Soil Temperature and Heat Flow

Hillel, pp. 309-316
Soil Temperature and Heat Flow

Soil Temperature:

Soil temperature is a factor of primary importance for many physical, chemical, and biological processes. It governs:

1) Evaporation and soil aeration
2) All kind of chemical processes and reactions within the soil
3) Biological processes such as seed germination, seedling emergence and growth, root development, microbial activity

Soil temperature varies in response to changes in radiant, thermal and latent energy exchange processes that take place preliminary through the soil surface.

The effects of these phenomena are propagated into the soil profile by a complex series of transport processes.
Soil Temperature and Heat Flow

There are three major heat transport processes in soils:

1.) Heat Conduction:
Conduction of heat through matter involves transfer of kinetic energy at the molecular level, where molecules in warmer regions vibrate rapidly resulting in collisions with, or excitation of, their colder "neighbors". Conduction is the primary mode of heat transfer in soils!

2.) Heat Radiation:
Emission of energy in form of electromagnetic waves. All bodies with temperatures above 0ºK emit energy according to the Stefan-Boltzmann law.

The Earth emits most of its radiation in a wavelength band between 0.5 and 30.0 micrometers (µm). Long-Wave Radiation
Soil Temperature and Heat Flow

**Heat Convection:**

Transfer or movement of thermal energy with “heat-carrying” mass (e.g., water or vapor).

The transfer of latent heat of vaporization during convection of water vapor is of particular importance. During transfer of water vapor in soils, heat in the form of latent heat of vaporization is absorbed from some locations during vaporization of liquid water, and is released in cooler locations during condensation.
Soil Temperature and Heat Flow

The most important features of heat transfer in soils may be cast in a form of heat conduction, although we realize that some processes are not strictly conduction. The vertical one dimensional flux density of heat \( J_H \) \([W \ m^{-2}]\) in soil is given by Fourier's law:

\[
J_H = -\lambda \frac{dT}{dz}
\]

\( \lambda \) is soil thermal conductivity \((J \ m^{-1} \ s^{-1} \ oC^{-1})\)

\( T \) is temperature in \( oC \)

\( z \) is soil depth

\( \lambda \) should be considered as the apparent soil thermal conductivity, as latent heat transfer cannot in practice be separated from conduction in moist soils.

\[
\lambda = \lambda^* + D_{vapor} \cdot L
\]

\( \lambda^* \) is the instantaneous thermal conductivity

\( D \) is the thermal vapor diffusivity

\( L \) is the latent heat of vaporization \((2.449 \ MJ/kg \ or \ 585 \ cal/g)\)
Soil Temperature and Heat Flow

Combining the heat flux equation with the equation for conservation of heat energy results in a general expression for soil heat flow where soil temperature may vary in time and space:

**Heat Flux**

\[ J_H = -\lambda \frac{dT}{dz} \]

**Conservation of heat energy**

\[ c_v \frac{\partial T}{\partial t} = \frac{\partial J_H}{\partial z} \]

\[ \rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) \]

- \( \rho_s \) is the soil bulk density
- \( c_s \) is the soil specific heat capacity (i.e., the amount of heat required to raise the temperature of a unit mass of moist soil by 1°C)
Soil Temperature and Heat Flow

The previous expression may be simplified by using the soil's volumetric heat capacity \( c_v = \rho_s c_s \), which may be approximated as (Brutsaert, 1982):

\[
c_v \approx 1.94 \cdot (1 - n - \phi) + 4.189 \cdot \theta_v + 2.50 \cdot \phi \quad \text{[MJ m}^{-3} °C^{-1}]\]

- \( n \) is the porosity of the soil
- \( \phi \) is the volume fraction of soil organic matter
- \( \theta_v \) is the volumetric water content

The general equation for soil heat flow including conservation of energy is exactly analogous to that for unsaturated nonsteady water flow.

