CAHSEE ON TARGET
SCHOOL/UNIVERSITY PARTNERSHIPS, UC DAVIS

ANSWER KEY
NUMBER SENSE

NAME:
Introduction to the CAHSEE

The CAHSEE stands for the California High School Exit Exam. The mathematics section of the CAHSEE consists of 80 multiple-choice questions that cover 53 standards across 6 strands. These strands include the following:

Number Sense (14 Questions)
Statistics, Data Analysis & Probability (12 Questions)
Algebra & Functions (17 Questions)
Measurement & Geometry (17 Questions)
Mathematical Reasoning (8 Questions)
Algebra 1 (12 Questions)

What is CAHSEE on Target?

CAHSEE on Target is a tutoring course specifically designed for the California High School Exit Exam (CAHSEE). The goal of the program is to pinpoint each student’s areas of weakness and to then address those weaknesses through classroom and small group instruction, concentrated review, computer tutorials and challenging games.

Each student will receive a separate workbook for each strand and will use these workbooks during their tutoring sessions. These workbooks will present and explain each concept covered on the CAHSEE, and introduce new or alternative approaches to solving math problems.

What is Number Sense?

Number Sense is the understanding of numbers and their relationships. The Number Sense Strand concepts that are tested on the CAHSEE can be divided into five major topics: Integers & Fractions; Exponents; Word Problems; Percents; and Interest. These topics are presented as separate units in this workbook.
Unit 1: Integers & Fractions  1.2 & 2.2

On the CAHSEE, you will be given several problems involving rational numbers (integers, fractions and decimals).

Integers are whole numbers; they include . . .

• positive whole numbers \{1, 2, 3, \ldots \}
• negative whole numbers \{-1, -2, -3, \ldots \} and
• zero \{0\}.

Positive and negative integers can be thought of as opposites of one another.

A. Signs of Integers

All numbers are signed (except zero). They are either positive or negative.

When adding, subtracting, multiplying and dividing integers, we need to pay attention to the sign (+ or -) of each integer.

Example: 5 \times -3 = ____

Example: -5 + 4 = ___

Example: -3 \times 12 = ____

Whether it’s written or not, every number has a sign:

Example: 5 means +5
Signed Numbers in Everyday Life 1.2

Signed numbers are used in everyday life to describe various situations. Often, they are used to indicate opposites:

**Altitude:** The elevator went up 3 floors (+3) and then went down 5 floors (-5).

**Weight:** I lost 20 pounds (-20) but gained 10 back (+10).

**Money:** I earned $60 (+60) and spent $25 (-25).

**Temperature:** The temperature rose 5 degrees (+5) and then fell 2 degrees (-2).

**Sea Level:** Jericho, the oldest inhabited town in the world, lies 853 feet below sea level (-853), making it the lowest town on earth. Mount Everest is the highest mountain in the world, standing at 8850 meters (+8850), nearly 5.5 miles above sea level.

Can you think of any other examples of how signed numbers are used in life?

Have students think of examples on their own or discuss in groups.

**Possible answers:**

- Sports scores (football team gained 4 yards: +4)
- Profit and loss (or assets and liabilities)
- Time (2005 BC versus 2005 AD)
i. Adding Integers

When adding two or more integers, it is very important to pay attention to the sign of each integer. Are we adding a positive or negative integer? We can demonstrate this concept with a number line.

Look at the two examples below. In the first example, we add a positive 3 (+3) to 2.

Example: 2 + 3 = 5

In this second example, we add a negative 3 (-3) to 2.

Example: 2 + (–3) = -1

As you can see, we get a very different answer in this second problem.

To add integers using a number line, begin with the first number in the equation. Place your finger on that number on the number line. Look at the value and sign of the second number: if positive, move to the right; if negative, move to the left. (If a number does not have a sign, this means it is positive.) With your finger, move the number of spaces indicated by the second number.

Example: 1 + (-2) = -1
On Your Own

1.2

-2 + (-3) = -5

-6 + (3) = -3

3 + (-6) = -3

-3 + 6 = +3
Rules for Adding Signed Numbers (without a Number Line)

A. Same Signs
- Find the sum
- Keep the sign

B. Different Signs
- Find the difference
- Keep the sign of the larger number (# with larger absolute value)

On Your Own

\[-8 + (-7) = \textcolor{red}{-15}\]  \[-8 + 7 = \textcolor{red}{-1}\]

\[-13 + (-9) = \textcolor{red}{-22}\]  \[(+13) + (+9) = \textcolor{red}{+22}\]

\[21 + (-21) = \textcolor{red}{0}\]  \[(-21) + 21 = \textcolor{red}{0}\]

\[-13 + 18 = \textcolor{red}{+5}\]  \[-18 + 13 = \textcolor{red}{-5}\]

Add -10 and -5: \textcolor{red}{-15}  Add (-10), (+4), and (-16): \textcolor{red}{-22}
ii. Subtracting Integers

We can turn any subtraction problem into an **addition** problem. Just **change the subtraction sign (−) to an addition sign (+)** and **change the sign of the second number**. Then solve as you would an addition problem.

**Example:** \(-2 - (+3) = \) 

Turn it from a subtraction problem to an addition problem; then change the sign of the second number:

\[-2 - (+3) = -2 + (-3)\]

Now solve as you would an addition problem.

We can show this on a number line. Place your finger on that number on the number line. Look at the value and sign of the second number: **if positive, move to the right; if negative, move to the left.** With your finger, move the number of spaces indicated by the second number:

Let's look at another problem:

**Example:** \(-2 - (-3) = -2 + (+3)\)

Answer: \(+1\)
On Your Own

6 – (+3) = 6 + (−3) = +3

3 – (−3) = 3 + (+3) = +6

−5 – (+1) = −5 + (−1) = −6

1 – (+1) = 1 + (−1) = 0
Rules for Subtracting Signed Numbers (without a Number Line)

**Add its opposite!** Draw the line and change the sign (of the second number), and follow the rules for addition.

**Example:** \(6 - (-4)\)

**Steps:**

- Draw the line (to turn the minus sign into a plus sign): \(6 \, \underline{+} \, \underline{+} \)
- Change the sign of the second number: \(6 \, + \, 4\)
- Now you have an addition problem. Follow the rules of adding numbers: \(6 \, + \, 4 \, = \, 10\)

**On Your Own:** Draw the line and change the sign. Then solve the addition problem.

\[
\begin{align*}
19 - (-13) & = +32 \\
-17 - (-15) & = -2 \\
34 - (-9) & = +43 \\
-18 - 14 & = -32 \\
-15 - (-35) & = +20 \\
13 - (+15) & = -2 \\
-13 - 15 & = -28 \\
-35 - (+35) & = -70 \\
\end{align*}
\]

Subtract (-15) from (20): \(+35\)

Subtract 4 from (-14): \(-18\)
Signed Numbers Continued

Look at the following problem:

Example: $1 - 3 + 5 = ____$

We can represent this problem on a number line:

We begin at 1, move 3 spaces backwards (to the left) and then 5 spaces forwards (to the right). We arrive at + 3.

When we are given a problem with three or more signed integers, we must work out, separately, the addition and subtraction for each integer pair:

$1 - 3 + 5 = (1 - 3) + 5$

$1 - 3 = -2$  Work out the addition or subtraction for the 1st 2 integers

$-2 + 5 = ____$  Take the answer from above & add it to the last integer.

On Your Own

1. $12 + 3 - 5 + 4 = 14$

2. $-3 + 5 - 2 + 3 = 3$

3. $4 - 6 + 3 - 2 = -1$
iii. Multiplying and Dividing with Signed Numbers 1.2

Multiplying
The product of two numbers with the same sign is positive.

Example: \(-5 \cdot -3 = 15\)

The product of two numbers with different signs is negative.

Example: \(-5 \cdot 3 = -15\)

Dividing
The quotient of two numbers with the same sign is positive.

Example: \(-15 \div -3 = 5\)

The quotient of two numbers with different signs is negative.

Example: \(-15 \div 3 = -5\)

On Your Own

\((+8) \cdot (-4) = -32\) \hspace{1cm} \((-7) \cdot (7) = -49\)

\((-8) \cdot (-8) = +64\) \hspace{1cm} \((+7)(+8) = +56\)

\(-36 \div (-3) = +12\) \hspace{1cm} \(-36 \div 3 = -12\)

Tutors: Play Bingo for Signed Numbers with your students (See the Tutor Supplement for Number Sense)
The absolute value of a number is its distance from 0. This distance is always expressed as a positive number, regardless if the number is positive or negative.

It is easier to understand this by examining a number line:

The absolute value of 5, expressed as \(|5|\), is 5 because it is 5 units from 0. We can see this on the number line above. The absolute value of -5, expressed as \(|-5|\), is also 5 because it is 5 units from 0. Again, look at the number line and count the number of units from 0.

On Your Own: Complete the chart. How far from zero is the number?

<table>
<thead>
<tr>
<th>Number</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-16</td>
</tr>
<tr>
<td></td>
<td>-115</td>
</tr>
<tr>
<td></td>
<td>342</td>
</tr>
<tr>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>-x</td>
</tr>
<tr>
<td></td>
<td>-100</td>
</tr>
</tbody>
</table>
Finding the Absolute Value of an Expression 2.5

On the CAHSEE, you may need to find the absolute value of an expression. To do this, ...

• Evaluate the expression within the absolute value bars.
• Take the absolute value of that result.
• Perform any additional operations outside the absolute value bars.

Example: $3 + |-4 - 3| = 3 + |-7| = 3 + 7 = 10$

On Your Own: Complete the chart.

| $5 \cdot |3 - 8|$ = | $5 \cdot |-5| = 5 \cdot 5$ = | 25 |
|---|---|---|
| $|15 + 6|$ = | $|21| = $ | 21 |
| $|-6 + 2|$ = | $|-4|$ = | 4 |
| $|1 - 3 + 2|$ = | $|0|$ = | 0 |
| $4 + |-6|$ = | $4 + 6$ = | 10 |
| $|-4| + |4|$ = | $4 + 4$ = | 8 |
| $|16| - |-16|$ = | $16 - 16$ = | 0 |
| $|-2| - |13|$ = | $2 - 13$ = | -11 |
| $|-2| - |-13|$ = | $2 - 13$ = | -11 |
Absolute Value Continued

While the absolute value of a number or expression will always be positive, the number \textit{between the absolute value bars} can be \textbf{positive} or \textbf{negative}.

\begin{center}
\begin{tikzpicture}
\node (example) at (0,0) {Example: $|8| = 8$ and $|8| = 8$};
\node at (-2,-2) {Positive Number};
\node at (-2,-3) {Negative Number};
\end{tikzpicture}
\end{center}

Notice that in each case, the expression is equal to +8.

You may be asked to identify these two possible values on the CAHSEE.

\textbf{Example:} If $|x| = 8$, what is the value of $x$?

For these types of problems, the answer consists of \textbf{two values}: the \textbf{positive} and \textbf{negative} value of the number.

In the example above, the two values for $x$ are \textbf{8 or -8}.

\textbf{On Your Own}

1. If $|y| = 225$, what is the value of $y$? \textbf{225 or -225}

2. If $|x| = 1,233$, what is the value of $x$? \textbf{1,233 or -1,233}

3. If $|m| = 18$, what is the value of $m$? \textbf{18 or -18}

4. If $|x| = 12$, what is the value of $x$? \textbf{12 or -12}

5. If $|y| = 17$, what is the value of $y$? \textbf{17 or -17}
C. Fractions

A fraction means a part of a whole.

**Example:** In the picture below, one of four equal parts is shaded:

![Fraction Example](image)

We can represent this as a fraction: \( \frac{1}{4} \)

Fractions are expressed as one number over another number: \( \frac{1}{4} \)

Every fraction consists of a **numerator** (the top number) and a **denominator** (the bottom number):

\[ \frac{A}{B} \quad \text{Numerator} \]
\[ \frac{A}{B} \quad \text{Denominator} \]

**Fractions mean division:** \( \frac{A}{B} = A \div B \)

\[ \frac{1}{4} = 1 \div 4 = 4\overline{1} = .25 \]

\[ \frac{4}{5} = 4 \div 5 = 5\overline{4} = .8 \]

\[ \frac{1}{2} = 1 \div 2 = 2\overline{1} = .5 \]
i. Adding & Subtracting Fractions 1.2 & 2.2

Same Denominator: Keep the denominator; add the numerators:

Example: $\frac{1}{4} + \frac{2}{4} = \underline{\quad}$

We can represent this problem with a picture: Begin with the first fraction, $\frac{1}{4}$, and add two more fourths ($\frac{2}{4}$):

![Fraction Pie Diagram]

We now have three-fourths of the whole shaded: $\frac{3}{4}$

On Your Own: Add the following fractions.

$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$\frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1 \text{ (1 whole number)}$

$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

Rule: When adding and subtracting fractions that have common denominators, we just add or subtract the numerators and keep the denominator. It gets trickier when the denominators are not the same.
Different Denominator \hspace{1cm} 1.2 & 2.2

Example: $\frac{1}{4} + \frac{3}{8}$

Let's represent this with a picture:

![Fraction Representation](image1)

The first picture shows one whole divided into four parts. One of these parts is shaded. We represent this as a fraction: $\frac{1}{4}$

The second picture shows one whole divided into eight parts. Three of these parts are shaded. We represent this as a fraction: $\frac{3}{8}$

In order to add these two fractions, we need to first divide them up into equal parts. The first picture is divided into fourths but the second is divided into eighths. We can easily convert the first picture into eighths by drawing two more lines (i.e. divide each fourth by half):

![Fraction Conversion](image2)
Now let's see how the **first fraction** would appear once it is divided into eighths:

![Diagram showing division into eighths]

We can see, from the above picture, that $\frac{1}{4}$ is equal to $\frac{2}{8}$.

Now that we have a common denominator (8), we can add the fractions: $\frac{2}{8} + \frac{3}{8}$. Just keep the denominator and add the numerators:

$$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

Answer: $\frac{5}{8}$

Let's look at another example: $\frac{2}{3} + \frac{4}{5}$

Can we add these two fractions in their current form? Explain.

