Transportation Engineering Module for Civil PE License

Separate Breadth (AM) & Depth (PM) Topics Per NCEES

A Concise & Comprehensive Summary of Construction Engineering Equations (Code & Non-Code), Tables, Charts, and Figures is Provided for a Quick Access in the Exam

Dr. Shahin A. Mansour, PE

First Edition
ISBN ?????????

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Current printing of this edition   1
Preface

Our next generation of Civil PE books were carefully developed by Dr. Shahin A. Mansour, Ph.D., P.E., a nationally recognized expert in PE Exam Preparation courses and founder of Professional Engineering Services, Inc., (PES). This generation is quickly becoming the Best in Class solution for engineering professionals working to pass upcoming California Board and NCEES exams. Each and every book in this comprehensive collection has been prepared with your readiness in mind and will serve as a powerful guide throughout your PE Exam Preparation studies in Civil PE, Seismic, and Surveying concentrations.

Dr. Mansour has taught and spoken to thousands of engineers and has been teaching PE, Seismic, Surveying, and EIT/FE classes for the past 22 years enabling his products, classes, and seminars to continuously evolve and adapt to the changing demands of this challenging field. His real-world experience and intercommunication with so many engineers throughout America has greatly improved his ability to craft concise and knowledgeable resources that work for professionals striving to pass their Professional Engineering Licensing Board Exam and other California special exams.

One of the primary advantages of PES’ preparation books is their compact and succinct format. The experiences and feedback of engineering students and professionals that have worked with these materials affirms our effective use of good, factual diagrams and illustrations to more fully explain concepts throughout each text. In fact, Dr. Mansour makes great use of this style of teaching through figures, comparative tables, charts, and illustrations to concisely convey important information to engineers preparing for the exam. The primary focus of each book in our collection is targeted on subjects that are directly relevant to the PE exam. This method provides substantial time savings to students that simply don’t have the wherewithal to digest leading competitive volumes, which typically exceed 1,000 pages.

Another strong asset of these next generation books is the separation of Breadth (a.m.) and Depth (p.m.) topics. Each Civil PE, Seismic, and Surveying book is organized in accordance with the most current exam specifications and has matured since their inception through the inputs, complaints, wishes, and other feedback given from course participants and colleagues for over 22 years. Recent changes in the format of the Civil PE exam have made it necessary to address the change from essay to multiple choice problems, separation of a.m. and p.m. sections, and the introduction of the construction module. The books’ improved organization is completed with a very detailed Table of Contents, which addresses both the a.m. and p.m. subjects and more, making them a favored resource for preparation as well as exam time.

The best part in all this growth and continuous improvement is the ability to apply this knowledge and experience to our supplemental products developed to give you the best head start for passing your Professional Engineering Licensing Board Exam. Every book, DVD, seminar, and accompanying Problems & Solutions workbooks are designed with all relevant codes, topics, board test plans, and NCEES exam specifications. And, these study materials and seminars, also manage to cover these fields of study in the same order as listed in the latest exam specifications for easy reference at-a-glance.

Not only are our next generation books written to be current and well-organized to save you time during exam preparation and test taking; they are also affordable and authored by a highly qualified and nationally recognized Civil Engineering Professor. Dr. Shahin A. Mansour, Ph.D., P.E. has helped thousands of students to pass their Professional Engineering Licensing Board Exam. His easy, step-by-step approach to solving problems has gained him popularity and a great reputation among students and professionals of all ages. We hope that you'll find the many resources at Professional Engineering Services, Inc. to be exceptional textbooks, DVDs, and seminars and as valuable a resource as we believe them to be.
Disclaimer
This publication is to help the candidates for the Transportation Module for PE Civil License. This publication expresses the opinion of the author. Every effort and care has been taken to ensure that all data, information, solutions, concepts, and suggestions are as accurate as possible. The author cannot assume or accept the responsibility or liability for errors in the data, information, solutions, concepts, and suggestions and the use of this material in preparation for the exam or using it during the exam.

Also, Professional Engineering Services (PES) and the author are in no way responsible for the failure of registrants in the exam, or liable for errors or omission in the solutions, or in the way they are interpreted by the registrants or others.

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PES and the author have made a substantial effort to ensure that the information in this publication is accurate. In the event that corrections or clarifications are needed, those will be posted on the PES website at http://www.passpe.com. PES and the author at their discretion, may or may not issue written errata. PES and the author welcome comments or corrections which can be emailed to info@passpe.com

How to Use This Book
This book was written to help you prepare for the PE-Civil. The following is a suggested strategy preparation for the exam:

1- Have all references organized before you start. Also, familiarize yourself with the comprehensive summary provided in this book.
2- Read the questions and ALL answers carefully and look for KEY WORDS in the question and the 4 possible answers.
3- Solve the easy questions first (ones that need no or minimum calculations) and record your answers on the answer sheet.
4- Work on questions that require lengthy calculations and record your answers on the answer sheet.
5- Questions that seem difficult or not familiar to you and may need considerable time in searching in your references should be left to the end
6- Never leave an answer blank
7- Remember the D³ rule:

DO NOT EXPECT THE EXAM TO BE EASY
DO NOT PANIC
DO NOT WASTE TOO MUCH TIME ON A SINGLE EASY, DIFFICULT, OR UNFAMILIAR PROBLEM

About the Author
Dr. Shahin A. Mansour, PE, has been teaching PE, Seismic, Surveying and EIT courses for the last 22 years. Dr. Mansour taught Civil Engineering Courses for 7 years at New Mexico State University (NMSU), Las Cruces, NM, USA. Also, he has been teaching Civil and Construction Engineering Courses (graduate & undergraduate) at CSU, Fresno, CA, for the last 20 years.

Dr. Mansour has helped thousands of students to pass their Professional Engineering Licensing Board Exam. His easy, step-by-step approach to solving problems has gained him popularity and a great reputation among students and professionals of all ages.
TABLE OF CONTENTS

Preface
Study Tips and Suggested Exam Strategy
NCEES New Format for the PE (Civil) Exam
List of References per NCEES
Summary of the Equations

Breadth (A.M.) Topics

1. Horizontal curves
2. Vertical curves
3. Sight distance
4. Superelevation
5. Vertical and/or horizontal clearances
6. Acceleration and deceleration

Depth (P.M.) Topics

1. Traffic capacity studies
2. Traffic signals
3. Speed studies
4. Intersection analysis
5. Traffic volume studies
6. Sight distance evaluation
7. Traffic control devices
8. Pedestrian facilities
9. Driver behavior and/or performance
10. Intersections and/or interchanges
11. Optimization and/or cost analysis (e.g., transportation route A or transportation route B)
12. Traffic impact studies
13. Capacity analysis (future conditions)
14. Roadside clearance analysis
15. Conflict analysis
16. Work zone safety
17. Accident analysis
18. Hydraulics
    1. Culvert design
    2. Open channel
19. Hydrology
    1. Hydrograph development and synthetic hydrographs
20. Engineering properties of soils and materials (e.g. index properties, identification of types of soils; suitable or unsuitable, boring logs) 
21. Soil mechanics analysis (e.g. Soil behavior, soil classification soil compaction) 
22. Engineering economics 
1. Value engineering and costing 
23. Construction operations and methods (e.g. erosion control measures, excavation / embankment) 
24. Pavement structures (e.g. flexible and rigid pavement design)

<table>
<thead>
<tr>
<th>Breadth (A.M.) Topics</th>
<th>Depth (P.M.) Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Topics</td>
<td>24 Topics</td>
</tr>
</tbody>
</table>

**Total Number of Topics = 30**
The National Council of Examiners for Engineering and Surveying
Principles and Practice of Engineering Examination
TRANSPORTATION Design Standards
Effective Beginning with the April 2011 Examinations

ABBREVIATION DESIGN STANDARD TITLE


4- AI  The Asphalt Handbook (MS-4), 2007, 7th edition, Asphalt Institute, Lexington, KY


7- PCA  Design and Control of Concrete Mixtures, 2002, 14th edition, Portland Cement Association, Skokie, IL.


Notes
1. Including all changes adopted through November 14, 2007.
NCEES Principles and Practice of Engineering CIVIL BREADTH and TRANSPORTATION DEPTH Exam Specifications Effective Beginning with the April 2008 Examinations

Â The civil exam is a breadth and depth examination. This means that examinees work the breadth (AM) exam and one of the five depth (PM) exams.

Â The five areas covered in the civil examination are construction, geotechnical, structural, transportation, and water resources and environmental. The breadth exam contains questions from all five areas of civil engineering. The depth exams focus more closely on a single area of practice in civil engineering.

Â Examinees work all questions in the morning session and all questions in the afternoon module they have chosen. Depth results are combined with breadth results for final score.

Â The exam is an 8-hour open-book exam. It contains 40 multiple-choice questions in the 4-hour AM session, and 40 multiple-choice questions in the 4-hour PM session.

Â The exam uses both the International System of Units (SI) and the US Customary System (USCS).

Â The exam is developed with questions that will require a variety of approaches and methodologies, including design, analysis, and application. Some problems may require knowledge of engineering economics.

Â The knowledge areas specified as examples of kinds of knowledge are not exclusive or exhaustive categories.

Â The specifications for the AM exam and the Transportation PM exam are included here. The design standards applicable to the Transportation PM exam are shown on the last page.

CIVIL BREADTH Exam Specifications

Approximate Percentage of Examination

20%

I. CONSTRUCTION

A. Earthwork Construction and Layout
   1. Excavation and embankment (cut and fill)
   2. Borrow pit volumes
   3. Site layout and control

B. Estimating Quantities and Costs
   1. Quantity take-off methods
   2. Cost estimating

C. Scheduling
   1. Construction sequencing
   2. Resource scheduling
   3. Time-cost trade-off

D. Material Quality Control and Production
   1. Material testing (e.g., concrete, soil, asphalt)

E. Temporary Structures
   1. Construction loads
II. GEOTECHNICAL

A. Subsurface Exploration and Sampling
   1. Soil classification
   2. Boring log interpretation (e.g., soil profile)
B. Engineering Properties of Soils and Materials
   1. Permeability
   2. Pavement design criteria
C. Soil Mechanics Analysis
   1. Pressure distribution
   2. Lateral earth pressure
   3. Consolidation
   4. Compaction
   5. Effective and total stresses
D. Earth Structures
   1. Slope stability
   2. Slabs-on-grade
E. Shallow Foundations
   1. Bearing capacity
   2. Settlement
F. Earth Retaining Structures
   1. Gravity walls
   2. Cantilever walls
   3. Stability analysis
   4. Braced and anchored excavations

III. STRUCTURAL

A. Loadings
   1. Dead loads
   2. Live loads
   3. Construction loads
B. Analysis
   1. Determinate analysis
C. Mechanics of Materials
   1. Shear diagrams
   2. Moment diagrams
   3. Flexure
   4. Shear
   5. Tension
   6. Compression
   7. Combined stresses
   8. Deflection
D. Materials
   1. Concrete (plain, reinforced)
   2. Structural steel (structural, light gage, reinforcing)
E. Member Design
   1. Beams
   2. Slabs
   3. Footings
IV. TRANSPORTATION

A. Geometric Design
   1. Horizontal curves
   2. Vertical curves
   3. Sight distance
   4. Superelevation
   5. Vertical and/or horizontal clearances
   6. Acceleration and deceleration

V. WATER RESOURCES AND ENVIRONMENTAL

20%

A. Hydraulics – Closed Conduit
   1. Energy and/or continuity equation (e.g., Bernoulli)
   2. Pressure conduit (e.g., single pipe, force mains)
   3. Closed pipe flow equations including Hazen-Williams, Darcy-Weisbach Equation
   4. Friction and/or minor losses
   5. Pipe network analysis (e.g., pipeline design, branch networks, loop networks)
   6. Pump application and analysis

B. Hydraulics – Open Channel
   1. Open-channel flow (e.g., Manning’s equation)
   2. Culvert design
   3. Spillway capacity
   4. Energy dissipation (e.g., hydraulic jump, velocity control)
   5. Stormwater collection (e.g., stormwater inlets, gutter flow, street flow, storm sewer pipes)
   6. Flood plains/floodways
   7. Flow measurement – open channel

C. Hydrology
   1. Storm characterization (e.g., rainfall measurement and distribution)
   2. Storm frequency
   3. Hydrographs application
   4. Rainfall intensity, duration, and frequency (IDF) curves
   5. Time of concentration
   6. Runoff analysis including Rational and SCS methods
   7. Erosion
   8. Detention/retention ponds

D. Wastewater Treatment
   1. Collection systems (e.g., lift stations, sewer networks, infiltration, inflow)

E. Water Treatment
   1. Hydraulic loading
   2. Distribution systems

Total 100%
A competent transportation engineer should have a basic knowledge in drainage, soils, and pavement design. Culvert design and pavement design are knowledges that have not been tested previously under the current Civil exam specifications. Beginning with the April 2010 exam, Section V of the Transportation module has been broadened to permit testing in these important transportation knowledges.

### Approximate Percentage of Examination

<table>
<thead>
<tr>
<th>Section</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Traffic Analysis</td>
<td>22.5%</td>
</tr>
<tr>
<td>II. Geometric Design</td>
<td>30%</td>
</tr>
<tr>
<td>III. Transportation Planning</td>
<td>7.5%</td>
</tr>
<tr>
<td>IV. Traffic Safety</td>
<td>15%</td>
</tr>
</tbody>
</table>

#### I. Traffic Analysis
- A. Traffic capacity studies
- B. Traffic signals
- C. Speed studies
- D. Intersection analysis
- E. Traffic volume studies
- F. Sight distance evaluation
- G. Traffic control devices
- H. Pedestrian facilities
- I. Driver behavior and/or performance

#### II. Geometric Design
- A. Horizontal curves
- B. Vertical curves
- C. Sight distance
- D. Superelevation
- E. Vertical and/or horizontal clearances
- F. Acceleration and deceleration
- G. Intersections and/or interchanges

#### III. Transportation Planning
- A. Optimization and/or cost analysis (e.g., transportation route A or transportation route B)
- B. Traffic impact studies
- C. Capacity analysis (future conditions)

#### IV. Traffic Safety
- A. Roadside clearance analysis
- B. Conflict analysis
- C. Work zone safety
- D. Accident analysis
V. Other Topics

A. Hydraulics
   1. Culvert design
   2. Open channel subcritical and supercritical flow

B. Hydrology
   1. Hydrograph development and synthetic hydrographs

C. Engineering properties of soils and materials (e.g. index properties, identification of types of soils; suitable or unsuitable soil. Boring logs)

D. Soil mechanics analysis (e.g. soil behavior, soil classification, soil compaction)

E. Engineering economics
   1. Value engineering and costing

F. Construction operations and methods (e.g., erosion control measures, excavation/embankment)

G. Pavement structures (e.g. flexible and rigid pavement design)
A CONCISE &
COMPREHENSIVE SUMMARY
Of
TRANSPORTATION
MODULE
EQUATIONS & TOPICS

Codes Equations, Non-Code Equations, Tables, Flow Charts, and Figures
Per NCEES List of Topics and Exam Specifications
I. Traffic Analysis  

22.5% (9 questions)
II. Geometric Design 30% (12 questions)
III. Transportation Planning  7.5% (3 questions)
### IV. Traffic Safety

15% (6 questions)
| V. Other Topics | 25% (10 questions) |
This page is left intentionally blank for additional summary of equations
This page is left intentionally blank for additional summary of equations
PART 1

Civil BREADTH (A.M.) Exam Specifications

TRANSPORTATION
Transportation Breadth (A.M.) Topics

Chapter A: Geometric Design

1. Horizontal curves
2. Vertical curves
3. Sight distance
4. Superelevation
5. Vertical and/or horizontal clearances
6. Acceleration and deceleration
Chapter A: Geometric Design

A.1 HORIZONTAL CURVES

A.1.1 Introduction:

Horizontal curves are required for connecting tangents in route surveying. Route surveying is the type of surveying to establish horizontal and vertical alignment for transportation facilities (highways, streets, railroads, etc.). Horizontal curves may be simple, compound, reverse, or spiral. Compound and reverse curves are treated as a combination of two or more simple curves, whereas the spiral curve is based on a varying radius.

![Types of Horizontal Curves](image)

Figure A-1 Types of Horizontal Curves

The sharpness of the curve is determined by the choice of the radius (R); large radius curves are relatively flat, whereas small radius curves are relatively sharp.

The following table explains the differences between types of horizontal curves.

<table>
<thead>
<tr>
<th>Table A-1 Types of Horizontal Curves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Circular Curves</strong></td>
</tr>
<tr>
<td>Simple</td>
</tr>
<tr>
<td>The simple curve is an arc of a circle. The radius of the circle determines the sharpness or flatness of the curve. The larger the radius, the flatter the curve. This type of curve is the most often used.</td>
</tr>
<tr>
<td><strong>Spiral Curves</strong></td>
</tr>
<tr>
<td>The spiral is a curve which has a varying radius. It is used on railroads and some modern highways. Its purpose is to provide a transition from the tangent to a simple curve or between simple curves in a compound curve</td>
</tr>
</tbody>
</table>
A.1.2 Horizontal Circular Curves:

Circular curves are common in highways and streets design. The route of a highway or a street is chosen to satisfy all design requirements (speed, sight distance, superelevation, etc.) with minimal social, environmental, and financial impact. The horizontal alignment of a transportation facility consists of series of straight lines (tangents) and circular curves as shown below.