In this case the change in heat flux \([\Delta J_{Hz}]\Delta x\Delta y\Delta t\) equals the change in heat content \([c_v \Delta T]\Delta x\Delta y\Delta z\), thus

\[
\Delta J_{Hz}/Dz = c_v \Delta T/\Delta t
\]
Soil Temperature and Heat Flow

If the soil is homogeneous, then the soil’s volumetric heat capacity $c_v$ and thermal conductivity $\lambda$ are independent of depth (constant with depth). This results in a simplified form of the heat flow equation.

$$\frac{\partial T}{\partial t} = D_H \frac{\partial^2 T}{\partial z^2}$$

$D_H$ is the soil thermal diffusivity:

$$D_H = \frac{\lambda}{c_v} \left[ \frac{L^2}{t} \right]$$

Analytical solutions to the simplified equation are available for a variety of heat sources, flow geometries, and for different boundary conditions [Carslaw and Jaeger, 1959].
Soil Thermal Properties – Heat Capacity

In addition to being requisite for calculation of heat flow, a discussion of individual soil thermal properties aids in understanding how the solid, liquid, and gas phases interact to influence the dynamics of heat energy in soils.

Heat Capacity:

The heat capacity per unit volume of soil is the quantity of heat needed to raise the temperature of a unit volume of soil by one degree C. Knowledge of the volume fractions ($\theta_i$) of the soil constituents: minerals-m, water-w, air-a, and organic matter-om, their densities ($\rho_i$), and specific heat capacities per unit mass ($c_i$), allows determination of volumetric heat capacity ($c_v$) for moist soils as:

$$c_v = \rho_m \theta_m c_m + \rho_w \theta_w c_w + \rho_a \theta_a c_a + \rho_{om} \theta_{om} c_{om}$$
Soil Thermal Properties – Heat Capacity

Typical values for $c_i$, $\rho_i$, and $c_v$ are:

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Specific Heat, $c_i$ [J kg$^{-1}$°C$^{-1}$]</th>
<th>Density [kg m$^{-3}$]</th>
<th>$c_v$ [MJ m$^{-3}$°C$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil minerals</td>
<td>733</td>
<td>2650</td>
<td>1.94</td>
</tr>
<tr>
<td>Soil organic matter</td>
<td>1926</td>
<td>1300</td>
<td>2.5</td>
</tr>
<tr>
<td>Water</td>
<td>4182</td>
<td>1000</td>
<td>4.18</td>
</tr>
<tr>
<td>Air</td>
<td>1005</td>
<td>1.2</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

The contribution of air to volumetric heat capacity of soil can generally be neglected.
Soil Thermal Properties – Thermal Conductivity

**Thermal Conductivity \( \lambda \):**
Thermal conductivity is defined as the amount of heat transferred through a unit area in unit time (heat flux density) under a unit temperature gradient. The soil thermal conductivity \( (\lambda) \) is dependent primarily upon the bulk density and the soil water content.

Increasing soil bulk density hence the contacts between solid particles increases the thermal conductivity.

The thermal conductivity also increases with increasing water content. Soil water improves the thermal contact between the soil particles, and replaces air which has 20 times lower thermal conductivity than water.
Soil Thermal Properties – Thermal Conductivity

Thermal Conductivities of Soil Constituents at 10 °C

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$\lambda$ [mcal (cm s °C )(^{-1})]</th>
<th>$\lambda$ [W m(^{-1}) °C(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>21</td>
<td>8.8</td>
</tr>
<tr>
<td>Soil minerals (avg.)</td>
<td>7</td>
<td>2.9</td>
</tr>
<tr>
<td>Soil organic matter</td>
<td>0.6</td>
<td>0.25</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>1.37</td>
<td>0.57</td>
</tr>
<tr>
<td>Ice (at 0°C)</td>
<td>5.2</td>
<td>2.2</td>
</tr>
<tr>
<td>Air</td>
<td>0.06</td>
<td>0.025</td>
</tr>
</tbody>
</table>
Soil Thermal Properties - Thermal Conductivity