**No, we need a common denominator**

To add two fractions, we need a common denominator. We must therefore convert the fractions to ones whose denominator is the same. We can use any common denominator, but it is much easier to use the **lowest common denominator**, or LCD. One way to find the LCD is to make a table and list, in order, the multiples of each denominator. *(Multiple means Multiply!)*
Finding the Lowest Common Denominator (LCD) 1.2 & 2.2

Look at the last problem again: \( \frac{2}{3} + \frac{4}{5} \)

Now list the **multiples** of each **denominator** until you reach a **common** number.

<table>
<thead>
<tr>
<th>Multiples of 3</th>
<th>Multiples of 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

The **lowest common denominator (LCD)** is the **first common number** in both columns: **15**. This will be the new denominator for both fractions.

Since we changed the **denominators**, we must also change the **numerators** so that each new fraction is **equivalent** (or equal) to the **original fraction**.

- Let’s start with the first original fraction: \( \frac{2}{3} \). Go back to the table. How many times did we **multiply** the denominator, 3, by itself? (Hint: How many rows did we go down in the first column?) \( 1^5 \)

- Since we multiplied the **denominator** (3) by \( 5 \) to get 15, we must also multiply the **numerator** (2) by \( 5 \).

  Our new fraction is \( \frac{10}{15} \)

- Now let’s look at the second fraction: \( \frac{4}{5} \). Since we multiplied the denominator (5) by 3, we do the same to the numerator: \( 4 \times 3 = 12 \).

  Our new fraction is \( \frac{12}{15} \)

- Now add the new fractions: \( \frac{10}{15} + \frac{12}{15} = \frac{22}{15} \)

We have an **improper fraction** because the numerator > the denominator. We must change it to a **mixed number**: \( \frac{22}{15} = 1 \frac{7}{15} \)
Let's look at another example:

**Example:** Add the following fractions: \( \frac{3}{4} + \frac{4}{5} \)

In order to add these fractions we must first find a common denominator. Make a table and list all of the **multiples** for each **denominator** until we reach a **common multiple**:

<table>
<thead>
<tr>
<th>Multiples of 4</th>
<th>Multiples of 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
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<tr>
<td>8</td>
<td>10</td>
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<tr>
<td>12</td>
<td>15</td>
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<tr>
<td>16</td>
<td>20</td>
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<td>20</td>
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</tbody>
</table>

We have a common denominator for both fractions: **20**. Since we changed the denominators for both fractions, we must also change the numerators so that each new fraction is **equivalent** to the original fraction.

Let’s begin with the first fraction: \( \frac{3}{4} = \frac{15}{20} \)

Now let’s proceed to the second fraction: \( \frac{4}{5} = \frac{16}{20} \)

Now both fractions have common denominators; add them:

\[
\frac{15}{20} + \frac{16}{20} = \frac{31}{20}
\]

If the sum is an **improper** fraction (i.e. numerator > denominator), we generally change it to a proper fraction: \( 1 \frac{11}{20} \)
On Your Own

Example: $\frac{3}{4} + \frac{5}{6}$

Step 1: Make a table and list the multiples of each denominator until you reach a common denominator:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
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<td>8</td>
<td>12</td>
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<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Step 2: Convert each fraction to an equivalent fraction:

$\frac{9}{12} + \frac{10}{12} = \frac{9+10}{12} = \frac{19}{12}$

Step 3: Add the fractions:

$\frac{9}{12} + \frac{10}{12} = \frac{9+10}{12} = \frac{19}{12}$

Note: If you end up with an improper fraction, be sure to convert it to a mixed number.

$\frac{19}{12} = 1 \frac{7}{12}$
Practice 1.2 & 2.2

\[
\frac{4}{5} + \frac{2}{5} = \frac{6}{5} = 1 \frac{1}{5}
\]

\[
\frac{7}{9} - \frac{3}{9} = \frac{4}{9}
\]

\[
\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20} = 1 \frac{3}{20}
\]

\[
\frac{2}{3} + \frac{5}{8} = \frac{31}{24} = 1 \frac{7}{24}
\]

\[
\frac{3}{4} - \frac{1}{6} = \frac{9 - 2}{12} = \frac{7}{12}
\]

\[
\frac{5}{8} - \frac{1}{2} = \frac{5 - 4}{8} = \frac{1}{8}
\]
Prime Factorization

Another way to find the lowest common denominator of two fractions is through prime factorization. First, let’s learn more about prime numbers:

**Prime Numbers:** A prime number has two distinct whole number factors: 1 and itself.

Note: 1 is not prime because it does not have two distinct factors.

**Example:** 6 is not prime because it can be expressed as 2 \( \times \) 3.

**Example:** 7 is prime because it can be expressed only as the product of two distinct factors: 1 \( \times \) 7.

Write the first 10 prime numbers:

\[
\begin{align*}
2 & \quad 3 & \quad 5 & \quad 7 & \quad 11 & \quad 13 & \quad 17 & \quad 19 & \quad 23 & \quad 29
\end{align*}
\]

**Composite Numbers**

A non-prime number is called a composite number. Composite numbers can be broken down into products of prime numbers:

**Example:** 4 = 2 \( \times \) 2

**Example:** 12 = 2 \( \times \) 6 = 2 \( \times \) 2 \( \times \) 3

**Example:** 66 = 6 \( \times \) 11 = 2 \( \times \) 3 \( \times \) 11

**Example:** 24 = 2 \( \times \) 12 = 2 \( \times \) 2 \( \times \) 2 \( \times \) 3

**Example:** 33 = 3 \( \times \) 11

**Example:** 125 = 5 \( \times \) 5 \( \times \) 5
**Practice:** Circle all of the *prime* numbers in the chart below:  

<p>| | | | | |</p>
<table>
<thead>
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<td>46</td>
<td>47</td>
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<td>50</td>
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</table>
Prime Factor Trees 2.2

We can find the prime factors of a number by making a factor tree:

**Example:** Find the prime factors of 18.

- Write your number: **18**
- Begin with the smallest prime number factor of 18 (i.e. the smallest prime number that divides evenly 18. This number is **2**.
- Draw two branches: **2** and the second factor: **9**.

```
  18
 / \  \
2   9
```

- Continue this process for each branch until you have no remaining composite numbers. The prime factors of 18 are **the prime numbers at the ends of all the branches**:

```
  18
 / \  \
2   9
    / \  \
   3 3
```

The prime factored form of 18 is **2 \cdot 3 \cdot 3**.
Example: Find the prime factors of 60 using the factor tree:

\[
\begin{array}{c}
60 \\
\downarrow \\
2 \quad 30 \\
\downarrow \\
2 \quad 15 \\
\downarrow \\
3 \quad 5
\end{array}
\]

The prime factors of 60 are the factors at the end of each branch: \(2, 2, 3,\) and \(5.\)

Helpful Guidelines:

• Start with the smallest numbers: first 2’s, then 3’s, and so on.
• If a number is even, it is divisible by 2.

Note: An even number ends in 0, 2, 4, 6, and 8.

Examples: \(124, 38, 46, 180, 112,\)

• If the digits of a number add up to a number divisible by 3, the number is divisible by 3.

Example: \(123\) can be divided evenly by 3 because if we add all of its digits, we get 6: \(1 + 2 + 3 = 6\)

Since the sum of the digits of 123 is divisible by 3, so too is 123.

• If a number ends in 0 or 5, it is divisible by 5.

Examples: \(25, 130, 125, 455,\)
On Your Own  
2.2

Find the prime factors of each number, using a factor tree:

\[
\begin{align*}
64 &\quad 48 \\
2 &\quad 2 \\
2 &\quad 2 \\
32 &\quad 24 \\
2 &\quad 2 \\
16 &\quad 12 \\
2 &\quad 2 \\
8 &\quad 6 \\
2 &\quad 2 \\
4 &\quad 3 \\
2 &\quad 2 \\
2 X 2 X 2 X 2 X 2 X 2 &\quad 2 X 2 X 2 X 2 X 3
\end{align*}
\]

\[
\begin{align*}
72 &\\
2 &\quad 36 \\
2 &\quad 18 \\
2 &\quad 9 \\
3 &\quad 3 \\
2 X 2 X 2 X 3 X 3
\end{align*}
\]

Tutors: For a fun reinforcement activity on prime numbers, play the Factor Game. (See the Tutor Supplement for Number Sense.)
Prime Factorization and the Lowest Common Denominator  2.2

On the CAHSEE, you will be asked to find the prime factored form of the lowest common denominator (LCD) of two fractions:

Example: Find the prime factored form for the lowest common denominator \( \frac{5}{6} + \frac{5}{9} \).

There are two methods we can use to solve this problem:

**Method I: Factor Tree and Pairing**

**Steps:**

- Make a factor tree for both denominators:

  \[
  \begin{array}{c}
  6 \\
  \downarrow \\
  2 \quad 3
  \end{array} 
  \quad 
  \begin{array}{c}
  9 \\
  \downarrow \\
  3 \quad 3
  \end{array}
  \]

- Pair up common prime factors:

  \[
  \begin{array}{c}
  6 = 3 \times 2 \\
  \downarrow \\
  9 = 3 \times 3
  \end{array}
  \quad \text{Count any common factor once!}
  \]

- Multiply the common factor (counted once) by all leftover (unpaired) factors:

  \[
  \text{LCD} = 3 \cdot 3 \cdot 2 = 18
  \]
Let's look at another example:

**Example:** Find the least common multiple of 72 and 24. Write the LCM in prime-factored form.

**Steps:**

- Make a factor tree for each number:
  - 72
    - 72
    - 36
      - 2
      - 18
        - 9
          - 3
          - 3
  - 24
    - 24
    - 12
      - 6
        - 3

- Pair off common factors:
  - 72 = 2 • 2 • 2 • 3 • 3
  - 24 = 2 • 2 • 2 • 3

- Multiply all common factors by all leftover (unpaired) factors:

  \[
  \text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72
  \]
On Your Own: Solve the following problems, using the factor tree/pairing method.

1. What is the prime factored form of the lowest common denominator of \(\frac{5}{9} + \frac{5}{12}\)?

\[
\begin{align*}
9 &= 3 \cdot 3 \\
12 &= 3 \cdot 2 \cdot 2 \\
\text{LCM} &= 3 \cdot 3 \cdot 2 \cdot 2 = 36
\end{align*}
\]

2. Find the least common multiple, in prime-factorization form, of 12 and 15.

\[
\begin{align*}
12 &= 3 \cdot 2 \cdot 2 \\
15 &= 3 \cdot 5 \\
\text{LCM} &= 3 \cdot 2 \cdot 2 \cdot 5 = 60
\end{align*}
\]

We will now look at the second method to find the prime factored from of the lowest common denominator (LCD) of two fractions.
Method II: Factor Tree and Venn Diagram

To illustrate this second method, let's return to the original problem:

Example: Find the prime factored form for the lowest common denominator of \( \frac{5}{6} + \frac{5}{9} \).

- Use the factor tree method to find the prime factored form of 6:

\[
\begin{array}{c}
6 \\
\downarrow \\
2 \\
\downarrow \\
3
\end{array}
\]

- Use the factor tree method to find the prime factored form of 9:

\[
\begin{array}{c}
9 \\
\downarrow \\
3 \\
\downarrow \\
3
\end{array}
\]

- Use a Venn diagram to find the prime-factored form of the lowest common denominator:

On the next page, we will learn how to fill out this diagram.
Venn diagrams are **overlapping circles** that help us compare and contrast the characteristics of different things. We can use them to find what is **common** to two items (where the circles overlap **in the middle**) and what is **different** between them (what is **outside** the overlap on either or both sides).

Here, we want to find out which prime factors are the same for two numbers and which factors are distinct, or different.

![Venn Diagram](image)

**Steps:**

- Since only one 3 is **common** to both numbers, we need to put it in the **middle**, where the two circles overlap:

![Updated Venn Diagram](image)
Now find the prime factors that are left for 6 and place them in the part of the circle for 6 that does **not overlap** with the circle for 9.

![Diagram](image)

Next, find the prime factors that are left for 9 and place them in the part of the circle that does **not overlap** with the circle for 6.

![Diagram](image)

The lowest common denominator for 6 and 9 is the **product** of **all** of the **numbers in the circles**:

$$2 \cdot 3 \cdot 3$$, which is equal to **18**.

**Note:** To write the LCD in **prime-factored form**, we do not carry out the multiplication; we just write the **prime numbers**:

**LDC of 6 and 9 = 2 \cdot 3 \cdot 3**
On Your Own

1. What is the prime factored form of the lowest common denominator of $\frac{1}{6}$ and $\frac{3}{10}$?

- Create separate prime factor trees for both denominators:

$\begin{array}{c}
6 \\
\big/ \\
\big/ \\
2 \ 3
\end{array}$

$\begin{array}{c}
10 \\
\big/ \\
\big/ \\
2 \ 5
\end{array}$

- Organize the prime factors of both denominators, using a Venn diagram:

$\begin{array}{c}
6 && \text{Both} && 10 \\
A && 2 && B \\
3 && 5 &&
\end{array}$

What is the LCD? 30

What is the LDD in prime-factored form? $2 \times 3 \times 5$
2. Find the prime factored form of the lowest common denominator for the following:

\[
\frac{5}{8} + \frac{11}{12}
\]

**Factor Trees:**

\[
\begin{align*}
8 & \quad 12 \\
\div & \quad \div \\
2 & \quad 2 \\
\div & \quad \div \\
2 & \quad 3 \\
\end{align*}
\]

**LCD:** \(24\)

**LCD in prime factored form:** \(2 \times 2 \times 2 \times 3\)
i. **Multiplying Fractions**

Whenever you are asked to find a fraction of a number, you need to multiply. In math, the word “of” means *multiply*.

**Example:** Find \( \frac{1}{2} \) of \( \frac{1}{2} \).

This is a multiplication problem. It means, “What is \( \frac{1}{2} \cdot \frac{1}{2} \)?”

We can represent the problem visually. Here is the first part of the problem: \( \frac{1}{2} \) of the circle has been shaded.

Taking \( \frac{1}{2} \) of a number means *dividing it by 2*.

