COORDINATED BEAMFORMING FOR THE MULTI-USER MIMO DOWNLINK USING FBMC/OQAM

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ABSTRACT
Due to the fact that the out-of-band radiation of filter bank based multi-carrier modulation (FB-MC) is significantly lower compared to orthogonal frequency division multiplexing with the cyclic prefix insertion (CP-OFDM), it is beneficial to choose FB-MC over CP-OFDM for asynchronous scenarios or to achieve an effective utilization of spectrum holes. It is known, however, the state-of-the-art solutions for filter bank based multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) based space division multiple access (SDMA) cannot be employed when the total number of receive antennas of the users exceeds the number of transmit antennas at the base station. This fact motivates the design of a transmission scheme that is able to alleviate this dimensionality constraint. Therefore, we develop an intrinsic interference mitigating coordinated beamforming (IIM-CBF) algorithm where the precoding matrix and the decoding matrix are computed jointly and iteratively. The simulations results show that FBMC/OQAM based multi-user MIMO downlink systems where the proposed IIM-CBF technique is employed achieve similar bit error rate (BER) performances as their CP-OFDM based counterparts while having a higher spectral efficiency.

Index Terms— FBMC/OQAM, multi-user MIMO downlink, coordinated beamforming

1. INTRODUCTION
Filter bank based multi-carrier modulation (FB-MC) is regarded as a promising alternative to orthogonal frequency division multiplexing with the cyclic prefix insertion (CP-OFDM). By using spectrally well-contained synthesis and analysis filter banks at the transmitter and at the receiver [1], [2], FB-MC has an agile spectrum. Therefore, it avoids a high level of out-of-band radiation which CP-OFDM suffers from. Moreover, in systems where filter bank based multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) is employed, the fact that the insertion of the CP is not required as in CP-OFDM based systems leads to a higher spectral efficiency. Due to these advantages, FBMC/OQAM based systems have received great research attention on their use in different contexts, such as cognitive radio [3] and professional mobile radio (PMR) networks [4], where an effective utilization of the available fragmented spectrum is required.

In FBMC/OQAM systems, the real and imaginary parts of each complex-valued data symbol are staggered by half of the symbol period [1], [5] such that the desired signal and the intrinsic interference are separated in the real domain and in the pure imaginary domain, respectively. For point-to-point MIMO FBMC/OQAM systems, a zero-forcing (ZF) based approach [6] has been developed to mitigate the intrinsic interference without assuming that the channel is flat fading. Based on the similar concept, the authors in [7] have adapted the conventional spatial Tomlinson Harashima precoder (STHP) to an FBMC/OQAM based multiple-input-single-output broadcast channel (MISO-BC) which results in a new non-linear precoder. It is known that non-linear precoders have a higher computational complexity compared to linear precoders. Moreover, the non-linear precoding technique in [7] is restricted to the case where each user is equipped with only a single receive antenna. On the other hand, a block diagonalization (BD) based linear precoder has been developed in [8] for the FBMC/OQAM based multi-user MIMO downlink with space division multiple access (SDMA). It adopts the central idea of BD [9] to mitigate the multi-user interference and then uses the ZF based approach [6] to deal with the intrinsic interference cancellation for the resulting equivalent single-user transmissions. This linear precoding scheme suffers from the dimensionality constraint that the total number of receive antennas of the users must not exceed the number of transmit antennas at the base station. Another linear precoder for FBMC/OQAM based multi-user downlink systems has a structure of a filter applied on each subcarrier and its two adjacent subcarriers at twice the symbol rate [10]. Also with the focus only on the scenario where the total number of transmit antennas at the base station is not smaller than the total number of receive antennas of the users, this scheme is able to suppress the multi-user interference and the intrinsic interference even when the channel is highly frequency selective. However, it only allows each user to have a single receive antenna, and consequently only one data stream can be transmitted to each user. For CP-OFDM based multi-user MIMO downlink systems, there have been some publications on coordinated beamforming techniques [11], [12] proposed to cope with the dimensionality constraint imposed on BD based precoding algorithms [9]. Inspired by these works, in this paper a coordinated beamforming based transmission scheme specifically for FBMC/OQAM based systems is developed to alleviate the same dimensionality constraint that all state-of-the-art transmission strategies for FBMC/OQAM based multi-user downlink settings suffer from. It achieves the mitigation of the multi-user interference as well as the intrinsic interference.

