding written text. The student is expected to:

(A) learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots, and base words;

(B) recognize directionality of English reading such as left to right and top to bottom;

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials;

(D) use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text;
(E) read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned;

(F) use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language;

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade level needs;

(H) read silently with increasing ease and comprehension for longer periods;

(I) demonstrate English comprehension and expand reading skills by employing basic reading skills such as demonstrating understanding of supporting ideas and details in text and graphic sources, summarizing text, and distinguishing main ideas from details commensurate with content area needs;

(J) demonstrate English comprehension and expand reading skills by employing inferential skills such as predicting, making connections between ideas, drawing inferences and conclusions from text and graphic sources, and finding supporting text evidence commensurate with content area needs; and

(K) demonstrate English comprehension and expand reading skills by employing analytical skills such as evaluating written information and performing critical analyses commensurate with content area and grade-level needs.

(5) Cross-curricular second language acquisition/writing. The ELL writes in a variety of forms with increasing accuracy to effectively address a specific purpose and audience in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. In order for the ELL to meet grade-level learning expectations across foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. For Kindergarten and Grade 1, certain of these student expectations do not apply until the student has reached the stage of generating original written text using a standard writing system. The student is expected to:

(A) learn relationships between sounds and letters of the English language to represent sounds when writing in English;

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary;

(C) spell familiar English words with increasing accuracy, and employ English spelling patterns and rules with increasing accuracy as more English is acquired;

(D) edit writing for standard grammar and usage, including subject-verb agreement, pronoun agreement, and appropriate verb tenses commensurate with grade-level expectations as more English is acquired;

(E) employ increasingly complex grammatical structures in content area writing commensurate with grade-level expectations, such as:

(i) using correct verbs, tenses, and pronouns/antecedents;

(ii) using possessive case (apostrophe s) correctly; and

(iii) using negatives and contractions correctly;

(F) write using a variety of grade-appropriate sentence lengths, patterns, and connecting words to combine phrases, clauses, and sentences in increasingly accurate ways as more English is acquired; and

(G) narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.
(d) Proficiency level descriptors.

(1) Listening, Kindergarten–Grade 12. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in listening. The following proficiency level descriptors for listening are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.

(A) Beginning. Beginning ELLs have little or no ability to understand spoken English in academic and social settings. These students:

(i) struggle to understand simple conversations and simple discussions even when the topics are familiar and the speaker uses linguistic supports such as visuals, slower speech and other verbal cues, and gestures;

(ii) struggle to identify and distinguish individual words and phrases during social and instructional interactions that have not been intentionally modified for ELLs; and

(iii) may not seek clarification in English when failing to comprehend the English they hear; frequently remain silent, watching others for cues.

(B) Intermediate. Intermediate ELLs have the ability to understand simple, high-frequency spoken English used in routine academic and social settings. These students:

(i) usually understand simple or routine directions, as well as short, simple conversations and short, simple discussions on familiar topics; when topics are unfamiliar, require extensive linguistic supports and adaptations such as visuals, slower speech and other verbal cues, simplified language, gestures, and preteaching to preview or build topic-related vocabulary;

(ii) often identify and distinguish key words and phrases necessary to understand the general meaning during social and basic instructional interactions that have not been intentionally modified for ELLs; and

(iii) have the ability to seek clarification in English when failing to comprehend the English they hear by requiring/requesting the speaker to repeat, slow down, or rephrase speech.

(C) Advanced. Advanced ELLs have the ability to understand, with second language acquisition support, grade-appropriate spoken English used in academic and social settings. These students:

(i) usually understand longer, more elaborated directions, conversations, and discussions on familiar and some unfamiliar topics, but sometimes need processing time and sometimes depend on visuals, verbal cues, and gestures to support understanding;

(ii) understand most main points, most important details, and some implicit information during social and basic instructional interactions that have not been intentionally modified for ELLs; and

(iii) occasionally require/request the speaker to repeat, slow down, or rephrase to clarify the meaning of the English they hear.

(D) Advanced high. Advanced high ELLs have the ability to understand, with minimal second language acquisition support, grade-appropriate spoken English used in academic and social settings. These students:

(i) understand longer, elaborated directions, conversations, and discussions on familiar and unfamiliar topics with occasional need for processing time and with little dependence on visuals, verbal cues, and gestures; some exceptions when complex academic or highly specialized language is used;
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(ii) understand main points, important details, and implicit information at a level nearly comparable to native English-speaking peers during social and instructional interactions; and

(iii) rarely require/request the speaker to repeat, slow down, or rephrase to clarify the meaning of the English they hear.