Now, if we take one-half of this again (divide it by 2 again), we get the following:

\[
\frac{1}{2} \text{ of } \frac{1}{2} \text{ is equal to } \frac{1}{4}.
\]

We end up with one-fourth of the circle.

**Note:** We also could have solved the above problem by multiplying the numerator by the numerator and the denominator by the denominator:

\[
\frac{\text{Numerator} \cdot \text{Numerator}}{\text{Denominator} \cdot \text{Denominator}} = \frac{1 \cdot 1}{2 \cdot 2} = \frac{1}{4}
\]
When working these problems out during the CAHSEE, you will need to apply this rule:

\[
\frac{\text{Numerator} \cdot \text{Numerator}}{\text{Denominator} \cdot \text{Denominator}}
\]

Look at the next problem:

Find \( \frac{1}{2} \) of 24. In math, we can write this as follows: \( \frac{1}{2} \cdot 24 \)

The first factor is a fraction and the second factor is a whole number. We can easily change the second factor to a fraction because any whole number can be expressed as a fraction by placing it over a 1:

\[ 24 = \frac{24}{1} \text{ because 24 means 24 ones.} \]

We can rewrite the problem as follows: \( \frac{1}{2} \cdot \frac{24}{1} \)

Now, just follow the rule for multiplying two fractions:

\[
\frac{\text{Numerator} \cdot \text{Numerator}}{\text{Denominator} \cdot \text{Denominator}}
\]

\[
\frac{1}{2} \cdot \frac{24}{1} = \frac{24}{2} = 12
\]

\textbf{Note:} Taking \( \frac{1}{2} \) of 24 means dividing 24 by 2.
Now look at the next example:

Example: \( \frac{24}{1} \cdot \frac{5}{6} = \) \_

There are two ways to solve this problem:

1. The hard way: Perform all operations

   - Multiply numerators: \( 24 \cdot 5 \)
   - Multiply denominators: \( 1 \cdot 6 \)
   - Divide new numerator by denominator: \( 120 \div 6 \)

   \[
   \frac{24 \cdot 5}{1 \cdot 6} = \frac{120}{6} = 20
   \]

2. The easy way: Simplify first, and then multiply:

   \[
   \frac{4}{1} \cdot \frac{5}{6} = \frac{20}{1} \quad \text{Simplify by dividing out common factors!}
   \]

Look at the following problems:

\[
\frac{533}{4} \cdot 4 = 533 \quad 3,435 \cdot \frac{9}{9} = 3,435 \quad 79 \cdot \frac{3}{3} = 79
\]

Do you need to work out these problems, or do you already know the answers? **No need to work out problems: in each case, there is a common factor that can be divided out in both the numerator & denominator.**

**Remember:** If you divide both a numerator and denominator by a common factor, you can make the problem much simpler to solve. So save yourself the time and work, and recognize these types of problems right away.
Look at the next set of problems:

\[
\frac{4}{12} \cdot \frac{6}{8} \quad \frac{8}{15} \cdot \frac{5}{12} \quad \frac{14}{21} \cdot \frac{3}{7}
\]

What do you notice about the above problems? **There are common factors in the numerators and denominators of each fraction.**

There is a lot of heavy multiplication involved in these problems. Is there a way to make your work easier? Explain:

We can simplify fractions by dividing out common factors before solving.

We can simplify these problems quite a bit before solving. This makes our job easier. Let's look at the first problem:

\[
\frac{4}{12} \cdot \frac{6}{8}
\]

We can divide out common factors in each fraction. These common factors become clear if we write each fraction as a product of prime factors. Let's begin with the first fraction:

\[
\frac{4}{12} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} \cdot 3} = \frac{1}{3}
\]

Now do the second fraction on your own:

\[
\frac{6}{8} = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{1}{2} \cdot \frac{1}{2} \cdot 2} = \frac{3}{4}
\]

Now let's multiply the two reduced fractions; but first, can we simplify anymore? **Yes** If so, simplify first, and then multiply:

\[
\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{4}
\]
On Your Own: Simplify and solve:

\[
\frac{8}{15} \cdot \frac{5}{12} = \frac{2}{9}
\]

\[
\frac{14}{21} \cdot \frac{3}{7} = \frac{2}{7}
\]

\[
\frac{3}{5} \cdot \frac{10}{21} = \frac{2}{7}
\]

\[
\frac{12}{15} \cdot \frac{5}{6} = \frac{2}{3}
\]

\[
36 \cdot \frac{2}{3} = 24
\]

\[
27 \cdot \frac{2}{9} = 6
\]

\[
\frac{2}{3} \cdot \frac{3}{2} = 1
\]

\[
\frac{10}{15} \cdot \frac{3}{2} = 1
\]

Note to Tutors: Be sure that students understand that they can cancel out factors shared between a numerator and any of the denominators:
ii. Dividing Fractions

When you divide something by a fraction, think, “How many times does the fraction go into the dividend?”

Example: \(3 \div \frac{1}{2}\)

This means, “How many times does \(\frac{1}{2}\) go into 3?”

We can represent this visually:

Answer: 6

Example: \(2 \div \frac{1}{8}\)

We can represent this visually:

Answer: 16
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1.2

On Your Own: Solve the next few problems, asking each time, “How many times does the fraction go into the whole number?”

\[ 3 \div \frac{1}{4} = 12 \]

\[ 3 \div \frac{1}{8} = 24 \]

\[ 4 \div \frac{1}{8} = 32 \]

Do you see a pattern? Explain. In each case, the answer is the product of the whole number and the fraction turned upside down, or inverted.
Reciprocals

As we saw in the previous exercise, each time we divide a whole number by a fraction, we get as our answer the product of the whole number and the reciprocal of the fraction.

Reciprocal means the flip-side, or inverse.

Example: The reciprocal of \( \frac{4}{5} \) is \( \frac{5}{4} \).

On Your Own: Find the reciprocal of each fraction:

\[
\begin{align*}
\frac{3}{4} & \rightarrow \frac{4}{3} \\
\frac{7}{9} & \rightarrow \frac{9}{7} \\
\frac{12}{5} & \rightarrow \frac{5}{12} \\
\frac{13}{1} & \rightarrow \frac{1}{13} \\
\frac{35}{53} & \rightarrow \frac{53}{35} \\
\frac{1}{12} & \rightarrow \frac{12}{1}
\end{align*}
\]

Now let's find the reciprocal of a whole number. We know that any whole number (or integer) can be expressed as a fraction by placing it over 1:

Example: \( 35 = \frac{35}{1} \)

The reciprocal is the fraction turned upside down, or inverted:

Example: The reciprocal of \( 35 \) is \( \frac{1}{35} \)

On Your Own: Find the reciprocal of each integer.

\[
\begin{align*}
1000 & \rightarrow \frac{1}{1000} \\
121 & \rightarrow \frac{1}{121} \\
173 & \rightarrow \frac{1}{173} \\
-18 & \rightarrow \frac{-1}{18} \\
-100 & \rightarrow \frac{-1}{100}
\end{align*}
\]
Now we are ready to divide a whole number by a fraction.

**Example:**

\[
2 \div \frac{1}{10} = \frac{2}{1} \cdot \frac{10}{1} = \frac{20}{1} = 20
\]

We can represent the above problem visually:

\[
2 \div \frac{1}{10}
\]

means . . .

If we count the number of little rectangles in the two big rectangles, we get **20**.

**On Your Own**

\[
3 \div \frac{1}{5} = 3 \cdot 5 = 15
\]

\[
6 \div \frac{1}{5} = 6 \cdot 5 = 30
\]

\[
5 \div \frac{1}{3} = 5 \cdot 3 = 15
\]

\[
2 \div \frac{1}{3} = 2 \cdot 3 = 6
\]

\[
4 \div \frac{1}{2} = 4 \cdot 2 = 8
\]
Simplifying Division Problems 1.2

Example: \[ \frac{3}{5} \div \frac{9}{10} \]

Remember the rule for dividing fractions:

**Rule:** When dividing fractions, multiply the first fraction by the reciprocal of the second fraction!

**Steps:**

- Multiplying the first fraction by the reciprocal of the second fraction, we get . . .

\[ \frac{3}{5} \cdot \frac{10}{9} \]

- We can simplify this problem by dividing out common factors:

\[ \frac{\frac{1}{3} \cdot \frac{10}{2}}{\frac{1}{5} \cdot \frac{9}{3}} \]

- Now, apply the rule for multiplication:

\[ \frac{\text{Numerator} \cdot \text{Numerator}}{\text{Denominator} \cdot \text{Denominator}} = \frac{1 \cdot 2}{1 \cdot 3} = \frac{2}{3} \]
On Your Own: Simplify and solve.

\[ \frac{1}{4} \div \frac{3}{8} = \frac{1}{4} \cdot \frac{8}{3} = \frac{2}{3} \]

\[ \frac{2}{5} \div \frac{3}{10} = \frac{2}{5} \cdot \frac{10}{3} = \frac{4}{3} = 1 \frac{1}{3} \]

\[ \frac{3}{7} \div \frac{9}{14} = \frac{3}{7} \cdot \frac{14}{9} = \frac{2}{3} \]

\[ \frac{5}{8} \div \frac{15}{24} = \frac{5}{8} \cdot \frac{24}{15} = \frac{3}{3} = 1 \]

\[ \frac{1}{10} \div \frac{13}{10} = \frac{1}{10} \cdot \frac{10}{13} = \frac{1}{13} \]
Unit Quiz: The following problems appeared on the CAHSEE.

1. \[ \frac{11}{12} - \left( \frac{1}{3} + \frac{1}{4} \right) = \]
   A. \( \frac{1}{3} \)
   B. \( \frac{3}{4} \)
   C. \( \frac{5}{6} \)
   D. \( \frac{9}{5} \)  

2. Which fraction is equivalent to \( \frac{5}{6} + \frac{7}{8} \)?
   A. \( \frac{35}{48} \)
   B. \( \frac{6}{7} \)
   C. \( \frac{20}{21} \)
   D. \( \frac{41}{24} \)  

3. What is the prime factored form for the lowest common denominator of the following: \( \frac{2}{9} + \frac{7}{12} \)?
   A. \( 3 \times 2 \times 2 \)
   B. \( 3 \times 3 \times 2 \times 2 \)
   C. \( 3 \times 3 \times 3 \times 2 \times 2 \)
   D. \( 9 \times 12 \)
4. Which of the following is the prime factored form of the lowest common denominator of $\frac{7}{10} + \frac{8}{15}$?

A. $5 \times 1$
B. $2 \times 3 \times 5$
C. $2 \times 5 \times 3 \times 5$
D. $10 \times 15$  

5. Which of the following numerical expressions results in a negative number?

A. $(-7) + (-3)$
B. $(-3) + (7)$
C. $(3) + (7)$
D. $(3) + (-7) + (11)$  

6. One hundred is multiplied by a number between 0 and 1. The answer has to be ____.

A. less than 0.
B. between 0 and 50 but not 25.
C. between 0 and 100 but not 50.
D. between 0 and 100.  

7. If $|x| = 3$, what is the value of $x$?

A. -3 or 0
B. -3 or 3
C. 0 or 3
D. -9 or 9  

Standard 2.2
8. What is the absolute value of -4?
   A. -4
   B. $-\frac{1}{4}$
   C. $\frac{1}{4}$
   D. 4

9. The winning number in a contest was less than 50. It was a multiple of 3, 5, and 6. What was the number?
   A. 14
   B. 15
   C. 30
   D. It cannot be determined

10. If $n$ is any odd number, which of the following is true about $n+1$?
    A. It is an odd number.
    B. It is an even number
    C. It is a prime number
    D. It is the same as $n-1$.

11. Which is the best estimate of 326 X 279?
    A. 900
    B. 9,000
    C. 90,000
    D. 900,000
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12. The table below shows the number of visitors to a natural history museum during a 4-day period.

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of Visitors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>597</td>
</tr>
<tr>
<td>Saturday</td>
<td>1115</td>
</tr>
<tr>
<td>Sunday</td>
<td>1346</td>
</tr>
<tr>
<td>Monday</td>
<td>365</td>
</tr>
</tbody>
</table>

Which expression would give the BEST estimate of the total number of visitors during this period?

A. $500 + 1100 + 1300 + 300$
B. $600 + 1100 + 1300 + 300$
C. $600 + 1100 + 1300 + 400$
D. $600 + 1100 + 1400 + 400$  
Standard 1.2 (& MR)

13. John uses $\frac{2}{3}$ of a cup of oats per serving to make oatmeal. How many cups of oats does he need to make 6 servings?

A. $2 \frac{2}{3}$  
B. 4  
C. $5 \frac{1}{3}$  
D. 9  
Standard 1.2

14. If $a$ is a positive number and $b$ is a negative number, which expression is always positive?

A. $a - b$  
B. $a + b$  
C. $a \times b$  
D. $a \div b$  
Standard 1.2
Unit 2: Exponents (1.1, 1.2, 2.1, 2.3, 2.4)

On the CAHSEE, you will be given several problems on exponents.

Exponents are a shorthand way of representing how many times a number is multiplied by itself.

Example: $9 \cdot 9 \cdot 9 \cdot 9$ can be expressed as $9^4$ since four 9's are multiplied together.

Base $\leftarrow 9^4 \rightarrow$ exponent

The number being multiplied is called the base.

The exponent tells how many times the base is multiplied by itself. $9^4$ is read as “9 to the 4th power,” or “9 to the power of 4.”

Let's look at another example: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

On Your Own

$2^3 = 8$ \hspace{1cm} $2^4 = 16$

$3^2 = 9$ \hspace{1cm} $3^3 = 27$

Power of 0

Any number raised to the 0 power (except 0) is always equal to 1.

Example: $100^0 = 1$

On Your Own

$7^0 = 1$ \hspace{1cm} $293^0 = 1$ \hspace{1cm} $(-131)^0 = 1$ \hspace{1cm} $47^0 = 1$
Power of 1

A number raised to the 1\textsuperscript{st} power (i.e., an exponent of 1) is always equal to that number.