![Horizontal and Vertical Alignments](image)

**Figure A-2** Horizontal (top) and Vertical (bottom) Alignments

A.1.3 Degree of Curve for Horizontal Circular Curves:

The degree of curve (for horizontal circular curves only) \( (D) \) defines the "sharpness" or "flatness" of the curve. There are two definitions for degree of curve, as follows:
Table A-2 Chord and Arc Definitions for Horizontal Circular Curves

<table>
<thead>
<tr>
<th>Chord Definition ((D_c))</th>
<th>Arc Definition ((D_a))</th>
</tr>
</thead>
<tbody>
<tr>
<td>The chord definition states that the degree of curve is the angle formed by two radii drawn from the center of the circle to the ends of a chord of 100 units long ((100 \text{ ft or } 100 \text{ m chord})). The chord definition is used primarily for civilian railroad construction and is used by the military for both roads and railroads.</td>
<td>The arc definition states that the degree of curve is the angle formed by two radii drawn from the center of the circle to the ends of an arc of 100 units long ((100 \text{ ft or } 100 \text{ m arc})). This definition is used primarily for highways and streets. Notice that the larger the degree of curve, the &quot;sharper&quot; the curve and the shorter the radius.</td>
</tr>
<tr>
<td>[ \sin \left( \frac{D_c}{2} \right) = \frac{50 \text{ ft}}{R} ] (A-1)</td>
<td>[ D_a = \frac{(360^\circ)(100 \text{ ft})}{2\pi R} = \frac{5729.578^\circ}{R} ] (A-2)</td>
</tr>
</tbody>
</table>

\[
R = \frac{50}{\sin(D/2)}
\]

\[
D_c = \frac{100}{2\pi R}
\]

\[
R = \frac{5729.58}{D}
\]

Since one meter equals 3.28084 ft, degrees of curve in the two systems (SI and English) differ by the same proportion. Therefore, a 1° curve in the foot system is the same curve as a 3.28084° curve in the metric system. And a 1° curve in the metric system is the same curve as a 0.3048° curve in the foot system. The following equation gives the relationship between the two systems.

\[ D_{\text{metric}} = D_{\text{foot}} \times 3.28 \] (A-3)

The following table shows the comparable values in both systems.

Table A-3 Degree-of-Curve Conversions

<table>
<thead>
<tr>
<th>Foot Definition ((D_{\text{foot}}))</th>
<th>Metric Definition ((D_{\text{metric}}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3.28</td>
</tr>
<tr>
<td>1.5</td>
<td>4.92</td>
</tr>
<tr>
<td>2.0</td>
<td>6.56</td>
</tr>
<tr>
<td>2.5</td>
<td>8.20</td>
</tr>
<tr>
<td>3.0</td>
<td>9.84</td>
</tr>
</tbody>
</table>
**Figure A-3 Circular Horizontal Curve Terminology**

- **Point of Intersection (PI):** the point at which the two tangents to the curve intersect
- **Delta Angle (Δ):** the angle between the tangents is also equal to the angle at the center of the curve
- **Back Tangent:** for a survey progressing to the right, it is the straight line that connects the PC (BC) to the PI
- **Forward Tangent:** for a survey progressing to the right, it is the straight line that connects the PI to the PT (EC)
- **Point of Curvature (PC) = Beginning of Curve (BC):** the beginning point of the curve
- **Point of Tangency (PT) = End of Curve (EC):** the end point of the curve
- **Tangent Distance (T):** the distance from the PC (BC) to PI or from the PI to PT (EC)
- **External Distance (E):** the distance from the PI to the middle point of the curve
- **Middle Ordinate (M):** the distance from the middle point of the curve to the middle of the chord (long chord) joining the PC (BC) and PT (EC)
- **Long Chord (C or LC):** the distance along the line joining the PC (BC) and the PT (EC)
- **Length of Curve (L):** the difference in stationing along the curve (arc length) between the PC (BC) and the PT (EC)
A.1.4 Geometry of Horizontal Circular Curves (Arc Definition):

\[ T = R \tan \frac{\Delta^o}{2} \]  
(A-4)

\[ LC = C = 2 \ R \sin \frac{\Delta^o}{2} = 2 \ T \cos(\Delta^o/2) \]  
(A-5)

\[ L_a = 2\pi \ R \left( \frac{\Delta^o}{360^o} \right) = R \ \Delta \text{ (radians)} = (100 \text{ ft}) \left( \frac{\Delta^o}{D_a} \right) \]  
(A-6)

\[ R \text{ (feet)} = \frac{5729.58}{D_a^o} \]  
(A-7)

\[ M = R \left( 1 - \cos \frac{\Delta^o}{2} \right) = \frac{C}{2} \tan \frac{\Delta^o}{4} = E \cos \frac{\Delta^o}{2} \]  
(A-8)

\[ E = R \left[ \frac{1}{\cos(\Delta^o/2)} - 1 \right] = R \left( \sec \frac{\Delta^o}{2} - 1 \right) = T \tan \frac{\Delta^o}{4} = R \tan \frac{\Delta^o}{2} \tan \frac{\Delta^o}{4} \]  
(A-9)

Notes:

1. \( \cos \frac{\Delta^o}{2} = \frac{R}{R + E} \) \text{ i.e. } \Delta = 2 \cos^{-1} \left( \frac{R}{R + E} \right) \text{ (from the geometry shown in the previous page)}

2. \( \text{versed sine (vers) } \tilde{Y} = \text{ vers (} \frac{\pi}{2} \text{)} = 1 \tilde{I} \cos (\frac{\pi}{2}) \)

3. \( \text{external secant (exsec) } \tilde{Y} = \text{ exsec (} \frac{\pi}{2} \text{)} = \sec (\frac{\pi}{2}) \tilde{I} = 1 \)

4. A common mistake is to determine the station of the \( \text{EC} \) by adding the \( T \) distance to the \( \text{PI} \). Although the \( \text{EC} \) is physically a distance of \( T \) from the \( \text{PI} \), the stationing (chainage) must reflect the fact that the centerline no longer goes through the \( \text{PI} \).

Table A-4 Arc and Chord Definitions Equations

<table>
<thead>
<tr>
<th>Arc Definition</th>
<th>Chord Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ T = R \tan \frac{\Delta^o}{2} ]  \text{ (A-4)}</td>
<td>Same</td>
</tr>
<tr>
<td>[ LC = C = 2 \ R \sin \frac{\Delta^o}{2} = 2 \ T \cos(\Delta^o/2) ]  \text{ (A-5)}</td>
<td>Same</td>
</tr>
<tr>
<td>[ L_a = 2\pi \ R \left( \frac{\Delta^o}{360^o} \right) = R \ \Delta \text{ (radians)} = (100 \text{ ft}) \left( \frac{\Delta^o}{D_a} \right) ]  \text{ (A-6)}</td>
<td>[ L_c = (100 \text{ m}) \left( \frac{\Delta^o}{D_c} \right) ]</td>
</tr>
<tr>
<td>[ R \text{ (feet)} = \frac{5729.58}{D_a^o} ]  \text{ (A-7)}</td>
<td>[ R \text{ (meter)} = \frac{5729.65}{D_c^o} ]</td>
</tr>
<tr>
<td>[ M = R \left( 1 - \cos \frac{\Delta^o}{2} \right) = \frac{C}{2} \tan \frac{\Delta^o}{4} = E \cos \frac{\Delta^o}{2} ]  \text{ (A-8)}</td>
<td>Same</td>
</tr>
<tr>
<td>[ E = R \left[ \frac{1}{\cos(\Delta^o/2)} - 1 \right] = R \left( \sec \frac{\Delta^o}{2} - 1 \right) = T \tan \frac{\Delta^o}{4} = R \tan \frac{\Delta^o}{2} \tan \frac{\Delta^o}{4} ]  \text{ (A-9)}</td>
<td>Same</td>
</tr>
</tbody>
</table>
A horizontal circular curve for a new conventional highway has the following data:
\( \Delta = 16^\circ38', \ R = 1000 \text{ ft}, \ \text{and PI Station at } 6 + 26.57 \)

**Sample Problem A-1: Horizontal Circular Curve Calculations**

**Find:** BC and EC stations are most nearly:

(A) 4 + 80.39, 7 + 72.75  
(B) 4 + 80.39, 7 + 70.70  
(C) 7 + 70.70, 4 + 80.39  
(D) 7 + 72.75, 4 + 80.39

**Solution:**

\[
T = R \frac{\Delta}{2} = 1000 \tan 8.3167^\circ = 146.18 \text{ ft} \\
L = 2\pi R \frac{\Delta}{360^\circ} = R\Delta (\text{radians}) = (100 \text{ ft}) \left( \frac{\Delta}{D} \right) \\
L = 2 \times 1000 \times \frac{16.6333}{360} = 290.31 \text{ ft} \\
L = 2 + 90.31 \text{ Sta.} \\
\]

PI at 6 + 26.57 \\
T 1 + 46.18

BC = 4 + 80.39 ≤  
+ L 2 + 90.31  
EC = 7 + 70.70 ≤

**Note:** Answer (A) is (PI Sta. + T), which is wrong (common mistake).

**Answer:** (B)

---

**Sample Problem A-2: Long Chord for Horizontal Circular Curve**

**Find:** The length of the long chord is most nearly:

(A) 150.49 ft  
(B) 189.30 ft  
(C) 289.29 ft  
(D) 572.49 ft

**Solution:**

\[
C = 2 R \sin \frac{\Delta}{2} = 2 T \cos (\Delta/2) = 2 \times 1000 \times \sin 8.3167^\circ = 289.29 \text{ ft} \leftarrow \text{ Answer: (C)} 
\]
Sample Problem A-3: External Distance for a Horizontal Circular Curve

Find: The external distance (E) for the given curve is most nearly:

(A) 10.23 ft
(B) 10.43 ft
(C) 10.52 ft
(D) 10.63 ft

Solution:

\[ E = R \left( \frac{1}{\cos(\Delta/2)} - 1 \right) = 1000 \left( \frac{1}{\cos(16.63/2)} - 1 \right) = 10.63 \text{ ft} \]

Answer: (D)

Sample Problem A-4: Middle Ordinate for a Horizontal Circular Curve

Find: The middle ordinate for the horizontal circular curve is most nearly:

(A) 10.23 ft
(B) 10.43 ft
(C) 10.52 ft
(D) 10.63 ft

Solution:

Refer to Figure A-3 for the definition of \( \hat{M} \hat{O} \)

\[ \text{Middle Ordinate (A to B)} = M = R \left( 1 - \cos \frac{\Delta}{2} \right) = \frac{C}{2} \tan \frac{\Delta}{4} = E \cos \frac{\Delta}{2} \Rightarrow \text{which relationship?} \]

\[ M = R \left( 1 - \cos \frac{\Delta}{2} \right) = 1000 \left( 1 - \cos 8.3167^\circ \right) = 10.52 \text{ ft} \]

Answer: (C)

Sample Problem A-5: Length a Horizontal Circular Curve

Given: A horizontal circular curve has a radius of 500 ft, and mid-ordinate of 16.5 ft.

Find: The length of the horizontal circular curve is most nearly:

(A) 257.61 ft
(B) 265.54 ft
(C) 315.45 ft
(D) 400.00 ft

Solution:

(Continued on next page)
Middle Ordinate (A to B) =

\[ M = R \left( 1 - \cos \frac{\Delta}{2} \right) = \frac{C}{2} \tan \frac{\Delta}{4} = E \cos \frac{\Delta}{2} \]

\( \iff \) select the appropriate relationship

\[ M = R \left( 1 - \cos \frac{\Delta}{2} \right) \]

\( 16.5 = 500 \left( 1 - \cos \frac{\Delta}{2} \right) \)

\( \bar{Y} \cos \frac{\Delta}{2} = 1 - \frac{M}{R} = 1 - \frac{16.5}{500} = 0.967 \)

\( \bar{Y} \frac{\Delta}{2} = \cos^{-1} (0.967) = 14.76^\circ \bar{Y} \alpha = 29.52^\circ \)

Curve Length (i.e. BC to EC):

\[ L = 2\pi R \frac{\Delta^\circ}{360^\circ} = R \Delta \text{ (radians) = (100 ft) } \left( \frac{\Delta}{D} \right) \]

\[ L = 2\pi R \frac{\Delta^\circ}{360^\circ} = 2\times\pi\times500\times\frac{29.52^\circ}{360^\circ} = 257.61 \text{ ft} \]

Answer: \( \text{[A]} \alpha \)
A.1.5 Deflection Angles, Central Angle, and Chord Calculations for Horizontal Circular Curves:

The deflection angle is defined as the angle between the tangent and a chord. The following two rules apply for the deflection angles for circular curves:

**Rule 1:** The deflection angle between a tangent and a chord is half the central angle subtended by the arc, i.e., the angle between the tangent $\widehat{BC-PI}$ and the chord $\widehat{PC-A}$ is $\frac{1}{2}$ the central angle $\widehat{BC-O-A}$, i.e., $\Delta$ & $2\Delta$.

**Rule 2:** The angle between two chords is $\frac{1}{2}$ the central angle subtended by the arc between the two chords, i.e., the angle $\widehat{A-BC-B}$ is $\frac{1}{2}$ the central angle $\widehat{A-O-B}$, i.e., $\beta$ & $2\beta$.

### Abbreviations:

- **BC** = Beginning of curve
- **PC** = Point of curvature
- **TC** = Tangent to curve
- **EC** = End of curve
- **PT** = Point of tangency
- **CT** = Curve to tangent

**Figure A-4** Deflection, Central Angles and Chord Calculations

\[
\text{deflection angle} = \left( \frac{\text{arc length}}{L} \right) \left( \frac{\Delta}{2} \right) \quad (A-10)
\]

\[
\text{Chord Length (BC to A)} = 2R \sin \alpha \quad (A-11)
\]

\[
\alpha = \frac{\text{arc length (BC to A)} \times 180^0}{2\pi R} \quad (A-12)
\]

\[
\frac{2\alpha}{\Delta} = \frac{\text{arc length (BC to A)}}{L} \quad (A-13)
\]
The following information is to be used for problems A-6 to A-8

A horizontal circular curve for a new urban highway has a radius of 2500 ft and a degree of curve (arc definition) of 2.29º. The beginning of the curve is at station 150 + 75 and the central angle is 30º. A drainage inlet (DI) is required to be staked out at station 154 + 60.

Sample Problem A-6: Deflection Angle Calculations for Circular Curve

**Find:** The deflection angle between the back tangent and the chord to the drainage inlet station is most nearly:

(A) 4.41º  
(B) 12.20º  
(C) 16.25º  
(D) 32.50º

**Solution:**

The deflection angle $\alpha$ is given by the following equation:

$$\alpha = \frac{\text{arc length (BC to A)} \times 180^\circ}{2\pi R}$$

Arc length = (DI Station $\tilde{I}$ BC Station) = $(154 + 60) \tilde{I} (150 + 75) = 3.85$ Sta. = 385 ft

$$\alpha = \frac{\text{arc length (BC to A)} \times 180^\circ}{2\pi R} = \frac{385 \text{ ft} \times 180^\circ}{2 \times \pi \times 2500 \text{ ft}} = 4.41^\circ$$

Answer: (A)
**Sample Problem A-7: Chord Calculations for Circular Curve**

**Find:** The length of the chord connecting the BC and the drainage inlet is most nearly:

(A) 175.30 ft  
(B) 254.52 ft  
(C) 325.15 ft  
(D) 384.47 ft

**Solution:**

Chord Length \((BC \text{ to } DI, \text{i.e. } BC \text{ to } A) = 2R \sin \alpha = 2 \times 2500 \text{ft} \times \sin 4.41^\circ = 384.47 \text{ft} \)

Answer: (D)

**Sample Problem A-8: Deflection Angle & Central Angle Relationship**

**Find:** The design calls for another DI at station 156 + 25. The central angle that should be turned to the right from the first DI is most nearly:

(A) 1.90º  
(B) 2.90º  
(C) 3.78º  
(D) 5.50º

**Solution:**

The following relationship could be established (by proportion):

\[
\frac{2\gamma}{\Delta} = \frac{\text{arc length } A \text{ to } B}{L}
\]

And the curve length is given by:

\[
L = 2\pi R \frac{\Delta}{360^\circ} = R\Delta(\text{radians}) = (100 \text{ft}) \left( \frac{\Delta}{D} \right)
\]

\[
= (100 \text{ft}) \left( \frac{30^\circ}{2.29^\circ} \right) = 1310.00 \text{ft}
\]

\[
\gamma = \Delta \times \frac{\text{arc length } A \text{ to } B}{L} = 30^\circ \times \frac{(156.25 - 154.60) \text{Sta.} \times 100 \text{ft}}{1310 \text{ft}} \frac{1}{\text{Sta.}} = 3.78^\circ
\]

Answer: (C)
A.1.6 Tangent Offset Calculations:

For short curves, when a total station instrument is not available, and for checking purposes, one of the four offset-type methods can be used for laying out circular curves. These methods are:

1. tangent offset-TO,
2. chord offset-CO,
3. middle ordinates-MO, and
4. ordinates from the long chord.

The four tangent offset method equations are given as follows:

\[
Y = R - \sqrt{R^2 - X^2} \tag{A-14}
\]

\[
Y = R - R \cos \theta = R (1 - \cos \theta) \tag{A-15}
\]

\[
\theta = \arcsin \frac{X}{R} \tag{A-16}
\]

\[
X = R \sin \theta \tag{A-17}
\]

**Figure A-5** Tangent Offset

<table>
<thead>
<tr>
<th>Sample Problem A-9: Tangent Offset Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given:</strong> In the diagram (Figure A-5), if the curve has a radius ( R = 800 ) feet and the distance ( X = 300 ) ft.</td>
</tr>
</tbody>
</table>

**Solution:**

\[
Y = R - \sqrt{R^2 - X^2}
\]

\[
Y = 800 - \sqrt{800^2 - 300^2} = 58.38 \text{ ft} \]
Sample Problem A-10: Geometrics of Circular Curves

**Given:** When set up the instrument at the BC of a simple curve.

**Find:** the angles to turn off the PI to hit the radius point is:

(A) The central angle plus the deflection angle
(B) Half the central angle
(C) The deflection angle
(D) Exactly right angle

**Solution:**

Radius point is another name for the center of a circle. The angle measured from the PI to the center is always 90°.

