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Find exact width of cutting a board into N equal pieces

January 10, 2012 in Email Questions | Permalink

Question:

A friend asked me to help him cut a 10” board into 10 equal parts factoring in for the loss due to the 1/8” blade kerf. We scratched our heads and wished we would have paid more attention in math class.

We knew that we would have to make 9 cuts to get the 10 pieces. So 9 * 1/8” = 1 1/8”. So to approximate the cutting width we subtracted 1/8” from each piece and cut the pieces 7/8” inches wide. This was not exact but fairly close.

My question is what math formula should I use to get the exact width of cutting a board into equal pieces factoring in the loss of the saw kerf? Thanks

Solution:

You logic will as you said get you pretty close to the correct cutting width.

Mathematically to calculate the cut width:

$10 \times x + 9(1/8) = 10$
$10 \times x = 10 - 9/8 = 80/8 - 9/8 = 71/8$
$x = 71/80$ which is very close to 7/8.

Many times in the wood shop a ruler is not the most accurate means of measurement unless our measurements are to the nearest 8th or 16th.

In order to get a more accurate measurement of 71/80 see this [diagram] for details.

It shows how to mark on a piece of paper a length of 71/80 using standard ruler and the concept of similar triangles from geometry.
See [article](http://www.jsommer.com) *Divide a line into N equal segments* later in this document for a brief explanation.

This technique is used many times to divide a line into a given number of equal segments.

In this case by multiplying by 10 and dividing into 10 equal pieces we get an accurate length of a decimal quantity.
Gear Math
June 30, 2010 in Uncategorized | Permalink

Here is a link to an article from Make Online Magazine with information about mathematics of Gears.

http://blog.makezine.com/archive/2010/06/make_your_own_gears.html
**Compass Rose**  
October 23, 2009 in Email Questions | Permalink

**Question:**

John... I want to make a 36” dia. compass rose with true north 8 points. I can’t find a plan that gives me the angles. Do you know of any information that would be helpful. I have been doing woodworking for a while, I am 74 and keep active in my shop but decided to make small projects and not furniture like I did. thanks Dan Lober

**Solution:**

Here is an image of an 8 pt compass rose. The angles are independent of the radius of the circle.

8pt Compass Rose
Also for other number of points:

<table>
<thead>
<tr>
<th>N</th>
<th>NbyE</th>
<th>NNE</th>
<th>NEbyN</th>
<th>NE</th>
<th>NEbyE</th>
<th>ENE</th>
<th>EbyN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.25</td>
<td>22.5</td>
<td>33.75</td>
<td>45</td>
<td>56.25</td>
<td>67.5</td>
<td>78.75</td>
</tr>
<tr>
<td>E</td>
<td>EbyS</td>
<td>ESE</td>
<td>SEbyE</td>
<td>SE</td>
<td>SEbyE</td>
<td>SSE</td>
<td>SbyE</td>
</tr>
<tr>
<td>90</td>
<td>101.25</td>
<td>112.5</td>
<td>123.75</td>
<td>135</td>
<td>146.25</td>
<td>157.5</td>
<td>168.75</td>
</tr>
<tr>
<td>S</td>
<td>SbyW</td>
<td>SSW</td>
<td>SWbyS</td>
<td>SW</td>
<td>SWbyW</td>
<td>WSW</td>
<td>WbyS</td>
</tr>
<tr>
<td>180</td>
<td>191.25</td>
<td>202.5</td>
<td>213.75</td>
<td>225</td>
<td>236.25</td>
<td>247.5</td>
<td>258.75</td>
</tr>
<tr>
<td>W</td>
<td>WbyN</td>
<td>WNW</td>
<td>NWbyW</td>
<td>NW</td>
<td>NWbyN</td>
<td>NNW</td>
<td>NbyW</td>
</tr>
<tr>
<td>270</td>
<td>281.25</td>
<td>292.5</td>
<td>303.75</td>
<td>315</td>
<td>326.25</td>
<td>337.5</td>
<td>348.75</td>
</tr>
</tbody>
</table>

Here is a link to some instructions on drawing a compass rose:
Rise and fall – degrees calculation
October 23, 2009 in Email Questions | Permalink

Question:
I don’t know if I’m saying this correctly and using the correct terms. I can’t find the answer, and am having trouble figuring it out for myself. It seems like there should be a very simple formula to calculate this.