Example: \(100^1 = 100\)

On Your Own

\(7^1 = 7\) \hspace{1cm} \(293^1 = 293\) \hspace{1cm} \((-131)^1 = -131\) \hspace{1cm} \(47^1 = 47\)

Power of 2 (Squares)

A number raised to the 2\textsuperscript{nd} power is referred to as the square of a number. When we square a whole number, we multiply it by itself.

Example: \(12^2 = 12 \times 12 = 144\)

The square of any whole number is called a perfect square.

Here are the first 3 perfect squares:

\(1^2 = 1 \times 1 = 1\) \hspace{1cm} \(2^2 = 2 \times 2 = 4\) \hspace{1cm} \(3^2 = 3 \times 3 = 9\)

On Your Own: Write the perfect squares for the following numbers:

\(4^2 = 16\) \hspace{1cm} \(5^2 = 25\) \hspace{1cm} \(6^2 = 36\) \hspace{1cm} \(7^2 = 49\)

\(8^2 = 64\) \hspace{1cm} \(9^2 = 81\) \hspace{1cm} \(10^2 = 100\) \hspace{1cm} \(11^2 = 121\)

\(20^2 = 400\) \hspace{1cm} \((2 - 8)^2 - (3 - 7)^2 = 20\) \hspace{1cm} \(3^2 + 5^2 = 36\)
Square Roots

The square root \( (\sqrt{\cdot}) \) of a number is one of its two equal factors.

Example: \( 8^2 = 64 \)

Any number raised to the second power (the power of 2) can be represented as a square. That’s why it’s called “squaring the number.”

The square above has 64 units. Each side (the length and width) is 8 units. The area of the square is determined by multiplying the length (8 units) by the width (8 units). The square root is the number of units in each of the two equal sides: 8

Note: 64 has a second square root: -8 (-8 \cdot -8 = +64). However, when we are asked to evaluate an expression, we always take the positive root.

Example: Find the square root of 36.

Answer: \( \sqrt{36} = 6 \)
On Your Own

1. \( \sqrt{25} = 5 \)

2. \( \sqrt{16} = 4 \)

3. \( \sqrt{100} = 10 \)

4. \( \sqrt{81} = 9 \)

5. \( \sqrt{49} = 7 \)

6. \( \sqrt{121} = 11 \)

7. \( \sqrt{400} = 20 \)

8. \( \sqrt{4} + \sqrt{9} = 5 \)

9. \( \sqrt{3^2 + 4^2} = 5 \)

10. Which is not a perfect square?

   A. 144
   B. 100
   C. 48
   D. 169
Power of 3 (Cubes) 1.2

A number with an exponent of 3 (or a number raised to the 3\textsuperscript{rd} power) is the **cube of a number**.

**Example:** \(5^3 = 5 \cdot 5 \cdot 5 = 125\)

The cube of a whole number is called a **perfect cube**.

**Cubes of Positive Numbers**

The cube of a positive number will always be a positive number.

\[
1^3 = 1 \cdot 1 \cdot 1 = 1 \quad 2^3 = 2 \cdot 2 \cdot 2 = 8
\]

**Cubes and Negative Numbers**

The cube of a negative number will always be a negative number.

\[
(-1)^3 = (-1)(-1)(-1) = -1 \quad (-2)^3 = (-2)(-2)(-2) = -8
\]

**On Your Own:** Write the perfect cubes for the following numbers?

\[
3^3 = 27 \quad 4^3 = 64 \quad 5^3 = 125
\]

\[
-3^3 = -27 \quad -4^3 = -64 \quad -5^3 = -125
\]
Cube Roots

The cube root of a number is one of its three equal factors. The cube root of a positive number will always be a positive number.

Example: What is the cube root of 27?

The cube root of 27 is written as $\sqrt[3]{27}$

To find the cube root of 27, ask, “What number multiplied by itself 3 times is equal to 27?”

$\_ \cdot \_ \cdot \_ = 27$

$3 \cdot 3 \cdot 3 = 27$, or $3^3 = 27$.

On Your Own:

$\sqrt[3]{8} = 2$  $\sqrt[3]{64} = 8$  $\sqrt[3]{1,000} = 10$  $\sqrt[3]{125} = 5$

Cube Roots of Negative Numbers

The cube root of a negative number will always be a negative number.

Example: $\sqrt[3]{-64}$

Ask, “What number multiplied by itself 3 times is equal to -64?”

$\_ \cdot \_ \cdot \_ = -64$

$-4 \cdot -4 \cdot -4 = -64$, or $(-4)^3 = -64$

On Your Own: $\sqrt[3]{-1,000} = -10$  $\sqrt[3]{-125} = -5$  $\sqrt[3]{-8} = -2$
Raising Fractions to a Power

When raising a fraction to an exponent, both the **numerator** (top number) and the **denominator** (bottom number) are raised to the exponent:

\[
\begin{array}{c}
\frac{2}{5} \\
\end{array}
\]

**Example:** \((2/5)^3 = 2 \cdot 2 \cdot 2 = \frac{8}{5 \cdot 5 \cdot 5} = \frac{8}{125}\)

On Your Own

\[
\begin{array}{ccc}
(1/4)^2 = & \frac{1}{16} & (1/2)^4 = \frac{1}{16} & (2/3)^3 = \frac{8}{27} \\
(2/5)^3 = & \frac{8}{125} & (4/7)^2 = \frac{16}{49} & (6/11)^2 = \frac{36}{121} \\
(3/4)^3 = & \frac{27}{64} & (5/12)^2 = \frac{25}{144} & (2/5)^2 = \frac{4}{25}
\end{array}
\]

Taking the Root of a Fraction

When taking the root of a fraction, we must take the root of both the numerator and denominator.

**Example:**

\[
\sqrt[3]{\frac{25}{81}} = \frac{\sqrt[3]{25}}{\sqrt[3]{81}} = \frac{5}{9}
\]

On Your Own

\[
\begin{array}{ccc}
\sqrt[3]{9/16} = & \frac{3}{4} & \sqrt[3]{25/100} = \frac{5}{10} = \frac{1}{2} \\
\sqrt[3]{36/81} = & \frac{6}{9} = \frac{2}{3} & \sqrt[3]{27/64} = \frac{3}{4}
\end{array}
\]

\[
\begin{array}{ccc}
\sqrt[3]{1/125} = & \frac{1}{5} & \sqrt[3]{81/121} = \frac{9}{11} \\
\sqrt[3]{1/9} = & \frac{1}{3} & \sqrt[3]{8/27} = \frac{2}{3}
\end{array}
\]
Raising Negative Numbers to a Power

When raising a negative number to a power, we are raising both the number and the negative sign. The answer may be positive or negative:

A. If the exponent is an even number, the answer will be a positive number (as in the example below) since a negative multiplied by a negative equals a positive.

Example: \((-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64\)

Note: \((-2)^6\) and \(-2^6\) are two different problems:
- \(-2^6\) tells us to multiply positive 2 by itself 6 times
  \((2)(2)(2)(2)(2)(2) = 64\)
  and then take the negative of that answer: \(-64\)
- \((-2)^6\) tells us to multiply -2 by itself 6 times:
  \((-2)(-2)(-2)(-2)(-2)(-2) = +64\)

B. If the exponent is an odd number, the answer will be a negative number (a positive multiplied by a negative equals a negative).

Example: \((-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32\)

On Your Own

\((-3)^2 = 9\) \hspace{1cm} \(-3^3 = -27\) \hspace{1cm} \((-2)^2 = 4\) \hspace{1cm} \((-2)^3 = -8\)

\(-4^2 = -16\) \hspace{1cm} \(-4^3 = -64\) \hspace{1cm} \((-5)^2 = 25\) \hspace{1cm} \(-5^2 = -25\)
Negative Exponents

On the CAHSEE, you may be given an expression with a negative exponent. When an exponent is negative, the expression represents a fraction:

Example: \(3^{-3} \text{ means } \frac{1}{3^3}\)

Notice that the exponent is now positive.

Remember: Any whole number can be written as a fraction by placing it over 1:

\(3^{-3} \text{ can also be written as } \frac{3^{-3}}{1}\)

We now flip, or invert the fraction and make the exponent positive.

\[\frac{3^{-3}}{1} \rightarrow \frac{1}{3^3}\]

In the above example, the numerator is equal to 1, and the denominator consists of the base and the (positive) exponent.

On Your Own: Invert the fraction and make the exponent positive.

\(3^{-2} = \frac{1}{9}\) \hspace{1cm} \(5^{-3} = \frac{1}{125}\) \hspace{1cm} \(2^{-3} = \frac{1}{8}\) \hspace{1cm} \(4^{-2} = \frac{1}{16}\)
Negative Exponents and Fractions  2.1

When raising a fraction to a negative exponent, just **invert** the entire fraction and make the **exponents positive**:  

**Example:** \( \frac{1}{3^{-3}} \)  

Here we have a fraction whose denominator consists of the base and a negative exponent. If we **invert the fraction**, the **exponent** becomes **positive**:  

\[
\frac{1}{3^{-3}} \rightarrow \frac{3^3}{1} \rightarrow 3^3 
\]

**On Your Own**  

1. \( \left( \frac{1}{3} \right)^{-2} = \left( \frac{3}{1} \right)^2 = 9 \)  
2. \( \left( \frac{2}{3} \right)^{-1} = \left( \frac{3}{2} \right)^1 = 1\frac{1}{2} \)  
3. \( \left( \frac{1}{8} \right)^{-2} = 64 \)  
4. \( \left( \frac{3}{5} \right)^{-2} = 25 \)  
5. \( \left( \frac{1}{3} \right)^{-3} = 27 \)
2.1

Multiplying Expressions Involving Exponents with a Common Base

On the CAHSEE, you may be asked to multiply expressions involving exponents.

**Example:** $3^5 \cdot 3^4$

In order to multiply expressions involving exponents, there must be a common base. In the above example, the base (3) is common to both terms.

When we have a common base, the rule for multiplying the expressions is simple: **keep the base and add the exponents:**

$$\text{Base}^5 \cdot \text{Base}^4 = \text{Base}^{5+4} \rightarrow 3^5 \cdot 3^4 = 3^{5+4} = 3^9$$

**On Your Own**

$$2^2 \cdot 2^8 = 2^{10}$$

$$3^1 \cdot 3^7 = 3^8$$

$$4^3 \cdot 4^2 = 4^5$$

$$4^4 \cdot 4^{-3} = 4^1 = 4$$

$$3^{-3} \cdot 3^3 = 3^0 = 1$$

$$6^3 \cdot 6^3 = 6^6$$
Note: In some cases, we may end up with a negative exponent. Remember to apply the rules for negative exponents: invert the fraction and make the exponent positive.

Example: \(3^2 \cdot 3^{-3} = 3^{2+(-3)} = 3^{-1} = \frac{3^{-1}}{1} = \frac{1}{3}\)

On Your Own

\[4^1 \cdot 4^{-3} = 4^{-2} = \frac{1}{16}\]

\[5^{-1} \cdot 5^{-1} = 5^{-2} = \frac{1}{25}\]

\[3^5 \cdot 3^{-8} = 3^{-3} = \frac{1}{27}\]

\[4^4 \cdot 4^{-7} = 4^{-3} = \frac{1}{64}\]

Dividing Expressions Involving Exponents with a Common Base

On the CAHSEE, you may be asked to divide expressions involving exponents.

For these types of problems, there must be a common base:

Example: \(\frac{3^5}{3^3}\)

To divide exponents with a common base, keep the base and subtract the exponent in the denominator from the exponent in the numerator:

\[\frac{Base^5}{Base^3} = Base^{5-3} = Base^2 \quad \frac{3^5}{3^3} = 3^{5-3} = 3^2\]
On Your Own

\[
\frac{3^3}{3^2} = \frac{3^1}{3^1} = 3 \\
\frac{4^5}{4^3} = \frac{4^2}{4^1} = 16
\]

\[
\frac{5^4}{5^2} = \frac{5^2}{5^1} = 25 \\
\frac{2^8}{2^3} = \frac{2^5}{2^1} = 32
\]

The next problem is a bit more complicated: \(\frac{3^{-2}}{3^{-3}}\)

Remember, when an exponent is negative, the expression is a fraction and the numerator (top number) is always equal to 1, while the denominator (bottom number) is the base. But, here, the problem is already a fraction, so we really have one fraction over another fraction. We will get back to the above problem in a moment, but first, let’s do a quick review of the rules for dividing fractions:

**Dividing Fractions**

To divide two fractions, we multiply the 1st fraction by the reciprocal of the 2nd fraction. This means that we invert the second fraction over, or invert it.

For example, the reciprocal of \(\frac{2}{5}\) is \(\frac{5}{2}\).

Let’s solve the following problem:

\[
\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \cdot \frac{3}{1} = \frac{9}{4}
\]

\(\frac{9}{4}\) is an improper fraction (numerator > denominator). We must change this to a mixed fraction (whole number and fraction): \(2 \frac{1}{4}\)
Now we are ready to tackle the earlier problem: \( \frac{3^{-2}}{3^{-3}} \).

We have **negative exponents** in both the numerator and the denominator. We can therefore rewrite each as fractions with **positive exponents**:

\[
3^{-2} = \frac{1}{3^2} \quad \text{and} \quad 3^{-3} = \frac{1}{3^3}
\]

We can now rewrite the original problem as follows: \( \frac{1}{3^2} \div \frac{1}{3^3} \).

Applying the rule for dividing two fractions, we **invert the second fraction** and **multiply**:

\[
\frac{1}{3^2} \cdot 3^3 = \frac{3^3}{3^2}
\]

Multiplying the numerator by the numerator, and the denominator by the denominator, we get \( \frac{3^3}{3^2} \).

Now we apply the rules for dividing exponents: the **base remains the same** and we **subtract** the exponent in the denominator from the exponent in the numerator:

\[
\frac{3^3}{3^2} = 3^{3-2} = 3^1 = 3
\]

**Shortcut!**

Since both exponents in the above example are negative, a quicker way to solve the problem is to just **invert** the fractions and **reverse the sign of each exponent**; then simplify:

\[
\frac{3^{-2}}{3^{-3}} = \frac{3^3}{3^2} = 3^{3-2} = 3^1 = 3
\]
Another Shortcut!