Answer: (D) 90°

Sample Problem A-11: Geometry of Horizontal Curves

**Given:** A perpendicular bisector of the long chord (LC) was drawn for a circular horizontal curve.

**Find:** This perpendicular bisector passes through which of the following:

(A) Radius point
(B) The Point of Intersection of the two tangents
(C) The centre of the curve
(D) All of the above

It is clear from the figure to the right that all of the above is the correct answer.

Answer: (D) All of the above
### Sample Problem A-12: Moving up on a curve

**Given:** When moving up on a horizontal curve to the right (i.e. set up anywhere on point on curve POC).

**Find:** The clockwise angle to turn off to hit the radius point is:

(A) Half the central angle.
(B) Deflection angle measured from PC (BC) to that point.
(C) Deflection angle measured from PC to that point plus 90°
(D) Deflection angle measured from PC to that point minus 90°

**Solution:**

From the Figure shown, the correct answer is C.

![Diagram of compound circular curves]

**Answer:** (C) $\theta$

### A.1.7 Compound Circular Curves:

Often two curves of different radii are joined together, as shown. The point connecting the two curves together is called point of compound curve or curvature (PCC). GH is a common tangent. Normally, the subscript 1 refers to the curve of smaller radius.

Compound curves should be used only for low-speed traffic routes, and in terrain where simple curves cannot be fitted to the ground without excessive construction cost.

![Figure A-6 Compound Circular Curve]

The following equations are used to solve the unknown parameters for a compound curve:
\[ \Delta_1 = \Delta - \Delta_2 \quad OR \quad \Delta_2 = \Delta - \Delta_1 \quad (A-18) \]

\[ t_1 = R_1 \tan \frac{\Delta_1}{2} \quad \& \quad t_2 = R_2 \tan \frac{\Delta_2}{2} \quad (A-19) \]

\[ GH = t_1 + t_2 \quad (A-20) \]

\[ VG = (\sin \Delta_2) \left( \frac{GH}{\sin \Delta} \right) \quad (A-21) \]

\[ VH = (\sin \Delta_1) \left( \frac{GH}{\sin \Delta} \right) \quad (A-22) \]

\[ T_1 = VA = VG + t_1 \quad (A-23) \]

\[ T_2 = VB = VH + t_2 \quad (A-24) \]

**Sample Problem A-13: Compound Curve**

**Given:** A compound curve with the following data: \( R_1 = 500 \) ft, \( R_2 = 800 \) ft, \( \Delta = 96^\circ 42'40'' \), \( \Delta_1 = 62^\circ 22'20'' \) & PI station = 16 + 30.12

**Find:** PC, PT & PCC stations

**Solution:**

\[ \Delta_2 = \Delta - \Delta_1 = 96^\circ 42'40'' \bar{1} 62^\circ 22'20'' = 34.3389^\circ \]

\[ t_1 = R_1 \tan \frac{\Delta_1}{2} \Rightarrow 500 \tan (31.1861^\circ) = 302.65 \text{ ft} \]

\[ t_2 = R_2 \tan \frac{\Delta_2}{2} \Rightarrow 800 \tan (17.1695^\circ) = 247.17 \text{ ft} \]

\[ GH = t_1 + t_2 = 302.65 + 247.17 = 549.82 \text{ ft} \]

\[ VG = (\sin \Delta_2) \left( \frac{GH}{\sin \Delta} \right) = (0.5641) \left( \frac{549.82}{0.9931} \right) = 312.31 \text{ ft} \]

\[ VH = (\sin \Delta_1) \left( \frac{GH}{\sin \Delta} \right) = (0.8859) \left( \frac{549.82}{0.9931} \right) = 490.49 \text{ ft} \]

\[ T_1 = VA = VG + t_1 = 312.28 + 302.65 = 614.93 \text{ ft} \]

\[ T_2 = VB = VH + t_2 = 490.49 + 247.17 = 737.66 \text{ ft} \]

\[ L_1 = 2\pi R_1 \frac{\Delta_1}{360} = R_1 \Delta_1 (\text{radians}) = 500(1.0886) = 544.30 \text{ ft} \]

\[ L_2 = 2\pi R_2 \frac{\Delta_2}{360} = R_2 \Delta_2 (\text{radians}) = 800(0.5993) = 479.46 \text{ ft} \]

PC Station = PI Sta. \( T_1 = (16 + 30.12) \bar{1} (6 + 14.93) = 10 + 15.19 \) \( \xi \)

PCC Station = PC Sta.+ \( L_1 = (10 + 15.19) + (5 + 44.30) = 15 + 59.49 \) \( \xi \)

PT Station = PCC Sta.+ \( L_2 = (15 + 59.49) + (4 + 79.46) = 20 + 38.95 \) \( \xi \)
A.1.8 Reverse Circular Curves:

Reverse curves are seldom used in highway or railway alignment. The instantaneous change in direction occurring at the point of reversed curvature (PRC) would cause discomfort and safety problems. Additionally, since the change in curvature is instantaneous, there is no room to provide superelevation transition from cross-slope right to cross-slope left. If reverse circular curves have to be used, the superelevation rate of change per station should be within the acceptable standards.

Reverse curves may have parallel or non-parallel tangents as shown below. As with compound curves, reverse curves have six independent parameters \((R_1, \Delta_1, T_1, R_2, \Delta_2, T_2)\); the solution technique depends on which parameters are unknown, and the techniques noted for compound curves will also provide the solution to reverse curve problems.

![Figure A-7 Reverse Curves: Parallel Tangents (top) and Non-Parallel Tangent (bottom)](image)
A.1.9 Spiral Curves:

Spiral curves (also known as transition curves) are used in highway and railroad horizontal alignment to overcome the abrupt change in direction that occurs when the alignment changes from tangent to circular curve, and vice versa. Figure A-8 illustrates how the spiral curve with a length of $\ell_s$” is inserted between tangent and circular curve alignment.

It can be seen that at the beginning of the spiral (T.S.= tangent to spiral) the radius of the spiral is the radius of the tangent line (infinitely large), and that the radius of the spiral curve decreases at a uniform rate until, at the point where the circular curve begins (S.C. = spiral to curve), the radius of the spiral equals the radius of the circular curve. In case of a spiral curve that connects two circular curves having different radii, there is an initial radius rather than an infinite value.

Spiral Curves Notation:

- T.S. : point of change from tangent to spiral
- S.C. : point of change from spiral to circle
- C.S. : point of change from circle to spiral
- S.T. : point of change from spiral to tangent
- $\tilde{\ell}_s$ : spiral arc length from T.S. to any point on the spiral
- $L_s$ : total length of spiral from T.S. to S.C.
- $L_c$ : length of circular curve
- LT : long tangent (Spiral)
- ST : short tangent (Spiral)
- $\varphi$ : total central angle of the circular curve
- $\varphi_c$ : central angle of circular arc of $L_c$ extending from the SC to the CS
- $Y$ : right Angle Distance from Tangent to S.C.
- $X$ : distance along tangent from T.S. to point at right angle to S.C.
- $T_s$ : total tangent distance = distance from PI to T.S. or S.T.
- $\Theta$ : central angle of spiral arc L
- $\theta_s$ : the spiral angle = central angle of spiral arc $L_s$
Figure A-9  Spiral Curve
Selected formulas for spiral curves:

Because the degree of curvature for a spiral increases from 0 at the T.S. to D (D_a) at the S.C., the rate of change of curvature of a spiral in degrees per station (K) is:

\[ K = \frac{100D}{L_s} \]  
(A-25)

The degree \( \hat{D}_p \)” and the radius “r” of the spiral curve at any point P along the curve are given as follows:

\[ D_p = \frac{l_s K}{100} \]  
(A-26)

\[ r = \frac{5729.58}{D_p} = (5729.58)(100) \frac{l_s \times K}{l_s} \]  
(A-27)

The radius at the S.C. is:

\[ R = \frac{5729.58}{D} = (5729.58)(100) \frac{l_s \times K}{l_s} \]  
(A-28)

The ratio of the two radii is:

\[ \frac{r}{R} = \frac{L_s}{l_s} \]  
(A-29)

The length of the spiral is:

\[ L_s = \frac{(200)(\Delta_s)}{D} \]  
(A-30)

The deflection angle is:

\[ \alpha_s = \frac{l_s^2}{L_s^2} \times \frac{\Delta_s}{3} \]  
(A-31)

The following equation, developed in 1909 by W. H. Shortt, is the basic expression used by some highway agencies for computing minimum length of a spiral transition curve:

\[ (L_s)_{min} = \frac{3.15V^3}{RC} \]  
(AASHTO Eq. 3-27, page 185)  
(A-32)

where

\[ V = \text{design speed, mph} \]
\[ R = \text{radius of circular curve, ft} \]
\[ C = \text{maximum rate of change in lateral acceleration, 1 to 3 ft/s}^3 \text{ is recommended by AASHTO for highways and 1 ft/s}^3 \text{ is generally accepted for railroad operation} \]
Table A-5 Minimum Length of Spiral Transition Curves

<table>
<thead>
<tr>
<th>Minimum Length of Spiral Transition Curves</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L = \frac{0.0214V^3}{RC} )</td>
<td>( L = \frac{3.15 V^3}{RC} )</td>
</tr>
<tr>
<td>where:</td>
<td></td>
<td>(3-27)</td>
</tr>
<tr>
<td>( L ) = minimum length of spiral, m;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V ) = design speed, km/h;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R ) = curve radius, m;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C ) = rate of increase of lateral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>acceleration, m/s^3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sample Problem A-14: Spiral Curve**

**Given:** Using 3 ft/s^3 for the rate of increase of lateral acceleration. The minimum spiral curve length transition for a new highway facility has a radius of 1200 ft and design speed of 50 mph is most nearly:

(A) 100 ft  
(B) 110 ft  
(C) 140 ft  
(D) 150 ft

**Solution:**

\[
L = \frac{3.15 V^3}{RC} = \frac{3.15 \times 50^3}{1200 \times 3} = 109.38 \text{ ft}
\]

where:
\( L \) = minimum length of spiral, ft;  
\( V \) = design speed, mph;  
\( R \) = curve radius, ft;  
\( C \) = rate of increase of lateral acceleration, ft/s^3

Answer: (B)

**Sample Problem A-15: Deflection Angle of a Spiral Curve**

**Given:** The coordinates of point P on a highway spiral relative to TS are 200 & 5 for x & y respectively.

**Find:** The deflection angle from TS to point P is most nearly:

(A) 00°28′38″  
(B) 01°25′55″  
(C) 02°50′00″  
(D) 04°05′02″

**Solution:**

\[
\tan \theta = \frac{5}{200} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{200}\right) = 1.4321°
\]

Answer: (B)
Desirable Length for spiral curves:

Based on recent operational studies, AASHTO Exhibit 3-37 (page 189) lists the desirable lengths of spiral transition curves. These lengths correspond to 2.0 s of travel time at the design speed of the roadway. The travel time has been found to be representative of the natural spiral path for most drivers.

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Metric</strong></td>
</tr>
<tr>
<td>Design speed (km/h)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
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<tr>
<td>40</td>
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<tr>
<td>50</td>
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<td>100</td>
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<td>110</td>
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<tr>
<td>120</td>
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<tr>
<td>130</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

Sample Problem A-16: Spiral Curve

**Given:** Using 3 ft/s³ for the rate of increase of lateral acceleration. The desirable spiral curve length transition for a new highway facility has a radius of 1200 ft and design speed of 50 mph is most nearly:

(A) 100 ft
(B) 110 ft
(C) 147 ft
(D) 161 ft

**Solution:**


Answer: (C)
Figure A-10 Curve with and without spiral transition. The sharp "corners" at the juncture of curve and straight line in the top view are quite obvious from the driver's seat. (Exhibit 3-35, page 186, AASHTO Geometric Design-Green Book, 2004 edition, 5th edition)
A.2 VERTICAL CURVES

A.2.1 Why Vertical Curves are Used?

Roads made up of a series of straight lines (or tangents) are not practical. To prevent abrupt changes in the vertical direction of moving vehicles, adjacent segments of differing grade are connected by a curve. This curve in the vertical plane is called a vertical curve.

The geometric curve used in vertical alignment design is the parabola curve. The parabola has these two desirable characteristics of:

1. a constant rate of change of grade \( r = \frac{g_2 - g_1}{L} \), which contributes to smooth alignment transition, and
2. ease of computation of vertical offsets, which permits easily computed curve elevations.

As a general rule, the higher the speed the road is designed for, the smaller the percent of grade that is allowed. For example, a road designed for a maximum speed of 30 miles per hour (mph) may have a vertical curve with the tangents to the curve arc having a grade as high as 6 to 8 percent. A road that is designed for 70 mph can have a vertical curve whose tangents have a grade of only 3 to 5 percent.

A.2.2 Vertical Curves Terminology:

![Figure A-11 Vertical Curve Terminology](image)

Vertical curves are used in highway and street vertical alignment to provide a gradual change between two adjacent grade lines. Some highway and municipal agencies introduce vertical curves at every change in grade-line slope, whereas other agencies introduce vertical curves into the alignment only when the net change in slope direction exceeds a specific value (e.g., 1.5% or 2%).
In Figure A-11, vertical curve terminology is introduced: $g_1$ is the slope (percent) of the entering grade line, $g_2$ is the slope of the exiting grade line, BVC is the beginning of the vertical curve, EVC is the end of the vertical curve, and PVI is the point of intersection of the two adjacent grade lines. The length of vertical curve ($L$) is the projection of the curve onto a horizontal surface and as such corresponds to plan distance.

### A.2.3 Types of Vertical Curves:

Vertical curves may be classified based on the location of the vertex with respect to the BVC and EVC as symmetrical and asymmetrical.

#### Table A-7 Types of Vertical Curves

<table>
<thead>
<tr>
<th>Symmetrical Vertical Curves</th>
<th>Asymmetrical Vertical Curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ The vertex is located at the half distance between the BVC &amp; EVC.</td>
<td>➢ The vertex is not located at the half distance between the BVC &amp; EVC.</td>
</tr>
<tr>
<td>➢ The equations used to solve the unknowns are based on parabolic formula.</td>
<td>➢ The equations used to solve the unknowns are based on parabolic formula (two vertical curves).</td>
</tr>
<tr>
<td>➢ They are used mostly in every project where no minimum vertical clearance or cover is needed.</td>
<td>➢ They used when a specific elevation at a certain station (point) is required and the grades of the grade lines are fixed.</td>
</tr>
<tr>
<td>➢ They are also called equal-tangent parabolic vertical curves.</td>
<td>➢ They are also called unequal-tangent parabolic vertical curves.</td>
</tr>
</tbody>
</table>

Another classification is crest and sag vertical curves.

#### Table A-8 Types of Vertical Curves

<table>
<thead>
<tr>
<th>Sag Vertical Curves ($g_2 &gt; g_1$)</th>
<th>Crest Vertical Curves ($g_2 &lt; g_1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ They can be symmetrical or asymmetrical.</td>
<td>➢ They can be symmetrical or asymmetrical.</td>
</tr>
<tr>
<td>➢ The equation used to solve the unknowns is a parabolic formula.</td>
<td>➢ The equation used to solve the unknowns is a parabolic formula.</td>
</tr>
<tr>
<td>➢ They connect downgrade (−) tangent to an upgrade (+) tangent (type III).</td>
<td>➢ They connect an upgrade (+) tangent to a downgrade (−) tangent (type I).</td>
</tr>
<tr>
<td>➢ The rate of grade change per station $r = \frac{g_2 - g_1}{L}$ is a positive quantity.</td>
<td>➢ The rate of grade change per station $r = \frac{g_2 - g_1}{L}$ is a negative quantity.</td>
</tr>
</tbody>
</table>
Figure A-12 Sag and Crest Vertical Curves

Figure A-13 Types of Vertical Curves

$G_1$ and $G_2$ = Tangent grades in percent
$A$ = Algebraic difference in grade
$L$ = Length of vertical curve

G1 and G2 = Tangent grades in percent
A = Algebraic difference in grade
L = Length of vertical curve
AASHTO lists FOUR requirements for vertical curves as follows:

1- safe (ample sight distances),
2- comfortable in operation (gravitational force versus vertical centripetal force)
3- pleasing in appearance (long versus short vertical curves), and
4- adequate for drainage (0.50% and 0.30% are minimum grades in snow country and other locations respectively)

A.2.4 Geometric Properties of the Parabola:

1. The difference in elevation between the BVC and a point on the $g_1$ grade line at a distance $x$ units (feet or meters) is $g_1 x$ ($g_1$ is expressed as a decimal).

![Figure A-11 (repeated) Geometric of a Parabola](image)

2. The tangent offset between the grade line and the curve is given by $ax^2$, where $x$ is the horizontal distance from the BVC (PVC); that is, tangent offsets are proportional to the squares of the horizontal distances.

3. The elevation of the curve at distance $x$ from the BVC is given by:

$$y = ax^2 + bx + c \quad \text{(general equation for a parabola)}$$

$$y_x = y_{BVC} + g_1 x + \frac{rx^2}{2} \quad \text{(A-33)}$$

$$r = \frac{g_2 - g_1}{L} \quad \text{(A-34)}$$

where: $x$ = the distance from BVC to a point on the curve

$r$ = rate of grade change per station

Note: The rate of vertical curvature $K = \frac{1}{r} = L/A = \frac{L}{|g_2 - g_1|}$ will be used in subsequent sections related to sag and crest vertical curves. $K =$ (Rate of Vertical Curvature) distance required to achieve a 1% change in grade.
4. The grade lines \((g_1 \text{ and } g_2)\) intersect midway between the BVC and the EVC; that is, \(BVC \text{ to } PVI = \frac{1}{2} L = PVI \text{ to } EVC. \) This is only true for symmetrical vertical curves.