This paper is organized as follows: Section 2 introduces the data model of an FBMC/OQAM based multi-user MIMO downlink system and briefly reviews the state-of-the-art transmission strategies. The proposed intrinsic interference mitigating coordinated beamforming (IIM-CBF) algorithm is described in detail in Section 3.
After that, numerical results are presented in Section 4, before the conclusions are drawn in Section 5.

2. DATA MODEL

In a multi-user MIMO downlink system where SDMA is employed, one base station equipped with $M_t^{(BS)}$ transmit antennas transmits to $Q$ users at the same time and on the same frequency. The number of receive antennas of all users severed simultaneously is then $M_r^{(tot)} = \sum_{q=1}^{Q} M_{r, q}$. Assuming that the channel on each subcarrier can be treated as flat fading [6], [8], the combined receive vector on the $k$th subcarrier and at the $n$th time instant is denoted by $y_k[n] = \left[ y_1^{(k)}[n] \ y_2^{(k)}[n] \ \cdots \ y_{Q,k}^{(k)}[n] \right]^T \in \mathbb{C}^{M_r^{(tot)} \times 1}$, where the received signals of all $Q$ users are stacked and can be represented by

$$y_k[n] = H_k[n] F_k[n] d_k[n] + \sum_{m=1}^{n-3} \sum_{\ell=k-1}^{k+1} H\ell[n] F\ell[n] c_{\ell} d_{\ell}[n] + n_k[n], \quad (\ell, i) \neq (k, n).$$

(1)

where $H_k[n] = \left[ H_{1,k}[n] \ H_{2,k}[n] \ \cdots \ H_{Q,k}[n] \right]^T$, \(2\)

and the matrices $H_{i,k}[n] \in \mathbb{C}^{M_k \times M_r^{(tot)}}$ represents the channel frequency response between the base station and the $q$th user, $q = 1, 2, \ldots, Q$. The data vector $d_k[n] \in \mathbb{C}^p$ with the total number of spatial streams $d = \sum_{q=1}^{Q} d_q[k]$ as an equivalent transmitted signal, (5) resembles


(5)

where $d_k[n]$ contains the real-valued desired signal and the pure imaginary interference

$$d_k[n] = d_k[n] + \sum_{m=1}^{n-3} \sum_{\ell=k-1}^{k+1} c_{\ell} d_{\ell}[n], \quad (\ell, i) \neq (k, n).$$

(6)

Considering $d_k[n]$ as an equivalent transmitted signal, (5) resembles the data model of a CP-OFDM based multi-user MIMO downlink system. Consequently, transmission strategies that have been developed for multi-user MIMO CP-OFDM downlink systems can be straightforwardly extended to their FBMC/OQAM based counterparts where only one additional step is required, i.e., taking the real part of the resulting signal after the multiplication by the decoding matrix

$$d_k[n] = \text{Re} \left\{ D_k[n] y_k[n] \right\},$$

(7)

where $D_k[n] \in \mathbb{C}^{M_r^{(tot)} \times d}$ is the combined block-diagonal decoding matrix on the $k$th subcarrier and at the $n$th time instant that contains the decoding matrices $D_{q,k}[n] \in \mathbb{C}^{M_{r,q} \times q, d}$, $q = 1, 2, \ldots, Q$, for the $Q$ users, respectively. It is worth noting that there is no cooperation among the users, and the decoding matrix for each user is computed separately.
2.2. BD based approach

In [8] a BD based precoding algorithm has been proposed for FBMC/OQAM based multi-user MIMO downlink systems, where $M_T^{(BS)} \geq M_R^{(tot)}$. First, the BD algorithm [9] is used to compute the first part of the precoding matrix $F_{q,k}[n]$, $q = 1, 2, \ldots, Q$, for the $Q$ users to mitigate the multi-user interference. By rendering $F_{q,k}[n]$ for the $q$th user to lie in the null space of all the other users' combined channel matrix, it is ensured that

$$H_{q,k}[n] \cdot F_{q,k}[n] = 0 \in \mathbb{C}^{M_{Tq} \times M_{Tq}^{(eq)}}, \quad g \neq q. \tag{8}$$