A third way to solve the problem is to apply the rule for dividing expressions with exponents: keep the base and subtract the exponent in the denominator from the exponent in the numerator:

\[
\frac{\text{Base}^{-2}}{\text{Base}^{-3}} = \text{Base}^{-2 - (-3)} = \text{Base}^{-2 + 3} \longrightarrow \text{Base}^1
\]

\[
\frac{3^{-2}}{3^{-3}} = 3^{-2 - (-3)} = 3^{-2 + 3} \longrightarrow 3^1 = 3
\]

On Your Own

\[
\frac{2^{-8}}{2^{-3}} = 2^{-5} = \frac{1}{32}
\]

\[
\frac{8^{-4}}{8^{-2}} = \frac{1}{64}
\]

\[
\frac{2^{-7}}{2^{-1}} = 2^{-6} = \frac{1}{64}
\]

\[
\frac{2^2}{2^3} = 2^3 = 32
\]

\[
\frac{2^{-2}}{2^3} = \frac{1}{32}
\]
Power Raised to a Power 2.3

When raising a power to a power, multiply the powers together:

Example: \((2^3)^2 = 2^{(3) \cdot (2)} = 2^6\)

This is easy to see if you expand the exponents:

\((2^3)^2 = \)

\((2^3)(2^3) =\)

\((2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2) =\)

\(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 =\)

\(2^6\)

On Your Own

\((y^3)^2 = y^6\) \hspace{1cm} \((2^4)^2 = 2^8\)

\((n^3)^3 = n^9\) \hspace{1cm} \((5^4)^3 = 5^{12}\)

\((2^2)^37 = 2^{74}\) \hspace{1cm} \((x^2y^3)^2 = x^4y^6\)
Square Roots of Non-Perfect Squares  

Remember that when we multiply a whole number by itself, we get a perfect square. And the square root of a perfect square is the factor that, when multiplied by itself, gave us the perfect square.

For example, the square root of 64 is 8 because $8 \cdot 8 = 64$.

But whole numbers that are not perfect squares still have square roots. However, their square roots are not whole numbers; rather they are **decimals** or **fractions** of whole numbers.

On the CAHSEE, you may be given a non-perfect square and asked to place its root between two **consecutive whole numbers**.

**Example:** Between what two consecutive whole numbers is $\sqrt{153}$?

**Solution:**

Think about our list of perfect squares. Refer to the chart on the next page.

Since 153 falls between **144** and **169** in our perfect squares list, the square root of 153 is between **12** and **13**. (Note: 12 and 13 are the square roots of 144 and 169 respectively).
Memorize these for the CAHSEE!

<table>
<thead>
<tr>
<th>Number</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
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<td>11</td>
<td>121</td>
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<tr>
<td>12</td>
<td>144</td>
</tr>
<tr>
<td>13</td>
<td>169</td>
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<tr>
<td>14</td>
<td>196</td>
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<td>15</td>
<td>225</td>
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<tr>
<td>16</td>
<td>256</td>
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<td>17</td>
<td>289</td>
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<td>18</td>
<td>324</td>
</tr>
<tr>
<td>19</td>
<td>361</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
</tr>
</tbody>
</table>
1. Between what two consecutive whole numbers is the square root of 17?
   **Answer: 4 & 5**

2. Between which two consecutive whole numbers is $\sqrt{200}$?
   **Answer: 14 & 15**

3. Between which two consecutive whole numbers is $\sqrt{130}$?
   **Answer: 11 and 12**

4. The square root of 140 is between which two numbers?
   **Answer: 11 & 12**

5. Between which two integers does $\sqrt{53}$ lie?
   **Answer: 7 and 8**
Roots and Exponents

On the CAHSEE, you may be given a variable that has been raised to a power and asked to find the base (the original number before it was raised).

Example: If \( x^2 = 25 \), find the value for \( x \).

Since the base \((x)\) is raised to the second power, we can find the value for \( x \) by taking the square root of \( x^2 \). Since we have an equation, we must also find the square root of 25 so that the two sides of the equation remain in balance.

\[
\sqrt{x^2} = \sqrt{25} \quad \rightarrow \quad x = 5
\]

You may also be given the root of a variable and asked to find the variable.

Example: \( \sqrt{x} = 5 \)

To solve this, we need to square both sides of the equation:

\[
(\sqrt{x})^2 = (5)^2 \quad \rightarrow \quad x = 25
\]

On Your Own: Find \( x \).

\[
\begin{align*}
x^2 &= 64 & x &= 8 & x^3 &= 8 & x &= 2 \\
x^3 &= \frac{1}{27} & x &= \frac{1}{3} & x^2 &= \frac{9}{16} & x &= \frac{3}{4} \\
x^3 &= \frac{8}{27} & x &= \frac{2}{3} & x^2 &= \frac{4}{25} & x &= \frac{2}{5} \\
\sqrt{x} &= 11 & x &= 121 & \sqrt[3]{x} &= 10 & x &= 1000 \\
\sqrt{x} &= 20 & x &= 400 & \sqrt{x} &= 9 & x &= 81
\end{align*}
\]
Scientific Notation

**Scientific Notation** is a way to express very small or very large numbers, using exponents.

**Example of a Very Big Number:** The distance of the earth from the sun is approximately 144,000,000,000 meters.

You can see that it can be tedious to write so many zeroes. This number can be expressed much more simply in scientific notation: \(1.44 \times 10^{11}\)

**Example of a Very Small Number:** An example using a very small number is the mass of a dust particle: 0.000000000 753 kg.

We can write this number in scientific notation as \(7.53 \times 10^{-10}\).

On the CAHSEE, you will need to . . .

- Read numbers in scientific notation
- Compare numbers in scientific notation
- Convert from **standard** notation (15,340) to **scientific** notation (1.534 \(\times 10^4\))
- Convert from **scientific** notation (2.36 \(\times 10^{-3}\)) to **standard** notation (.00236)
Scientific Notation is a special type of exponent expression: the base is always 10 and it is raised to a positive or negative power.

A number written in scientific notation consists of four parts:

i. $4.95 \times 10^{-2}$
   - a number (n) greater than or equal to 1 and less than 10

ii. $4.95 \times 10^{-2}$
    - a multiplication sign

iii. $4.95 \times 10^{-2}$
    - the base, which is always 10

iv. $4.95 \times 10^{-2}$
    - a positive or negative exponent
### Examples

<table>
<thead>
<tr>
<th>Correct Scientific Notation?</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>$1 \leq n &lt; 10$</td>
</tr>
<tr>
<td>No</td>
<td>$n &gt; 10$</td>
</tr>
</tbody>
</table>

**Remember:** For an expression to be written in correct scientific notation, the number ($n$) that appears before the base must be greater or equal to 1 and less than 10.

**On Your Own:** Check all expressions in correct scientific notation:

- $3.2 \times 10^{13}$
- $23.6 \times 10^{12}$
- $5.788 \times 10^3$
- $5.788 \times 10^{-3}$
- $57.88 \times 10^2$
- $2.36 \times 10^3$
- $2.36 \times 10^2$
- $0.0236 \times 10^8$
- $0.236 \times 10^7$
- $2.3 \times 10^7$
- $2.3 \times 10^{-3}$
- $0.23 \times 10^{-2}$
## Converting to Scientific Notation: 1.1

**Example:** Write in scientific notation: 3,860,000

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
</table>
| 1. | Convert to a number between 1 and 10.  
   How: Place decimal point such that there is one non-zero digit to the left of the decimal point: **3.86** |
| 2. | Multiply by a power of 10:  
   How: Count number of decimal places that the decimal has "moved" from the original number. This will be the exponent of the 10.  
   
   \[
   \begin{array}{ccccccc}
   3 & 8 & 6 & 0 & 0 & 0 & 0 \\
   \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
   6 & 5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]
   We have moved 6 places so the number (3.86) is multiplied by \(10^6\) |
| 3. | If the original number was less than 1, the exponent is negative; if the original number was greater than 1, the exponent is positive.  
   3,860,000 > 1, so the exponent is positive. |

**Answer:** \(3.86 \times 10^6\)
**On Your Own:** Express in correct scientific notation.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>39,400</td>
<td>$3.94 \times 10^4$</td>
</tr>
<tr>
<td>0.0000394</td>
<td>$3.94 \times 10^{-5}$</td>
</tr>
<tr>
<td>394</td>
<td>$3.94 \times 10^2$</td>
</tr>
<tr>
<td>39,400,000</td>
<td>$3.94 \times 10^7$</td>
</tr>
<tr>
<td>39.4</td>
<td>$3.94 \times 10^1$</td>
</tr>
<tr>
<td>0.394</td>
<td>$3.94 \times 10^{-1}$</td>
</tr>
<tr>
<td>3,940</td>
<td>$3.94 \times 10^3$</td>
</tr>
<tr>
<td>0.00394</td>
<td>$3.94 \times 10^{-3}$</td>
</tr>
<tr>
<td>3.94</td>
<td>$3.94 \times 10^0$</td>
</tr>
<tr>
<td>0.000394</td>
<td>$3.94 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Place the following numbers in order, from smallest to largest:

$3.35 \times 10^0, \ 7.4 \times 10^{-2}, \ 1.6 \times 10^{-1}, \ 4.33 \times 10^3, \ 7.45 \times 10^{-3}$

$7.45 \times 10^{-3}, \ 7.4 \times 10^{-2}, \ 1.6 \times 10^{-1}, \ 3.35 \times 10^0, \ 4.33 \times 10^3$
Converting from Scientific Notation to Standard Form: 1.1

A. Positive Exponents

- If exponent is positive, move decimal point to the right.
- The exponent will determine how many decimals to move.

Example: \(3.45 \times 10^2 = 345\) (Move to the right 2 places)

B. Negative Exponents

- If exponent is negative, move decimal point to the left.
- The exponent will determine how many decimals to move.

Example: \(3.45 \times 10^{-2} = .0345\) (Move to the left 2 places)

On Your Own: Express in standard form:

a. \(3.45 \times 10^{-8}\)
   - Since the exponent is negative, we move to the left.
   - Since the exponent is 8, we move to the left 8 places.
   Answer: \(0.0000000345\) (8 places to the left)

b. \(5.3 \times 10^3\)  \(5,300\)

c. \(5.3 \times 10^{-3}\)  \(0.0053\)

d. \(7.98 \times 10^{-4}\)  \(0.000798\)

e. \(7.98 \times 10^5\)  \(798,000\)

Tutors: For a good spiraling activity on scientific notation, play the "Scientific Match Up Game" (See Supplement.)
1. \((3)^{-2} = \frac{1}{9}\)

2. \((-3)^{2} = 9\)

3. \(- (3)^{2} = -9\)

3. \((-3)^{1} = -3\)

4. \((3)^{1} = 3\)

5. \((3)^{-1} = \frac{1}{3}\)

6. \((3)^{0} = 1\)

7. \((-3)^{0} = 1\)

8. \(- (3)^{0} = -1\)

9. \(3^{5} \times 3^{1} = 3^{6}\)
10. \((3^1)^2 = 3^2\)

11. \(3 \cdot 10^{-1} = .3\)

12. \(3^{-5} \cdot 3^1 = 3^{-4}\)

13. \(3^3 \cdot 3^{-6} = 3^{-3}\)

14. \(\sqrt[3]{64} = 4\)

15. \(\sqrt[4]{16} = 2\)

16. \(\sqrt{\frac{25}{100}} = \frac{5}{10} = \frac{1}{2}\)

17. \(4^4 \times 4^0 = 4^4\)

18. \(\frac{3^{-6}}{3^{-8}} = 3^2\)

19. \(4^{-4} \times 4^4 = 4^0 = 1\)
20. Which shows the number 34,600,000 written in scientific notation?

A. $346 \times 10^5$

B. $34.6 \times 10^6$

C. $3.46 \times 10^7$

D. $3.46 \times 10^{-7}$

E. $0.346 \times 10^{-8}$

21. \(\frac{3^5}{3^{-1}} = 3^6\)

22. \(\frac{3^{-5}}{3^1} = 3^{-6}\)

23. \(\frac{4^4}{4^{-1}} = 4^5\)

25. \(\frac{4^{-5}}{4^{-3}} = 4^{-2}\)

26. \(5^3 \cdot 5^{-2} = 5^{1} = 5\)
Unit Quiz: The following problems appeared on the CAHSEE.

1. \( \left\{ \frac{3}{4} \right\}^3 = \) 
   A. \( \frac{9}{12} \)  
   B. \( \frac{9}{16} \)  
   C. \( \frac{27}{32} \)  
   D. \( \frac{27}{64} \)  
   Standard 1.2

2. Solve for \( x \): \( x^3 = \frac{1}{8} \) 
   A. \( x = 2 \)  
   B. \( x = 3 \)  
   C. \( x = \frac{1}{2} \)  
   D. \( x = \frac{1}{3} \)  
   Standard 2.4

3. Which number equals \( (2)^{-4} \) 
   A. -8  
   B. \( -\frac{1}{16} \)  
   C. \( \frac{1}{16} \)  
   D. \( \frac{1}{8} \)  
   Standard 2.1
4. $\frac{10^{-2}}{10^{-4}} = _____$
   A. $10^{-6}$
   B. $10^{-2}$
   C. $10^2$
   D. $10^8$  \[ \text{Standard 2.1} \]

5. Between which two integers does $\sqrt{76}$ lie?
   A. 7 and 8
   B. 8 and 9
   C. 9 and 10
   D. 10 and 11  \[ \text{Standard 2.4} \]

6. The square of a whole number is between 1500 and 1600. The number must be between:
   A. 30 and 35
   B. 35 and 40
   C. 40 and 45
   D. 45 and 50  \[ \text{Standard 2.4} \]

7. The square root of 150 is between which two numbers?
   A. 10 and 11
   B. 11 and 12
   C. 12 and 13
   D. 13 and 14  \[ \text{Standard 2.4} \]
8. The radius of the earth’s orbit is 150,000,000,000 meters. What is this number in scientific notation?
   A. 1.5 \times 10^{11}
   B. 1.5 \times 10^{11}
   C. 15 \times 10^{10}
   D. 150 \times 10^9

9. 3.6 \times 10^2 =
   A. 3.600
   B. 36
   C. 360
   D. 3,600

10. (3^8)^2 =
    A. 3^4
    B. 3^6
    C. 3^{10}
    D. 3^{16}

11. 4^3 \times 4^2 =
    A. 4^5
    B. 4^6
    C. 16^5
    D. 16^6

12. (x^2)^4 =
    A. x^6
    B. x^8
    C. x^{16}
    D. x^{24}
Unit 3: Multi-Step Word Problems

Some problems involve more than one step. These are called multi-step problems. On the CAHSEE you can expect to get at least a few multi-step problems.