5. The curve lies midway between the PVI and the midpoint of the chord; that is, \(\Delta C B = B \cap PVI = d_o\) which can be calculated as follows:
   
   either:
   
   
   \[
   d_o = \frac{1}{2} (\text{difference in elevation of PVI and mid-chord elevation})
   \]
   
   \[
   = \frac{1}{2} (\text{elevation of BVC + elevation of EVC})
   \]
   
   Or:
   
   \[
   d_o = \frac{|g_2 - g_1|L}{8} = \frac{AL(sta.)}{8} = \frac{AL(feet)}{800}
   \] (A-35)

6. The slope \(S\), in percentage, of the tangent to the curve at any point on the curve is given by the following formula:

   \[
   S = g_1 - \frac{x(g_1 - g_2)}{L}
   \] (A-36)

7. The distance \(D\) in feet from Vertex to \(P'\)is given as:

   \[
   D = \frac{100(Y_{H} - Y_{P'})}{(g_1 - g_2)}
   \] (A-37)

8. The distance between the curve and the grade line (tangent) \(d\) is given as:

   \[
   d = \text{offset} = \frac{rx^2}{2} = \frac{x^2(g_2 - g_1)}{200L}
   \] (L curve length in feet) (A-38)
A.2.5 High and Low Points on Vertical Curves:

The locations of curve high and low points are important for drainage considerations; for example, on curbed streets catch basins or drainage inlets (DI) must be installed precisely at the drainage low point. Also, the location of low and high points are required for the vertical clearance calculations between a bridge and a roadway. The minimum cover over a utility pipe would require the determination of low points along the profile of the road.

![Sag Vertical Curve in Relation to Minimum Clearance and Cover](image)

**Figure A-14  Sag Vertical Curve in Relation to Minimum Clearance and Cover**

From equation (A-33), equation the slope \( \frac{dy}{dx} \) to zero and solving for \( X \):

\[
g_1 + rX = 0
\]

\[
X = \frac{-g_1}{r} = \frac{-g_1L}{g_2 - g_1} = \frac{g_1L}{g_1 - g_2}
\]

(A-39)  
(A-40)

Where \( X \) is the distance from BVC to the low or high points.

It should be noted that the distance \( X \) (upper case \( X \)) in equation (A-40) is different from distance \( x \) (lower case \( x \)) in all other equations i.e. it is a unique point (high or low) on vertical curves.
The following information is to be used for problems A-17 to A-21

A sag vertical curve has a length (L) of 300 ft. The entering and exiting grades are $g_1 = -3.2\%$ and $g_2 = +1.8\%$ respectively. The PVI is located at station 30 + 30 and has an elevation of 485.92 ft.

**Sample Problem A-17: Low Point on a Vertical Curve**

**Find:** The station of the low point is most nearly:

- (A) 30 + 72.00
- (B) 30 + 30.00
- (C) 29 + 00.00
- (D) 28 + 72.00

**Solution:**

$$X = \frac{-g_1}{g_2 - g_1} = \frac{-g_1L}{g_1 - g_2} = \frac{-(3.2)(3)}{(+1.8) - (-3.2)} = 1.92 \text{ Sta.} = 192.00 \text{ ft}$$

This means that the low point is located at a distance of 192.00 ft from BVC i.e., at Station = [(30 + 30.00) - (1 + 50.00)] + (1 + 92.00) = 30 + 72.00

**Notes:**

1) All **distances** used to locate a low or a high point or used to calculate an elevation of any point on a vertical curve are **measured from BVC**.

2) The point on the curve at the PVI station is not necessarily to be the low or high points.

**Answer:** (A)

**Sample Problem A-18: Elevation of the Low Point on a Vertical Curve**

**Find:** The elevation of the low point is most nearly:

- (A) 480.65 ft
- (B) 487.65 ft
- (C) 493.55 ft
- (D) 498.24 ft

(continued on next page)
Solution:

\[ y_x = y_{BVC} + g_1 x + \frac{rx^2}{2} \]

\[ = [485.92 + (1.5)(3.2)] + (-3.2)(1.92) + \left(\frac{1.8 - (-3.2)}{3.00}\right)\left(\frac{1.92^2}{2}\right) = 487.65 \text{ ft} \text{ @ Sta 30 + 72.00} \]

Answer: (B) $\varepsilon$

Sample Problem A-19: Elevation of the Mid-Chord on a Vertical Curve

Find: Which of the following is NOT true for symmetrical vertical curves:

(A) The elevation of the mid-chord point is the average of the elevations of the BVC and EVC
(B) The vertical curve lies midway between the PVI and the midpoint of the chord
(C) The low point on a sag vertical curve is always at the PVI station
(D) All of the above

Solution:

(A) is true
(B) is true
(C) is NOT true. This statement is true only if \( g_1 = g_2 \) and the vertical curve is symmetrical i.e., PVI at L/2

Answer: (C) $\varepsilon$

Sample Problem A-20: Elevation of the Mid-Chord on a Vertical Curve

Find: The elevation of point A (mid-chord point) is most nearly:

(A) 490.72 ft
(B) 488.62 ft
(C) 489.00 ft
(D) 489.67 ft

(continued on next page)
Solution:
mid-chord elevation (point A) = \( \frac{1}{2} \) (Elevation of BVC + Elevation of EVC)

\[
= \frac{1}{2} \{(485.92 + 1.5 \times 3.2) + (485.92 +1.5 \times 1.8)\} = 489.67 \text{ ft}
\]

Answer: (D)

Sample Problem A-21: Middle Ordinate Distance for a Vertical Curve

Find: The middle ordinate distance for the given vertical curve is most nearly:

(A) 1.275 ft  
(B) 1.375 ft  
(C) 1.675 ft  
(D) 1.875 ft

Solution:
The curve lies midway between the PVI and the midpoint of the chord; that is, A to B = B to PVI = \( d_0 \) which can be calculated as follows:

a) mid-chord elevation (point A) = \( \frac{1}{2} \) (Elevation of BVC + Elevation of EVC)

\[
= \frac{1}{2} \{(485.92 + 1.5 \times 3.2) + (485.92 +1.5 \times 1.8)\} = 489.67 \text{ ft}
\]

\( d_0 = \frac{1}{2} \) (difference in elevation of PVI and mid-chord elevation)

\[
= \frac{1}{2} \ (489.67 - 485.92) = 1.875 \text{ ft} \leftarrow
\]

b) \( d_0 \) (middle ordinate distance) = \( \frac{|g_1 - g_2| L}{8} \)

\[
= \frac{|-3.2 - (+1.8)| \times 3}{8} = 1.875 \text{ ft} \leftarrow
\]

(L = curve length in stations & \( |g_1 - g_2| \) = absolute value in percent)

Answer: (B)

Note: the middle ordinate (m = \( d_0 \)) may be written as: \( \frac{|g_1 - g_2| L}{800} \) (L in feet)
A.2.7 Asymmetrical (Unsymmetrical) Vertical Curves:

Asymmetrical vertical curves; also called unequal tangent vertical curves, are encountered in practice where certain limitations are exist as fixed tangent grades, vertical clearance or minimum cover. The vertex for unequal tangent curve is not in the middle between the BVC and EVC. An unequal tangent vertical curve is simply a pair of equal tangent curves, where the EVC of the first is the BVC of the second. This point is called CVC, point of compound vertical curvature. The same basic equation for the symmetrical curves will be used for the unsymmetrical curves.

![Image](figure-a-15_unsymmetrical_vertical_curve.png)

**Figure A-15 Unsymmetrical Vertical Curve**

<table>
<thead>
<tr>
<th>Sample Problem A-22: Unsymmetrical Vertical Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given:</strong> g₁ = 2 %, g₂ = +1.6 %,, BCV, V and EVC stations are 83 + 00, 87 + 00 and 93 + 00 respectively. Elevation at V = 743.24 ft. See Figure A.17</td>
</tr>
</tbody>
</table>

**Solution:**

The following steps will be followed:

**Step 1:**

\[ Y_{BVC} = 743.24 + 4(2.00) = 751.24 \text{ ft} \]
\[ Y_A = 743.24 + 2(2.00) = 747.24 \text{ ft} \]
\[ Y_{EVC} = 743.24 + 6(1.60) = 752.84 \text{ ft} \]
\[ Y_B = 743.24 + 3(1.60) = 748.04 \text{ ft} \]

\[ (\text{Grade})_{AB} = \frac{748.04 - 747.24}{5} = +0.16 \% \]

\[ Y_{CVC} = 747.24 + 2(0.16) = 747.56 \text{ ft} \]

**Step 2:**

\[ r_1 = \frac{0.16 - (-2.00)}{4} = +0.54\% / \text{sta} \]
\[ r_2 = \frac{1.60 - 0.16}{6} = +0.24\% / \text{sta} \]

(continued on next page)
### Sample Problem A-22: Unsymmetrical Vertical Curve (cont.)

<table>
<thead>
<tr>
<th>Station</th>
<th>x (sta.)</th>
<th>$g_1x$</th>
<th>$rx^2/2$</th>
<th>$y_x$</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; Difference</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>83 + 00 (BVC)</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>751.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>84 + 00</td>
<td>1</td>
<td>$\mathcal{C}2.00$</td>
<td>0.27</td>
<td>749.51</td>
<td>$\mathcal{C}1.73$</td>
<td>0.54</td>
</tr>
<tr>
<td>85 + 00</td>
<td>2</td>
<td>$\mathcal{C}4.00$</td>
<td>1.08</td>
<td>748.32</td>
<td>$\mathcal{C}1.19$</td>
<td>0.54</td>
</tr>
<tr>
<td>86 + 00</td>
<td>3</td>
<td>$\mathcal{C}6.00$</td>
<td>2.43</td>
<td>747.67</td>
<td>$\mathcal{C}0.65$</td>
<td>0.54</td>
</tr>
<tr>
<td>87 + 00 (CVC)</td>
<td>4</td>
<td>$\mathcal{C}8.00$</td>
<td>4.32</td>
<td>747.56</td>
<td>$\mathcal{C}0.11$</td>
<td></td>
</tr>
<tr>
<td>88 + 00</td>
<td>1</td>
<td>0.16</td>
<td>0.12</td>
<td>747.84</td>
<td>0.28</td>
<td>0.24</td>
</tr>
<tr>
<td>89 + 00</td>
<td>2</td>
<td>0.32</td>
<td>0.48</td>
<td>748.36</td>
<td>0.52</td>
<td>0.24</td>
</tr>
<tr>
<td>90 + 00</td>
<td>3</td>
<td>0.48</td>
<td>1.08</td>
<td>749.12</td>
<td>0.76</td>
<td>0.24</td>
</tr>
<tr>
<td>91 + 00</td>
<td>4</td>
<td>0.64</td>
<td>1.92</td>
<td>750.12</td>
<td>1.00</td>
<td>0.24</td>
</tr>
<tr>
<td>92 + 00</td>
<td>5</td>
<td>0.80</td>
<td>3.00</td>
<td>751.36</td>
<td>1.24</td>
<td>0.24</td>
</tr>
<tr>
<td>93 + 00 (EVC)</td>
<td>6</td>
<td>0.96</td>
<td>4.32</td>
<td>752.84</td>
<td>1.48</td>
<td></td>
</tr>
</tbody>
</table>
A.3 SIGHT DISTANCES

Sight distance, in the context of road design, is how far a driver can see before the line of sight is blocked by a hill crest, or an obstacle on the inside of a horizontal curve or intersection i.e. **sight distance is the continuous length of highway ahead visible to the driver.** Insufficient sight distance can have implications for the safety or operations of a roadway or intersection.

**Table A-9 Four Types of Sight Distances**

(For Any Given Speed (say 50 mph) → PSD > DSD > SSD → 1835 ft > 465 ft A-1030 ft E > 425 ft)

<table>
<thead>
<tr>
<th>1- Stopping Sight Distance (SSD)</th>
<th>2- Passing Sight Distance (PSD)</th>
<th>3- Decision Sight Distance (DSD)</th>
<th>4- Intersection (Corner) Sight Distance (CSD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Used on multilane highways and on 2-lane roads when passing sight distance is not economically obtainable.</td>
<td>● Used where an opposing lane can provide passing opportunities (two-lane roads)</td>
<td>● Used at major decision points e.g. off-ramp noses, branch connections, ..</td>
<td>● Used at intersections</td>
</tr>
</tbody>
</table>

**Stopping Sight Distance (ALL ROADS)**

**Passing Sight Distance (TWO-LANE ROADS)**

**Figure A-16 Stopping and Passing Sight Distances**
A.3.1 Stopping Sight Distance:

Stopping sight distance is the distance traveled while the vehicle driver perceives a situation requiring a stop, realizes that stopping is necessary, applies the brake, and comes to a stop. Actual stopping distances are also affected by road conditions, the mass of the car, the incline of the road, and numerous other factors. For design, a conservative distance is needed to allow a vehicle traveling at design speed to stop before reaching a stationary object in its path. Typically the design sight distance allows a below-average driver to stop in time to avoid a collision. In the United States (AASHTO Green Book), the driver's eye is assumed to be 42 inches (3.5 feet) above the pavement, and the object height is 24 inches (2 feet-about the height of vehicle taillights).

Stopping Sight Distance $d =$

- distance traversed during the brake reaction time + distance to brake the vehicle to a stop

\[ d = \text{distance traversed during the brake reaction time} + \text{distance to brake the vehicle to a stop} \]

![Diagram of Stopping Sight Distance](image)

**Figure A-17** Stopping Sight Distance (Driver’s Eye Height =3.5’ and Object Height= 2.0’)

The perception-reaction time for a driver is often broken down into the four components that are assumed to make up the perception reaction time. These are referred to as the PIEV time or process.

**PIEV Process:**

i) Perception the time to see or discern an object or event,

ii) Intellection the time to understand the implications of the object’s presence or event,

iii) Emotion the time to decide how to react, and

iv) Volition the time to initiate the action, for example, the time to engage the brakes.

Human factors research defined the required perception-reaction times as follows:

- Design 2.5 sec
- Operations/control 1.0 sec

These perception reaction times were based on observed behavior for the 85th percentile driver; that is, 85% of drivers could react in that time or less. More recent research has shown these times to be conservative for design.
A.3.1.1 Stopping Sight Distance On A Flat Grade:

The following equations will be used to calculate the stopping distance on a flat grade facilities:

\[
d = 0.278 Vt + 0.039 \frac{V^2}{a}
\]

where:
- \( V \) = design speed, km/h;
- \( a \) = deceleration rate, m/s^2 (3.4 m/s^2)

\[
d = 1.47 Vt + 1.075 \frac{V^2}{a}
\]

where:
- \( V \) = design speed, mph;
- \( a \) = deceleration rate, ft/s^2 (11.2 ft/s^2)

The above equations are used to generate the following table for wet-pavement conditions.

### Table A-10 Stopping Sight Distance Equations on Flat Grades

<table>
<thead>
<tr>
<th>Stopping Sight Distance (SSD)</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d = 0.278 Vt + 0.039 \frac{V^2}{a} )</td>
<td>( d = 1.47 Vt + 1.075 \frac{V^2}{a} )</td>
</tr>
</tbody>
</table>

### Table A-11 Stopping Sight Distance on Flat Grades

<table>
<thead>
<tr>
<th>Design speed (km/h)</th>
<th>Brake reaction distance (m)</th>
<th>Braking distance on level (m)</th>
<th>Stopping sight distance Calculated (m)</th>
<th>Design (m)</th>
<th>Design speed (mph)</th>
<th>Brake reaction distance (ft)</th>
<th>Braking distance on level (ft)</th>
<th>Stopping sight distance Calculated (ft)</th>
<th>Design (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>13.9</td>
<td>4.6</td>
<td>18.5</td>
<td>20</td>
<td>15</td>
<td>55.1</td>
<td>21.6</td>
<td>76.7</td>
<td>80</td>
</tr>
<tr>
<td>30</td>
<td>20.9</td>
<td>10.3</td>
<td>31.2</td>
<td>35</td>
<td>20</td>
<td>73.5</td>
<td>38.4</td>
<td>111.9</td>
<td>115</td>
</tr>
<tr>
<td>40</td>
<td>27.8</td>
<td>18.4</td>
<td>46.2</td>
<td>50</td>
<td>25</td>
<td>91.9</td>
<td>60.0</td>
<td>151.9</td>
<td>155</td>
</tr>
<tr>
<td>50</td>
<td>34.8</td>
<td>28.7</td>
<td>63.5</td>
<td>65</td>
<td>30</td>
<td>110.3</td>
<td>86.4</td>
<td>196.7</td>
<td>200</td>
</tr>
<tr>
<td>60</td>
<td>41.7</td>
<td>41.3</td>
<td>83.0</td>
<td>85</td>
<td>35</td>
<td>128.6</td>
<td>117.6</td>
<td>246.2</td>
<td>250</td>
</tr>
<tr>
<td>70</td>
<td>48.7</td>
<td>56.2</td>
<td>104.9</td>
<td>105</td>
<td>40</td>
<td>147.0</td>
<td>153.6</td>
<td>300.6</td>
<td>305</td>
</tr>
<tr>
<td>80</td>
<td>55.6</td>
<td>73.4</td>
<td>129.0</td>
<td>130</td>
<td>45</td>
<td>165.4</td>
<td>194.4</td>
<td>359.8</td>
<td>360</td>
</tr>
<tr>
<td>90</td>
<td>62.6</td>
<td>92.9</td>
<td>155.5</td>
<td>160</td>
<td>50</td>
<td>183.8</td>
<td>240.0</td>
<td>423.8</td>
<td>425</td>
</tr>
<tr>
<td>100</td>
<td>69.5</td>
<td>114.7</td>
<td>184.2</td>
<td>185</td>
<td>55</td>
<td>202.1</td>
<td>290.3</td>
<td>492.4</td>
<td>495</td>
</tr>
<tr>
<td>110</td>
<td>76.5</td>
<td>138.8</td>
<td>215.3</td>
<td>220</td>
<td>60</td>
<td>220.5</td>
<td>345.5</td>
<td>566.0</td>
<td>570</td>
</tr>
<tr>
<td>120</td>
<td>83.4</td>
<td>165.2</td>
<td>248.6</td>
<td>250</td>
<td>65</td>
<td>238.9</td>
<td>405.5</td>
<td>644.4</td>
<td>645</td>
</tr>
<tr>
<td>130</td>
<td>90.4</td>
<td>193.8</td>
<td>284.2</td>
<td>285</td>
<td>70</td>
<td>257.3</td>
<td>470.3</td>
<td>727.6</td>
<td>730</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>75</td>
<td>275.6</td>
<td>539.9</td>
<td>815.5</td>
<td>820</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>80</td>
<td>294.0</td>
<td>614.3</td>
<td>908.3</td>
<td>910</td>
</tr>
</tbody>
</table>