I am building a ‘veranda’ for my tortoise, which will go over his dog house, from a 4’ x 8’ sheet of plywood and 2×4’s. I want it to be at an angle so that the rain will run off. So I thought I would make it lower one inch for every 16 inches. Eight feet is 96 inches, and 16 goes into 96 six times, so one side will be 4’ tall and the other 3’6″ tall.

What I need is a formula that will take ”rise and fall“ (?) of X and Y (x=16, y=1) and convert it to the degrees that I can set my table saw to so that I can cut the correct angle for the 2×4’s.

The resulting overall length won’t actually be 8’ because of the angling, so I don’t know if that means anything, or needs to be taken into account, or what.

Attached and inserted is a diagram image.

Solution:
To find the angle you need to use some trigonometry.

In your diagram we have a right triangle with a hypotenuse of 96” (8 ft) and an adjacent leg of 4” (difference between 4’ and 3’ 6″). Since you know the rise and fall is a ratio of 1 to 16 you can find the angle by computing the arc cosine of 1/16 using a calculator [this ratio is what determines the angle, so if the length is not 8ft but you still want this slope for the over hang then still use the 1 to 16 ratio].
For example, if you go to this site http://www.carbidedepot.com/formulas-trigright.asp you will find a basic right triangle calculator. Enter 16 for side c and 1 for side a (or could use 96 and 4) then click calculate and the site will return the angle you requested which in this case is 86.42 degrees.

If you cut a smaller board in the same ratio as your bigger project, such as 1” tall and 16” long in the form of the right triangle then you will have a template for the angle you need without doing any mathematics. This is because of the mathematics’ property of similar triangles which is the basis of trigonometry – the ratio of sides and angles are the same as you make a triangle bigger and smaller using proportional increase or reduction in size.

This link to Wolfram-Alpha http://www.wolframalpha.com/input/?i=right+triangle+slope will give you information on equations of the right triangle.
Curved chest top
October 23, 2009 in Email Questions, Uncategorized | Permalink

This question builds on the previous post which shows how to calculate the center of a circle given a chord and distance from the chord to the circle (called the Saggita).

Question:

Hi,
i’m working on a wood chest and the lid i wanted it arched but i have no idea at what angle and HOW MANY pieces of wood i need to complete the lid.
My chest lid BASE measure 27 inches and the height at the middle point it should be 7″.... the stripes are about 1/2″ thick and 3″ wide. HOW do i do it?

PS: a small draft is attached to the message!
Thank you so much!

William!
Solution:

Here is a link to a GeoGebra simulation: http://www.jsommer.com/geogebra/ArcLength.html. I created to illustrate how to answer the question. Below is a picture from William with some additional lines I added to help to illustrate how the mathematical solution does apply to his original question. **Note** – the height stated as 17” is incorrect, should be 7”. 17” is more than half the length of the chord.
Here is a snapshot of a Google Spreadsheet I created to work out the calculations with his specific information. Note D1 reference is for value of PI.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Conditions</strong></td>
<td><strong>Spreadsheet Formulas</strong></td>
<td></td>
</tr>
<tr>
<td>Chord length</td>
<td>27</td>
<td>input</td>
</tr>
<tr>
<td>Height (bulge)</td>
<td>7</td>
<td>input</td>
</tr>
<tr>
<td><strong>Calculations of Arc</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radius of circle for arc</td>
<td>16.52</td>
<td>$(B4^2 + (B3/2)^2)/(2*B4)$</td>
</tr>
<tr>
<td>Angle of arc (radians)</td>
<td>1.91</td>
<td>$(2*asin((B3/2)/B6))$</td>
</tr>
<tr>
<td>Angle of arc (degrees)</td>
<td>109.63</td>
<td>$(180/D1)*B7$</td>
</tr>
<tr>
<td>Length of Arc</td>
<td>31.51</td>
<td>$B7/(2<em>D1)</em>(2<em>D1</em>B6)$</td>
</tr>
<tr>
<td><strong>Section of Arc</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle of arc (degrees)</td>
<td>11</td>
<td>input angle of arc section</td>
</tr>
<tr>
<td>Angle of arc (radians)</td>
<td>0.19</td>
<td>B11/(D1/180)</td>
</tr>
<tr>
<td>Len of Arc</td>
<td>3.17</td>
<td>$(B12/(2<em>D1))</em>(2<em>D1</em>B6)$</td>
</tr>
<tr>
<td>Small Chord length</td>
<td>3.17</td>
<td>$(2*B6)*sin(B12/2)$</td>
</tr>
<tr>
<td>Diff of error of arc &amp; chord</td>
<td>0.00</td>
<td>B13-B14</td>
</tr>
<tr>
<td># of chords(boards) needed</td>
<td>9.98</td>
<td>B9/B14</td>
</tr>
<tr>
<td>Trapezoid Angle beta</td>
<td>84.5</td>
<td>$(180-B11)/2$</td>
</tr>
<tr>
<td>Trapezoid Angle gamma</td>
<td>95.5</td>
<td>180-B17</td>
</tr>
<tr>
<td>Cutting angle 90 - beta</td>
<td>5.5</td>
<td>90-B17</td>
</tr>
</tbody>
</table>
Chest Top Calculations