Consequently, the received signal of the $q$th user is expressed as

$$y_{q,k}[n] = H_{q,k}[n]F_{q,k}[n]G_{q,k}d_{q,k}[n] + \sum_{i=m-3}^{n+3} \sum_{\ell=k-1}^{k+1} H_{q,e}[i]F_{q,e}[i]G_{q,e}d_{q,e}[i] + n_{q,k}[n], \quad (\ell, i) \neq (k, n), \tag{9}$$

where $H_{q,k}[n]F_{q,k}[n] \in \mathbb{C}^{M_{Tq} \times M_{Tq}^{(eq)}}, \quad q = 1, 2, \ldots, Q$, can be treated as the equivalent channels for parallel single-user transmissions. Then the second part of the precoding matrix is computed for each user such that the intrinsic interference is canceled after taking the real part of the received signal of each user [8], i.e.,

$$\text{Im} \left\{ H_{q}F_{q}G_{q} \right\} = 0 \in \mathbb{C}^{M_{Tq} \times d_q}, \tag{10}$$

where $F_{q}G_{q}$ represents the precoding matrix for the $q$th user on each subcarrier and at each time instant, and $H_{q}$ denotes the channel matrix for the $q$th user on the same subcarrier and at the same time instant. Note that from now on, the time and frequency indices are ignored as the precoding is performed on a per-subcarrier basis.

This approach outperforms the straightforward extension of the CP-OFDM case in the sense that it is able to tolerate a certain level of the frequency selectivity of the channel. However, it suffers from the dimensionality constraint that the number of transmit antennas at the base station has to be larger than or equal to the total number of receive antennas of the users, i.e., $M_T^{(BS)} \geq M_R^{(tot)}$.

3. COORDINATED BEAMFORMING

We propose an IIM-CBF algorithm to iteratively and jointly compute the precoding matrix and the decoding matrix for the downlink of multi-user MIMO FBMC/OQAM systems where $M_T^{(BS)} \geq M_R^{(tot)}$. Let us first define an equivalent combined channel matrix after the multiplication by the decoding matrix at the user terminals as

$$H_{e} = \begin{bmatrix} D_{1}^{\top}H_1 \\ D_{2}^{\top}H_2 \\ \vdots \\ D_{d_q}^{\top}H_Q \end{bmatrix} \in \mathbb{C}^{d \times M_T^{(BS)}}. \tag{11}$$

Unlike the coordinated beamforming schemes in [11] or [12], the decoding matrices $D_{q} \in \mathbb{R}^{M_{Tq} \times d_q}, \quad q = 1, 2, \ldots, Q$, are forced to be real-valued. The IIM-CBF algorithm is described in detail as follows:

- **Step 1:** Initialize the decoding matrices $D_{q}^{(0)} \in \mathbb{R}^{M_{Tq} \times d_q}$ ($q = 1, 1, \ldots, Q$), set the iteration index $p$ to zero, and set a threshold $\varepsilon$ for the stopping criterion. If the current subcarrier is the first one, the decoding matrices are generated randomly; otherwise, set the decoding matrices as those calculated for the previous subcarrier [11].

- **Step 2:** Set $p \leftarrow p + 1$ and calculate the equivalent channel matrix $H_{e}^{(p)}$ in the $p$th iteration as

$$H_{e}^{(p)} = \begin{bmatrix} H_{e1}^{(p)\top} \\ H_{e2}^{(p)\top} \\ \cdots \\ H_{eQ}^{(p)\top} \end{bmatrix}^{\top}, \tag{12}$$

where $H_{e}^{(p)} = D_{q}^{(p-1)\top}H_{q}$ is the equivalent channel matrix for the $q$th user in the $p$th iteration.

- **Step 3:** Calculate the precoding matrices $F_{q}^{(p)}$ ($q = 1, 1, \ldots, Q$) in the $p$th iteration to cancel the multi-user interference based on the BD algorithm [9]. For the $q$th user, define a matrix $H_{eq}^{(p)} \in \mathbb{C}^{(d-d_q) \times M_T^{(BS)}}$ as

$$H_{eq}^{(p)} = \begin{bmatrix} H_{eq1}^{(p)\top} \\ H_{eq2}^{(p)\top} \\ \cdots \\ H_{eqQ}^{(p)\top} \end{bmatrix}^{\top},$$

which contains the equivalent channel matrices of all the other users that are calculated in Step 2. The precoding matrix $F_{q}^{(p)}$ for the $q$th user in the $p$th iteration is obtained as

$$F_{q}^{(p)} = V_{q}^{(p,0)} \in \mathbb{C}^{(d-d_q) \times M_T^{(BS)}},$$

where $V_{q}^{(p,0)}$ contains the last $M_{Tq}^{(eq)}$ right singular vectors that form an orthonormal basis for the null space of $H_{eq}^{(p)}$ [9]. To this end, the multi-user MIMO downlink transmission is decoupled into parallel equivalent single-user MIMO transmissions that will be considered in the following steps.