Note: Brake reaction distance predicated on a time of 2.5 s; deceleration rate of 3.4 m/s^2 [11.2 ft/s^2] used to determine calculated sight distance.
Sample Problem A-23: Stopping Sight Distance on a Flat Grade

**Given:** Verify the stopping sight distance for a speed of 60 mph and a flat grade.

**Solution**

\[ d = 1.47 Vt + 1.075 \frac{V^2}{a} \]  

AASHTO (3-2)

\[ d = 1.47 (60)(2.5) + 1.075 \frac{60^2}{11.2} = 220.50 + 345.54 = 566.04 \text{ ft versus 566.0 ft given in AASHTO} \]

A.3.1.2 Stopping Sight Distance On a Grade:

When a highway is on a grade, the equation for **braking distance** should be modified as follows:

**Table A-12** Braking Sight Distance Equations on a Grade  

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ d = \frac{V^2}{2.54 \left( \frac{a}{9.81} \pm G \right)} ]</td>
<td>[ d = \frac{V^2}{30 \left( \frac{a}{32.2} \pm G \right)} ] (3-3)</td>
</tr>
</tbody>
</table>

Where:

- \( G = \) percent of grade divided by 100;
- \( V = \) design speed, km/h;
- \( a = \) deceleration rate, m/s\(^2\) (3.4 m/s\(^2\))

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ d = 0.278 Vt + \frac{V^2}{2.54 \left( \frac{a}{9.81} \pm G \right)} ]</td>
<td>[ d = 1.47 Vt + \frac{V^2}{30 \left( \frac{a}{32.2} \pm G \right)} ] (3-3 mod.)</td>
</tr>
</tbody>
</table>

where:

- \( t = \) brake reaction time, 2.5 s;
- \( G = \) percent of grade divided by 100;
- \( V = \) design speed, km/h;
- \( a = \) deceleration rate, m/s\(^2\) (3.4 m/s\(^2\))

Stopping Sight Distance \( d = \) distance traversed during brake reaction time + distance to brake the vehicle to a stop

**Table A-13** Stopping Sight Distance Equations on a Grade
As can be seen from the above equations that, the stopping distances needed on upgrades are shorter than the one on level roadways; and those on down grades are longer.

**Table A-14** Stopping Sight Distance on Grades  

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design speed (km/h)</strong></td>
<td><strong>Design speed (mph)</strong></td>
</tr>
<tr>
<td><strong>Stopping sight distance (m)</strong></td>
<td><strong>Stopping sight distance (ft)</strong></td>
</tr>
<tr>
<td>Downgrades</td>
<td>3%</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>32</td>
</tr>
<tr>
<td>40</td>
<td>50</td>
</tr>
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<td>66</td>
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<td>60</td>
<td>87</td>
</tr>
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<td>70</td>
<td>110</td>
</tr>
<tr>
<td>80</td>
<td>136</td>
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<td>164</td>
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<tr>
<td>100</td>
<td>194</td>
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<tr>
<td>110</td>
<td>227</td>
</tr>
<tr>
<td>120</td>
<td>263</td>
</tr>
<tr>
<td>130</td>
<td>302</td>
</tr>
</tbody>
</table>

| **Design speed (km/h)** | **Design speed (mph)** |
| **Stopping sight distance (m)** | **Stopping sight distance (ft)** |
| Upgrades | 3% | 6% | 9% | 3% | 6% | 9% | Upgrades | 3% | 6% | 9% | 3% | 6% | 9% |
| 20 | 20 | 20 | 19 | 18 | 18 | 15 | 80 | 82 | 85 | 75 | 74 | 73 |
| 30 | 32 | 35 | 31 | 30 | 29 | 20 | 116 | 120 | 126 | 109 | 107 | 104 |
| 40 | 50 | 50 | 45 | 44 | 43 | 25 | 158 | 165 | 173 | 147 | 143 | 140 |
| 50 | 66 | 70 | 61 | 59 | 58 | 30 | 205 | 215 | 227 | 200 | 184 | 179 |
| 60 | 87 | 92 | 80 | 77 | 75 | 35 | 257 | 271 | 287 | 237 | 229 | 222 |
| 70 | 110 | 116 | 100 | 97 | 93 | 40 | 315 | 333 | 354 | 289 | 278 | 269 |
| 80 | 136 | 144 | 123 | 118 | 114 | 45 | 378 | 400 | 427 | 344 | 331 | 320 |
| 90 | 164 | 174 | 148 | 141 | 136 | 50 | 446 | 474 | 507 | 405 | 388 | 375 |
| 100 | 194 | 207 | 174 | 167 | 160 | 55 | 520 | 553 | 593 | 469 | 450 | 433 |
| 110 | 227 | 243 | 203 | 194 | 186 | 60 | 598 | 638 | 686 | 538 | 515 | 495 |
| 120 | 263 | 281 | 234 | 223 | 214 | 65 | 682 | 728 | 785 | 612 | 584 | 561 |
| 130 | 302 | 323 | 267 | 254 | 243 | 70 | 771 | 825 | 891 | 690 | 658 | 631 |

**Sample Problem A-24: Stopping Sight Distance on a Grade**

**Given:** Verify the stopping sight distance for a speed of 60 mph and a down grade of 6%.

**Solution:**

\[
d = 1.47Vt + \frac{V^2}{30 \left( \frac{a}{32.2} \pm G \right)}
\]

\[
d = 1.47(60)(2.5) + \frac{60^2}{30 \left( \frac{11.2}{32.2} - 6/100 \right)} = 637.42 \text{ ft} \quad \text{versus 638 ft (above table)}
\]
Sample Problem A-25: Stopping Sight Distance on a Grade

**Given:** The stopping sight distance for a speed of 60 mph and a down grade of 4.5% is most nearly:

(A) 638 ft  
(B) 628 ft  
(C) 617 ft  
(D) 598 ft

**Solution:**

\[ d = 1.47Vt + \frac{V^2}{30 \left( \frac{a}{32.2} \right) \pm G} \quad \text{AASHTO (3-3 mod.)} \]

\[ d = 1.47(60)(2.5) + \frac{60^2}{30 \left( \frac{11.2}{32.2} - \frac{4.5}{100} \right)} = 616.77 \text{ ft} \quad \text{Answer: (C) \xi} \]

versus 618 ft (linear interpolation between 3% and 6% from the above table).

Sample Problem A-26: Stopping Sight Distance on a Grade

**Given:** The stopping sight distance for a speed of 62.5 mph and an upgrade of 9% is most nearly:

(A) 495 ft  
(B) 528 ft  
(C) 561 ft  
(D) 631 ft

**Solution:**

\[ d = 1.47Vt + \frac{V^2}{30 \left( \frac{a}{32.2} \right) \pm G} \quad \text{AASHTO (3-3 mod.)} \]

\[ d = 1.47(62.5)(2.5) + \frac{62.5^2}{30 \left( \frac{11.2}{32.2} + \frac{9}{100} \right)} = 527.08 \text{ ft} \quad \text{Answer: (B) \xi} \]

versus 528 ft (linear interpolation between 60 mph and 65 mph from the above table).

**Note:** Design speed normally reported in a 5 mph increments.
A.3.1.3 Variations for Trucks:

The recommended stopping sight distances given in the previous table and the equations are based on passenger car operation and do not explicitly consider design for truck operation. Trucks as a whole, especially the larger and heavier units, need longer stopping distances for a given speed than passenger vehicles. However, there is one factor that tends to balance the additional braking lengths for trucks with those for passenger cars. The truck driver is able to see substantially farther beyond vertical sight obstructions because of the higher position of the seat in the vehicle.
A.3.2 Passing Sight Distance:

Passing sight distance is the minimum sight distance required for the driver of one vehicle to pass another vehicle safely and comfortably. Passing must be accomplished assuming an oncoming vehicle comes into view and maintains the design speed, without reduction, after the overtaking maneuver is started.

Per AASHTO Standards, the minimum passing sight distance for two-lane highways is determined as the sum of the following four distances:

\[ d_1 = \text{Distance traversed during perception and reaction time during the initial acceleration to the point of encroachment on the left lane.} \]

\[ d_2 = \text{Distance traveled while the passing vehicle occupies the left lane.} \]

\[ d_3 = \text{Distance between the passing vehicle at the end of its maneuver and the opposing vehicle.} \]

\[ d_4 = \text{Distance traversed by the opposing vehicle for two-thirds of the time the passing vehicle occupies the left lane, or } \frac{2}{3} \text{ of } d_2. \]

**Figure A-18 Elements of Passing Sight Distance for Two-Lane Highways**

### Table A-15  Elements of Safe **Passing Sight Distance** for Design of two-Lane Highways  

<table>
<thead>
<tr>
<th>Component of passing maneuver</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Speed range (km/h)</td>
<td>Speed range (mph)</td>
</tr>
<tr>
<td></td>
<td>50-65 66-80 81-95 96-110</td>
<td>30-40 40-50 56-60 60-70</td>
</tr>
<tr>
<td>Average passing speed (km/h)</td>
<td>56.2 70.0 84.5 99.8</td>
<td>34.9 43.8 52.6 62.0</td>
</tr>
<tr>
<td>Initial maneuver</td>
<td>a = average acceleration(^a)</td>
<td>2.25 2.30 2.37 2.41</td>
</tr>
<tr>
<td></td>
<td>t(_1) = time (sec)(^a)</td>
<td>3.6 4.0 4.3 4.5</td>
</tr>
<tr>
<td></td>
<td>d(_1) = distance traveled</td>
<td>45 66 89 113</td>
</tr>
<tr>
<td>Occupation of left lane:</td>
<td>t(_2) = time (sec)(^a)</td>
<td>9.3 10.0 10.7 11.3</td>
</tr>
<tr>
<td></td>
<td>d(_2) = distance traveled</td>
<td>145 195 251 314</td>
</tr>
<tr>
<td>Clearance length:</td>
<td>d(_3) = distance traveled(^a)</td>
<td>30 55 75 90</td>
</tr>
<tr>
<td>Opposing vehicle:</td>
<td>d(_4) = distance traveled</td>
<td>97 130 168 209</td>
</tr>
<tr>
<td>Total dist. = d(_1) + d(_2) + d(_3) + d(_4)</td>
<td>317 446 583 726</td>
<td>1040 1468 1918 2383</td>
</tr>
</tbody>
</table>

\(^a\)For consistent speed relation, observe values adjusted slightly.

**Note:** In the metric portion of the table, speed values are in km/h, acceleration rates in km/h/s, and distances are in meters. In the U.S. customary portion of the table, speed values are in mph, acceleration rates in mph/sec, and distances are in feet.

### Table A-16  Passing Sight Distance for Design of Two-Lane Highways  

<table>
<thead>
<tr>
<th>Design speed (km/h)</th>
<th>Assumed speeds (km/h)</th>
<th>Passing sight distance (m)</th>
<th>Rounded for design</th>
<th>Design speed (mph)</th>
<th>Assumed speeds (mph)</th>
<th>Passing sight distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passed vehicle</td>
<td>Passing vehicle</td>
<td>From Exhibit 3-6</td>
<td>Rounded for design</td>
<td>Passed vehicle</td>
<td>Passing vehicle</td>
</tr>
<tr>
<td>30</td>
<td>29</td>
<td>44</td>
<td>200</td>
<td>200</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>40</td>
<td>36</td>
<td>51</td>
<td>266</td>
<td>270</td>
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<td>22</td>
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<td>345</td>
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<td>66</td>
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<td>74</td>
<td>482</td>
<td>485</td>
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<td>45</td>
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<td>670</td>
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<td></td>
<td></td>
<td>80</td>
<td>58</td>
</tr>
</tbody>
</table>

\(^a\)Exhibit 3-6 is the graph showing the total passing sight distance and its components (d\(_1\), d\(_2\), d\(_3\), and d\(_4\) for two-lane highways
Per ASSHTO, the minimum passing sight distances presented in Exhibit 3-7 (page 124) are generally *conservative* for modern vehicles. Notice that the difference in speed between the passed and passing vehicles which is *10 mph*.

---

**Figure A-19** Total Passing Sight Distance and Its Components for Two-Lane Highways (Exhibit 3-6 AASHTO Geometric Design-Green Book, 2004 edition, 5th edition)
Sample Problem A-27: Passing Sight Distance on a Grade

**Given:** The passing sight distance for a design speed of 55 mph assumed the passed vehicle is at 44 mph is most nearly:

(A) 1835 ft  
(B) 1985 ft  
(C) 2135 ft  
(D) 2285 ft

**Solution:**
Using AASHTO Exhibit 3-7 (Table A-19), the passing sight distance is 1985 ft.

Answer: (B)

A.3.3 Length of Crest Vertical Curves and Sight Distances:

Minimum lengths of crest vertical curves based on sight distance criteria generally are satisfactory from the standpoint of safety, comfort, and appearance.

Table A-17 Crest Vertical Curve Length and Sight Distances  

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where $S$ is less than $L$, $(S &lt; L)$</td>
<td>Where $S$ is less than $L$, $(S &lt; L)$</td>
</tr>
<tr>
<td>$L = \frac{AS^2}{100\left(\sqrt{2h_1} + \sqrt{2h_2}\right)^2}$</td>
<td>$L = \frac{AS^2}{100\left(\sqrt{2h_1} + \sqrt{2h_2}\right)^2}$ (3-41)</td>
</tr>
<tr>
<td>Where $S$ is greater than $L$, $(S &gt; L)$</td>
<td>Where $S$ is greater than $L$, $(S &gt; L)$</td>
</tr>
<tr>
<td>$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$</td>
<td>$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$ (3-42)</td>
</tr>
</tbody>
</table>

where:

$L = \text{length of vertical curve, m; }$  
$S = \text{sight distance, m; }$  
$A = \text{algebraic difference in grades, percent }$  
$h_1 = \text{height of eye above roadway surface, m; }$  
$h_2 = \text{height of object above roadway surface, m}$

$L = \text{length of vertical curve, ft; }$  
$S = \text{sight distance, ft; }$  
$A = \text{algebraic difference in grades, percent }$  
$h_1 = \text{height of eye above roadway surface, ft; }$  
$h_2 = \text{height of object above roadway surface, ft}$
Figure A-20 Parameters Considered in Determining the Length of a Crest Vertical Curve to Provide Sight Distances
When the height of eye and the height of object are 3.5 ft and 2.0 ft respectively, are used for *stopping sight distance*, the equations become:

![Stopping Sight Distance](image-url)

**Table A-18** Crest Vertical Curve Length and *Stopping Sight Distance* for Heights of 3.5 ft (eye) and 2.0 ft (object)  

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
</table>
| Where $S$ is less than $L$,  
$L = \frac{AS^2}{658}$  
where: 
$L =$ length of vertical curve, m;  
$S =$ sight distance, m;  
$A =$ algebraic difference in grades, percent | Where $S$ is less than $L$ ($S < L$),  
$L = \frac{AS^2}{2158}$  
(3-43) |
| Where $S$ is greater than $L$,  
$L = 2S - \frac{658}{A}$ | Where $S$ is greater than $L$ ($S > L$),  
$L = 2S - \frac{2158}{A}$  
(3-44) |

When the height of eyes of the two drivers is 3.5 ft is used for *passing sight distance*, the equations become:

**Table A-19** Crest Vertical Curve Length and *Passing Sight Distance* for Height of 3.5 ft (eye) for Both Drivers  

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
</table>
| Where $S$ is less than $L$,  
$L = \frac{AS^2}{864}$  
where: 
$L =$ length of vertical curve, m;  
$S =$ sight distance, m;  
$A =$ algebraic difference in grades, percent | Where $S$ is less than $L$,  
$L = \frac{AS^2}{2800}$  
(3-45) |
| Where $S$ is greater than $L$,  
$L = 2S - \frac{864}{A}$ | Where $S$ is greater than $L$,  
$L = 2S - \frac{2800}{A}$  
(3-46) |

where:
$L =$ length of vertical curve, ft;  
$S =$ sight distance, ft;  
$A =$ algebraic difference in grades, percent
### Table A-20 Design Controls for Stopping Sight Distance on Crest Curves

<table>
<thead>
<tr>
<th>Design Speed (km/h)</th>
<th>Stopping sight distance (m)</th>
<th>Metric</th>
<th>Rate of vertical curvature, K&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated</td>
<td>Design</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>35</td>
<td>1.9</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
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</tr>
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<td>73.6</td>
<td>74</td>
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<td>250</td>
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<td>124</td>
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</table>

<table>
<thead>
<tr>
<th>Design Speed (mph)</th>
<th>Stopping sight distance (ft)</th>
<th>US Customary</th>
<th>Rate of vertical curvature, K&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated</td>
<td>Design</td>
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<td>820</td>
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</tr>
<tr>
<td>80</td>
<td>910</td>
<td>383.7</td>
<td>384</td>
</tr>
</tbody>
</table>

<sup>a</sup> Rate of vertical curvature, K, is the length of curve per percent algebraic difference in interesting grades (A).

\[ K = \frac{L}{A} = \frac{1}{r} \]

### Table A-21 Design Controls for Passing Sight Distance on Crest Curves

<table>
<thead>
<tr>
<th>Design Speed (km/h)</th>
<th>Passing sight distance (m)</th>
<th>Metric</th>
<th>Rate of vertical curvature, K*&lt;sup&gt;a&lt;/sup&gt; design</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>200</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>270</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Speed (mph)</th>
<th>Passing sight distance (ft)</th>
<th>US Customary</th>
<th>Rate of vertical curvature, K*&lt;sup&gt;a&lt;/sup&gt; design</th>
</tr>
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<tbody>
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<td>45</td>
<td>1625</td>
<td>943</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1835</td>
<td>1203</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>1985</td>
<td>1407</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>2135</td>
<td>1628</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>2285</td>
<td>1865</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>2480</td>
<td>2197</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>2580</td>
<td>2377</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>2680</td>
<td>2565</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Rate of vertical curvature, K, is the length of curve per percent algebraic difference in interesting grades (A).

\[ K = \frac{L}{A} = \frac{1}{r} \]
Figure A-21  Design Controls for **Crest Vertical Curves**-Open Road Conditions (Exhibit 3-71, page 271, AASHTO Geometric Design-Green Book, 2004 edition, 5th edition)

The following key points should be considered in relation to AASHTO Exhibit 3-71:
1- The vertical lines represent the minimum vertical curve length;
2- The minimum vertical curve length \( L \) (meters) = 0.6\( V \), where \( V \) is in kilometers per hour (\( L_{\min} = 0.6V \)); and
3- The minimum vertical curve length \( L \) (feet) = 3\( V \), where \( V \) is in miles per hour (\( L_{\min} = 3V \)).

