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Lesson 1 Reteach

Rates

A ratio that compares two quantities with different kinds of units is called a **rate**. When a rate is simplified so that it has a denominator of 1 unit, it is called a **unit rate**.

**Example 1**

**DRIVING** Alita drove her car 78 miles and used 3 gallons of gas. What is the car's gas mileage in miles per gallon?

Write the rate as a fraction. Then find an equivalent rate with a denominator of 1.

\[
\frac{78 \text{ mi}}{3 \text{ gal}} = \frac{78 \text{ mi}}{3 \text{ gal}} = \frac{78 \text{ mi} \div 3}{3 \text{ gal} \div 3} = \frac{26 \text{ mi}}{1 \text{ gal}}
\]

The car's gas mileage, or unit rate, is 26 miles per gallon.

**Example 2**

**SHOPPING** Joe has two different sizes of boxes of cereal from which to choose. The 12-ounce box costs $2.54, and the 18-ounce box costs $3.50. Which box costs less per ounce?

Find the unit price, or the cost per ounce, of each box. Divide the price by the number of ounces.

\[
\begin{align*}
\text{12-ounce box} & \quad \frac{2.54}{12 \text{ oz}} \approx 0.21 \text{ per ounce} \\
\text{18-ounce box} & \quad \frac{3.50}{18 \text{ oz}} \approx 0.19 \text{ per ounce}
\end{align*}
\]

The 18-ounce box costs less per ounce.

**Exercises**

Find each unit rate. Round to the nearest hundredth if necessary.

1. 18 people in 3 vans
   \[
   \frac{6 \text{ people}}{1 \text{ van}}
   \]

2. $156 for 3 books
   \[
   \frac{$52}{1 \text{ book}}
   \]

3. 115 miles in 2 hours
   \[
   \frac{57.5 \text{ mi}}{1 \text{ h}}
   \]

4. 8 hits in 22 games
   \[
   \frac{0.36 \text{ hit}}{1 \text{ game}}
   \]

5. 65 miles in 2.7 gallons
   \[
   \frac{24.07 \text{ mi}}{1 \text{ gal}}
   \]

6. 2,500 Calories in 24 hours
   \[
   \frac{104.17 \text{ C}}{1 \text{ h}}
   \]

Choose the lower unit price.

7. $12.95 for 3 pounds of nuts or $21.45 for 5 pounds of nuts
   \[
   \frac{$21.45}{5 \text{ lb}}
   \]

8. A 32-ounce bottle of apple juice for $2.50 or a 48-ounce bottle for $3.84.
   \[
   \frac{$2.50}{32 \text{ oz}}
   \]
Lesson 1 Skills Practice