I also found a nice simple calculator for circle, arcs and chords which can also be helpful.

http://www.1728.com/circsect.htm
Many times you need to calculate the radius of a circular arc for a given chord width and height (distance from chord to the top of the arc) – See diagram below:

On the diagram w represents the width of the circular arc (chord) and d is the height (distance from chord to top of the arc). Given both of these we can calculate the radius of the circle needed to create the arc from using the formula on the diagram designated as r.

Here is an example of a calculator to compute the radius:

<table>
<thead>
<tr>
<th>Entries need to be same units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord Width (w) =</td>
</tr>
<tr>
<td>Arc Height (d) =</td>
</tr>
</tbody>
</table>

The formula is derived using Analytical Geometry. The chord and arc are drawn on the XY axis with one end of the arc at point (0,0) and the other end at (w,0). The center of the arc will be at point (w/2, d). Using the standard equation of the circle and algebra we can derive the formula for the radius needed to create the arc. Below shows the derivation of the formula.
Woodworking and Mathematics

\[(x-h)^2 + (y-k)^2 = r^2\] general equation of a circle with center at \((h,k)\)

A: \(h^2 + k^2 = r^2\) for point \((0,0)\)

B: \((w-h)^2 + k^2 = r^2\) for point \((w,0)\)

C: \(\left(\frac{w}{2} - h\right)^2 + (d-k)^2 = r^2\) for point \((w/2, d)\)

Solve equation B for \(h\)

\[w^2 - 2wh + h^2 + k^2 = r^2\] using result of equation A for \(h^2 + k^2 = r^2\)

\[w^2 - 2wh + r^2 = r^2\]

\[w^2 - 2wh = 0\]

\[w(w - 2h) = 0\]

\[w = 0\] or \(w = 2h\)

\[h = \frac{w}{2}\]

Solve equation C for \(k\)

\[\frac{w^2}{4} - \frac{wh}{2} + \frac{k^2}{4} - 2dk + k^2 = h^2 + k^2\] expand equation and substitute \(r^2\) for \(h^2 + k^2\)

\[w^2 - w\left(\frac{w}{2}\right) + d^2 - 2dk = 0\] simplify and substitute \(\frac{w}{2}\) for \(h\)

\[-\frac{w^2}{4} + d^2 - 2dk = 0\]

\[d^2 - \frac{w^2}{4} = 2dk\]

\[k = \frac{4d^2 - w^2}{8d}\]

center of circle: \(\left(\frac{w}{2}, \frac{4d^2 - w^2}{8d}\right)\)

radius: \(r = d + (-1) \left(\frac{4d^2 - w^2}{8d}\right)\)

\[d + \frac{|w^2 - 4d^2|}{8d}\]

\[\frac{3d^2 + w^2 - 4d^2}{8d}\]

\[r = \frac{4d^2 + w^2}{8d}\]
Compound Miters
August 20, 2007 in Reference, Uncategorized | Permalink

Compound miter cuts involve making a miter angle cut and a blade tilt cut for projects such as crown molding, hexagon shaped container with sides tilted out, etc. Some of these cuts require not only a standard miter angle (angle 1 in illustration) in a vertical direction but also a horizontal angled cut called a bevel (angle 2).

I have found several very good sites which are good resources with tools, explanations and mathematics for compound miter cuts and application.

This site has a Advanced Box Cutting calculator to determine the proper cutting angles for polygons http://www.josephfusco.org/Calculators/Advanced_Box_Cutter.html. And here is the page for the mathematics of Dihedral and Equal Angle Polygons (regular) http://www.josephfusco.org/Articles/Dihedral/Dihedral1.html.

A site with a calculator specific for a pyramid. http://www.cleavebooks.co.uk/scol/calpyr.htm

The same author also has good explanations, tables and a calculator for cutting crown molding http://www.josephfusco.org/Articles/Crown_Moulding/crownscript.html. For additional information and instructions on Crown Molding see Alter Eagle site http://altereagle.com/How_to_install.html.