(2) Speaking, Kindergarten–Grade 12. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in speaking. The following proficiency level descriptors for speaking are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.

(A) Beginning. Beginning ELLs have little or no ability to speak English in academic and social settings. These students:

(i) mainly speak using single words and short phrases consisting of recently practiced, memorized, or highly familiar material to get immediate needs met; may be hesitant to speak and often give up in their attempts to communicate;

(ii) speak using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts;

(iii) lack the knowledge of English grammar necessary to connect ideas and speak in sentences; can sometimes produce sentences using recently practiced, memorized, or highly familiar material;

(iv) exhibit second language acquisition errors that may hinder overall communication, particularly when trying to convey information beyond memorized, practiced, or highly familiar material; and

(v) typically use pronunciation that significantly inhibits communication.

(B) Intermediate. Intermediate ELLs have the ability to speak in a simple manner using English commonly heard in routine academic and social settings. These students:

(i) are able to express simple, original messages, speak using sentences, and participate in short conversations and classroom interactions; may hesitate frequently and for long periods to think about how to communicate desired meaning;

(ii) speak simply using basic vocabulary needed in everyday social interactions and routine academic contexts; rarely have vocabulary to speak in detail;

(iii) exhibit an emerging awareness of English grammar and speak using mostly simple sentence structures and simple tenses; are most comfortable speaking in present tense;

(iv) exhibit second language acquisition errors that may hinder overall communication when trying to use complex or less familiar English; and

(v) use pronunciation that can usually be understood by people accustomed to interacting with ELLs.

(C) Advanced. Advanced ELLs have the ability to speak using grade-appropriate English, with second language acquisition support, in academic and social settings. These students:

(i) are able to participate comfortably in most conversations and academic discussions on familiar topics, with some pauses to restate, repeat, or search for words and phrases to clarify meaning;

(ii) discuss familiar academic topics using content-based terms and common abstract vocabulary; can usually speak in some detail on familiar topics;
(iii) have a grasp of basic grammar features, including a basic ability to narrate and describe in present, past, and future tenses; have an emerging ability to use complex sentences and complex grammar features;

(iv) make errors that interfere somewhat with communication when using complex grammar structures, long sentences, and less familiar words and expressions; and

(v) may mispronounce words, but use pronunciation that can usually be understood by people not accustomed to interacting with ELLs.

(D) Advanced high. Advanced high ELLs have the ability to speak using grade-appropriate English, with minimal second language acquisition support, in academic and social settings. These students:

(i) are able to participate in extended discussions on a variety of social and grade-appropriate academic topics with only occasional disruptions, hesitations, or pauses;

(ii) communicate effectively using abstract and content-based vocabulary during classroom instructional tasks, with some exceptions when low-frequency or academically demanding vocabulary is needed; use many of the same idioms and colloquialisms as their native English-speaking peers;

(iii) can use English grammar structures and complex sentences to narrate and describe at a level nearly comparable to native English-speaking peers;

(iv) make few second language acquisition errors that interfere with overall communication; and

(v) may mispronounce words, but rarely use pronunciation that interferes with overall communication.

(3) Reading, Kindergarten-Grade 1. ELLs in Kindergarten and Grade 1 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. The following proficiency level descriptors for reading are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction and should take into account developmental stages of emergent readers.

(A) Beginning. Beginning ELLs have little or no ability to use the English language to build foundational reading skills. These students:

(i) derive little or no meaning from grade-appropriate stories read aloud in English, unless the stories are:

   (I) read in short “chunks;”

   (II) controlled to include the little English they know such as language that is high frequency, concrete, and recently practiced; and

   (III) accompanied by ample visual supports such as illustrations, gestures, pantomime, and objects and by linguistic supports such as careful enunciation and slower speech;

(ii) begin to recognize and understand environmental print in English such as signs, labeled items, names of peers, and logos; and

(iii) have difficulty decoding most grade-appropriate English text because they:

   (I) understand the meaning of very few words in English; and

   (II) struggle significantly with sounds in spoken English words and with sound-symbol relationships due to differences between their primary language and English.
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(B) Intermediate. Intermediate ELLs have a limited ability to use the English language to build foundational reading skills. These students:

(i) demonstrate limited comprehension (key words and general meaning) of grade-appropriate stories read aloud in English, unless the stories include:

(I) predictable story lines;

(II) highly familiar topics;

(III) primarily high-frequency, concrete vocabulary;

(IV) short, simple sentences; and

(V) visual and linguistic supports;

(ii) regularly recognize and understand common environmental print in English such as signs, labeled items, names of peers, logos; and

(iii) have difficulty decoding grade-appropriate English text because they:

(I) understand the meaning of only those English words they hear frequently; and

(II) struggle with some sounds in English words and some sound-symbol relationships due to differences between their primary language and English.