- **Step 4:** Define a matrix $\tilde{H}_{eq}^{(p)} \in \mathbb{R}^{d \times 2M_{Tq}^{(eq)}}$ for the $q$th user based on its equivalent channel matrix $H_{eq}^{(p)}F_{q}^{(p)}$ after the cancellation of the multi-user interference

$$\tilde{H}_{eq}^{(p)} = \begin{bmatrix} \text{Im} \left\{ H_{eq}^{(p)}F_{q}^{(p)} \right\} & \text{Re} \left\{ H_{eq}^{(p)}F_{q}^{(p)} \right\} \end{bmatrix}. \tag{13}$$

- **Step 5:** Calculate the precoding matrix $G_{q}^{(p)} = G_{q1}^{(p)}G_{q2}^{(p)}$ for the $q$th user in the $p$th iteration. First, we perform the singular value decomposition (SVD) of $\tilde{H}_{eq}^{(p)}$ and obtain $V_{q1}^{(p,0)} \in \mathbb{R}^{2M_{Tq}^{(eq)} \times M_{eq}}$ as containing the last $M_{Tq}^{(eq)}$ right singular vectors that form an orthonormal basis for the null space of $H_{eq}^{(p)}$, where $r_{q}$ denotes the rank of $\tilde{H}_{eq}^{(p)}$.

Hence, $G_{q1}^{(p)} \in \mathbb{C}^{M_{Tq}^{(eq)} \times M_{eq}}$ can be obtained via

$$V_{q1}^{(p,0)} = \begin{bmatrix} \text{Re} \left\{ G_{q1}^{(p)} \right\} \\ \text{Im} \left\{ G_{q1}^{(p)} \right\} \end{bmatrix} \in \mathbb{R}^{2M_{Tq}^{(eq)} \times M_{eq}}, \tag{14}$$

such that (10) is fulfilled to achieve the mitigation of the intrinsic interference.

Now we define the following equivalent channel matrices after canceling the intrinsic interference for the $q$th user in the $p$th iteration

$$\tilde{H}_{eq}^{(p)} = \begin{bmatrix} \text{Re} \left\{ H_{eq}^{(p)}F_{q}^{(p)}G_{q1}^{(p)} \right\} \end{bmatrix} \in \mathbb{R}^{d \times M_{eq}}. \tag{15}$$

Further calculate the SVD of $\tilde{H}_{eq}^{(p)}$ and define $V_{q2}^{(p,1)} \in \mathbb{R}^{M_{eq} \times d_q}$ as containing the first $d_q$ right singular vectors. Then $G_{q2}^{(p)}$ is obtained as $G_{q2}^{(p)} = V_{q2}^{(p,1)}$.

- **Step 6:** Update the decoding matrix for each user based on the real-valued equivalent channel matrix where the processing at
In this section, we evaluate the bit error rate (BER) performance of a CP-OFDM based system with the same configuration as in the first example. Similar observations can be obtained from other examples, e.g., based on ZF or maximum ratio combining (MRC), where LoCCoBF is employed. For both the proposed IIM-CBF and direct extension (cf. Section 2.1) of the LoCCoBF algorithm [11] originally designed for the case of CP-OFDM, the transmission strategy proposed in [8] and briefly reviewed in Section 2.2 cannot be employed. Fig. 1 shows the BER curves of two schemes in a multi-user MIMO downlink system where the IIM-CBF scheme is employed and the FBMC/OQAM based system suffers from a performance degradation due to the frequency selectivity of the channel. By assuming that the channel stays constant across the neighboring subcarriers, the multi-user interference and the intrinsic interference cannot be eliminated even for high signal-to-noise-ratios.

Let us further look at a three-user scenario, where the three users are equipped with three receive antennas each, and two data streams are transmitted to each of the three users. The base station has also $M_{T}^{(BS)} = 8$ transmit antennas. The other parameters are the same as in the first example. Similar observations can be obtained from Fig. 2 as in Fig. 1.