This is a link to a Dr. Math article about Pyramid Construction http://www.mathforum.org/library/drmath/view/56462.html in which he includes some of the mathematics and a good way to visualize a compound angle and its mathematics.
Taper Jig
May 31, 2007 in Email Questions, Uncategorized | Permalink

Original Question:
I am constructing a taper jig to use with my ShopSmith woodworking machine. Is there an already derived formula for conversion of taper (inches per foot) directly to degrees and vice versa. I used the tangent of the opposite side (taper) divided by one foot (12 inches, adjacent side of triangle) and then look up the degrees for the derived tangent for my right triangle. This is the answer but I thought perhaps you already have a quick and easy formula for this. Your help is appreciated. I really like your website. It has much useful information.
Answer:
This is an illustration of a typical taper jig used on a table saw.

The pictures below are pages describing the mathematics of a taper jig and suggestion how to add a measuring device to set the jig taper directly in inches per foot.
Taper Jig Mathematics

Taper jig is adjustable at point A so angle \( \theta \ (\angle BAC) \) is variable.

Since \( \overline{AB} \parallel \overline{DF} \) and \( \overline{AC} \parallel \overline{DE} \ (\parallel \rightarrow \text{parallel}) \)
then \( \angle BAC = \angle EDF \)

and \( \angle ABC = \angle DEF = 90^\circ \)

then \( \triangle ABC \cong \triangle DEF \) \( (\cong \text{ similar}) \)

From properties of similar triangles:

\[ \frac{BC}{AE} = \frac{EF}{DE} \]

from trigonometry

\[ \tan \theta = \frac{BC}{AB} = \frac{EF}{DE} \]

so you can find the angle of the jig using trig tables or \( \tan^{-1} \) function on a calculator.

Let \( T = \frac{EF}{DE} \). We will call \( T \) the taper ratio.

John Sommer

Taper Jig
Instead of setting the jig for a given angle we may want to set the taper as \( T \) inches per foot.

Suppose we wanted a taper of \( 1 \) inch per foot. That would mean if we wanted to taper our board up to 18" on the board the saw cut would be 1 ½" from edge.

\[
T = \frac{\frac{18}{12}}{\frac{18}{12} + \frac{16}{12}} = \frac{1}{14}
\]

If we wanted to know the angle

\[
\tan \theta = T = \frac{1}{14} \Rightarrow \theta = 4.8^\circ
\]

Instead of trying to set the angle for the desired taper we can use properties of similar triangles to create a set of graduations on BC and read our desired taper \( T \) directly.
Since \[ \frac{BC}{AB} = \frac{EF}{BE} = T \] (see figure 1)

we can set up BC bar to measure the desired taper.

Point B is fixed and C varies by taper needed.

Set \( AB \) equal to an integral multiple of 12 inches (calculations are easier).
Let multiple be \( M \).

Thus \( AB = M \) ft. \( \Rightarrow T = \frac{BC\text{ inches}}{M\text{ ft.}} = \frac{BC}{M} \frac{1}{12} \text{ ft.} \)

Suppose \( M = 2 \). We would then set the graduations on \( BC \) to be \( \frac{1}{M} = \frac{1}{2} \) of actual measurement. Thus 2" on \( BC \) would be marked as 1 on the graduation for \( BC \).
Then we can read \( T \) directly on \( BC \).

To get a taper of 1½" per foot we would adjust taper jig so \( BC = 1\frac{1}{2} \) on our graduation which is 3" actual.
Reader’s response.

Thought you might like to see my completed Taper Jig (picture1, picture2). I used a 24 inch long board next to the work instead of a 12 inches one, so my separation of the “V” is in 1/2 inch increments instead of quarters of an inch.

i.e. 1/4 inch per foot on 12 inches is same as 1/2 per foot on 24 inch length.

I used tangent to calculate the angles in degrees for each setting. My scale goes from 1/4 inch per foot to 2 1/2 inches per foot and from 1.16 to 11.57 degrees. Note the taper in inches per foot is the left column and the degrees are in the right column.

Again, thank you so much for your help. I think I was on the right track, just need a little reassurance and coaching.
Woodworking and Mathematics
Original Question:

My deck has a curve in it. It’s like a half circle, except its depth is not equal to its width. From a bird’s eye view, the curve is 72” wide and 48” deep. The deck posts are already set.