Thermal Properties of Soils and Snow (Hillel, 1998)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Porosity $f$</th>
<th>Volumetric wetness $\theta$</th>
<th>Thermal conductivity ($10^{-3}$ cal/cm sec °C)</th>
<th>Volumetric heat capacity $C_v$ (cal/cm sec °C)</th>
<th>Damping depth (diurnal) $d$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.4</td>
<td>0.0</td>
<td>0.7</td>
<td>0.3</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.2</td>
<td>4.2</td>
<td>0.5</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>5.2</td>
<td>0.7</td>
<td>14.3</td>
</tr>
<tr>
<td>Clay</td>
<td>0.4</td>
<td>0.0</td>
<td>0.6</td>
<td>0.3</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.2</td>
<td>2.8</td>
<td>0.5</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>3.8</td>
<td>0.7</td>
<td>12.2</td>
</tr>
<tr>
<td>Peat</td>
<td>0.8</td>
<td>0.0</td>
<td>0.14</td>
<td>0.35</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.4</td>
<td>0.7</td>
<td>0.75</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.8</td>
<td>1.2</td>
<td>1.15</td>
<td>5.4</td>
</tr>
<tr>
<td>Snow</td>
<td>0.95</td>
<td>0.05</td>
<td>0.15</td>
<td>0.05</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.2</td>
<td>0.32</td>
<td>0.2</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.5</td>
<td>1.7</td>
<td>0.5</td>
<td>9.7</td>
</tr>
</tbody>
</table>

$^a$ After van Wijk and de Vries (1963).
Soil Thermal Properties - Thermal Conductivity

Thermal Properties of Soils (Hillel, 1998)

Graphs showing the relationship between heat capacity and water content for different soil types.

- Heat capacity, HC (Cal. g^-1 °C^-1)
- Heat conductivity, K (mcal.cm^-1 °C^-1)

Soil Types: Sand, Clay, Peat
Soil Thermal Properties – Thermal Conductivity

Soil thermal conductivity as a function of water content:

[Graph showing thermal conductivity as a function of water content for different soil types like sand, clay, and peat.]
Measurement of Thermal Conductivity:

The thermal conductivity of a soil may be measured by means of a transient-heat probe consisting of a needle encasing a heater and thermocouple temperature sensor.

The sensor is embedded in the presumed homogenous soil where the needle approximates an infinitely long linear source of heat.

Heat is generated for a short time by application of a constant electric current through the heater, while measuring its temperature rise with respect to time using the thermocouple.
The radial flow of heat from a linear heat source may be described using a variation of the simplified heat flow equation for a radial coordinate system.

\[
\frac{\partial T}{\partial t} = D_H \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)
\]

- \(T\) is temperature in [°C]
- \(D_H\) is the thermal diffusivity in [m²/s]
- \(t\) is time in [s]
- \(r\) is radial distance from the heating source in [m]

When a long, electrically heated probe of strength \(q\) (W/m) is inserted into a medium, the rise in temperature from an initial temperature \(T_0\) at some distance \(r\) is:

\[
T - T_0 = \frac{q}{4 \pi \lambda} \ln (t + t_0) + d, \quad \frac{r^2}{4Dt} \ll 1
\]

where \(d\) is a constant, and \(t_0\) is a time correction which may be ignored for probes of 0.1 cm diameter or less, and for \(t > 60\) s.
Soil Thermal Properties – Thermal Conductivity

A linear regression of $T$ vs. $\ln(t)$ yields a line from which the thermal conductivity may be estimated as:

$$\lambda = \frac{q}{4\pi \cdot \text{slope of the line}}$$
Soil Thermal Properties – Thermal Diffusivity

**Thermal Diffusivity $D_H$:**

Thermal diffusivity is the ratio of the thermal conductivity $\lambda$ to the volumetric heat capacity $c_v$.

Thermal diffusivity at first increases rapidly with increasing water content, then decreases at a slower rate. This behavior results from the fact that while heat capacity $c_v$ increases linearly with water content, $\lambda$ increases most rapidly at low water contents.
Soil Thermal Properties - Thermal Diffusivity

**Thermal Diffusivity $D_H$:**

Thermal diffusivity may be estimated from measurements of $\lambda$ and $c_v$, or it may be measured directly.