Rates

Find each unit rate. Round to the nearest hundredth if necessary.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$112 in 8 hours</td>
<td>$14 per h</td>
</tr>
<tr>
<td>2.</td>
<td>150 miles in 6 gallons</td>
<td>25 mi per gal</td>
</tr>
<tr>
<td>3.</td>
<td>49 points in 7 games</td>
<td>7 points per game</td>
</tr>
<tr>
<td>4.</td>
<td>105 students in 3 classes</td>
<td>35 students per class</td>
</tr>
<tr>
<td>5.</td>
<td>120 problems in 5 hours</td>
<td>24 problems per h</td>
</tr>
<tr>
<td>6.</td>
<td>3 accidents in 12 months</td>
<td>0.25 accident per mo</td>
</tr>
<tr>
<td>7.</td>
<td>6 eggs in 7 days</td>
<td>0.86 egg per day</td>
</tr>
<tr>
<td>8.</td>
<td>8 batteries in 3 months</td>
<td>2.67 batteries per mo</td>
</tr>
<tr>
<td>9.</td>
<td>122 patients in 4 weeks</td>
<td>30.5 patients per wk</td>
</tr>
<tr>
<td>10.</td>
<td>51 gallons in 14 minutes</td>
<td>3.64 gal per min</td>
</tr>
<tr>
<td>11.</td>
<td>$8.43 for 3 pounds</td>
<td>$2.81 per lb</td>
</tr>
<tr>
<td>12.</td>
<td>357 miles in 6.3 hours</td>
<td>56.67 mi per h</td>
</tr>
<tr>
<td>13.</td>
<td>25 letters in 4 days</td>
<td>6.25 letters per day</td>
</tr>
<tr>
<td>14.</td>
<td>$99 for 12 CDs</td>
<td>$8.25 per CD</td>
</tr>
<tr>
<td>15.</td>
<td>5 breaks in 8 hours</td>
<td>0.63 break per h</td>
</tr>
<tr>
<td>16.</td>
<td>3 trips in 14 months</td>
<td>0.21 trip per mo</td>
</tr>
<tr>
<td>17.</td>
<td>2 pay raises in 3 years</td>
<td>0.67 raise per yr</td>
</tr>
<tr>
<td>18.</td>
<td>7 errors in 60 minutes</td>
<td>0.12 error per min</td>
</tr>
<tr>
<td>19.</td>
<td>15 pounds in 6 weeks</td>
<td>2.5 lb per wk</td>
</tr>
<tr>
<td>20.</td>
<td>8 commercials in 15 minutes</td>
<td>0.53 commercial per min</td>
</tr>
<tr>
<td>21.</td>
<td>8 glasses every 24 hours</td>
<td>0.33 glass per h</td>
</tr>
<tr>
<td>22.</td>
<td>13 feet in 5 steps</td>
<td>2.6 ft per step</td>
</tr>
</tbody>
</table>

Choose the lower unit price.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>23.</td>
<td>$4.99 for 6 cans or $7.99 for 10 cans</td>
<td>$7.99 for 10 cans</td>
</tr>
<tr>
<td>24.</td>
<td>$21.50 for 4 pounds of lunch meat or $15.10 for 3 pounds of lunch meat</td>
<td>$15.10 for 3 pounds</td>
</tr>
</tbody>
</table>
Lesson 2 Reteach

Complex Fractions and Unit Rates

Fractions like \( \frac{2}{3} \) are called complex fractions. **Complex fractions** are fractions with a numerator, denominator, or both that are also fractions.

Example 1

Simplify \( \frac{2}{3} \) or \( \frac{9}{10} \).

A fraction can also be written as a division problem.

\[
\frac{2}{3} = 2 \div \frac{3}{4}
\]

Write the complex fraction as a division problem.

\[
\frac{2}{3} = \frac{2}{1} \times \frac{4}{3}
\]

Multiply by the reciprocal of \( \frac{3}{4} \), which is \( \frac{4}{3} \).

\[
\frac{2}{3} = \frac{8}{3} \text{ or } 2 \frac{2}{3}
\]

Simplify.

So, \( \frac{2}{3} \) is equal to \( 2 \frac{2}{3} \).

Exercises

Simplify.

1. \( \frac{3}{1} \) or \( 9 \)
2. \( \frac{5}{7} \) or \( 1 \frac{2}{3} \)
3. \( \frac{4}{5} \) or \( 20 \)
4. \( \frac{2}{9} \) or \( 4 \frac{1}{2} \)
5. \( \frac{1}{4} \) or \( \frac{5}{4} \) or \( 1 \frac{1}{4} \)
6. \( \frac{10}{8} \) or \( 11 \frac{3}{7} \)
7. \( \frac{3}{7} \) or \( 1 \frac{2}{5} \)
8. \( \frac{1}{6} \) or \( \frac{1}{5} \)
9. \( \frac{4}{9} \) or \( \frac{8}{10} \)
10. \( \frac{3}{5} \) or \( 2 \)

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Lesson 2 Skills Practice

Complex Fractions and Unit Rates

Simplify.