There are posts at the point where the curve begins and ends and there are two additional posts set at equal intervals, dissecting the curve into three parts.

To cut the top and bottom 3.5” wide rails (2x4 lumber on the straight rail) I will use 2x6 lumber and draw a pattern to produce the three sections of curved railing.

To cut the rail cap (5.5” wide, or a 2x6 on the straight rail cap) I will use a 2x8 or 2x10 to draw a pattern to produce the three sections of curved 5.5” wide rail cap.

I can create the curve for the railing in two ways:

1. Transfer the curve of the facia board (that covers the deck framing’s joist ends) to the deck rails?
2. Calculate the railing curve, as dictated by the post placement and curve dimensions.

Here is my question:

How do I do the easiest of the two methods?

Or, is there an easier way to calculate the rail curves so I can draw my patterns?
Answer:

Diagram of the representation of the curved deck railing:

Calculations:

**GIVEN:**
Width of curve of deck 72"
Depth of curve 48"
Width of rail 5.5"

**LEGEND:**
R = radius of deck curve. [37.5"
]
a = angle of arc of 3 equal sections of deck rail. [70.8°]
V = added length for rail width
Z = length of cord of rail. [43.45" + 2* 9.04"
]
W = minimum width of board needed to cut curve. [6.93" + width of the rail 5.5"
]
V = additional width to W (to draw pattern for curve) [6.75"]
Calculation Diagram

Calculate R:
\[ 48 \cdot Y = R \]
\[ 36^2 + Y^2 = R^2 \]
\[ 36^2 + (48 - R)^2 = R^2 \]
thus \( R = 37.5 \)

Angles:
\[ \sin b^\circ = 36 / 37.5 \text{ thus } b^\circ = 73.74^\circ \]
\[ b + b + a + a/2 + a/2 + a = 360 \text{ thus } a = 70.8^\circ \]

Calculate W & Z:
\[ \cos (a/2) = X / R \text{ thus } X = \cos (70.8 / 2) \cdot 37.5 = 30.57 \]
\[ W + X = R \text{ thus } W = 37.5 - 30.57 = 6.93 \]
\[ \sin (a/2) -(Z / 2) / R \text{ thus } Z = 2 \cdot \sin(70.8 / 2) \cdot 37.5 - 43.45 \]

Blow Up of Green Triangle

to find added width for board to cut rail curve:
\[ 5.5 / V = \cos (70.8 / 2) \]
\[ V = 5.5 / \cos(35.4) = 6.75^\circ \]

5.5

\[ \sqrt{2} \]

V
I’m into building bird feeders (see picture) and need to know if there is a formula to calculate the length and the width of the pie shaped pieces that make up the roof, I would like to keep the roof about 16” wide and about 5” high using 10 pie or (pyramid) shaped pieces per roof. If you have a better or easier way to compute the sizes I’m all ears.
Answer:

A "birds eye" view of the bird house. The house has ten sides with radius of 8".

Angle AOB will be 36 degrees (360/10). We can calculate AB as follows:
Bisect AOB with a perpendicular to AB. This will also bisect AB.

\[
\sin 18 = \frac{BC}{8} = \frac{AC}{8}
\]

\[
BC = 8 \times \sin 18 = 2.4721
\]

\[
AB = 2 \times BC = 4.9443\text{"}
\]

which will be the length of the base of each triangle forming the roof.

The side view of the bird feeder.
OT is the inside height which was given to be 5\".
The base is 8\" (half the diameter given – 16\")

\[ AT^2 = OA^2 + OT^2 = 64 + 25 = 89 \]
\[ AT = 9.434\" \]

\[ \tan (\text{Angle AOT}) = \frac{5}{8} \text{ thus} \]
\[ \text{Angle AOT} = 32 \text{ degrees} \]

**Reader’s Response:**

Thank You so much John, When I saw your trig formula for figuring this out for me a light when on and my math days started coming back, this will help me enormously, Thank You again. –Kenneth
Pole Barn Angle Brace
May 31, 2007 in Email Questions, Uncategorized | Permalink

Question:
I’m constructing a pole barn. Think of a wall as consisting of several rectangles in a row, each defined by an overhead 2×6 and a bottom 2×6 and a vertical 2×6 on each side. I am placing a 2×6 “angle brace” in each rectangle from the bottom left side to the top right side. Is there an easy way to obtain the needed cut angles at the top and bottom of the “angle brace” 2×6?