One method for measuring $D_H$ directly is based on placing a heat source having constant temperature ($T_s$) in contact with the surface of a soil column having constant cross sectional area and insulated sides.

The temperature response at a parallel plane at distance $x$ from the heat source and the initial ambient temperature is measured.

$D_H$ can be estimated from the solution of the simplified heat flux equation (see next slide).
A solution of the simplified heat flux equation by for this setup is given (Carslaw and Jaeger, 1959):

\[
\frac{\partial T}{\partial t} = D_H \frac{\partial^2 T}{\partial z^2}
\]

**Simplified heat flux equation**

**Complementary Error Function**

\[
\text{erfc}(u) = 2\pi^{-1/2} \int \exp(-u^2) \, du
\]

Tabulated values of \text{erfc}(u) may be found in mathematical handbooks, similar to finding values of log, sin, cos, etc., or can be calculated using approximate solutions. Most of the spreadsheet programs (e.g., Excel) have \text{erf} and \text{erfc} as built-in functions.

The above solution is applicable only if the soil column is sufficiently long so that the temperature at the other face of the column remains unchanged at \(T_0\).
Soil Thermal Properties – Thermal Diffusivity

An explicit solution for the thermal diffusivity equation can be derived from observations of temperature versus time:

\[
\frac{T - T_0}{T_s - T_0} = \text{erfc} \left[ \frac{x}{\sqrt{4D_H t}} \right]
\]

1.) Calculate the variable \( p \) for a given time \( t \):

\[
p = \left( -\ln \left[ \frac{0.5 \cdot (T - T_0)}{(T_s - T_0)} \right] \right)^{1/2}
\]

2.) Approximate the argument \( u \) of the \( \text{erfc}(u) \):

\[
u = p - \frac{1.881796 + 0.9425908p + 0.0546028p^3}{1 + 2.356868p + 0.3087091p^2 + 0.0937563p^3 + 0.0219104p^4}
\]

3.) Calculate thermal diffusivity as:

\[
D_H = \frac{x^2}{u^2 \cdot 4t}
\]
Soil Thermal Properties – Thermal Diffusivity

Example: Thermal Diffusivity and Heat Capacity

Given: A soil with $\theta_v = 0.3 \, \text{m}^3/\text{m}^3$, $\rho_b = 1350 \, \text{kg/m}^3$ and thermal conductivity $\lambda = 2 \, \text{J/(m s °C)}$. Find the thermal diffusivity $D_H$.

1.) First we calculate the volumetric heat capacity $c_v$, neglecting organic matter and air contribution.

$$c_v \approx 1.94 \cdot (1 - n - \varphi) + 4.189 \cdot \theta_v + 2.50 \cdot \varphi \quad \text{[MJ m}^{-3} \, \text{°C}^{-1}]$$

$$c_v = 1.94 \left(1 - \left[1 - \frac{1350}{2650}\right]\right) + 4.19 \cdot 0.3 = 2.245 \, \text{MJ m}^{-3} \, \text{°C}^{-1}$$

2.) Then we calculate thermal diffusivity as:

$$D_H = \frac{\lambda}{c_v} = \frac{2}{2.245 \cdot 10^6} = 8.907 \cdot 10^{-7} \, \text{m}^2 \, \text{s}^{-1} = 0.077 \, \text{m}^2 \, \text{d}^{-1}$$
Diurnal and Seasonal Variations in Soil Temperature

Hillel, pp. 323-334
Diurnal and Seasonal Variations in Soil Temp.

In the field, soil thermal regime is characterized by periodic changes in response to the natural periodicity in atmospheric conditions controlling energy inputs to the soil surface.

There is a diurnal (daily) cycle as well as a superimposed seasonal cycle. These diurnal and seasonal cycles are perturbed by irregular meteorological events including cloudiness, warm and cold fronts, precipitation, etc.
Diurnal and Seasonal Variations in Soil Temp.