1. \(\frac{1}{2} - \frac{5}{2} = 2\frac{1}{2}\)

2. \(\frac{4}{5} - \frac{32}{5} = 6\frac{2}{5}\)

3. \(\frac{4}{3} - \frac{32}{3} = 10\frac{2}{3}\)

4. \(\frac{10}{5} - \frac{24}{12} = 2\)

5. \(\frac{8}{3} - \frac{32}{3} = 10\frac{2}{3}\)

6. \(\frac{1}{5} - \frac{2}{7} = \frac{2}{10}\)

7. \(\frac{2}{5} - \frac{9}{10} = \frac{1}{2}\)

8. \(\frac{8}{9} - \frac{2}{45} = \frac{2}{20}\)

9. \(\frac{5}{6} - \frac{5}{72} = \frac{1}{12}\)

10. \(\frac{3}{8} - \frac{9}{14} = \frac{9}{12}\)

11. \(\frac{7}{9} - \frac{1}{18} = \frac{1}{14}\)

12. \(\frac{6}{7} - \frac{14}{49} = \frac{45}{49}\)

13. \(\frac{8}{11} - \frac{10}{11} = \frac{2}{5}\)

14. \(\frac{30}{5} - \frac{42}{7} = \frac{6}{7}\)

15. \(\frac{6}{7} - \frac{2}{49} = \frac{25}{9}\)

16. \(\frac{15}{5} - \frac{27}{9} = \frac{26}{9}\)

17. \(\frac{1}{3} - \frac{3}{8} = \frac{25}{24}\)

18. \(\frac{2}{3} - \frac{36}{25} = \frac{25}{36}\)
Lesson 3 Reteach

Convert Unit Rates

Unit ratios and their reciprocals can be used to convert rates. Sometimes you have to multiply more than once.

**Example**
The speed limit on the interstate is 65 miles per hour. How many feet per minute is the speed limit?

Because the unit of miles must divide out, use the unit ratio \( \frac{5,280 \text{ ft}}{1 \text{ mi}} \) because the unit of miles is in the denominator. Use \( \frac{1 \text{ h}}{60 \text{ min}} \) to convert from hours to minutes.

\[
\frac{65 \text{ mi}}{1 \text{ h}} = \frac{65 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}}
\]

\[
= \frac{65 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}}
\]

\[
= \frac{65 \cdot 5,280 \text{ ft} \cdot 1}{1 \cdot 1 \cdot 60 \text{ min}} = 343,200 \text{ ft} \quad \text{or} \quad 5,720 \frac{\text{ft}}{1 \text{ min}}
\]

The speed limit is 5,720 feet per minute.

**Exercises**

Convert each rate.

1. \( 10 \text{ mi/h} = \frac{880}{1} \text{ ft/min} \)

2. \( 35 \text{ cm/sec} = \frac{21}{1} \text{ m/min} \)

3. \( 4.5 \text{ mi/h} = \frac{6.6}{1} \text{ ft/sec} \)

4. **WALK** Tina walks at a rate of 180 feet per minute. How many feet per second does Tina walk? \( 3 \text{ ft/s} \)

5. **TRAVELING** A car is traveling at a rate of 55 miles per hour. How many feet per hour does the car travel? \( 290,400 \frac{\text{ft}}{1 \text{ h}} \)
Lesson 3 Skills Practice

Convert Unit Rates

Complete. Round to the nearest tenth if necessary.

1. \(660 \text{ ft/min} = \underline{11} \text{ ft/s}\)
2. \(25 \text{ mi/h} \approx \underline{36.7} \text{ ft/s}\)

3. \(32 \text{ gal/min} = \underline{7,680} \text{ qt/h}\)
4. \(425 \text{ ft/h} = \underline{85} \text{ in./min}\)

5. \(0.5 \text{ L/s} = \underline{1,800,000} \text{ mL/h}\)
6. \(60 \text{ ft/s} \approx \underline{0.7} \text{ mi/min}\)

7. \(3.4 \text{ mi/h} = \underline{5.0} \text{ ft/sec}\)
8. \(2.1 \text{ yd/min} = \underline{0.1} \text{ ft/s}\)

9. \(5.6 \text{ lb/gal} = \underline{89.6} \text{ oz/gal}\)
10. \(4 \text{ m/h} = \underline{6.7} \text{ cm/min}\)