Answer:
**Math to calculate cutting angle for diagonal brace:**

Given following information:

- A = width of rectangle
- B = height of rectangle
- C = length of diagonal
- $\theta^\circ = \text{angle diagonal makes with bottom}$

Want to calculate C and $\theta^\circ$

- $C = \sqrt{A^2 + B^2}$ [Pythagorean theorem]
- $\tan \theta^\circ = \frac{A}{B}$ and
- $\theta^\circ = \arctan \left( \frac{A}{B} \right)$

**Example:**

If A = 96" and B = 85" then

- $C = \sqrt{96^2 + 85^2} = 128.22"$
- And
- $\theta^\circ = \arctan \left( \frac{96}{85} \right) = 48.45^\circ$
Original Question:
I was wondering if you could tell me how a scale ruler works. I can see the obvious in the feet part of the ruler, but can’t figure out how to read the fraction of a inch. The whole numbers I get, the fractions is the concept giving me trouble. Thanks for any help you can give me on this.

Answer:

Architect’s scale

From Wikipedia, the free encyclopedia.

An architect’s scale is a specialized ruler. It is used in making or measuring from reduced scale drawings, such as blueprints. It is marked with a range of calibrated (scales).

The scale was traditionally made of wood but for accuracy and longevity the material used should be dimensionally stable and durable. Today they are now more commonly made of rigid plastic or aluminum. Depending on the number of different scales to be accommodated architect’s scales may be flat or shaped with a cross-section of an equilateral triangle.

United States and Imperial units

In the United States, and prior to metrification in Britain, Canada and Australia, architect’s scales are/were marked as a ratio of x inches-to-the-foot. For example one inch measured from a drawing with a scale of “one-inch-to-the-foot” is equivalent to one foot in the real world (a scale of 1:12) whereas one inch measured from a drawing with a scale of “two-inches-to-the-foot” is equivalent to six inches in the real world (a scale of 1:6).

Typical scales used in the United States are:

- Full scale, with inches divided into sixteenths of an inch

  The following scales are generally grouped in pairs using the same dual-numbered index line:

  - three-inches-to-the-foot (1:4) / one-and-one-half-inch-to-the-foot (1:8)
  - two-inches-to-the-foot (1:6) / one-inch-to-the-foot (1:12)
  - three-quarters-inch-to-the-foot (1:16) / three-eighths-inch-to-the-foot (1:32)
  - one-half-inch-to-the-foot (1:24) / one-quarter-inch-to-the-foot (1:48)
  - one-eighths-inch-to-the-foot (1:96) / one-sixteenths-inch-to-the-foot (1:192)

From http://www.tpub.com/content/engineering/14069/css/14069_75.htm
Standard scales on an architect's scale ruler.

Notice that all scales except the 16th scale are actually two scales that read from either left to right or right to left. When reading a scale numbered from left to right, notice that the numerals are located closer to the outside edge (top of the ruler). On scales that are numbered from right to left, the numerals are located closer to the inside edge (middle of the ruler).

Architect's scales are divided (only the main divisions are marked throughout the length) with the only subdivided interval being an extra interval below the 0-ft mark. These extra intervals
are divided into 12ths. To make a scale measurement in feet and inches, lay off the number of feet on the main scale and add the inches on the subdivided extra interval. However, notice that the 16th scale is fully divided with its divisions being divided into 16ths. Now let’s measure off a distance of 1 ft 3 in. to see how each scale is read and how the scales compare to one another. Since the graduations on the 16th scale are subdivided into 16ths, we will have to figure out that 3 in. actually is 3/12 or 1/4 of a foot. Changing this to 16ths, we now see we must measure off 4/16ths to equal the 3-in. measurement. Note carefully the value of the graduations on the extra interval, which varies with different scales. On the 3 in. = 1 ft scale, for example, the space between adjacent graduations represents one-eighth in. On the 3/32 in. = 1 ft scale, however, each space between adjacent graduations represents 2 in.

Example with fractions:

For a harder example let’s look closer at the ¼ (left) and 3/8 (right) scale on the ruler. For this example we will use the 3/8 scale. Reading from right to left 0, 1 (14 on ¼ scale), 2, 3 (12 on ¼ scale) etc. Each of the larger marks are 3/8 units in length (which could be feet, inches, etc. depending on your major unit of measurement). The further graduated portion of the ruler on the far right side (from 0 marking to last mark before 3/8) is divided into 12 equal divisions. 12 equal divisions are used so you can easily measure fractional portions of the unit.