Seasonal Cycle

[Graph showing temperature variations over seasons at different depths (0.1m and 1.0m).]
Diurnal and Seasonal Variations in Soil Temp.

There are three fundamental characteristics related to diurnal and annual soil thermal regimes.

1) We observe diurnal and annual temperature cycles in response to the fluctuating (cyclic) inputs of solar radiation.

2) The incoming solar radiation energy is utilized to heat as it travels down the soil profile. That means the available energy decreases with depth. Thus we observe the phenomenon of amplitude damping, or a reduction in the magnitude of these temperature cycles with increasing depth.

3) Because it takes time for heat to travel into and out of the soil, there is a delay in the time at which any specific location on the temperature cycle reaches a given point in the soil, and this time lag becomes more pronounced with increasing distance.
Diurnal and Seasonal Variations in Soil Temp.

Amplitude damping and time lag:

![Graph showing soil temperature variations over time at different depths.](image-url)
Mathematical Description of the Soil Thermal Regime

It is common to assume that soil temperature oscillates as a pure harmonic (sinusoidal) function of time about some mean value of temperature. We can represent the surface temperature (z=0) as a function of time, \( T(0,t) \), by:

\[
T(0,t) = \bar{T} + A_0 \sin \omega t
\]

\[
A_0 = \frac{(T_{\text{max}} - T_{\text{min}})}{2}
\]

\[
\omega = \frac{2\pi}{P}
\]

Durnal: \( P = 24 \text{ hr} = 86400 \text{ sec} \)

Anual: \( P = 12 \text{ months} = 365 \text{ days} \)

\( \bar{T} \) is the mean temperature at the soil surface in \([\degree C]\)

\( A_0 \) is the amplitude of temperature fluctuation in \([\degree C]\)

\( \omega \) is the angular frequency \([\text{s}^{-1}]\)

\( P \) is the period of oscillation

This equation is valid for the soil surface. At very large depth the temperature is constant (i.e., no variations with time).
The characteristic damping depth $d$ is the depth at which the amplitude $A_z$ decreases to the fraction $1/e$ ($1/2.718=0.37$) of the surface amplitude $A_0$. 

\[ d = \sqrt{\frac{2D_H}{\omega}} = \sqrt{\frac{PD_H}{\pi}} \]

Note that the 8 in ‘(t-8)‘ of the solution on the previous slide is an offset in hours to the sine function to obtain maximum soil surface temperature at 2 p.m., which is consistent with many field measurements. This value may be adjusted based on measurements to obtain optimal agreement.
Mathematical Description of the Soil Thermal Regime

An expression for temperature variations as a function of time and depth is given as a solution of the simplified heat flow equation assuming constant volumetric heat capacity $c_v$ and thermal conductivity $\lambda$ throughout the soil profile:

$$\frac{\partial T}{\partial t} = D_H \frac{\partial^2 T}{\partial z^2}$$

Simplified heat flow equation

$$T(z, t) = \bar{T} + A_0 e^{-\frac{z}{d}} \sin \left[ \omega (t - 8) - \frac{z}{d} \right] \quad 0 \leq z < \infty$$

$\bar{T}$ is the mean temperature at the soil surface in [°C]

$A_0$ is the amplitude of temperature fluctuation in [°C]

$\omega$ is the angular frequency [s$^{-1}$]

$z$ is the depth below surface expressed as positive number [m]

$d$ is the characteristic damping depth [m]
Mathematical Description of the Soil Thermal Regime

\[
T(z,t) = \bar{T} + A_0 e^{-z/d} \sin \left[ \omega (t - 8) - \frac{z}{d} \right] \quad 0 \leq z < \infty
\]

From above equation one can see that:

1) The amplitude at each depth is reduced to \( A_z = A_0 e^{-z/d} \)

2) There is an increase in phase (time) lag with depth equal to \( z/d \)

3) The average temperature and the period are the same for all depths

In-situ measurements of soil temperature with time and depth may be used in combination with above equation to infer soil thermal properties.