4. SIMULATION RESULTS

In this section, we evaluate the bit error rate (BER) performance and the convergence behavior of the proposed IIM-CBF technique. For all examples, the number of subcarriers is 1024, and the total bandwidth is 10 MHz. In case of CP-OFDM, the length of the CP is set to 1/4 of the symbol period. The ITU Ped-A channel [16] is adopted. Moreover, the PHYDYAS prototype filter [13] with the overlapping factor $K = 4$ is employed. The data symbols are drawn from a 16 QAM constellation. Perfect channel state information is assumed at the transmitter and at the receiver is assumed.

First, we consider a multi-user MIMO downlink setting where the base station equipped with $M_{T}^{(BS)} = 8$ transmit antennas serves two users simultaneously, i.e., $Q = 2$. Each user has five receive antennas, and the number of data streams transmitted to each user is three. Note that for such a $M_{T}^{(tot)} > M_{T}^{(BS)}$ configuration, the transmission strategy proposed in [8] and briefly reviewed in Section 2.2 cannot be employed. Fig. 1 shows the BER curves of two schemes for the FBMC/OQAM based system, i.e., the IIM-CBF scheme presented in Section 3 and a direct extension (cf. Section 2.1) of the LoCCoBF algorithm [11] originally designed for the case of CP-OFDM. For the purpose of comparison, we also present the BER performance of a CP-OFDM based system with the same configuration where LoCCoBF is employed. For both the proposed IIM-CBF technique and LoCCoBF, $s$ for the stopping criterion is set to $10^{-5}$, and the maximum number of iterations is 50. It can be observed that the performance of the FBMC/OQAM based multi-user MIMO downlink system where the IIM-CBF scheme is employed is slightly better than its CP-OFDM based counterpart due to the fact that no insertion of the CP is required. The other transmission scheme for the FBMC/OQAM based system suffers from a performance degradation due to the frequency selectivity of the channel. By assuming that the channel stays constant across the neighboring subcarriers, the multi-user interference and the intrinsic interference cannot be eliminated even for high signal-to-noise-ratios.

In addition, Fig. 3 illustrates the complimentary cumulative distribution function (CCDF) of the number of iterations required for

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3 Other receivers, e.g., based on ZF or maximum ratio combining (MRC), can also be employed.
the proposed IIM-CBF technique to achieve the convergence. The three-user scenario used for Fig. 2 is considered. By comparison, we also plot the same set of results for a four-user case, i.e., $Q = 4$. The base station has eight transmit antennas, and each of the four users is equipped with three receive antennas and is sent to two data streams. Therefore, the total number of the receive antennas of the users $M_{R}^{\text{(tot)}} = 12$, and all spatial degrees of freedom are exploited. It can be observed that for the three-user scenario the IIM-CBF algorithm converges within 6 iterations in almost all of the cases. As the number of users and consequently the total number of receive antennas increase, the number of iterations needed for the convergence becomes slightly larger. Nevertheless, the convergence is achieved within 10 iterations. Moreover, we can see from the comparison of IIM-CBF for the FBMC/OQAM based system and LoCoBF for the case of CP-OFDM that the number of iterations required for the convergence for both schemes are similar. Hence, compared to the CP-OFDM based multi-user MIMO downlink setting, employing such a coordinated beamforming technique in the FBMC/OQAM based system where $M_{R}^{\text{(tot)}} > M_{R}^{\text{(BS)}}$ does not result in an increased number of iterations for the convergence. Only the processing dedicated to the elimination of the intrinsic interference contributes to a slight additional complexity.

5. CONCLUSION

In this contribution, we have proposed a coordinated beamforming based transmission strategy, called IIM-CBF, for the downlink of FBMC/OQAM based multi-user MIMO systems. By employing an iterative procedure to jointly compute the precoding matrix and the decoding matrix, the dimensionality constraint that the total number of receive antennas of the users must not exceed the number of transmit antennas of the base station is alleviated. Moreover, the proposed IIM-CBF technique does not rely on the assumption that the channel is flat fading. It can be observed in the numerical results that in FBMC/OQAM based multi-user MIMO downlink settings where the state-of-the-art transmission schemes cannot be used due to the dimensionality constraint, the IIM-CBF algorithm developed in this work is able to achieve a satisfactory performance in terms of BER with a moderate additional complexity.

6. REFERENCES