### Sample Problem A-28: Stopping Sight Distance (SSD) on a Grade

**Given:** What is the length of the vertical curve to provide stopping sight distance (SSD) of 650 feet? The grades of the curve are + 4\% ascending grade, and − 2\% descending grade.

- (A) 940 ft
- (B) 1180 ft
- (C) 1710 ft
- (D) 1910 ft

**Solution:**

From the given grades, the vertical curve is a crest

- Assume that \( S < L \) and using equation (3-43)

\[
L = \frac{AS^2}{2158} = \frac{6 \times 650^2}{2158} = 1174 \text{ ft} > SSD = 650 \text{ ft} \quad \text{OK} \quad (3-43)
\]

where:
- \( L \) = length of vertical curve, ft;
- \( S \) = sight distance, ft;
- \( A \) = algebraic difference in grades, percent

**Note:** If \( S > L \) is assumed and equation (3-44) is used, the following result will be obtained:

\[
L = 2S - \frac{2158}{A} = 2 \times 650 - \frac{2158}{6} = 940.33 \text{ ft} > 650 \text{ ft} \quad S \quad \text{Not Good} \quad (3-44)
\]

**Answer:** (B)

**Question:** Can we use Figure A-21 (AASHTO Exhibit 3-71) to solve the problem? **YES**

**Hint:** For SSD = 650 ft \( \checkmark \) \( V \) \( \checkmark \) 65 mph (Exhibit 3-1, page 112, Table A-11)
A.3.4 Length of Sag Vertical Curves and Sight Distances:

There are four different criteria for establishing lengths of sag vertical curves as follows:

1- headlight sight distance,
2- passenger comfort,
3- drainage control, and
4- general appearance

Figure A-22 Stopping Sight Distance (SSD) for Sag Vertical Curves

The following equation show the relationships between $S$, $L$, and $A$, using $S$ as the distance between the vehicle and point where the 1-degree ($1^\circ$) upward angle of the light beam intersects the surface of the roadway.

<table>
<thead>
<tr>
<th>Design Controls for Sag Vertical Curves</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>When $S$ is less than $L$ ($S &lt; L$),</td>
<td>$L = \frac{AS^2}{200[0.6 + S(\tan 1^\circ)]}$</td>
<td>$L = \frac{AS^2}{200[2.0 + S(\tan 1^\circ)]}$ (3-47)</td>
</tr>
<tr>
<td>or,</td>
<td>$L = \frac{AS^2}{120 + 3.5S}$</td>
<td>$L = \frac{AS^2}{400 + 3.5S}$ (3-48)</td>
</tr>
<tr>
<td>When $S$ is greater than $L$ ($S &gt; L$),</td>
<td>$L = 2S - \frac{200[0.6 + S(\tan 1^\circ)]}{A}$</td>
<td>$L = 2S - \frac{200[2.0 + S(\tan 1^\circ)]}{A}$ (3-49)</td>
</tr>
<tr>
<td>or,</td>
<td>$L = 2S - \left(\frac{120 + 3.5S}{A}\right)$</td>
<td>$L = 2S - \left(\frac{400 + 3.5S}{A}\right)$ (3-50)</td>
</tr>
</tbody>
</table>

where:

- $L$ = length of sag vertical curve, m;
- $S$ = light beam distance, m;
- $A$ = algebraic difference in grades, percent

where:

- $L$ = length of sag vertical curve, ft;
- $S$ = light beam distance, ft;
- $A$ = algebraic difference in grades, percent
The effect on passenger comfort of the change in vertical curve direction is greater on sag than on crest vertical curves because gravitational and centripetal forces are in opposite directions, rather than in the same direction.

The following table gives the sag vertical curve length taken into consideration the comfort of the driver of sag curves.

| Table A-23 Length of Sag Vertical Curves Considering Comfort*  
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Metric</strong></td>
</tr>
<tr>
<td>$L = \frac{AV^2}{395}$</td>
</tr>
<tr>
<td>where: $L =$ length of sag vertical curve, m; $A =$ algebraic difference in grades, Percent; $V =$ design speed, km/h</td>
</tr>
<tr>
<td>where: $L =$ length of sag vertical curve, ft; $A =$ algebraic difference in grades, Percent; $V =$ design speed, mph</td>
</tr>
</tbody>
</table>

*The length of sag vertical curves are based on centripetal acceleration does not exceed 1 ft/s$^2$ [0.3 m/s$^2$]

| Table A-24 Design Controls for Sag Vertical Curves  
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Metric</strong></td>
</tr>
<tr>
<td>Design Speed (km/h)</td>
</tr>
<tr>
<td>Design Controls for Sag Curves</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>70</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>90</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>110</td>
</tr>
<tr>
<td>120</td>
</tr>
<tr>
<td>130</td>
</tr>
<tr>
<td>75</td>
</tr>
</tbody>
</table>

$^a$ Rate of vertical curvature, $K$, is the length of curve per percent algebraic difference in interesting grades (A). $K = L/A = 1/r$
The following key points should be considered in relation to AASHTO Exhibit 3-74:

Figure A-23  Design Controls for **Sag Vertical Curves**-Open Road Conditions
1- The vertical lines represent the minimum vertical curve length;
2- The minimum vertical curve length \( L \) (meters) = 0.6\( V \), where \( V \) is in kilometers per hour \( (L_{\text{min}} = 0.6V) \); and
3- The minimum vertical curve length \( L \) (feet) = 3\( V \), where \( V \) is in miles per hour \( (L_{\text{min}} = 3V) \).

### Sample Problem A-29: Vertical Curve Length and SSD

**Given:** What is the length of the vertical curve to provide stopping sight distance (SSD) of 650 feet? The grades of the curve are \( -4\% \) ascending grade, and \( +2\% \) descending grade.

- (A) 950 ft
- (B) 1180 ft
- (C) 1710 ft
- (D) 1910 ft

**Solution:**

From the given grades, the vertical curve is sag

➢ Assume that \( S < L \) and using equation (3-48)

\[
A = \text{algebraic difference in grades, percent} = b G_1 \, i \quad G_2 \quad b = b \, i \, i \, (2) b = 6\%
\]

\[
L = \frac{A S^2}{400 + 3.5 S} = \frac{6 \times 650^2}{400 + 3.5 \times 650} = 947.66 \text{ ft} > 650 \text{ ft} \quad \hat{\text{OK}} \quad (3-48)
\]

where:

- \( L \) = length of vertical curve, ft;
- \( S \) = sight distance, ft;
- \( A \) = algebraic difference in grades, percent

**Notes:**

1- If \( S > L \) is assumed and equation (3-50) is used, the following result will be obtained:

\[
L = 2S - \left( \frac{400 + 3.5 S}{A} \right) = 2 \times 650 - \left( \frac{400 + 3.5 \times 650}{6} \right) = 854.42 \text{ ft} \quad (3-50)
\]

\[
\text{But} \quad 854.42 \text{ ft} > \text{SSD} = 650 \text{ ft} \rightarrow N.G.
\]

2- Check the curve length for comfort: \( L = \frac{A V^2}{46.5} = \frac{6 \times 65^2}{46.5} = 545.2 \text{ ft} < 854.42 \text{ ft} \quad (3-51) \)

**Answer:** (A)

**Question:** Can we use Fig. A-22 (AASHTO Exhibit 3-74, page 275) to solve the problem? **YES**

**Hint:** For SSD = 650 ft \( \hat{V} \) \( \hat{V} \) \( \hat{V} \) 65 mph (Exhibit 3-1, Table A-11)
A.3.5 Decision Sight Distance:

Decision sight distance is used when drivers must make decisions more complex than stop or don't stop. It is longer than stopping sight distance to allow for the distance traveled while making a more complex decision. The decision sight distance is "distance required for a driver to detect an unexpected or otherwise difficult-to-perceive information source or hazard in a roadway environment that may be visually cluttered, recognize the hazard or its threat potential, select an appropriate speed and path, and initiate and complete the required maneuver safely and efficiently."

In the United States (AASHTO Green Book), the driver's eye is assumed to be 42 inches (3.5 feet) above the pavement, and the object height is 24 inches (2 feet-about the height of vehicle taillights). There are five maneuvers as follows:

1- Avoidance Maneuver A: Stop on rural road $t = 3.0$ s
2- Avoidance Maneuver B: Stop on urban road $t = 9.1$ s
3- Avoidance Maneuver C: Speed/path/direction change on rural road $t$ varies between 10.2 and 11.2 s
4- Avoidance Maneuver D: Speed/path/direction change on suburban road $t$ varies between 12.1 and 12.9 s
5- Avoidance Maneuver E: Speed/path/direction change on urban road $t$ varies between 14.0 and 14.5 s

**Table A 25- Decision Sight Distance Equations for Avoidance Maneuvers A and B**

<table>
<thead>
<tr>
<th>DSD for Maneuvers A &amp; B</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 0.278 Vt + 0.039 \frac{V^2}{a}$</td>
<td>$d = 1.47 Vt + 1.075 \frac{V^2}{a}$ (3-4)</td>
<td></td>
</tr>
</tbody>
</table>

Where:
- $t =$ pre-maneuver time, s (see types of maneuvers above)
- $V =$ design speed, km/h;
- $a =$ deceleration rate, m/s$^2$ (3.4 m/s$^2$)

**Table A 26- Decision Sight Distance Equations for Avoidance Maneuvers C, D, and E**

<table>
<thead>
<tr>
<th>DSD for Maneuvers C, D &amp; E</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 0.278 Vt$</td>
<td>$d = 1.47 Vt$ (3-5)</td>
<td></td>
</tr>
</tbody>
</table>

Where:
- $t =$ pre-maneuver time, s (see types of maneuvers above);
- $V =$ design speed, km/h

- $V =$ design speed, mph

Where:
- $t =$ pre-maneuver time, s (see types of maneuvers above);
- $V =$ design speed, mph
### Table A-27 Decision Sight Distance

<table>
<thead>
<tr>
<th>Decision Sight Distance (DSD)</th>
<th>Metric</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Design Speed (km/h)</th>
<th>Design Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avoidance maneuver</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>Avoidance maneuver</td>
</tr>
<tr>
<td>50</td>
<td>70</td>
<td>155</td>
<td>145</td>
<td>170</td>
<td>195</td>
<td>30</td>
<td>220</td>
</tr>
<tr>
<td>60</td>
<td>95</td>
<td>195</td>
<td>170</td>
<td>205</td>
<td>235</td>
<td>35</td>
<td>275</td>
</tr>
<tr>
<td>70</td>
<td>115</td>
<td>235</td>
<td>200</td>
<td>235</td>
<td>275</td>
<td>40</td>
<td>330</td>
</tr>
<tr>
<td>80</td>
<td>140</td>
<td>280</td>
<td>230</td>
<td>270</td>
<td>315</td>
<td>45</td>
<td>395</td>
</tr>
<tr>
<td>90</td>
<td>170</td>
<td>325</td>
<td>270</td>
<td>315</td>
<td>360</td>
<td>50</td>
<td>465</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>370</td>
<td>315</td>
<td>355</td>
<td>400</td>
<td>55</td>
<td>535</td>
</tr>
<tr>
<td>110</td>
<td>235</td>
<td>420</td>
<td>330</td>
<td>380</td>
<td>430</td>
<td>60</td>
<td>610</td>
</tr>
<tr>
<td>120</td>
<td>265</td>
<td>470</td>
<td>360</td>
<td>415</td>
<td>470</td>
<td>65</td>
<td>695</td>
</tr>
<tr>
<td>130</td>
<td>305</td>
<td>525</td>
<td>390</td>
<td>450</td>
<td>510</td>
<td>70</td>
<td>780</td>
</tr>
</tbody>
</table>

- Avoidance Maneuver A: Stop on rural road $t = 3.0$ s
- Avoidance Maneuver B: Stop on urban road $t = 9.1$ s
- Avoidance Maneuver C: Speed/path/direction change on rural road $t$ varies between 10.2 and 11.2 s
- Avoidance Maneuver D: Speed/path/direction change on suburban road $t$ varies between 12.1 and 12.9 s
- Avoidance Maneuver E: Speed/path/direction change on urban road $t$ varies between 14.0 and 14.5 s

### Sample Problem A-31: Decision Sight Distance

**Given:** The decision sight distance for a speed of 50 mph and a maneuver B is most nearly:

- (A) 435 ft
- (B) 690 ft
- (C) 800 ft
- (D) 910 ft

**Solution:**

**Using AASHTO equation (3-4):**

$$d = 1.47 \frac{Vt}{a} + 1.075 \frac{V^2}{a}$$  \hspace{1cm} (3-4)

For maneuver B: $t$ is 9.1 s

$$d = 1.47 \frac{(50)(9.1)}{a} + 1.075 \frac{50^2}{11.2} = 908.81 \text{ ft}$$

**Using Table:** 910 ft (from the above table)

Answer: (D) 910 ft
A.3.4 Intersection Sight Distance:

Intersection sight distance is the decision needed to safely proceed through an intersection. The distance needed depends on the type of traffic control at the intersection, and the maneuver (left turn, right turn, or proceeding straight).

Table A-28 Case A: Moving Vehicle to Safely Cross or Stop Railroad Crossing

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_H = AV_v t + \frac{BV_v^2}{a} + D + d_e)</td>
<td>(d_H = \frac{AV_v t}{V_T} + \frac{BV_v^2}{a} + D + d_e)</td>
</tr>
<tr>
<td>(d_T = \frac{V_T}{V_v} [(A) V_v t + \frac{BV_v^2}{a} + 2D + L + W])</td>
<td>(d_T = \frac{V_T}{V_v} [(A) V_v t + \frac{BV_v^2}{a} + 2D + L + W])</td>
</tr>
</tbody>
</table>

where:
- \(A = \) constant = 0.278
- \(B = \) constant = 0.039
- \(d_H = \) sight-distance leg along the highway allows a vehicle proceeding to speed \(V_v\) to cross tracks even though a train is observed at a distance \(d_T\) from the crossing or to stop the vehicle without encroachment of the crossing area (m)
- \(d_T = \) sight-distance leg along the railroad tracks to permit the maneuvers described as for \(d_H\) (m)
- \(V_v = \) speed of the vehicle (km/h)
- \(V_T = \) speed of the train (km/h)
- \(t = \) perception/reaction time, which is assumed to be 2.5 s (This is the same value used in Chapter 3 to determine the stopping sight distance.)
- \(a = \) driver deceleration, which is assumed to be 3.4 m/s\(^2\) (This is the same value used in Chapter 3 to determine the stopping sight distance.)
- \(D = \) distance from the stop line or front of the vehicle to the nearest rail, which is assumed to be 4.5 m
- \(d_e = \) distance from the driver to the front of the vehicle, which is assumed to be 2.4 m
- \(L = \) length of vehicle, which is assumed to be 20 m
- \(W = \) distance between outer rails (for a single track, this value is 1.5 m)

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A = ) constant = 1.47</td>
<td>(d_H = \frac{AV_v t}{V_T} + \frac{BV_v^2}{a} + D + d_e)</td>
</tr>
<tr>
<td>(B = ) constant = 1.075</td>
<td>(d_T = \frac{V_T}{V_v} [(A) V_v t + \frac{BV_v^2}{a} + 2D + L + W])</td>
</tr>
</tbody>
</table>
| \(d_H = \) sight-distance leg along the highway allows a vehicle proceeding to speed \(V_v\) to cross tracks even though a train is observed at a distance \(d_T\) from the crossing or to stop the vehicle without encroachment of the crossing area (ft)
| \(d_T = \) sight-distance leg along the railroad tracks to permit the maneuvers described as for \(d_H\) (ft)
| \(V_v = \) speed of the vehicle (mph)
| \(V_T = \) speed of the train (mph)
| \(t = \) perception/reaction time, which is assumed to be 2.5 s (This is the same value used in Chapter 3 to determine the stopping sight distance.)
| \(a = \) driver deceleration, which is assumed to be 11.2 ft/s\(^2\) (This is the same value used in Chapter 3 to determine the stopping sight distance.)
| \(D = \) distance from the stop line or front of the vehicle to the nearest rail, which is assumed to be 15 ft
| \(d_e = \) distance from the driver to the front of the vehicle, which is assumed to be 8 ft
| \(L = \) length of vehicle, which is assumed to be 65 ft
| \(W = \) distance between outer rails (for a single track, this value is 5 ft)
Figure A-24  Case A: Moving Vehicle to Safety or Stop at Railroad Crossing
Table A-29 Case B: Departure of Vehicle from Stopped Position to Cross Single Railroad Track  