Example: The following problem appeared on the CAHSEE.

The five members of a band are getting new outfits. Shirts cost $12 each, pants cost $29 each, and boots cost $49 a pair. What is the total cost of the new outfits for all the band members?

To solve this kind of problem, we must follow some basic steps:

A. First, determine what the question asks: **Total cost for all band members**

B. Write down all of the numerical information given in the problem:

- 5 members in band
- Shirts @ $12 each
- Pants @ $29 each
- Boots @ $49 each

C. Determine the operations required to solve the problem. In other words, what do we do with all of the numbers listed in step 2?

- Multiply each item bought by 5 since there are 5 members and each item is required for each member:
  
  Shirts: 12 X 5 = 60
  Pants: 29 X 5 = 145
  Boots: 49 X 5 = 245

- Add it all up (listing biggest numbers first):
  
  \[
  \begin{array}{c}
  245 \\
  145 \\
  + 60 \\
  \hline
  450
  \end{array}
  \]

**Answer:** The total cost of the band’s outfits is $450.
1.2

On Your Own

1. Derrick wants to buy a sweater that costs $46. If he has $22 saved up and earns $12 a week in allowance, how long will it take before he has enough money to buy the sweater?

Steps:

A. What does the question ask: **How long before he can afford to buy the sweater?**

B. Write down all of the information that is important:

   a. Sweater costs $46
   b. $22 to begin
   c. + $12 a week

C. Determine the operations required to solve the problem and then apply these operations to solve the problem.

   • The sweater costs $46, but he already has $22. How much more money does he need? Which operation is required to answer this question? **Subtraction**

   Solve:
   
   $46 \text{ (total needed)} - 22 \text{ (saved)} = 24 \text{ more needed}$

   • Now that we know how much more money Derrick needs, all we have to do is to figure out how many weeks it will take to earn this amount. Which operation is required to answer this question? **Division**

   Solve:
   
   $24 \div 12 = 2$

Answer: Derrick can buy the sweater in 2 weeks.
2. Uncle Bernie took his three nieces to the movies. Each niece ordered a small popcorn, a large soda, and a chocolate bar. If a small order of popcorn costs $4, a large soda costs $3, and a chocolate bar costs $1.50, how much did Uncle Bernie spend on snacks?

Answer:

\[(4 \times 3) + (3 \times 3) + (1.50 \times 3) = 12 + 9 + 4.50 = $25.50\]

3. Cynthia wants to buy a pair of jeans that cost $56, including tax. If she earns $10.50 each week for allowance and spends $3.50 per week on bus fare to and from her dance lessons, what is the fewest number of weeks that it will take Cynthia to save enough money to buy the jeans?

Step 1: \[10.50 - 3.50 = 7.00\]

Step 2: \[56 \div 7 = 8\]

Answer: She will save enough money in 8 weeks.
Sometimes there may be information in the problem that you don’t need. It may be there to confuse you. Whenever you come to information that is extraneous (i.e., don’t need it, don’t want it), cross it out:

Example: Daphne, Cynthia, and Rachel went to the movies on October 21. October 21 fell on a Friday. The movie began at 8:00 p.m. They each bought a bucket of popcorn and a snickers bar. If each movie ticket costs $8.00, a bucket of popcorn costs $4.00, and a snickers bar costs $2.50, how much money did they spend altogether?

Steps:

• Cross out any information that you don’t need. Don’t just ignore it — cross it out.

Daphne, Cynthia, and Rachel went to the movies on October 21. October 21 fell on a Friday. The movie began at 8:00 p.m. They each bought a bucket of popcorn and a snickers bar. If each movie ticket costs $8.00, a bucket of popcorn costs $4.00, and a snickers bar costs $2.50, how much money did they spend altogether?

• Write down all of the information that is important:

3 people
Tickets $8.00 each
Popcorn $4.00 each
Snickers $2.50 each

• Figure out how much one person spent:

\[ 8 + 4 + 2.50 = \$14.50 \]

• Figure out how much all three people spent:

\[ 14.50 \cdot 3 = \$43.50 \]
On Your Own

1. Matthew bought a used car for $800. The car was 15 years old. He wrote a check for $520 and gave the salesman $125 in cash. The rest he promised to pay at the end of the week, when he would be receiving a paycheck for $385. How much does he owe on the car?

• **Cross out** any information that you don’t need. Don’t just ignore it — cross it out. That way you will be sure that you don’t accidentally slip it in later.

Matthew bought a used car for $800. The car was 15 years old. He wrote a check for $520 and gave the salesman $125 in cash. The rest he promised to pay at the end of the week, when he would be receiving a paycheck for $385. How much does he owe on the car?

• Now **write** down all of the information that is **important**:

  - Car costs $800
  - Gave $520 (check)
  - Gave $125 (cash)

• How much did Matthew already pay?  \(520 + 125 = 645\)

• How much more does he owe?  \(800 - 645 = 155\)

2. Mrs. Brown took her four children out to the pizza party. She and her children each ordered a small pepperoni pizza and a large soda. A small vegetarian pizza costs $4.50, while a small pepperoni pizza costs $5.25. Small sodas cost $2.00, medium sodas cost $2.50, and large sodas cost $3.25. How much did Mrs. Brown spend at the pizza party?

\[(5 \times 5.25) + (5 \times 3.25) = 42.50\]
Unit Quiz

1. Martine bought 3 cans of soda for 65¢ each, 2 pretzels for $1.25 each and a slice of pizza for $1.75. She paid with a $20 bill. How much change should she get back from the cashier?

   Answer: $13.80

2. Joy spent 25% of her weekly paycheck to help her sister buy a new dress. If the dress costs $235 and her paycheck is $450, how much does she have left for the week?

   Answer: $337.50

3. If it costs $150 to feed a family of four for the week, how much will it cost to feed a family of six?

   Answer: $37.50 \times 6 = $225.00

4. Adrienne has $166 left in her checking account and $1300 in her savings account. Each week she earns $175 as a cashier at the Five and Dime Store. She is planning on buying a set of dishes for her best friend’s wedding shower. The set costs $500. If she does not take any money out of her savings account, what is the fewest number of weeks that she must work in order to buy the dishes?

   Answer: 2 weeks (She needs $334 more; in 2 weeks she will have earned $350.)
Unit 4: Percent (1.3, 1.6, 1.7)

On the CAHSEE, you will have many problems that involve percent. (Note: Some of these will be word problems.)

Percent of a Number 1.3

Percent, written as %, literally means "out of 100." Any number expressed as a percent stands for a fraction.

Example: Five percent (or 5%) means 5 out of 100. As a fraction, this is written as 5/100, which can be reduced to 1/20. This can also be expressed as a decimal: .05 (read as five hundredths).

Example: Seventy-five percent (75%) means 75 out of 100, or 75/100. This fraction can be reduced to 3/4. As a decimal, 75% would be written as 0.75, which means 75 hundredths.

Converting from Percent to Decimal

To change a percent to a decimal, divide the percent value by 100: move the decimal point two places to the left.

Examples:
17% = .17
3% = .03
80% = .80 (or .8)
125% = 1.25
.8% = .008
3.4% = .034
46% = 0.46
Converting from Decimal to Percent

To change a decimal to a percent, move the decimal point two places to the right. (Multiply by 100!)

Examples:

\[ .34 = 34\% \quad .09 = 9\% \quad 2.3 = 230\% \quad .6 = 60\% \quad 0.125 = 12.5\% \]

Practice: Fill in the following chart. Reduce fractions to lowest terms.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{8}{100} = \frac{2}{25} ]</td>
<td>0.08</td>
<td>8%</td>
</tr>
<tr>
<td>[ \frac{35}{100} = \frac{7}{20} ]</td>
<td>.35</td>
<td>35%</td>
</tr>
<tr>
<td>[ \frac{80}{100} = \frac{4}{5} ]</td>
<td>0.8</td>
<td>80%</td>
</tr>
<tr>
<td>[ \frac{9}{100} ]</td>
<td>.09</td>
<td>9%</td>
</tr>
<tr>
<td>[ \frac{95}{100} = \frac{19}{20} ]</td>
<td>.95</td>
<td>95%</td>
</tr>
<tr>
<td>[ \frac{9}{10} ]</td>
<td>.9</td>
<td>90%</td>
</tr>
<tr>
<td>[ \frac{84}{100} = \frac{21}{25} ]</td>
<td>0.84</td>
<td>84%</td>
</tr>
<tr>
<td>[ \frac{60}{100} = \frac{3}{5} ]</td>
<td>.6</td>
<td>60%</td>
</tr>
<tr>
<td>[ \frac{1}{4} ]</td>
<td>.25</td>
<td>25%</td>
</tr>
</tbody>
</table>
There are two methods for solving percent problems. The first is setting up a proportion.

**Method 1: Proportion**

A *proportion* is two equivalent ratios, written as fractions. In any proportion, the *product of the means* is equal to the *product of the extremes*:

\[
\frac{3}{5} = \frac{6}{10} \quad \Rightarrow \quad 3 \cdot 10 = 5 \cdot 6
\]

We see that this is true: \(3 \cdot 10 = 30\) and \(5 \cdot 6 = 30\)

The product of the means is equal to the product of the extreme.

We can solve a percent problem by setting up a proportion. Here is the proportion used to solve percent problems:

\[
\frac{\text{Part}}{\text{Whole}} = \frac{x}{100}
\]

This proportion may be translated as follows: The part is to the whole as *what number* is to 100?"

And, since in a proportion, the *product of the means* is equal to the *product of the extremes*, the following is also true:

\[
\text{Whole} \cdot x = \text{Part} \cdot 100
\]

This *relationship* will *always be true for a proportion*. Since we multiply diagonally across the proportion, people often use the term "cross multiplying" for short (since it can be encumbering to keep saying, "The product of the means is equal to the product of the extremes." You can use the term "cross-multiplication" if you like; just remember the concept that is behind it.
Let's solve a percent problem together, using the proportion method:

Example: 23 is what percent of 50?

\[
\frac{23}{50} = \frac{x}{100}
\]

50x = 2300

Divide to solve for x.

\[x = 46\]

Answer: \(\frac{23}{50}\) is equal to 46%

On Your Own: Solve for x.

1. \(\frac{x}{20} = \frac{5}{100}\)

100x = 100

\[x = 1\]

Answer: 1%
2. \( \frac{4}{25} = \frac{x}{100} \)

\[
25x = 400 \\
x = 16
\]

Answer: 16%

3. \( \frac{18}{20} = \frac{x}{100} \)

\[
20x = 1800 \\
x = 90
\]

Answer: 90%

4. \( \frac{7}{10} = \frac{x}{100} \)

\[
10x = 700 \\
x = 70
\]

Answer: 70%
Let's look at more examples of percent problems solved with the proportion method:

**Example:** 18 is what percent of 50?

- Set up a proportion:

\[
\frac{\text{Part}}{\text{Whole}} = \frac{x}{100}
\]

**Note:** The term 50 is the "whole" and 18 is the "part":

\[
\frac{18}{50} = \frac{x}{100}
\]

- Cross multiply: 50x = 1800

- Isolate x by dividing by 50: 
  \[x = \frac{1800}{50} = 36\]

**Example:** What is 25% of $60?

- Set up a proportion: 
  \[
  \frac{x}{60} = \frac{25}{100}
  \]

**Note:** The term 60 is the "whole" and the problem asks for 25% of this whole:

- Cross multiply: 
  \[100x = 1500\]

- Find x: 
  \[x = \frac{1500}{100} = 15\]
1. 25% of what number is 15?

   • Set up a proportion: \( \frac{15}{x} = \frac{25}{100} \)

   **Note:** The term 15 is the "part" and the problem asks for 25% of the whole:

   • Cross multiply: 25x = 1500
   • Find x: \( x = 60 \)

2. 30 is what percent of 50?

   \[
   \frac{30}{50} = \frac{x}{100} \\
   x = 60
   \]

3. What is 45% of 90?

   \[
   \frac{x}{90} = \frac{45}{100} \\
   x = 40.5
   \]

4. 30% of what number is 60?

   \[
   \frac{60}{x} = \frac{30}{100} \\
   x = 200
   \]
Method 2: Translate & Compute

In this method, we translate the problem into an algebraic equation and then solve. Let's look at the previous problems again, this time using this second method:

**Example:** 18 is what percent of 50?

- **Translate:**
  
  \[ 18 \text{ is what percent of } 50? \]
  
  \[ 18 = \frac{x}{100} \cdot 50 \]

- **Compute:**
  
  \[ 18 = \frac{50x}{100} \]
  
  \[ 18 = \frac{150x}{200} \implies \text{Simplify all fractions} \]
  
  \[ 18 = \frac{x}{2} \]
  
  \[ 18 \cdot 2 = x \cdot 2 \]
  
  \[ x = 36 \]

  **Answer:** 18 is **36%** of 50.

**Note:** This method can also be used as a complement to Method 1 and as a way to verify the answers you arrived at using Method 1.
On Your Own

1. What is 25% of $60?

   • Translate:
     \[ \text{What is 25 percent of 60?} \]
     \[ x = \frac{25}{100} \cdot 60 \]

   • Compute:
     \[ x = \frac{25}{100} \cdot 60 = \frac{1}{4} \cdot 60 \]
     \[ x = 15 \]
     Answer: 15 is 25% of 60.

2. 25% of what number is 15?

