Strategies that Promote Number Sense and Reasoning

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mathsolutions.com/presentations

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Strategies that Promote Number Sense and Reasoning

Standards call for increased attention to number sense.

– What does that mean?
– What does it look like in the classroom?
– How can all students have access to developing their sense of numbers?
What has influenced my thinking and growth?

• Work at Math Solutions
• National Council of Teachers of Mathematics (NCTM)
• National Assessment of Educational Progress (NAEP)
• Common Core State Standards
What has influenced my thinking and growth?

- Work at Math Solutions
- National Council of Teachers of Mathematics (NCTM)
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- Common Core State Standards
What fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$

B. $\frac{1}{6}$

C. $\frac{2}{2}$

D. $\frac{1}{5}$
What fraction has a value closest to \( \frac{1}{2} \)?

A. \( \frac{5}{8} \)---25%

B. \( \frac{1}{6} \)---6%

C. \( \frac{2}{2} \)---41%

D. \( \frac{1}{5} \)---26%

Omitted---2%
476 ÷ 5

A. 85 R1
C. 95 R 1
E. 96
D. 135 R 1
476 ÷ 5

A. 85 R1------11%

C. 95 R 1------59%

E. 96---------12%

G. 135 R 1-----16%

Omitted----2%
What has influenced my thinking and growth?

• Work at Math Solutions
• National Council of Teachers of Mathematics (NCTM)
• National Assessment of Educational Progress (NAEP)
• Common Core State Standards
Common Core State Standards

“Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from.”
## Comparing Standards

<table>
<thead>
<tr>
<th>New York – 3rd grade</th>
<th>CCSS– 3rd grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use a variety of strategies to add and subtract 3-digit numbers (with and without regrouping).</td>
<td>Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</td>
</tr>
</tbody>
</table>
8 Standards for Mathematical Practice

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning
So, what is number sense?
Number sense refers to an intuitive feeling for numbers and their various uses and interpretations, an appreciation for various levels of accuracy when computing, the ability to detect arithmetical errors, and a common-sense approach to using numbers.

Howden, McIntosh, Reys, and Reys
Number Sense Develops Over Time
Strategies to Build Students’ Number Sense

• Model different methods for computing.
• Ask students regularly to calculate mentally.
• Have class discussions about strategies for computing.
• Make estimation an integral part of computing.
• Question students about how they reason numerically.
• Pose numerical problems that have more than one possible answer.
Double 47

- Model different methods for computing.
- Ask students regularly to calculate mentally.
- Have class discussions about strategies for computing.
Double 38

Use a strategy you heard that you didn’t use the first time.

What was it like to use a different strategy? What thinking did you do to be able to use ado in using a different strategy?

Triple 38
How Far Away?

1. Sketch a number line and locate on it 0, 25, 50, 75, 100

2. Create all the possible 2 digit numbers using the three digits that were rolled: 2, 6, 3.

3. Place the numbers on the number line using 0, 25, 50, 75, and 100 as benchmark numbers.

4. Determine how far away each number is from 100?
How might an activity like this help students build their number sense?

What would you want to observe and assess as students engage in this activity?
## My Numbers

<table>
<thead>
<tr>
<th>1.5</th>
<th>10</th>
<th>24670</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>78729</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of miles on my odometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sisters I have</td>
</tr>
<tr>
<td>How old my car is</td>
</tr>
<tr>
<td>Number of cats I’d like to have</td>
</tr>
<tr>
<td>My zip code</td>
</tr>
<tr>
<td>Number of years I lived in California</td>
</tr>
<tr>
<td>My Numbers</td>
</tr>
<tr>
<td>----------------------------</td>
</tr>
<tr>
<td>1.5  10  24670</td>
</tr>
<tr>
<td>2   78729  6</td>
</tr>
<tr>
<td>Number of miles on my odometer----<strong>24,670</strong></td>
</tr>
<tr>
<td>Number of sisters I have-----<strong>2</strong></td>
</tr>
<tr>
<td>How old my car is</td>
</tr>
<tr>
<td>Number of cats I’d like to have</td>
</tr>
<tr>
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</table>
Dart Board Problem
Dart Board Problem 1

• Betty throws 3 darts in the outermost ring, one in the next ring, and two in the ring next to the center. What is her score?

• Solve the problem individually, then share your strategies and solutions with a partner.
Dart Board Problem 2

Betty throws 6 darts and earns a score of 150. Where might her darts have landed?

- Solve the problem individually, then share your strategies and solutions with a partner.
# Comparing Standards

## New York – 3rd grade

Use a variety of strategies to add and subtract 3-digit numbers (with and without regrouping).

## CCSS– 3rd grade

Use place value understanding and properties of operations to perform multi-digit arithmetic.  
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
From the classroom

- 3rd grade class from South Shades Crest Elementary in Hoover, AL
- Number Talk about:

  \[ 38 + 37 \]

Focus for Viewing Video

What evidence do you find that the teacher offers instructional experiences to support students in developing:

• fluency in adding using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, and
### New York – 5th grade

Use a variety of strategies to divide three-digit numbers by one- and two-digit numbers.

### CCSS – 5th grade

Perform operations with multi-digit whole numbers and with decimals to hundredths.

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
- Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
From the classroom

• 5th grade class from South Shades Crest Elementary in Hoover, AL
• Number Talk about:

496 ÷ 8

Watch Video Clip
Grade 5 Number Talk: 496 ÷ 8

Watch the following video clip to see what mathematics students use to solve this problem mentally.
Students with number sense...

- naturally decompose numbers;
- develop and use benchmarks as referents;
- use the relationships among operations and their knowledge of the base-ten number system to solve problems;
- estimate a reasonable result for a problem; and
- have a disposition to make sense of numbers, problems, and results.

NCTM’s Principals and Standards for School Mathematics- Standards 2000
Tell Me All You Can
2/3 + 3/4
Tell Me All You Can
75 X 12
Exploring Tens and Ones on the 1—100 Chart

Creating Balancing Number Puzzles
Balancing Number Puzzles

What does this remind you of?
What do you know?

64

[Blank]

100
How would you balance this puzzle?

```
100   32
     /|
    / \
```

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What is a possible solution for this puzzle?
Find a possible solution

200

[Diagram with two blank boxes branched from 200]
Questions

Questions such as the following can help students to focus on numerical reasoning skills and routines:

• Why do you think that?
• Why does your answer make sense?
• How do you know when you have an answer?
• Is there only one solutions? How do you know?
• Will your strategy work with every number? Every similar situation? Why do you think so?
• Did anyone think about this differently?

From *Enriching Your Math Curriculum Grade 5* by Lainie Schuster, Math Solutions Publications
Reflection

What are you taking from this session that will help you support students in developing their number sense?

Why do you think focusing on number sense is important to student learning and achievement?