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_T = AV_T \left( \frac{V_G}{a_1} + \frac{L + 2D + W - d_a + J}{V_G} \right))</td>
<td>(d_T = AV_T \left( \frac{V_G}{a_1} + \frac{L + 2D + W - d_a + J}{V_G} \right))</td>
</tr>
</tbody>
</table>

\(A = \text{constant} = 0.278\) \(A = \text{constant} = 1.47\)

\(d_T = \text{sight distance leg along railroad tracks to permit the maneuvers described as for } d_H \text{ (formula) (m)}\) \(d_T = \text{sight distance leg along railroad tracks to permit the maneuvers described as for } d_H \text{ (formula) (ft)}\)

\(V_T = \text{speed of train (km/h)}\) \(V_T = \text{speed of train (mph)}\)

\(V_G = \text{maximum speed of vehicle in first gear, which is assumed to be 2.7 m/s}\) \(V_G = \text{maximum speed of vehicle in first gear, which is assumed to be 8.8 ft/s}\)

\(a_1 = \text{acceleration of vehicle in first gear, which is assumed to be 0.45 m/s}^2\) \(a_1 = \text{acceleration of vehicle in first gear, which is assumed to be 1.47 ft/s}^2\)

\(L = \text{length of vehicle, which is assumed to be 20 m}\) \(L = \text{length of vehicle, which is assumed to be 65 ft}\)

\(D = \text{distance from stop to nearest rail, which is assumed to be 4.5 m}\) \(D = \text{distance from stop to nearest rail, which is assumed to be 15 ft}\)

\(J = \text{sum of perception and time to activate clutch or automatic shift, which is assumed to be 2.0 s}\) \(J = \text{sum of perception and time to activate clutch or automatic shift, which is assumed to be 2.0 s}\)

\(W = \text{distance between outer rails for a single track, this value is 1.5m}\) \(W = \text{distance between outer rails for a single track, this value is 5 ft}\)

\(d_a = \frac{V_G^2}{2a_1}\) or distance vehicle travels while accelerating to maximum speed in first gear

\(\frac{V_G^2}{2a_1} = \frac{(2.7)^2}{(2)(0.45)} = 8.1m\) or distance vehicle travels while accelerating to maximum speed in first gear

\(\frac{V_G^2}{2a_1} = \frac{(8.8)^2}{(2)(1.47)} = 26.3 \text{ ft}\)
Figure A-25 Case B: Departure of Vehicle from Stopped Position to Cross Single Railroad Track
Sample Problem A-32: Triangle Sight Distance

Given: A vehicle is approaching a railroad track at a speed of 45 mph. Assume AASHTO-recommended design values for perception–reaction time. The stop line is located 15 ft from the near side rail, and the driver is located 5 ft back from the front bumper of the vehicle. The required sight triangle distance (ft) along the highway for a vehicle to stop at the stop line for an approaching train is most nearly:

(A) 360  
(B) 380  
(C) 440  
(D) 560

Solution:

Using AASHTO Equation 9-3 (page 734):

\[ d_H = AV_t + \frac{BV^2}{a} + D + d_e = (1.47)(45)(2.5) + \frac{1.075(45)^2}{11.2} + 15 + 5 = 380 \text{ ft} \]  

Alternate Solution (short cut):

Using AASHTO Exhibit 3-1(page 112):

Stopping sight distance (SSD) for 45 mph = 360 ft (design SSD)  
The total distance = SSD + D + d_e = 360 + 15 + 5 = 380 ft

Answer: (B)
A.4 SUPERELEVATION

A.4-1 Introduction:

According to the laws of mechanics, when a vehicle travels on a curve it is forced outward by centrifugal force.

The purpose of superelevation or banking of curves is to counteract the centripetal acceleration produced as a vehicle rounds a curve.

On a superelevated highway, the centrifugal force is resisted by the vehicle weight component parallel to the superelevated surface and side friction between the tires and pavement. It is impractical to balance centrifugal force by superelevation alone, because for any given curve radius a certain superelevation rate is exactly correct for only one driving speed. At all other speeds there will be a side thrust either outward or inward, relative to the curve center, which must be offset by side friction.

If the vehicle is not skidding, these forces are in equilibrium as represented by the following equation, which is used to design a curve for a comfortable operation at a particular speed:

\[
\text{Centrifugal Force} = e + f = \frac{0.067V^2}{R} = \frac{V^2}{15R} \quad (A-41)
\]

where:
- \(e\) = Superelevation slope in feet per foot
- \(e_{\text{max}}\) = Maximum superelevation rate for a given condition
- \(f\) = Side friction factor
- \(R\) = Curve radius in feet
- \(V\) = Velocity in miles per hour

The parameters that affect the superelevation formula are shown in the following table which are derived from the law of mechanics:

<table>
<thead>
<tr>
<th>Table A-30 Formula for Vehicle Operation on a Curve</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.01e + f = \frac{v^2}{gR} = \frac{0.079V^2}{R} = \frac{V^2}{127R}]</td>
<td>[0.01e + f = \frac{v^2}{gR} = \frac{0.067V^2}{R} = \frac{V^2}{15R}]</td>
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Where:
- \(e\) = rate of roadway superelevation, percent;
- \(f\) = side friction (demand) factor;
- \(v\) = vehicle speed, m/s;
- \(g\) = gravitational constant, 9.81 m/s²
- \(V\) = design speed, mph;
- \(R\) = radius of curve measured to a vehicle's center of gravity, m

Where:
- \(e\) = rate of roadway superelevation, percent;
- \(f\) = side friction (demand) factor;
- \(v\) = vehicle speed, ft/s;
- \(g\) = gravitational constant, 32.2 ft/s²
- \(V\) = design speed, mph;
- \(R\) = radius of curve measured to a vehicle's center of gravity, ft
Side friction factors tabulated on by AASHTO for design purposes. AASHTO, A Policy on Geometric Design of Highways and Streets, states, "In general, studies show that the maximum side friction factors developed between new tires and wet concrete pavements range from about 0.5 at 20 mph to approximately 0.35 at 60 mph." The design side friction factors are, therefore, about one-third the values that occur when side skidding is imminent.

\[ \alpha = \text{Ball Bank Indicator angle} \]
\[ \rho = \text{Body roll angle} \]
\[ \phi = \text{Superelevation angle} \]
\[ \theta = \text{Centripetal acceleration angle} \]

**Figure A-26** Geometry for Ball-Bank Indicator (Top: Manual & Electronic Ball Bank) (Exhibit 3-9, page 134, AASHTO Geometric Design-Green Book, 2004 edition, 5th edition)

**Table A -31** Minimum Radius Using \( e_{\text{max}} \) and \( f_{\text{max}} \) (Page 146- AASHTO Geometric Design-Green Book, 2004 edition, 5th edition)

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<td>[ R_{\text{min}} = \frac{V^2}{15(0.01e_{\text{max}} + f_{\text{max}})} ] (3-10)</td>
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### Table A-32 Minimum Radius Using Limiting Values of e and f


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### Table A-32 Minimum Radius Using Limiting Values of $e$ and $f$ (Continued)

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<td>10.0</td>
<td>0.11</td>
<td>0.21</td>
<td>1341.3</td>
<td>1340</td>
</tr>
<tr>
<td>130</td>
<td>10.0</td>
<td>0.08</td>
<td>0.18</td>
<td>739.3</td>
<td>739</td>
<td>70</td>
<td>10.0</td>
<td>0.10</td>
<td>0.20</td>
<td>1633.3</td>
<td>1630</td>
</tr>
<tr>
<td>80</td>
<td>10.0</td>
<td>0.08</td>
<td>0.18</td>
<td>1973.7</td>
<td>1970</td>
<td>75</td>
<td>10.0</td>
<td>0.09</td>
<td>0.19</td>
<td>1973.7</td>
<td>1970</td>
</tr>
<tr>
<td>140</td>
<td>10.0</td>
<td>0.08</td>
<td>0.18</td>
<td>2370.4</td>
<td>2370</td>
<td>80</td>
<td>10.0</td>
<td>0.08</td>
<td>0.18</td>
<td>2370.4</td>
<td>2370</td>
</tr>
</tbody>
</table>

Part I- Civil Breadth (Transportation A.M.) - Chapter A: Geometric Design 87
Sample Problem A-33: Superelevation Rate

Given: A horizontal curve on a two-lane rural highway has the following characteristics:
- Design speed, $V = 60$ mph
- Radius (minimum safe) = 1,091 ft
- Coefficient of side friction = 0.12

The rate of superelevation required for this curve is almost nearly:

(A) 6%
(B) 8%
(C) 10%
(D) 33%

Solution:

- **Using AASHTO Equation 3-10 (Note: $e_{\text{max}}$ in the following equation is a percentage):**

$$R_{\text{min}} = \frac{V^2}{15(0.01e_{\text{max}} + f_{\text{max}})}$$

\[
1091 = \frac{60^2}{15(0.01e_{\text{max}} + 0.12)}
\]

Solve the above equation for $e_{\text{max}}$ gives 10%

- **Using Minimum Radius Using Limiting Values of $e$ and $f$ Table**


Look for a radius of 1090 ft (1090.9 ft) and $V$ of 60 mph, the corresponding superelevation rate is 10% for a maximum friction of 0.12

Answer: (C)
A.4-2 Superelevation Runoff and Superelevation Diagram:

As mentioned before, the purpose of superelevation or banking of curves is to counteract the centripetal acceleration produced as a vehicle rounds a curve. The term itself comes from railroad practice, where the top of the rail is the profile grade. In curves, the profile grade line follows the lower rail, and the upper rail is said to be "superelevated." Since most railways are built to a standard gage, the superelevations are given as the difference in elevation between the upper and lower rail. In the case of highways, somewhat more complicated modifications of the cross section are required, and, because widths vary, superelevation is expressed as a slope.

Consider the force diagram in Figure A-27. If the vehicle is traveling around a curve with a radius $R$ at a constant speed $v$, there will be a radial acceleration toward the center of the curve (toward the left in the diagram) of $v^2/R$, which will be opposed by a force of:

\[
\text{Force} = \text{mass} \times \text{acceleration} = m \times a = W/g \times v^2/R
\]  

(A-42)

Other forces acting on the vehicle are its weight $W$ and the forces exerted against the wheels by the roadway surface. These forces are represented by two components: the normal forces $N_1$ and $N_2$ and the lateral forces $F_1$ and $F_2$. For highway vehicles $F_1$ and $F_2$ are friction forces, so that:

\[
F_1 \leq \mu N_1 \\
F_2 \leq \mu N_2
\]

where $\mu$ or $f$ is the coefficient of friction between tires and roadway.

![Figure A-27 Force Diagram for Superelevation](image-url)
In order to construct the superelevation diagram, the following parameters must be determined:

1- Axis of Rotation
2- Superelevation Transition

1- Axis of Rotation

(a) Undivided Highways:

For undivided highways the axis of rotation for superelevation is usually the centerline of the roadbed. However, in special cases such as desert roads where curves are preceded by long relatively level tangents, the plane of superelevation may be rotated about the inside edge of traveled way to improve perception of the curve. In flat country, drainage pockets caused by superelevation may be avoided by changing the axis of rotation from the centerline to the inside edge of traveled way.

(b) Ramps and Freeway-to-freeway Connections:

The axis of rotation may be about either edge of traveled way or centerline if multilane. Appearance and drainage considerations should always be taken into account in selection of the axis of rotation.

(c) Divided Highways.

(i) Freeways-Where the initial median width is 65 feet or less, the axis of rotation should be at the centerline.

Where the initial median width is greater than 65 feet and the ultimate median width is 65 feet or less, the axis of rotation should be at the centerline, except where the resulting initial median slope would be steeper than 10: 1. In the latter case, the axis of rotation should be at the ultimate median edges of traveled way.

Where the ultimate median width is greater than 65 feet, the axis of rotation should normally be at the ultimate median edges of traveled way. To avoid sawtooth on bridges with decked medians, the axis of rotation, if not already on centerline, should be shifted to the centerline.

(ii) Conventional Highways-The axis of rotation should be considered on an individual project basis and the most appropriate case for the conditions should

2- Superelevation Transition:

The superelevation transition generally consists of the crown runoff and the superelevation runoff as shown on Figure A-28.

A superelevation transition should be designed to satisfy the requirements of safety, comfort and pleasing appearance. The length of superelevation transition should be based upon the combination of superelevation rate and width of rotated plane.
Runoff. Two-thirds of the superelevation runoff should be on the tangent and one-third within the curve. This results in two-thirds of the full superelevation rate at the beginning or ending of a curve. This may be altered as required to adjust for flat spots or unsightly sags and humps, or when conforming to existing roadway.

Figure A-28 Superelevation Diagram and Roadway Cross Sections

Figure A-29 Superelevation Transition (Elements) for a Curve to the right
A.5 VERTICAL AND/OR HORIZONTAL CLEARANCES

A.5.1 Vertical Curves Under or Over An Obstruction:
Sometimes it necessarily to construct a vertical curve under a structure or over a utility pipe. The equation shown below will give the curve length in stations to meet certain cover or vertical clearance.

\[ L = 2 \left( b + \frac{2y}{\Delta G} \right) + 4 \sqrt{\frac{b \times y}{\Delta G} + \left( \frac{y}{\Delta G} \right)^2} \]  

where:
- \( L \) = length of vertical curve (stations)
- \( b \) = distance from vertex to point (stations)
- \( y \) = tangent offset to curve (ft)
- \( \Delta G \) = the absolute value of \( g_2 - g_1 \)

A.5.2 Sight Distance at Undercrossing:
Sight distance on the highway through a grade separation should be at least as long as the minimum stopping sight distance and preferably longer.
The following two tables show two cases for calculating the sag vertical curve length:

**Case 1:** Sight distance > curve length, i.e. \( S > L \)

**Case 2:** Sight distance < curve length, i.e. \( S < L \)

### Table A-33 Case 1 - Sight Distance Greater than the Sag Vertical Curve Length (\( S > L \))


<table>
<thead>
<tr>
<th>Vertical Clearance for Sag Vertical Curve S&gt;L</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ L = 2S - \frac{800 \left( C - \left( \frac{h_1 + h_2}{2} \right) \right)}{A} ]</td>
<td>[ L = 2S - \frac{800 \left( C - \left( \frac{h_1 + h_2}{2} \right) \right)}{A} ] (3-52)</td>
<td></td>
</tr>
</tbody>
</table>

Where:
- \( L \) = length of vertical curve, m;
- \( S \) = sight distance, m;
- \( A \) = algebraic difference in grades, percent
- \( C \) = vertical clearance, m;
- \( h_1 \) = height of eye, m;
- \( h_2 \) = height of object, m

### Table A-34 Case 2 - Sight Distance Less than the Sag Vertical Curve Length (\( S < L \))


<table>
<thead>
<tr>
<th>Vertical Clearance for Sag Vertical Curve S&lt;L</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ L = \frac{AS^2}{800 \left( C - \left( \frac{h_1 + h_2}{2} \right) \right)} ]</td>
<td>[ L = \frac{AS^2}{800 \left( C - \left( \frac{h_1 + h_2}{2} \right) \right)} ] (3-53)</td>
<td></td>
</tr>
</tbody>
</table>

where:
- \( L \) = length of vertical curve, m;
- \( S \) = sight distance, m;
- \( A \) = algebraic difference in grades, percent
- \( C \) = vertical clearance, m;
- \( h_1 \) = height of eye, m;
- \( h_2 \) = height of object, m
Using an eye height of 8.0 ft [2.4 m] for a **truck driver** and an object 2.0 ft [0.6 m] for the taillights of a vehicle, the following equations for the sag vertical curve length can be derived:

### Table A-35 Case 1- Sight Distance Greater than the Sag Vertical Curve Length ($S > L$)


<table>
<thead>
<tr>
<th>Vertical Clearance for Sag Vertical Curve, $S &gt; L$</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 2S - \frac{800(C - 1.5)}{A}$</td>
<td>$L = 2S - \frac{800(C - 5)}{A}$ (3-54)</td>
<td></td>
</tr>
<tr>
<td>Where:</td>
<td>Where:</td>
<td></td>
</tr>
<tr>
<td>$L$ = length of vertical curve, m;</td>
<td>$L$ = length of vertical curve, ft;</td>
<td></td>
</tr>
<tr>
<td>$S$ = sight distance, m;</td>
<td>$S$ = sight distance, ft;</td>
<td></td>
</tr>
<tr>
<td>$A$ = algebraic difference in grades, percent</td>
<td>$A$ = algebraic difference in grades, percent</td>
<td></td>
</tr>
<tr>
<td>$C$ = vertical clearance, m</td>
<td>$C$ = vertical clearance, ft</td>
<td></td>
</tr>
</tbody>
</table>

### Table A-36 Case 2- Sight Distance Less than the Sag Vertical Curve Length ($S < L$)


<table>
<thead>
<tr>
<th>Vertical Clearance for Sag Vertical Curve, $S &lt; L$</th>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = \frac{AS^2}{800(C - 1.5)}$</td>
<td>$L = \frac{AS^2}{800(C - 5)}$ (3-55)</td>
<td></td>
</tr>
<tr>
<td>Where:</td>
<td>Where:</td>
<td></td>
</tr>
<tr>
<td>$L$ = length of vertical curve, m;</td>
<td>$L$ = length of vertical curve, ft;</td>
<td></td>
</tr>
<tr>
<td>$S$ = sight distance, m;</td>
<td>$S$ = sight distance, ft;</td>
<td></td>
</tr>
<tr>
<td>$A$ = algebraic difference in grades, percent</td>
<td>$A$ = algebraic difference in grades, percent</td>
<td></td>
</tr>
<tr>
<td>$C$ = vertical clearance, m;</td>
<td>$C$ = vertical clearance, ft;</td>
<td></td>
</tr>
</tbody>
</table>
**Sample Problem A-35: vertical curve length UNDER a pipe**

**Given:** Two grade lines intersect at station 10 + 50 at an elevation of 80.40 ft. The entering grade is -3.00% and the exiting grade is +3.500%. A pipeline crosses the grade line at right angles at station 8 + 90.00. The outside diameter of the pipe is 3.0 ft and the elevation measured at the top of the pipe is 114.50 ft.