11. \(42 \text{ cm/s} = \underline{25.2} \text{ m/min}\)
12. \(4,500 \text{ ft/h} = \underline{1.3} \text{ ft/s}\)

13. \(8 \text{ mi/h} = \underline{704} \text{ ft/min}\)
14. \(900 \text{ cm/h} = \underline{15} \text{ cm/min}\)

15. JOGGING Jarin jogs at a rate of 7.5 miles per hour. How many miles per minute does Jarin jog? \(0.125 \text{ mi/min}\)

16. BUCKETS Alonzo fills buckets at a rate of 6 gallons per minute. What is the rate in pints per hour? \(2,880 \text{ pints per hour}\)
Lesson 4 Reteach

Proportional and Nonproportional Relationships

Two related quantities are proportional if they have a constant ratio between them. If two related quantities do not have a constant ratio, then they are nonproportional.

Example 1
The cost of one CD at a record store is $12. Create a table to show the total cost for different numbers of CDs. Is the total cost proportional to the number of CDs purchased?

<table>
<thead>
<tr>
<th>Number of CDs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>$12</td>
<td>$24</td>
<td>$36</td>
<td>$48</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Total Cost}}{\text{Number of CDs}} = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \frac{48}{4} = $12 \text{ per CD}
\]

Divide the total cost for each by the number of CDs to find a ratio. Compare the ratios.

Since the ratios are the same, the total cost is proportional to the number of CDs purchased.

Example 2
The cost to rent a lane at a bowling alley is $9 per hour plus $4 for shoe rental. Create a table to show the total cost for each hour a bowling lane is rented if one person rents shoes. Is the total cost proportional to the number of hours rented?

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>$13</td>
<td>$22</td>
<td>$31</td>
<td>$40</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Total Cost}}{\text{Number of Hours}} \rightarrow \frac{13}{1} \text{ or } \frac{22}{2} \text{ or } 11 \frac{31}{3} \text{ or } 10.34 \frac{40}{4} \text{ or } 10
\]

Divide each cost by the number of hours.

Since the ratios are not the same, the total cost is nonproportional to the number of hours rented with shoes.

Exercises

1. PICTURES A photo developer charges $0.25 per photo developed. Is the total cost proportional to the number of photos developed? Yes

<table>
<thead>
<tr>
<th>Number of Photos</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>0.25</td>
<td>0.50</td>
<td>0.75</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Total Cost}}{\text{Number of Hours}} \rightarrow \frac{0.25}{1} = \frac{0.50}{2} = \frac{0.75}{3} = \frac{1.00}{4} = $0.25 \text{ per photo}
\]

2. SOCCER A soccer club has 15 players for every team, with the exception of two teams that have 16 players each. Is the number of players proportional to the number of teams? No

<table>
<thead>
<tr>
<th>Number of Teams</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Players</td>
<td>16</td>
<td>32</td>
<td>47</td>
<td>62</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Number of Teams}}{\text{Number of Players}} \rightarrow \frac{16}{1} = \frac{32}{2} \neq \frac{47}{3} \neq \frac{62}{4}
\]
Lesson 4 Skills Practice

Proportional and Nonproportional Relationships

For Exercises 1–3, use the table of values. Write the ratios in the table to show the relationship between each set of values.

1. Number of Hours | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Amount Earned</td>
<td>$15</td>
<td>$30</td>
<td>$45</td>
<td>$60</td>
</tr>
<tr>
<td>Ratios</td>
<td>$15 \div 1$ or $15 \div 15$</td>
<td>$30 \div 2$ or $15 \div 15$</td>
<td>$45 \div 3$ or $15 \div 15$</td>
<td>$60 \div 4$ or $15 \div 15$</td>
</tr>
</tbody>
</table>

2. Number of Packages | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>$11</td>
<td>$20</td>
<td>$29</td>
<td>$38</td>
</tr>
<tr>
<td>Ratios</td>
<td>$11 \div 1$ or $11 \div 1$</td>
<td>$20 \div 2$ or $10 \div 10$</td>
<td>$29 \div 3$ or $9.67 \div 3$</td>
<td>$38 \div 4$ or $9.5 \div 4$</td>
</tr>
</tbody>
</table>