(C) Advanced. Advanced ELLs have the ability to use the English language, with second language acquisition support, to build foundational reading skills. These students:

(i) demonstrate comprehension of most main points and most supporting ideas in grade-appropriate stories read aloud in English, although they may still depend on visual and linguistic supports to gain or confirm meaning;

(ii) recognize some basic English vocabulary and high-frequency words in isolated print; and

(iii) with second language acquisition support, are able to decode most grade-appropriate English text because they:

(I) understand the meaning of most grade-appropriate English words; and

(II) have little difficulty with English sounds and sound-symbol relationships that result from differences between their primary language and English.

(D) Advanced high. Advanced high ELLs have the ability to use the English language, with minimal second language acquisition support, to build foundational reading skills. These students:

(i) demonstrate, with minimal second language acquisition support and at a level nearly comparable to native English-speaking peers, comprehension of main points and supporting ideas (explicit and implicit) in grade-appropriate stories read aloud in English;

(ii) with some exceptions, recognize sight vocabulary and high-frequency words to a degree nearly comparable to that of native English-speaking peers; and

(iii) with minimal second language acquisition support, have an ability to decode and understand grade-appropriate English text at a level nearly comparable to native English-speaking peers.

(4) Reading, Grades 2-12. ELLs in Grades 2-12 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. The following proficiency level descriptors for reading are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.
(A) Beginning. Beginning ELLs have little or no ability to read and understand English used in academic and social contexts. These students:

(i) read and understand the very limited recently practiced, memorized, or highly familiar English they have learned; vocabulary predominantly includes:

   (I) environmental print;

   (II) some very high-frequency words; and

   (III) concrete words that can be represented by pictures;

(ii) read slowly, word by word;

(iii) have a very limited sense of English language structures;

(iv) comprehend predominantly isolated familiar words and phrases; comprehend some sentences in highly routine contexts or recently practiced, highly familiar text;

(v) are highly dependent on visuals and prior knowledge to derive meaning from text in English; and

(vi) are able to apply reading comprehension skills in English only when reading texts written for this level.

(B) Intermediate. Intermediate ELLs have the ability to read and understand simple, high-frequency English used in routine academic and social contexts. These students:

(i) read and understand English vocabulary on a somewhat wider range of topics and with increased depth; vocabulary predominantly includes:

   (I) everyday oral language;

   (II) literal meanings of common words;

   (III) routine academic language and terms; and

   (IV) commonly used abstract language such as terms used to describe basic feelings;

(ii) often read slowly and in short phrases; may re-read to clarify meaning;

(iii) have a growing understanding of basic, routinely used English language structures;

(iv) understand simple sentences in short, connected texts, but are dependent on visual cues, topic familiarity, prior knowledge, pretaught topic-related vocabulary, story predictability, and teacher/peer assistance to sustain comprehension;

(v) struggle to independently read and understand grade-level texts; and

(vi) are able to apply basic and some higher-order comprehension skills when reading texts that are linguistically accommodated and/or simplified for this level.

(C) Advanced. Advanced ELLs have the ability to read and understand, with second language acquisition support, grade-appropriate English used in academic and social contexts. These students:

(i) read and understand, with second language acquisition support, a variety of grade-appropriate English vocabulary used in social and academic contexts:

   (I) with second language acquisition support, read and understand grade-appropriate concrete and abstract vocabulary, but have difficulty with less commonly encountered words;
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(II) demonstrate an emerging ability to understand words and phrases beyond their literal meaning; and

(III) understand multiple meanings of commonly used words;

(ii) read longer phrases and simple sentences from familiar text with appropriate rate and speed;

(iii) are developing skill in using their growing familiarity with English language structures to construct meaning of grade-appropriate text; and

(iv) are able to apply basic and higher-order comprehension skills when reading grade-appropriate text, but are still occasionally dependent on visuals, teacher/peer assistance, and other linguistically accommodated text features to determine or clarify meaning, particularly with unfamiliar topics.

(D) Advanced high. Advanced high ELLs have the ability to read and understand, with minimal second language acquisition support, grade-appropriate English used in academic and social contexts. These students:

(i) read and understand vocabulary at a level nearly comparable to that of their native English-speaking peers, with some exceptions when low-frequency or specialized vocabulary is used;

(ii) generally read grade-appropriate, familiar text with appropriate rate, speed, intonation, and expression;

(iii) are able to, at a level nearly comparable to native English-speaking peers, use their familiarity with English language structures to construct meaning of grade-appropriate text; and

(iv) are able to apply, with minimal second language acquisition support and at a level nearly comparable to native English-speaking peers, basic and higher-order comprehension skills when reading grade-appropriate text.