Example: Need to mark of 3 5/8” on the 3/8 scale. The 3” mark is 13 mark from the right hand 0 mark. To get 5/8” we use the further graduations on the far right side. There are a total of 12 markings (looking from 0 to the right). To represent 5/8 of 12 (5/8 * 12), [5/(2*4)] * (3*4) = (5*3)/2 = 15/2 = 7 ½. In order to mark off 5/8” on the 3/8 scale we move to mark 7 and half way to mark 8 for 7 ½. diagram for the illustration.

Example: 3 5/8 on a 3/8 scale ruler

To mark 5/8 on the 3/8 scale ruler:
Each large division on the 3/8 scale ruler is 3/8 of one unit (a unit could be a foot, inch, meter, etc.) The finer divisions of the 3/8 ruler are divided such that 12 parts = one unit.
To find 5/8 of the one unit we take 5/8 times 12.
5/8 * 12/1 = 15/2 = 7 1/2

Diagram which may help to explain how to calculate the multiplication of fraction 5/8 of 12. I used an applet from http://www.arcytech.org/java/fractions/fractions.html to create the illustration.
Reader’s response.

Thanks very much for the information. I was doing fine until it came to dividing fractions. I got lost when it started talking about dividing 3/8 by 12. Could you please help me with fraction multiplication?

Thanks for trying to help out…I need to do alot of work to learn how to be better at math…but I do appreciate all you have done here. It’s good to know there are people like yourself who want to help others in need…thanks
Woodworking and Mathematics

Golden Ratio
May 29, 2007 in Reference, Uncategorized | Permalink

As a wood worker you have many opportunities to design furniture, cabinets, houses, etc. How many times have you wondered about the best way to determine the dimensions and proportions that would look just right. The ancient Greeks, Phidias in particular, have some help for you. We need to first look at something called the Divine Proportion.

"Geometry has two great treasures! One is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel."

--Johannes Kepler [1571-1630]

What is the most aesthetically pleasing way to divide a line. In half, thirds, quarters? Art theorists speak of a "dynamic symmetry". From the ancient Greeks to Western art, the answer is the Divine proportion.

A line divided in a divine proportion is divided such that the ratio of the length of the line to the longer segment equals the ratio of the longer segment to the shorter one.

\[
\frac{AB}{CB} = \frac{CB}{AC}
\]

Refer to the figure below. It turns out that this ratio is always equal to 1.6180339887... (close to 1 5/8). This number is commonly called the Divine proportion, or Phi (after the Greek sculpture Phidias who utilized this proportion in his work).

This proportion has been found in many areas of nature. From growth patterns in flowers and plants to the rise and fall of the market for a stock analyst. If you are interested in studying more about this ratio from the mathematician’s perspective, I suggest you start with the study of Fibonacci numbers, which starts with the story of multiplying rabbits.

Right now, I would like to move onto the two dimensional form of the Divine proportion – the Golden Rectangle. Here are the basic steps to construct the golden rectangle.

The construction starts with creation of a square of any size.
Next step is to divide the square in half.

Mark point G on the same line as the bottom of the square such that FB = FG. One way to do this is to use a compass to draw an arc of a circle with center at F and radius of FB. Point G is the intersection of the arc and extension of line CD.
Complete a rectangle with the intersection of top of square AB and perpendicular extension of Point G. We then have rectangle AHGC.

5. \( \frac{AH}{AC} = \frac{CG}{HG} = \frac{\sqrt{5} + 1}{2} \)

\( \Phi = \frac{\sqrt{5} + 1}{2} \approx 1.61803398... \)

I removed the intermediate construction lines to show our resulting rectangle and the ratio of the long to short side. The sides of the Golden Rectangle are the same ratio as the Divine Proportion. The Golden Rectangle can be used to help design furniture which is not only functional but pleasing to the eye.
Other golden ratio constructions.

Here are examples of the Golden Ratio in other geometric figures.
Triangles and Quadrilaterals

Golden Ratio
\[ \frac{AB}{AG} = \frac{AG}{GB} = \phi \]

Golden Rectangle
\[ \frac{AB}{AD} = \frac{CD}{BC} = \phi \]

Golden Spiral Curve
ABCD Golden Rectangle
\[ \frac{AC_1}{AD} = \frac{CG_2}{BC} = \phi \]

Golden Rhombus
\[ \frac{PC}{\phi B} = \phi \]

Golden Triangle
\[ \frac{AC}{AB} = \frac{BC}{AB} = \phi \]

Golden Root 5 Rectangle
\[ \phi = \frac{1 + \sqrt{5}}{2} \]