3. Number of Classrooms | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Students</td>
<td>24</td>
<td>48</td>
<td>72</td>
<td>92</td>
</tr>
<tr>
<td>Ratios</td>
<td>$24 \div 1$ or $24 \div 1$</td>
<td>$48 \div 2$ or $24 \div 12$</td>
<td>$72 \div 3$ or $24 \div 8$</td>
<td>$92 \div 4$ or $23 \div 4$</td>
</tr>
</tbody>
</table>

For Exercises 4–8 use the table of values. Write proportional or nonproportional.

4. Number of Hours | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Amount Earned</td>
<td>$0.99</td>
<td>$1.98</td>
<td>$2.97</td>
<td>$3.96</td>
</tr>
<tr>
<td>proportional</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Number of Hours | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Amount Earned</td>
<td>$17.25</td>
<td>$35.50</td>
<td>$50.75</td>
<td>$70</td>
</tr>
<tr>
<td>nonproportional</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Number of Hours | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pages Read in Book</td>
<td>37</td>
<td>73</td>
<td>109</td>
<td>145</td>
</tr>
<tr>
<td>nonproportional</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Number of Lunches | 1 | 2 | 3 | 4
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>$2.75</td>
<td>$5.50</td>
<td>$8.25</td>
<td>$11</td>
</tr>
<tr>
<td>proportional</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Fred is ordering pies for a family reunion. Each pie costs $4.50. For orders smaller than a dozen pies, there is a $5 delivery charge. Is the cost proportional to the number of pies ordered? Use a table of values to explain your reasoning.

<table>
<thead>
<tr>
<th>Number of Pies</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>$9.50</td>
<td>$14.00</td>
<td>$18.50</td>
<td>$23.00</td>
</tr>
</tbody>
</table>
| \[ \frac{\text{Total Cost}}{\text{Number of Pies}} = \frac{9.50}{1} \neq \frac{14.00}{2} \] nonproportional
Example

**LIGHT** The speed of light is about 186,000 miles per second. What is the approximate speed of light in feet per second?

**Understand**
We know that the speed of light is about 186,000 miles per second. We need to find the speed of light in feet per second.

**Plan**
To solve the problem, write an expression that converts miles per second to feet per second. Then divide out common units.

**Solve**
One mile is 5,280 feet.

\[
\frac{186,000 \text{ mi}}{1 \text{ sec}} \times \frac{5,280 \text{ feet}}{1 \text{ mi}} = \frac{982,080,000 \text{ feet}}{1 \text{ sec}}
\]

So, the speed of light is about 982,080,000 feet per second.

**Check**
You know that if light travels about 186,000 miles per second, the number of feet that it travels in the same amount of time will have a higher value.

**Exercises**

1. **SWIMMING** Noah is training for a swim meet and swims 500 meters each day. How many kilometers does Noah swim in a week if he trains every day? **3.5 km**
Skills Practice

Problem-Solving Investigation: The Four-Step Plan

Solve.

1. LIFE SCIENCE An adult has about 5 quarts of blood. About how many fluid ounces is this? 160 fl oz

2. COFFEE In Switzerland, the average amount of coffee consumed per year is 1,089 cups per person. About how many cups is this per week? Express your answer to the nearest whole cup. 21 cups

3. SKYSCRAPERS A skyscraper is 0.493 kilometers tall. What is the height of the skyscraper in meters? 493 m

4. NUMBERS A number is multiplied by 5. Then 8 is added to the product. The result is 63. What is the number? 11

5. MONEY Fernando has $1.20 in change in his pocket. If he has an equal number of nickels, dimes, and quarters, how many of each does he have? 3 of each coin

6. FRUIT Ling places 3 apples and 2 oranges into each fruit basket she makes. If she has used 18 apples and 12 oranges, how many fruit baskets has she made? 6 baskets
Lesson 5 Reteach

**Graph Proportional Relationships**

A way to determine whether two quantities are proportional is to graph them on a coordinate plane. If the graph is a straight line through the origin, then the two quantities are proportional.