(5) Writing, Kindergarten-Grade 1. ELLs in Kindergarten and Grade 1 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. The following proficiency level descriptors for writing are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction and should take into account developmental stages of emergent writers.

(A) Beginning. Beginning ELLs have little or no ability to use the English language to build foundational writing skills. These students:

(i) are unable to use English to explain self-generated writing such as stories they have created or other personal expressions, including emergent forms of writing (pictures, letter-like forms, mock words, scribbling, etc.);

(ii) know too little English to participate meaningfully in grade-appropriate shared writing activities using the English language;

(iii) cannot express themselves meaningfully in self-generated, connected written text in English beyond the level of high-frequency, concrete words, phrases, or short sentences that have been recently practiced and/or memorized; and

(iv) may demonstrate little or no awareness of English print conventions.

(B) Intermediate. Intermediate ELLs have a limited ability to use the English language to build foundational writing skills. These students:
(i) know enough English to explain briefly and simply self-generated writing, including emergent forms of writing, as long as the topic is highly familiar and concrete and requires very high-frequency English;

(ii) can participate meaningfully in grade-appropriate shared writing activities using the English language only when the writing topic is highly familiar and concrete and requires very high-frequency English;

(iii) express themselves meaningfully in self-generated, connected written text in English when their writing is limited to short sentences featuring simple, concrete English used frequently in class; and

(iv) frequently exhibit features of their primary language when writing in English such as primary language words, spelling patterns, word order, and literal translating.

(C) Advanced. Advanced ELLs have the ability to use the English language to build, with second language acquisition support, foundational writing skills. These students:

(i) use predominantly grade-appropriate English to explain, in some detail, most self-generated writing, including emergent forms of writing;

(ii) can participate meaningfully, with second language acquisition support, in most grade-appropriate shared writing activities using the English language;

(iii) although second language acquisition support is needed, have an emerging ability to express themselves in self-generated, connected written text in English in a grade-appropriate manner; and

(iv) occasionally exhibit second language acquisition errors when writing in English.

(D) Advanced high. Advanced high ELLs have the ability to use the English language to build, with minimal second language acquisition support, foundational writing skills. These students:

(i) use English at a level of complexity and detail nearly comparable to that of native English-speaking peers when explaining self-generated writing, including emergent forms of writing;

(ii) can participate meaningfully in most grade-appropriate shared writing activities using the English language;

(iii) although minimal second language acquisition support may be needed, express themselves in self-generated, connected written text in English in a manner nearly comparable to their native English-speaking peers.

(6) Writing, Grades 2-12. ELLs in Grades 2-12 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. The following proficiency level descriptors for writing are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.

(A) Beginning. Beginning ELLs lack the English vocabulary and grasp of English language structures necessary to address grade-appropriate writing tasks meaningfully. These students:

(i) have little or no ability to use the English language to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) lack the English necessary to develop or demonstrate elements of grade-appropriate writing such as focus and coherence, conventions, organization, voice, and development of ideas in English; and

(iii) exhibit writing features typical at this level, including:
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(I) ability to label, list, and copy;

(II) high-frequency words/phrases and short, simple sentences (or even short paragraphs) based primarily on recently practiced, memorized, or highly familiar material; this type of writing may be quite accurate;

(III) present tense used primarily; and

(IV) frequent primary language features (spelling patterns, word order, literal translations, and words from the student's primary language) and other errors associated with second language acquisition may significantly hinder or prevent understanding, even for individuals accustomed to the writing of ELLs.

(B) Intermediate. Intermediate ELLs have enough English vocabulary and enough grasp of English language structures to address grade-appropriate writing tasks in a limited way. These students:

(i) have a limited ability to use the English language to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) are limited in their ability to develop or demonstrate elements of grade-appropriate writing in English; communicate best when topics are highly familiar and concrete, and require simple, high-frequency English; and

(iii) exhibit writing features typical at this level, including:

(I) simple, original messages consisting of short, simple sentences; frequent inaccuracies occur when creating or taking risks beyond familiar English;

(II) high-frequency vocabulary; academic writing often has an oral tone;

(III) loosely connected text with limited use of cohesive devices or repetitive use, which may cause gaps in meaning;

(IV) repetition of ideas due to lack of vocabulary and language structures;

(V) present tense used most accurately; simple future and past tenses, if attempted, are used inconsistently or with frequent inaccuracies;

(VI) undetailed descriptions, explanations, and narrations; difficulty expressing abstract ideas;

(VII) primary language features and errors associated with second language acquisition may be frequent; and

(VIII) some writing may be understood only by individuals accustomed to the writing of ELLs; parts of the writing may be hard to understand even for individuals accustomed to ELL writing.