   • Translate:
     \[ \text{25 percent of what number is 15?} \]
     \[ \frac{25}{100} \cdot x = 15 \]

   • Compute:
     \[ \frac{25}{100} (x) = 15 \]
     \[ \frac{x}{4} = 15 \]
     \[ x = 60 \]
3. 15% of what number is 30?

\[ \frac{15}{100} x = 30 \]

\[ x = 200 \]

4. 35 is what percent of 80?

\[ 35 = \frac{x}{100} (80) \]

\[ x = 43.75 \]

5. What is 15% of 90?

\[ x = \frac{15}{100} (90) \]

\[ x = 13.5 \]

6. 15 is what percent of 80?

\[ 15 = \frac{x}{100} (80) \]

\[ x = 18.75 \]
**Practice:** Use either method to solve the percent problems below. 1.3

1. On average, every 16 out of 200 students study Calculus. What percent study Calculus?

\[
\frac{16}{200} = \frac{x}{100}
\]

\[
200x = 1600
\]

\[
x = 8 \quad \text{Answer: 8%}
\]

2. If 150 people were surveyed in the 2004 presidential elections and 90 of those people said that they were going to vote for John Kerry, find the percent of Kerry supporters in the sample population?

\[
\frac{90}{150} = \frac{x}{100}
\]

\[
x = 60 \quad \text{Answer: 60%}
\]

3. The Brighton Movie Theater sells the following candies at their snack bar: Snickers Candy Bars, Peanut Clusters, O’Henry Bars, M & M’s, and Milky Way Bars. Currently, there are 10,000 candies in stock. The following chart below shows a percentage breakdown of each type of candy in stock. Find the actual number of each type of candy:

<table>
<thead>
<tr>
<th>Candy</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snickers: 5%</td>
<td>500</td>
</tr>
<tr>
<td>Peanut Clusters: 15%</td>
<td>1500</td>
</tr>
<tr>
<td>O’Henry: 20%</td>
<td>2000</td>
</tr>
<tr>
<td>M &amp; M’s: 25%</td>
<td>2500</td>
</tr>
<tr>
<td>Milky Way Bars: 35%</td>
<td>3500</td>
</tr>
</tbody>
</table>

4. What is the fractional equivalent of 95% (reduced to lowest terms)?

\[
\text{Answer: } \frac{19}{20}
\]
5. Four months of the year have 30 days. Which percentage most closely represents the months that do not have 30 days?
   A. 33%
   B. 44%
   C. 66%
   D. 75%

   **Answer:** C. 66%

6. What is the percentage equivalent of \( \frac{12}{15} \)?

   **Answer:** 80%

7. In Mr. Martin’s class, 9 of the 27 students in Mr. Martin’s class received a B+ or higher on the Algebra quiz. What percent of the students received a grade of B or lower?

   **Answer:** 66% (or round up to 67%)

8. What is \( \frac{4}{5} \) expressed as a percent?

   **Answer:** 80%

9. What is 0.80 expressed as a fraction (in simplest terms)?

   **Answer:** \( \frac{4}{5} \)
Percent Increases & Decrease 1.6

Often, when we compare one entity across time (such as changes in population or the price of a particular item), we express these changes in terms of percent. The percent of change is the ratio of the amount of change to the original amount.

<table>
<thead>
<tr>
<th>Ratio: How much it went up or down</th>
<th>The original amount</th>
</tr>
</thead>
</table>

An easy way to solve percent increase & percent decrease problems is to set up a proportion that consists of two ratios, the one above and a second one for the percent. Remember that “percent” is always a ratio and the denominator of that ratio is always 100.

So the second ratio looks like this: \[
\frac{\text{What}}{\text{Percent}} = \frac{x}{100}
\]

If you set up a proportion using these two ratios, you get the following:

<table>
<thead>
<tr>
<th>How much it went up or down = (x)</th>
<th>The original amount = 100</th>
</tr>
</thead>
</table>

All you need to do after that is cross multiply and isolate the \(x\) value.

Let’s look at an example on the next page.
Example: Jimmy got a raise from $6.00 to $8.00 per hour. This represents a raise of what percent?

Steps:

• Find out how much it went up or down and place this number over the original amount:

<table>
<thead>
<tr>
<th>2</th>
<th>It went up $2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Original amount is $6.00</td>
</tr>
</tbody>
</table>

• Set up proportion:

$$\frac{2}{6} = \frac{x}{100}$$

• Cross multiply:

$$6x = 200$$

• Solve for x:

$$\frac{6x}{6} = \frac{200}{6}$$

$$x = \frac{200}{6} = \frac{33.333 \ldots}{3}$$

• Express answer as a percent: 33%

Make sure you use the original amount as the denominator!
**Alternative Method**

You can also solve the previous problem by setting up an equation:

\[
\text{Change} = \text{What Percent of Original Amount}
\]

Let's look at the problem again:

**Example:** Jimmy got a raise from $6.00 to $8.00 per hour. This represents a raise of what percent?

Now plug the values in the equation:

\[
2 = \frac{x}{100} \cdot 6
\]

Now solve:

\[
2 = \frac{x}{100} \quad (6)
\]

\[
2 = \frac{3x}{50}
\]

\[
100 = 3x
\]

\[
x = 33.33
\]
1. Tasty Delight raised their prices on ice cream sundaes from $5.00 to $7.00. This represents an increase of what percent?

- Set up proportion: \( \frac{2}{5} = \frac{x}{100} \)
- Cross multiply: \( 5x = 200 \)
- Solve for \( x \): \( x = 40 \)
- Express answer as a percent: \( 40\% \)

**Note:** Be sure to use the original amount as the denominator!

Now solve the above problem using the alternative method:

<table>
<thead>
<tr>
<th>Plug in values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change = What Percent of Original Amount</td>
</tr>
<tr>
<td>( \downarrow ) 2 = ( \frac{x}{100} ) ( \downarrow ) 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solve:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2 = \frac{5x}{100} )</td>
</tr>
<tr>
<td>( 200 = 5x )</td>
</tr>
<tr>
<td>( 40 = x )</td>
</tr>
</tbody>
</table>
2. A shirt that costs $40 in 1999 costs $60 in 2005. What is the percent increase? (Use either method to solve.)

Solve:

Percent Increase: **50% Increase**

What would be a trick answer on the CAHSEE? **33%**

3. Elizabeth’s basketball card collection increased in value from $500 to $1,000. What is the percent increase? (Use either method to solve.)

Solve:

Percent Increase: **100% Increase**

What would be a trick answer on the CAHSEE? **50%**
4. Jordy’s basketball card collection decreased in value from $1,000 to $500. What is the percent decrease?

Solve:

Percent Decrease: 50% Decrease

What would be a trick answer on the CAHSEE? 100% 

5. Last year Andrea had 36 students in her class. This year she only has 27. What is the percent decrease?

Solve:

Percent Decrease: 25% Decrease

What would be a trick answer on the CAHSEE? 33% (or 75%)

6. If a shirt that costs $80 last year is worth only 75% as much this year, what is the current value of the shirt?

Solve:

Answer: $60
Price Discounts

Stores will often sell items for a discounted sales price. The store will
discount an item by a percent of the original price. To find the amount
of a discount (in dollars), simply multiply the original price by the
percent discount.

Example: An item, which originally cost $20, was discounted by 25%.
Find the discount in dollars?

Steps:

• Translate the problem into math:

\[
25\% \text{ of } \$20 = \frac{25}{100} \cdot 20
\]

• Calculate:

\[
\frac{25}{100} \cdot 20 = \$5.00 \text{ (or } \frac{1}{4} \cdot 20 = 5)\]

The item was sold for $5.00 less than its original price.

Terms you may see for discounted items:

• 50% Off
• Save 50%
• Discounted by 50%
On Your Own

1. A transistor radio is normally sold for $80 at Karter Electric Goods. This week, it is being offered at a 20% discount. How much cheaper is the radio this week?

Answer: $16.00 cheaper

2. A dress, which sold for $80 last week, is on sale for 20% off. This represents a discount of how much (in dollars)?

Answer: $16

3. At Peppy’s Pizza, a small pepperoni pizza normally sells for $6.00. This week, the store is offering a 25% discount on all small pizzas. How much cheaper is the pizza this week?

Answer: $1.50
**Note:** Many price discount questions on the CAHSEE ask you to find the **new price, not the amount of discount**. This involves one more important step.

**Example:** An item originally cost $20 and was discounted by 25%. What was the new sales price?

**Steps:**

- Translate the problem into math: \( \frac{25}{100} \times 20 \)

- Calculate the discount in dollars:

\[
\frac{25}{100} \times 20 = 5.00 \text{ (or } \frac{1}{4} \times 20 = 5)\]

The item was sold for **$5.00 less** than its original price.

- Finally, to find the new sales price, subtract the amount of discount from the original price:

\[
20.00 - 5.00 = 15.00
\]

**CAHSEE Alert!** Don’t forget this last step. If this were an actual item on the exam, $5.00 (the amount of discount) would probably be one of the answer choices. If you are working out a problem that has multiple steps, remember to do all of the steps to get the right answer.
1. A dress, which sold for $80 last week, is on sale for 20% off. What is the new price of the dress?

Solve:

New Price: $64
What would be a trick answer on the CAHSEE? $16.00

2. At Peppy’s Pizza, a small pepperoni pizza normally sells for $6.00. This week, the store is offering a 25% discount on all small pizzas. How much does a small pepperoni pizza cost this week?

Solve:

New Price: $4.50
What would be a trick answer on the CAHSEE? $1.50

3. Rain boots regularly sell for $70 a pair. They are currently on sale for 40% off. What is the sale price of the boots?

Solve:

New Price: $42.00
What would be a trick answer on the CAHSEE? $28.00

Tutors: To reinforce or spiral this concept, play the Discount Game with your students (Supplement)
Markups

Stores buy items from a wholesaler or a distributor and increase the price when they sell them to consumers. The increase in price provides money for the operation of the store and the salaries of people who work in the store. A store may have a rule that the price of a certain type of item needs to be increased by a certain percentage. This percentage is called the markup.

A. Two-Step Method

1. Find the markup in dollar amount:

   Original Cost ● Percent of Markup

2. Add this dollar amount to the original price.

   Markup in $ + Original Price

Example: A merchant buys an item for $4.00 and marks it up by 25%. How much does he charge for the item?

1. Find the markup in dollars:

   $4.00 ● .25 = $1.00 Or . . .
   4 ● \( \frac{1}{4} \) = 1

2. Add this dollar amount to the original price:

   $4.00 + 1.00 = $5.00

CAHSEE Alert! Like the discount problems, be careful not to forget the last step. In the above problem, a probable answer choice would be $1.00. Don’t be fooled! Read the question carefully.
B. One-Step Method

A faster way to calculate the sales price is to make the original cost equal to 100%.

Example: A merchant buys an item for $4.00 and marks it up by 25%. How much does he charge for the item?

Since the markup is 25%, the customer pays 125% of the original cost. Multiply the original cost by 125% (or 1.25):

\[
\begin{align*}
\$4.00 \cdot 1.25 &= \\
(4 \cdot 1) + (4 \cdot .25) &= \\
4 + 1 &= \$5.00 & \text{The merchant charges } \$5.00 \text{ for the item.}
\end{align*}
\]

CAHSEE Alert! Some questions on the CAHSEE may ask for the dollar amount of the markup, not the final sales price.

Example: Harry’s Bargain Basement has a 20% markup on all its goods. If the manufacturer price of irons is $16, how much extra does the customer pay for each iron?

Solve: $3.20

Compare with this problem:

Harry’s Bargain Basement has a 20% markup on all its goods. If the manufacturer price of irons is $16, how much does the customer pay for an iron?

Solve: $16.00 + $3.20 = $19.20

Note: On the CAHSEE, be sure to read the question carefully to determine whether it is asking for the markup or final sales price.
**CAHSEE on Target**  
UC Davis, School/University Partnerships  
Answer Key to Student Workbook: Number Sense Strand

**1.7**

**On Your Own:** Read each problem carefully and determine what the question is asking. Then solve the problem.

1. The original cost of a dress is $12.00. (This is the amount that the store paid the manufacturer.) The store marks up all their items by 20%. How much does the store charge for the dress?

   **Answer:** $14.40

2. Bill’s Auto Supplies buys tires for $80. If the store sells its tires for $100, what is its percent markup?

   **Answer:** 25%

3. All items at Bargain Slim’s have been marked up by 40%. If the store paid $12 for each CD, how much does the customer pay?

   **Answer:** $16.80

4. A stainless steel refrigerator is bought for $500 and then marked up by 100%. What is the new price of the refrigerator?

   **Answer:** $1,000
Commissions

Sales commissions are often paid to employees who sell merchandise or products. Commissions serve to motivate salespersons to sell a lot.

A commission is generally a percentage of the total sales made by a salesperson. To find the commission, just multiply the value of the total sales by the commission rate. It is that simple!

**Example:** A salesman receives a 10% commission on all sales. If he sells $1500 worth of merchandise, how much does he earn in commission?

\[
$1,500 \times 0.10 = $150 \\
\text{or} \\
$1500 \times \frac{10}{100} = 15 \times 10 = $150
\]

**On Your Own**

1. Sarah is a real estate agent. She earns 12% commission on every house she sells. Sarah recently sold a house for $400,000. What was her commission?

   **Answer:** $48,000

2. Ronald is a salesman in the men’s department at Bloomingdale’s Department Store. He earns 15% on all sales. His total sales for the month of August came to $80,000. How much did he earn in commission?

   **Answer:** $12,000

3. Alvin Ray sells used cars at Kaplan’s Auto Dealer. His commission rate is 30%. What was his commission on the used Audi he sold for $36,000?