**Example 1**

A racquetball player burns 7 Calories a minute. Determine whether the number of Calories burned is proportional to the number of minutes played by graphing on the coordinate plane.

**Step 1**

Make a table to find the number of Calories burned for 0, 1, 2, 3, and 4 minutes of playing racquetball.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories Burned</td>
<td>0</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
</tr>
</tbody>
</table>

**Step 2**

Graph the ordered pairs on the coordinate plane. Then connect the ordered pairs.

The line passes through the origin and is a straight line. So, the number of Calories burned is proportional to the number of minutes of racquetball played.

**Exercise**

1. Shontell spends $7 a month plus $0.10 per minute. Determine whether the cost per month is proportional to the number of minutes by graphing on the coordinate plane.
Lesson 5 Skills Practice

Graph Proportional Relationships

Determine whether the relationship between the two quantities shown in each table are proportional by graphing on the coordinate plane.

1. Volume of a Cube

<table>
<thead>
<tr>
<th>Side Length (ft)</th>
<th>Volume (ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

nonproportional

2. DVD Rental

<table>
<thead>
<tr>
<th>Number of DVDs</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

nonproportional

3. Gallons of Gas Used Per Hour

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Gallons of Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

proportional
Lesson 6 Reteach

Solve Proportional Relationships

A proportion is an equation that states that two ratios are equivalent. To determine whether a pair of ratios forms a proportion, use cross products. You can also use cross products to solve proportions.

Example 1
Determine whether the pair of ratios \( \frac{20}{24} \) and \( \frac{12}{18} \) form a proportion.

Find the cross products.
\[
\frac{20}{24} \rightarrow 24 \cdot 12 = 288 \\
\frac{12}{18} \rightarrow 20 \cdot 18 = 360
\]

Since the cross products are not equal, the ratios do not form a proportion.

Example 2
Solve \( \frac{12}{30} = \frac{k}{70} \).

\[
\frac{12}{30} = \frac{k}{70}
\]

Write the equation.

\[
12 \cdot 70 = 30 \cdot k
\]

Find the cross products.

\[
840 = 30k
\]

Multiply.

\[
\frac{840}{30} = \frac{30k}{30}
\]

Divide each side by 30.

\[
28 = k
\]

Simplify.

The solution is 28.

Exercises

Determine whether each pair of ratios forms a proportion.

1. \( \frac{17}{10}, \frac{12}{5} \) no
2. \( \frac{6}{9}, \frac{12}{18} \) yes
3. \( \frac{8}{12}, \frac{10}{15} \) yes

4. \( \frac{7}{15}, \frac{13}{32} \) no
5. \( \frac{7}{9}, \frac{49}{63} \) yes
6. \( \frac{8}{24}, \frac{12}{28} \) no

7. \( \frac{4}{7}, \frac{12}{71} \) no
8. \( \frac{20}{35}, \frac{30}{45} \) no
9. \( \frac{18}{24}, \frac{3}{4} \) yes

Solve each proportion.

10. \( \frac{x}{5} = \frac{15}{25} \) 3
11. \( \frac{3}{4} = \frac{12}{c} \) 16
12. \( \frac{6}{9} = \frac{10}{r} \) 15

13. \( \frac{16}{24} = \frac{z}{15} \) 10
14. \( \frac{5}{8} = \frac{s}{12} \) 7.5
15. \( \frac{14}{t} = \frac{10}{11} \) 15.4

16. \( \frac{w}{6} = \frac{2.8}{7} \) 2.4
17. \( \frac{5}{y} = \frac{7}{16.8} \) 12
18. \( \frac{x}{18} = \frac{7}{36} \) 3.5
Lesson 6 Skills Practice