**Find:** The sag vertical curve that passes 16.50 ft below the bottom of the pipe. Express the answer in full station.

(A) 17.00  
(B) 17.89  
(C) 18.00  
(D) 18.89

**Solution:**

\[
L = 2 \left( A + \frac{2(y)}{\Delta G} \right) + 4 \sqrt{\frac{b(y)}{\Delta G} + \left( \frac{y}{\Delta G} \right)^2} \quad (A-32)
\]

- \( b = \) distance from vertex to point (stations) = (10 + 50) \( \bar{\imath} \) (8 + 90) = 1.6 Sta.
- \( \Delta G = \) the absolute value of \( g_2 \bar{\imath} g_1 = 3.5 + 3.0 = 6.5 \% \)
- \( y = \) tangent offset to curve (ft) = Difference in elevation between the proposed curve and the tangent at the pipe location.  
  - curve elevation \( \bar{\imath} \) tangent elevation  
  - = \( 114.5 \bar{\imath} 3 \bar{\imath} 16.5 \bar{\imath} (80.4 + 3.00 \times 1.6) = 9.80 \) ft

\[
L = 2 \left( 1.6 + \frac{2(9.8)}{6.5} \right) + 4 \sqrt{\frac{1.6(9.8)}{6.5} + \left( \frac{9.8}{6.5} \right)^2} = 17.89 \text{ Sta.}
\]

Here the minimum vertical clearance is 16.5 ft and if a 1800 ft (18 Sta. rounded to full station) curve is selected, the minimum clearance required will be reduced. Therefore, the curve length should be 1700 ft (17 Sta.) to ensure that the pipe will have at least a clearance of 16.5 ft.

**Answer is A**
Sample Problem A-36: vertical curve length OVER a pipe

**Given:** Two grade lines intersect at station 10 + 50 at an elevation of 80.40 ft. The entering grade is $\bar{I} = 3.00\%$ and the exiting grade is $+3.50\%$. A pipeline crosses the grade line at right angles at station 8 + 90.00. The outside diameter of the pipe is 3.0 ft and the elevation measured at the top of the pipe is 83.20 ft.

**Find:** Determine the vertical curve length rounded to the nearest full station that will provide the 60.0 inches clearance (cover) over the pipe.

(A) 8.00
(B) 8.94
(C) 9.00
(D) 9.84

**Solution:**

\[ L = 2\left(A + \frac{2(y)}{\Delta G}\right) + 4\sqrt{\frac{b(y)}{\Delta G} + \left(rac{y}{\Delta G}\right)^2} \quad (A-32) \]

- \( b \) = distance from vertex to point (stations) = (10 + 50) $\bar{I}$ (8 + 90) = 1.6 Sta.
- \( \Delta G \) = the absolute value of $g_2\bar{I} - g_1 = 3.5 + 3.0 = 6.5\%$
- \( y \) = tangent offset to curve (ft) = Difference in elevation between the proposed curve and the tangent at the pipe location.
- Curve elevation $\bar{I}$ = Tangent elevation
  = (83.2 + 60 inches $\times$ ft/12 inches) $\bar{I}$ (80.4 + 3.00 $\times$1.6) = 3.00 ft

\[ L = 2\left(1.6 + \frac{2(3)}{6.5}\right) + 4\sqrt{\frac{1.6(3)}{6.5} + \left(\frac{3}{6.5}\right)^2} = 8.94 \text{ Sta.} \]

Here the minimum cover is 60 inches (5 ft) and if a 800 ft (8 Sta.) curve is selected, the minimum cover required will be reduced.

Therefore, the curve length should be 900 ft (9 Sta.) to ensure that the pipe will have at least a cover of 60 inches (5 ft).

**Answer:** (C) ☑

*NOTE: This is opposite to the previous sample problem where a smaller curve was selected compared to the calculated value to ensure minimum vertical clearance*
A.5.2 Horizontal Sight Distance:

Figure A-32 Diagram Illustrating Components for Determining Horizontal Sight Distance (Exhibit 3-54, page 227, AASHTO Geometric Design-Green Book, 2004 edition, 5th edition)


<table>
<thead>
<tr>
<th>Metric</th>
<th>US Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ HSO = R \left[ 1 - \cos \frac{28.65S}{R} \right] ]</td>
<td>[ HSO = R \left[ 1 - \cos \frac{28.65S}{R} \right] ] (3-38)</td>
</tr>
<tr>
<td>where:</td>
<td>where:</td>
</tr>
<tr>
<td>( S ) = Stopping sight distance, m</td>
<td>( S ) = Stopping sight distance, ft</td>
</tr>
<tr>
<td>( R ) = Radius of curve, m</td>
<td>( R ) = Radius of curve, ft</td>
</tr>
<tr>
<td>( HSO ) = Horizontal sightline offset, m</td>
<td>( HSO ) = Horizontal sightline offset, ft</td>
</tr>
</tbody>
</table>
Sample Problem A-37: Horizontal Sight Distance

Given: A two lane conventional highway is designed for 50 mph with a 850 ft centerline radius and 12 ft lanes. The property owner adjacent to the highway wants to build a house on his land.

Find: The house should be placed at from the edge of pavement at distance (in feet) of:

(A) 21  
(B) 26  
(C) 27  
(D) 32

Solution:

The house should be placed away from the sight distance as required per AASHTO Exh. 3-54.

\[
HSO = R \left[ \left(1 - \cos \frac{28.65S}{R} \right) \right]
\]

where:

\( S \) = Stopping sight distance, ft  
\( R \) = Radius of curve, ft  
\( HSO \) = Horizontal sightline offset, ft

Note: \( R \) is measured to the centerline of the inside lane.

\( R = 850 \, \text{ft} - 12 \, \text{ft}/2 = 844 \, \text{ft} \)

\( S = 425 \, \text{ft} \) (AASHTO Exhibit 3-1, page 112)

\[
HSO = R \left[ \left(1 - \cos \frac{28.65 \times 425}{844} \right) \right] = 844 \left[ \left(1 - \cos \frac{28.65 \times 425}{844} \right) \right] = 26.61 \, \text{ft}
\]

The distance from the edge of pavement = \( HSO - \frac{1}{2} \) (lane width) = \( 26.61 \, \text{ft} - 6 \, \text{ft} = 20.61 \, \text{ft} \)

Answer: (A) 21
A.6 ACCELERATION AND DECELERATION

A.6.1 Definitions and Terms Related to Acceleration and Deceleration:

Figure A-33  Typical Gore Area Characteristics
1. \( L_a \) IS THE REQUIRED ACCELERATION LENGTH AS SHOWN IN EXHIBIT 10-70 OR AS ADJUSTED BY EXHIBIT 10-71.

2. POINT (A) CONTROLS SPEED ON THE RAMP. \( L_a \) SHOULD NOT START BACK ON THE CURVATURE OF THE RAMP UNLESS THE RADIUS EQUALS 300 m [1000 ft] OR MORE.

3. \( L_g \) IS REQUIRED GAP ACCEPTANCE LENGTH. \( L_g \) SHOULD BE A MINIMUM OF 90 TO 150 m [300 to 500 ft] DEPENDING ON THE NOSE WIDTH.

4. THE VALUE OF \( L_a \) OR \( L_g \), WHICHEVER PRODUCES THE GREATER DISTANCE DOWNSTREAM FROM WHERE THE NOSE EQUAL 0.6 m [2 ft], IS SUGGESTED FOR USE IN THE DESIGN OF THE RAMP ENTRANCE.

Figure A-34 Typical Single Lane Entrance Ramps
1. $L_a$ IS THE REQUIRED ACCELERATION LENGTH AS SHOWN IN EXHIBIT 10-70 OR AS ADJUSTED BY EXHIBIT 10-71.
2. POINT (A) CONTROLS SPEED ON THE RAMP. $L_a$ SHOULD NOT START BACK ON THE CURVATURE OF THE RAMP UNLESS THE RADIUS EQUALS 300 m [1000 ft] OR MORE.
3. $L_g$ IS REQUIRED GAP ACCEPTANCE LENGTH. $L_g$ SHOULD BE A MINIMUM OF 90 TO 150 m [300 to 500 ft] DEPENDING ON THE NOSE WIDTH.
4. THE VALUE OF $L_a$ OR $L_g$, WHICHEVER PRODUCES THE GREATER DISTANCE DOWNSTREAM FROM WHERE THE NOSE EQUAL 0.6 m [2 ft], IS SUGGESTED FOR USE IN THE DESIGN OF THE RAMP ENTRANCE.

Figure A-35 Typical Two-Lane Entrance Ramps
### Metric

<table>
<thead>
<tr>
<th>Highway</th>
<th>Speed reached, $V_a$ (km/h)</th>
<th>Stop condition</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design speed, $V$ (km/h)</td>
<td>and initial speed, $V'_a$ (km/h)</td>
<td>0</td>
<td>20</td>
<td>28</td>
<td>35</td>
<td>42</td>
<td>51</td>
<td>63</td>
<td>70</td>
</tr>
<tr>
<td>50</td>
<td>37</td>
<td>60</td>
<td>50</td>
<td>30</td>
<td>ũ</td>
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<td>ũ</td>
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</tr>
<tr>
<td>60</td>
<td>45</td>
<td>95</td>
<td>80</td>
<td>65</td>
<td>45</td>
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<tr>
<td>70</td>
<td>53</td>
<td>150</td>
<td>130</td>
<td>110</td>
<td>90</td>
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<td>ũ</td>
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<tr>
<td>80</td>
<td>60</td>
<td>200</td>
<td>180</td>
<td>165</td>
<td>145</td>
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<td>90</td>
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<td>260</td>
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<td>205</td>
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<td>125</td>
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<td>100</td>
<td>74</td>
<td>345</td>
<td>325</td>
<td>305</td>
<td>285</td>
<td>255</td>
<td>205</td>
<td>110</td>
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<td>110</td>
<td>81</td>
<td>430</td>
<td>410</td>
<td>390</td>
<td>370</td>
<td>340</td>
<td>290</td>
<td>200</td>
<td>125</td>
</tr>
<tr>
<td>120</td>
<td>88</td>
<td>545</td>
<td>530</td>
<td>515</td>
<td>490</td>
<td>460</td>
<td>410</td>
<td>325</td>
<td>245</td>
</tr>
</tbody>
</table>

**Note:** Uniform 50:1 to 70:1 tapers are recommended where lengths of acceleration lanes exceed 400 m.

### US Customary

<table>
<thead>
<tr>
<th>Highway</th>
<th>Speed reached, $V_a$ (mph)</th>
<th>Stop condition</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design speed, $V$ (mph)</td>
<td>and initial speed, $V'_a$ (mph)</td>
<td>0</td>
<td>14</td>
<td>18</td>
<td>22</td>
<td>26</td>
<td>30</td>
<td>36</td>
<td>40</td>
<td>44</td>
</tr>
<tr>
<td>30</td>
<td>23</td>
<td>180</td>
<td>140</td>
<td>ũ</td>
<td>ũ</td>
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<td>ũ</td>
<td>ũ</td>
<td>ũ</td>
<td>ũ</td>
</tr>
<tr>
<td>35</td>
<td>27</td>
<td>280</td>
<td>220</td>
<td>160</td>
<td>ũ</td>
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<tr>
<td>40</td>
<td>31</td>
<td>360</td>
<td>300</td>
<td>270</td>
<td>210</td>
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<tr>
<td>45</td>
<td>35</td>
<td>560</td>
<td>490</td>
<td>440</td>
<td>380</td>
<td>280</td>
<td>160</td>
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</tr>
<tr>
<td>50</td>
<td>39</td>
<td>720</td>
<td>660</td>
<td>610</td>
<td>550</td>
<td>450</td>
<td>350</td>
<td>130</td>
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<td>1160</td>
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</table>

**Note:** Uniform 50:1 to 70:1 tapers are recommended where lengths of acceleration lanes exceed 1,300 ft

$V = $ design speed of highway (km/h)  
$V_a = $ average running speed on highway (km/h)  
$V'_a = $ average running speed on entrance curve (km/h)

*Figure A-36* Minimum Acceleration Lengths for Entrance Terminals with Flat Grades of Two Percent or Less ($G \leq 2\%$)  
### Metric

<table>
<thead>
<tr>
<th>Highway design speed, $V$ (km/h)</th>
<th>Speed reached, $V_a$ (km/h)</th>
<th>Stop condition</th>
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</table>

$V = $ design speed of highway (km/h)  
$V_a = $ average running speed on highway (km/h)  
$V_N = $ design speed of exit curve (km/h)  
$V_a' = $ average running speed on exit curve (km/h)

### US Customary

<table>
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<tr>
<th>Highway design speed, $V$ (mph)</th>
<th>Speed reached, $V_a$ (mph)</th>
<th>Stop condition</th>
<th>15</th>
<th>20</th>
<th>25</th>
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<td>575</td>
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<td>440</td>
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</tbody>
</table>

$V = $ design speed of highway (mph)  
$V_a = $ average running speed on highway (mph)  
$V_N = $ design speed of exit curve (mph)  
$V_a' = $ average running speed on exit curve (mph)

### Figure A-37

Minimum Acceleration Lengths for Entrance Terminals with Flat Grades of Two Percent or Less ($G \leq 2\%$)  
<table>
<thead>
<tr>
<th>All Speeds</th>
<th>3 to 4% upgrade</th>
<th>3 to 4% downgrade</th>
<th>5 to 6% upgrade</th>
<th>5 to 6% downgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design speed of highway (km/h)</td>
<td>0.9</td>
<td>1.2</td>
<td>0.8</td>
<td>1.35</td>
</tr>
<tr>
<td>All Speeds</td>
<td>0.9</td>
<td>1.2</td>
<td>0.8</td>
<td>1.35</td>
</tr>
<tr>
<td>Design speed of highway (mph)</td>
<td>0.9</td>
<td>1.2</td>
<td>0.8</td>
<td>1.35</td>
</tr>
</tbody>
</table>

**Figure A-38** Speed Change Lane Adjustment Factors as a Function of Grade (G > 2%)
Sample Problem A-38: Acceleration and Deceleration Length

**Given:** A single lane entrance ramp joins a tangent section of freeway mainline as a parallel-type entrance. The entrance ramp design speed is 30 mph, and the highway speed is 70 mph. The grade of the ramp is +1.0% and the grade of the freeway at the merging point is 2%.

**Find:** The minimum acceleration length L (ft) needed for the entrance is most nearly:

- (A) 1,350
- (B) 1,390
- (C) 1,420
- (D) 1,580

**Solution:**

ASSHTO (*Policy on Geometric Design of Highways and Streets, 2004*) Exhibit 10-70

**Enter the table with:**

- \( V \) (highway design speed) = 70 mph
- Entrance ramp design speed = 30 mph

**Answer:**

Acceleration Length L (ft) = 1350

Answer: (A)  

---

Sample Problem A-39: Acceleration and Deceleration Length

**Given:** A single lane entrance ramp joins a tangent section of freeway mainline as a parallel-type entrance. The entrance ramp design speed is 30 mph, and the highway speed is 70 mph. The grade of the ramp is -4.0%.

**Find:** The minimum acceleration length L (ft) needed for the entrance is most nearly:

- A) 1,350
- B) 1,120
- C) 910
- D) 810

**Solution:**

ASSHTO (*Policy on Geometric Design of Highways and Streets, 2004*) Exhibit 10-70

**Enter the table with:**

- \( V \) (highway design speed) = 70 mph
- Entrance ramp design speed = 30 mph

**Answer:**

Acceleration Length L (ft) = 1350

Adjustment factor per AASHTO Exhibit 10-71 = 0.60

Adjusted acceleration length = 1350 ft \( \times \) 0.60 = 810 ft

Answer: (D)  

---
### Sample Problem A-40: Acceleration and Deceleration Length

<table>
<thead>
<tr>
<th>Given: A single lane entrance ramp joins a tangent section of freeway mainline as a parallel-type entrance. The entrance ramp design speed is 30 mph, and the freeway mainline speed is 70 mph. The grade of the ramp is +4.0%.</th>
<th>Find: The minimum acceleration length ( L ) (ft) needed for the entrance is most nearly:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A) 1,350</td>
</tr>
<tr>
<td></td>
<td>B) 1,580</td>
</tr>
<tr>
<td></td>
<td>C) 2,160</td>
</tr>
<tr>
<td></td>
<td>D) 2,970</td>
</tr>
</tbody>
</table>

**Solution:**

ASSHTO (*Policy on Geometric Design of Highways and Streets, 2004*) Exhibit 10-70

**Enter the table with:**

- \( V \) (highway design speed) = 70 mph
- Entrance ramp design speed = 30 mph

**Answer:**

Acceleration Length \( L \) (ft) = 1350 ft

Adjustment factor per AASHTO Exhibit 10-71 = 1.60

Adjusted acceleration length = 1350 ft \( \times \) 1.60 = 2160 ft

Answer: (C) 2160 ft