(C) Advanced. Advanced ELLs have enough English vocabulary and command of English language structures to address grade-appropriate writing tasks, although second language acquisition support is needed. These students:

(i) are able to use the English language, with second language acquisition support, to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) know enough English to be able to develop or demonstrate elements of grade-appropriate writing in English, although second language acquisition support is particularly needed when topics are abstract, academically challenging, or unfamiliar; and

(iii) exhibit writing features typical at this level, including:
(I) grasp of basic verbs, tenses, grammar features, and sentence patterns;
partial grasp of more complex verbs, tenses, grammar features, and sentence patterns;

(II) emerging grade-appropriate vocabulary; academic writing has a more academic tone;

(III) use of a variety of common cohesive devices, although some redundancy may occur;

(IV) narrations, explanations, and descriptions developed in some detail with emerging clarity; quality or quantity declines when abstract ideas are expressed, academic demands are high, or low-frequency vocabulary is required;

(V) occasional second language acquisition errors; and

(VI) communications are usually understood by individuals not accustomed to the writing of ELLs.

(D) Advanced high. Advanced high ELLs have acquired the English vocabulary and command of English language structures necessary to address grade-appropriate writing tasks with minimal second language acquisition support. These students:

(i) are able to use the English language, with minimal second language acquisition support, to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) know enough English to be able to develop or demonstrate, with minimal second language acquisition support, elements of grade-appropriate writing in English; and

(iii) exhibit writing features typical at this level, including:

(I) nearly comparable to writing of native English-speaking peers in clarity and precision with regard to English vocabulary and language structures, with occasional exceptions when writing about academically complex ideas, abstract ideas, or topics requiring low-frequency vocabulary;

(II) occasional difficulty with naturalness of phrasing and expression; and

(III) errors associated with second language acquisition are minor and usually limited to low-frequency words and structures; errors rarely interfere with communication.

(e) Effective date. The provisions of this section supersede the ESL standards specified in Chapter 128 of this title (relating to Texas Essential Knowledge and Skills for Spanish Language Arts and English as a Second Language) upon the effective date of this section.

Source: The provisions of this §74.4 adopted to be effective December 25, 2007, 32 TexReg 9615.
Notes
The image of the wooden abacus on the front cover was purchased from iStockphoto.com.

Abaci, or counting frames, have been employed for solving basic mathematical problems since the 2nd century BC. Though abaci were commonly made of wood, modern abaci have beads made from resin, plastic, glass, or stone.

We chose this image of a traditional wooden abacus for its warm tones and its historical association to mathematics.

This book was designed using two type families—Myriad and Times New Roman—and was produced using Adobe InDesign CS4.

Dana Center Staff:
Senior editor Rachel Jenkins and senior designer Phil Swann handled editing and design/layout. Steve Engler proofread the book. Jenkins and Swann comanaged production.
Algebra II Assessments: Performance Tasks Aligned to the Texas Standards (2nd edition, with revised TEKS; 2007)

Intended for: Algebra II teachers

Algebra II Assessments includes 38 performance tasks in eight categories linked to the Algebra II Texas Essential Knowledge and Skills:


This resource provides teachers with performance tasks that help define what the TEKS require of students in Algebra II, which is becoming increasingly important as a rigorous mathematics course for all students. Also included are connections to the TEKS and TAKS, guiding questions, model solutions, and sample student work to help you understand what your students know about Algebra II content. 376 pages.

Price (book): $40.00
Price (CD): $15.00

Geometry Assessments: Performance Tasks Aligned to the Texas Standards (2nd edition, with revised TEKS; 2007)

Intended for: Geometry teachers

Geometry Assessments includes 43 performance tasks in seven categories linked to the Texas Essential Knowledge and Skills: coordinate geometry; patterns, conjecture, and proof; properties and relationships of geometric figures; area, perimeter, and volume; solids and nets; congruence; similarity.

This resource provides teachers with performance tasks, including connections to the TEKS and the TAKS. The resource also provides guiding questions, model solutions, and sample student work to help you understand what your students know about Geometry content.

Price (book): $40.00
Price (CD): $15.00

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The Dana Center’s professional development offers focused learning opportunities for individuals, small groups, campuses, and districts.

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For descriptions of our professional development courses for leaders and teachers, mathematics educators, and science educators—and to sign up online—please go to www.utdanacenter.org/pd.

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