We can use the ratio of two amplitudes \( (A_1 \text{ and } A_2) \) measured at different depths \( (z_1 \text{ and } z_2) \) to infer soil thermal diffusivity \( K_H \):

\[
D_H = \frac{\pi (z_1 - z_2)^2}{P \left( \ln \left[ \frac{A_1}{A_2} \right] \right)^2}
\]
Mathematical Description of the Soil Thermal Regime

We also can use the difference between two time lags to infer soil thermal diffusivity $K_{HH}$. The argument in the sine function for the attainment of max temperature should equal $\pi/2$, thus:

$$D_H = \frac{P(z_1 - z_2)^2}{4\pi(t_1 - t_2)^2}$$

Values of $D_H$ resulting from these two estimation methods may differ due to violation of assumptions made in the derivation of the solution, e.g. the soil may be nonuniform, the measured temperature wave may not be well represented by a sine function, etc.
Example-I: Diurnal Variations in Soil Temperature

The thermal diffusivity \( D_H \) is 0.003 m\(^2\)/hr, the average temperature of the soil is 20 °C, and the daily amplitude at the soil surface is 20 °C. Find the temperature at a depth of \( z = -0.2 \) m and at \( t = 8 \) hr.

1.) First we calculate the characteristic damping depth \( d \) according to:

\[
d = \sqrt{2 \frac{D_H}{\omega}} = \sqrt{\frac{PD_H}{\pi}}
\]

\[
d = \left( \frac{24 \cdot 0.003}{3.14} \right)^{1/2} = 0.151 \text{ m}
\]

2.) Then we plug in the appropriate values and obtain:

\[
T(z,t) = \bar{T} + A_0 e^{-\frac{z}{d}} \sin \left[ \frac{\omega(t - 8) - \frac{z}{d}}{\omega} \right] \quad 0 \leq z < \infty
\]

\[
T(-0.2,8) = 20 + 20e^{(-0.2/0.151)} \sin \left[ \frac{2\pi(8-8)}{24} - \frac{0.2}{0.151} \right] = 14.8 \degree C
\]

Note that we have to use radians for computation of the sine function.
Example-II: Estimation of Thermal Diffusivity

Given are the data in the figure below and the following information for the first day: the maximum and minimum temperatures at 20 and 100 mm were (40.0, 18.7) and (33.6, 23.4) °C, respectively. Their respective occurrence times were 1500 and 1800 hrs for the maximum temperature, 0700 and 0900 hrs for the minimum.

Find the thermal diffusivity ($D_H$) for the soil using the time lag and the amplitude damping methods.
Soil Temperature - Example-II

Example-II: Estimation of Thermal Diffusivity - Time Lag Method

We observe that the time difference between the maximum temperatures at -0.02 m and -0.10 m was \( (t_1-t_2)=3.0 \text{ hrs} \). Using \( P=24 \) hrs, we find:

\[
D_H = \frac{P(z_1-z_2)^2}{4\pi(t_1-t_2)^2}
\]

\[
D_H = \frac{24 (-0.02 + 0.10)^2}{4\pi \cdot 3^2} = 0.00136 \text{ m}^2 \text{ hr}^{-1} = 0.03269 \text{ m}^2 \text{ day}^{-1}
\]

Damping Depth Method

We compute amplitudes at -0.02 m and – 0.1 m depths.

\[
A_1 = \frac{40.0 - 18.7}{2} = 10.65 \degree \text{C}
\]

\[
A_2 = \frac{33.6 - 23.4}{2} = 5.10 \degree \text{C}
\]
And calculate thermal diffusivity as:

\[
D_H = \frac{\pi (z_1 - z_2)^2}{P(\ln[A_1 - A_2])^2}
\]

\[
D_H = \frac{\pi (-0.02 + 0.10)^2}{24 \cdot \left( \ln \left[ \frac{10.65}{5.10} \right] \right)^2} = 0.00154 \text{ m}^2 \text{ hr}^{-1} = 0.0371 \text{ m}^2 \text{ day}^{-1}
\]