Golden Brick
Surface Area of Sphere
\[ S = 4\pi r^2 \]

Diagonal across 2
\[ A + B = C \]

Total Surface Area
\[ 15 : 4\phi \]

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Circle, Ellipse and Pentagon - a very golden shape

1. Golden Circles
   \[ \phi \]
   \[ \frac{\text{Radius}}{\phi} = \frac{1}{2} \]
   \[ C = \pi \phi \]

2. Golden Ellipse
   \[ \frac{\text{Major Axis}}{\text{Minor Axis}} = \phi \]
   \[ A = \pi \phi \]

3. Golden Pentagon
   Concentric Circles:
   \[ \text{Outer radius} = a \]
   \[ \text{Next} = \frac{a}{\phi} \]
   \[ \text{Next} = \frac{a}{\phi^2} \]
   \[ \text{Next} = \frac{a}{\phi^3} \]
   etc.

Each triangle formed with vertices of a Pentagon is a Golden Triangle.

\[ a = \frac{a + b}{\phi} = \frac{a + b + c}{\phi^2} \]
\[ a = \phi \]
\[ b = \phi^2 \]
\[ c = \phi^3 \]
all equal \( \phi \)
Divide a line into N equal segments

May 27, 2007 in Tips, Uncategorized | 1 comment

The standard method of dividing a line into N equal segments uses the properties derived from similar triangles where corresponding sides are in proportion to each other. This site http://www.mathopenref.com/constdividesegment.html has a very nice illustration using Java to demonstrate how this can be done.

Below is an excerpt from First Six Books of Euclid which has his geometrical explanation of how to divide a line into equal segments.

(15.) From a given point to draw a line cutting two given parallel lines, so that the difference of its segments may be equal to a given line.

Let $AB$, $CD$ be the given parallels, and $P$ the given point. From $P$ draw any line $PB$, meeting the given lines in $B$ and $E$. Make $EF = EP$, and draw $FG$ parallel to $AB$. With any point $O$ as centre, and radius equal to the given line, describe a circle cutting $GF$ in $H$. Join $OH$, and draw $PGA$ parallel to it. $PGA$ will be the line required.

Since $PE$ is equal to $EF$, $:\therefore PI = IG$; and $AG$ is equal to the difference of $AI$ and $IP$, the segments of $PA$; and $AG = OH =$ the given line.

This diagram at the bottom demonstrates the Glad method to divide a line into regular partitions. http://jwilson.coe.uga.edu/emt668/EMT668.Folders.F97/Waggener/Units/Partitions/partitions.htm
Introduction to Fractions
May 27, 2007 in Tips, Uncategorized | Permalink

Fractions are encountered just about every time you step into the shop. Here is a link to some pages that go over the basics of fraction math http://www.sosmath.com/algebra/fraction/frac1/frac1.html. It is from S.O.S Math web site, which is a free resource of math review material. You will find a topic such as fractions broken down into logical sections with very good explanations and problems to check your understanding.

What follows are a few tools to make working with fractions easier in your shop.

First, are two tables you can print out to keep in your shop. They show addition and subtraction for the most common fractions used by woodworkers. From 1/2 to n/16 each table has all combinations of addition or subtraction. To use the table for addition, pick your first fraction from the top row, find the second fraction in the first column then read across until the column and row intersect (see red line on table for this example). Follow same process for subtraction using the subtraction table. For example; 3/4 – 3/8 is computed by finding 3/4 in the top row and 3/8 in the left column. Read the answer on the intersection. (see blue line on the table).

For example: 3/4 + 3/8 = 1 1/8 and 3/4 – 3/8 = 3/8

### Fraction’s Addition Table

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Second, I have included a diagram of an easy to make shop calculator to add and subtract fractions using two standard rulers. It is made by using two regular rulers one above another. To add two fractions, slide the top ruler until its left edge lines up with the first number. In the picture below the first number is 3/8. Find the second number on the top ruler, say 5/16, then read the sum on the ruler below. The picture shows two examples: 3/8 + 5/16 = 1 11/16; and 3/8 + 7/8 = 1 ¼.

To subtract two fractions, find the first fraction on the bottom ruler. Move the top ruler until the second fraction is directly above the first. On the picture look at 1 1/4 on the bottom (yellow ruler), above it (on the aqua ruler) you will find 7/8. Now go the the left end of the top rule and read the answer on the bottom ruler. 1 1/4 – 7/8 = 3/8.