   **Answer:** $10,800
Unit Quiz: The following questions appeared on the CAHSEE.

1. At a recent school play, 504 of the 840 seats were filled. What percent of the seats were empty?
   A. 33.6%
   **B. 40%**
   C. 50.4%
   D. 60%

   **Standard 1.3**

2. Some of the students attend school 180 of the 365 days in a year. About what part of the year do they attend school?
   A. 18%
   **B. 50%**
   C. 75%
   D. 180%

   **Standard 1.3**

3. What is the fractional equivalent of 60%?
   A. \( \frac{1}{6} \)
   B. \( \frac{3}{6} \)
   C. \( \frac{3}{5} \)
   D. \( \frac{2}{3} \)

   **Standard 1.3**
4. If Freya makes 4 of her 5 free throws in a basketball game, what is her free throw shooting percentage?
   A. 20%
   B. 40%
   **C. 80%**
   D. 90%  
   **Standard 1.3**

5. Between 6:00 AM and noon, the temperature went from 45° to 90°. By what percentage did the temperature increase between 6:00 AM to noon?
   A. 45%
   B. 50%
   C. 55%
   **D. 100%**  
   **Standard 1.6**

6. The price of a calculator has decreased from $12.00 to $9.00. What is the percent of decrease?
   A. 3%
   **B. 25%**
   C. 33%
   D. 75%  
   **Standard 1.6**

7. The cost of an afternoon movie ticket last year was $4.00. This year an afternoon movie ticket costs $5.00. What is the percent increase of the ticket from last year to this year?
   A. 10%
   B. 20%
   **C. 25%**
   D. 40%  
   **Standard 1.6**
8. A pair of jeans regularly sells for $24.00. They are on sale for 25% off. What is the sales price of the jeans?
   A. $6.00
   B. $18.00
   C. $20.00
   D. $30.00     Standard 1.7

9. A CD player regularly sells for $80. It is on sale for 20% off. What is the sales price of the CD player?
   A. $16
   B. $60
   C. $64
   D. $96     Standard 1.7

10. Mr. Norris is paid a 5% commission on each house that he sells. What is his commission on a house that he sells for $125,000?
    A. $625
    B. $6,250
    C. $62,500
    D. $625,000     Standard 1.7
On the CAHSEE, you may be asked several questions on interest. These questions will cover both simple interest and compound interest.

**Introduction to Interest**

Did you know that money can make more money? Whenever money is invested or borrowed, additional funds, called interest, are charged for the use of that money for a certain period of time. When the money is paid back, both the principal (amount of money that was borrowed) and the interest are due.

If you invest money in the bank, the bank is borrowing the money and the interest is paid to you. On the other hand, when you take out a loan, you are borrowing the money and you must pay the interest.

Interest can be simple or compound:

**Simple interest** is generally used when borrowing or investing money for **short periods of time**.

**Compound interest** is generally used when borrowing or investing money for **longer periods of time**. We will learn about compound interest later.

**Interest depends on three things:**

1. **Principle** (**P**): The amount you invest or borrow; principle is expressed in dollars.

2. **Interest Rate** (**R**): How much it costs you to borrow the money or how much you gain by investing your money; this rate is always expressed as a percent in the problem, although you may convert the rate to a decimal during computation.

3. **Time** (**T**): How long you borrow the money or how long you invest your money; this time is always expressed in **number of years**.
Converting to Years

When calculating interest, time is expressed in years. If the period of time is given in months, you must first convert it to the number of years.

Examples:

- 6 months is $\frac{1}{2}$ year, or .5 year.
- 18 months is $\frac{3}{2}$ years, or 1.5 years.

Working with Improper Fractions

We will see that, when calculating interest, it is easiest to work with improper fractions than with mixed numbers.

Mixed Numbers: A mixed number consists of both a whole integer and a fraction.

- $1 \frac{1}{4}$ is a mixed number because it consists of a whole number (1) and a fraction $\frac{1}{4}$.

Improper Fractions: An improper fraction is one in which the numerator is greater than the denominator.

- $\frac{5}{4}$ is an improper fraction because the numerator (5) is greater than the denominator (4).

Tutors: For reinforcement on converting improper fractions to mixed numbers, play "Four in a Row." (See the Tutor Supplement for Number Sense.)
Converting Mixed Numbers to Improper Fractions

To convert a mixed number to an improper fraction, follow these steps:

1. Multiply the whole number by the denominator of the fraction.
2. Add the numerator of the fraction to the product found above.
3. Place the result over the fraction's denominator.

**Example:** Convert \(\frac{1}{4}\) to an improper fraction.

- Multiply the whole number by the denominator of the fraction:
  \[1 \cdot 4 = 4\]
- Add the numerator of the fraction to the product found in Step 1:
  \[4 + 1 = 5\]
- Place the result over the fraction's denominator:
  \[\frac{5}{4}\]

On Your Own: Convert \(\frac{3}{5}\) to a mixed fraction.

\[\frac{1}{3} = \frac{3}{5}\]
**Practice:** Express each time interval below in years. Express as both a fraction (**reduced to lowest terms**) and a decimal.

**Note:** Be sure to covert any mixed number to an **improper** fraction.

<table>
<thead>
<tr>
<th>Months</th>
<th>Years: Fraction</th>
<th>Years: Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>$\frac{1}{4}$</td>
<td>.25</td>
</tr>
<tr>
<td>4 months</td>
<td>$\frac{1}{3}$</td>
<td>.33</td>
</tr>
<tr>
<td>8 months</td>
<td>$\frac{2}{3}$</td>
<td>.67</td>
</tr>
<tr>
<td>27 months</td>
<td>$2\frac{1}{4} = \frac{9}{4}$</td>
<td>2.25</td>
</tr>
<tr>
<td>15 months</td>
<td>$1\frac{1}{4} = \frac{5}{4}$</td>
<td>1.25</td>
</tr>
<tr>
<td>9 months</td>
<td>$\frac{3}{4}$</td>
<td>.75</td>
</tr>
<tr>
<td>21 months</td>
<td>$1\frac{3}{4} = \frac{7}{4}$</td>
<td>1.75</td>
</tr>
<tr>
<td>1 year and 8 months</td>
<td>$1\frac{2}{3} = \frac{5}{3}$</td>
<td>1.67</td>
</tr>
</tbody>
</table>
Converting to Percents & Decimals

The interest rate is expressed as a **percent** or as a **decimal**.

**Examples:**

\[
5\% = \frac{5}{100} = 0.05 \\
8\frac{1}{2}\% = \frac{8.5}{100} = 0.085
\]

**On Your Own:** Express as both a percent \( \left( \frac{x}{100} \right) \) and a decimal.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Percent</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>12%</td>
<td>( \frac{12}{100} )</td>
<td>.12</td>
</tr>
<tr>
<td>20%</td>
<td>( \frac{20}{100} )</td>
<td>.2</td>
</tr>
<tr>
<td>3%</td>
<td>( \frac{3}{100} )</td>
<td>.03</td>
</tr>
<tr>
<td>18%</td>
<td>( \frac{18}{100} )</td>
<td>.18</td>
</tr>
<tr>
<td>9\frac{1}{2}%</td>
<td>( \frac{9.5}{100} )</td>
<td>.095</td>
</tr>
<tr>
<td>12\frac{1}{2}%</td>
<td>( \frac{12.5}{100} )</td>
<td>.125</td>
</tr>
</tbody>
</table>
Solving Simple Interest Problems

To solve simple interest problems, just apply the formula:

\[
\text{Principle} \times \text{Rate} \times \text{Time}
\]

We can abbreviate this as follows: \(P \times R \times T\)

Note: Be sure to learn this formula for the CAHSEE!

One Step-Problems

One-step problems ask you to find the interest (in dollars and cents) earned (from an investment) or owed (on a loan). Just apply the formula for interest:

\[
\text{Principle} \times \text{Rate} \times \text{Time}
\]

Note: Be sure to convert all terms to their correct units:

- Rate in % or decimal
- Time in years

Example: $500 invested for 6 months in an account paying 7% interest. How much is earned in interest?

To solve, simply plug the correct values into the equation and do the computation:

\[
\text{Principle} \times \text{Rate} \times \text{Time} = \frac{500 \times 0.07 \times 0.5}{100 \times 2} = \$17.50
\]

\[
\text{OR}
\]

\[
\frac{500 \times 7 \times 1}{100 \times 2} = \$17.50
\]
As we mentioned earlier, when solving interest problems, it is easier to work with improper fractions than with mixed numbers.

**Example:** Shawn invests $4,000 at 16%. How much does he earn in 15 months?

Let’s solve this **problem by expressing 15 months as an improper fraction.** *(Note: It is much easier to multiply with improper fractions than with mixed numbers.)*

There are 12 months in 1 year; we have 15 months:

\[
\frac{15}{12} \quad \text{Improper fraction: Denominator > numerator}
\]

We can reduce this fraction: \( \frac{15}{12} = \frac{5}{4} \)

Now let’s solve the problem:

\[
\begin{align*}
P & \quad \bullet \quad R & \quad \bullet \quad T \\
4000 & \quad \bullet \quad \frac{16}{100} & \quad \bullet \quad \frac{5}{4} \\
4,000 & \quad \bullet \quad \frac{416}{100} & \quad \bullet \quad \frac{5}{4} \\
40 \times 20 & = 800
\end{align*}
\]
On Your Own: Plug the correct values and solve.

1. $1,500 is borrowed at an interest rate of 3 percent for 20 years. How much is earned in interest?

\[ P = $1500 \]
\[ R = 3\% = .03 \]
\[ T = 20 \]

\[ P \times R \times T = 1500 \times .03 \times 20 = $900 \]

Or \[ 1500 \times \frac{3}{100} \times 20 = $900 \]

2. $5,000 is invested for 24 months in an account paying 6% interest. How much is earned in interest.

\[ P = $5,000 \]
\[ R = 6\% = .06 \]
\[ T = 2 \]

\[ P \times R \times T = 5000 \times .06 \times 2 = 600 \]

Or \[ 5000 \times \frac{6}{100} \times 2 = $600 \]

3. Drew earns 6% in simple interest. If he invests $8,000 in a bank account, how much interest will he have earned after 18 months?

Answer: $720

Note: In each of the above questions, we are asked to find the interest earned, rather than the value of the entire investment. We will now learn to add the interest to the principle.
Two-Step Problems

On the CAHSEE, you may be asked to find the value of the entire investment. For these problems, there is one additional step.

Example: Marianne invested $5,000 at an annual interest rate of 8 ½ percent. How much will her investment be worth in two years?

• Find the amount of interest earned:

\[ P \cdot R \cdot T = 5,000 \cdot \frac{8.5}{100} \cdot 2 = \$850 \]

• Add the interest to the principal to get the value of the investment:

\[ 5,000 + 850 = \$5,850 \]

On Your Own:

1. Rachel invests $3,000 at 12%. How much will her investment be worth in 15 months?

• Plug the values into the formula and compute interest earned:

\[ P \cdot R \cdot T = 3,000 \cdot \frac{12}{100} \cdot \frac{5}{4} = \$450 \]

• Add interest to principal:

\[ 3,000 + 450 = 3,450 \]
2. Amy has a bank account that pays an annual interest rate of 7% (simple interest). If she has $7,000 in principal, how much interest will she earn this year?

**Answer:** $490

2. Denise earns 8% in simple interest each year. If she now has $900 in her savings account, what will be the value of her savings account in six months?

**Answer:**

A. \(900 \times \frac{8}{100} \times \frac{1}{2} = \$36\)

B. \(36 + 900 = \$936\)

3. Emily has invested $15,000 at Chase Manhattan Bank. If her current rate of interest is 8%, how much interest will she have earned in nine months?

**Answer:** \(15,000 \times \frac{8}{100} \times \frac{3}{4} = \$900\)

4. Refer back to the previous problem. What will be the total value of Emily’s investment after nine months?

**Answer:** \(15,000 + 900 = \$15,900\)
Compound Interest

While simple interest is paid once per year, compound interest can be paid twice a year (semi-annually), four times a year (quarterly) or even monthly!

Example: Peter invests $500 in a savings account. The bank pays 10% annual interest, compounded twice a year. What is the value of Peter’s investment after one year?

Steps:

• How much does Peter earn after 6 months? Not 10% because that is what he earns annually. 10% annual interest compounded twice a year means that half of the interest is paid after 6 months (half of the year) and the other half is paid at the end of the year. Since six months is one-half of a year, Peter only earns half of 10%, or 5%, after six months.

  Calculate 5% of $500: \[ \frac{5}{100} \times 500 = 25 \]

• Add this to the principle to find the total value of his investment after six months:

  \[ 500 + 25 = 525 \]

• For the next six months, Peter will earn 5% on $525. Calculate the interest:

  \[ 5\% \times 525 = 26.25 \]

• Add this amount to the value of his investment after one year:

  \[ 26.25 + 525.00 = 551.25 \]
CAHSEE Tip: Since the math section of the CAHSEE uses a multiple-choice format, you can automatically rule out certain choices on compound interest problems:

- We know that compound interest is always greater than simple interest; therefore, you can cross out any answers that are less than or equal to the amount calculated for simple interest.

- At the same time, the answer will be only slightly greater than the amount obtained under simple interest since CAHSEE compound interest problems will generally be limited to one year (the difference between the interest earned under simple and compound interest gets bigger with each year); therefore, you can cross out answers that are significantly greater than that obtained under simple interest.

See if you can apply this strategy for the following two problems.

1. Ellie invested $3,000 in a savings account that pays an annual interest rate of 6% compounded twice a year. How much will she have in the bank after one year?

   A. $3,000.00  
   B. $3,180.00  
   C. **$3,182.70**  
   D. $3,600

2. Drew has invested $10,000 at Bank of America. His current rate of interest is 5%, compounded twice a year. How much interest will he have earned in one year?

   A. $10,756.25  
   **B. $10,506.25**  
   C. $10,256.25  
   D. $10,500
Unit Quiz: The following questions appeared on the CAHSEE.

1. Sally puts $200 in a bank account. Each year the account earns 8% simple interest. How much interest will she earn in three years?
   A. $16.00
   B. $24.00
   C. $48.00
   D. $160.00  Standard 1.7

2. Mr. Yee invested $2000 in a savings account that pays an annual interest rate of 4% compounded twice a year. If Mr. Yee does not deposit or withdraw any money, how much will he have in the bank after one year?
   A. $2,080.00
   B. $2,080.80
   C. $2,160.00
   D. $2,163.20  Standard 1.7

Note: See if you can solve this problem by applying the multiple-choice strategy for compound interest problems.

Tutors: For some good spiraling activities on all number sense skills, play the two versions of "I have...Who has...?"