Solve Proportional Relationships

Determine whether each pair of ratios form a proportion.
1. \( \frac{5}{8}, \frac{2}{3} \) no
2. \( \frac{7}{3}, \frac{14}{6} \) yes
3. \( \frac{6}{8}, \frac{9}{12} \) yes
4. \( \frac{16}{9}, \frac{11}{6} \) no
5. \( \frac{55}{10}, \frac{12}{2} \) no
6. \( \frac{6}{8}, \frac{15}{20} \) yes
7. \( \frac{5}{9}, \frac{15}{27} \) yes
8. \( \frac{3}{18}, \frac{11}{66} \) yes
9. \( \frac{7}{11}, \frac{15}{23} \) no
10. \( \frac{9}{13}, \frac{13}{17} \) no
11. \( \frac{3}{42}, \frac{5}{70} \) yes
12. \( \frac{6}{7}, \frac{36}{49} \) no

Solve each proportion.
13. \( \frac{4}{12} = \frac{y}{9} \) 3
14. \( \frac{6}{18} = \frac{4}{c} \) 12
15. \( \frac{7}{z} = \frac{84}{12} \) 1
16. \( \frac{5}{10} = \frac{8}{w} \) 16
17. \( \frac{x}{9} = \frac{4}{15} \) 2.4
18. \( \frac{6}{20} = \frac{y}{5} \) 1.5
19. \( \frac{5}{9} = \frac{6}{r} \) 10.8
20. \( \frac{8}{n} = \frac{10}{7} \) 5.6
21. \( \frac{d}{5} = \frac{12}{80} \) 0.75
22. \( \frac{y}{5} = \frac{13}{10} \) 6.5
23. \( \frac{2}{28} = \frac{p}{33} \) 2.5
24. \( \frac{11}{l} = \frac{100}{11} \) 1.21
25. \( \frac{1.2}{m} = \frac{3}{5} \) 2
26. \( \frac{0.9}{0.5} = \frac{a}{10} \) 18
27. \( \frac{3}{7} = \frac{k}{4.2} \) 1.8
28. \( \frac{6.3}{x} = \frac{18}{5} \) 1.75
29. \( \frac{3.6}{9} = \frac{b}{0.5} \) 0.2
30. \( \frac{14}{1.5} = \frac{4.2}{y} \) 0.45
Lesson 7 Reteach

**Constant Rate of Change**

A rate of change is a rate that describes how one quantity changes in relation to another. A constant rate of change is the rate of change of a linear relationship.

**Example 1**
Find the constant rate of change for the table.

<table>
<thead>
<tr>
<th>Students</th>
<th>Number of Textbooks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

The change in the number of textbooks is 15. The change in the number of students is 5.

\[
\frac{\text{change in number of textbooks}}{\text{change in number of students}} = \frac{15 \text{ textbooks}}{5 \text{ students}}
\]

The number of textbooks increased by 15 for every 5 students.

\[
\frac{15}{5} = 3 \text{ textbooks per student}
\]

So, the number of textbooks increases by 3 textbooks per student.

**Example 2**
The graph represents the number of T-shirts sold at a band concert. Use the graph to find the constant rate of change in number per hour.

To find the rate of change, pick any two points on the line, such as (8, 25) and (10, 35).

\[
\frac{\text{change in number}}{\text{change in time}} = \frac{(35 - 25)}{(10 - 8)} = \frac{10}{2} = 5 \text{ T-shirts per hour}
\]

**Exercises**
Find the each constant rate of change.

1. | Side Length | Perimeter |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

4 units of perimeter per unit of side length

2. | Distance Traveled |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>72</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>88</td>
</tr>
<tr>
<td>96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 A.M.</td>
</tr>
<tr>
<td>12 P.M.</td>
</tr>
<tr>
<td>2 P.M.</td>
</tr>
</tbody>
</table>

4 miles per hour
Lesson 7 Skills Practice

Constant Rate of Change

Find the constant rate of change for each table.

1. Time Spent Mowing (h) | Money Earned ($)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
</tr>
</tbody>
</table>

$10 per hour

2. Time | Temperature (°F)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00</td>
<td>60</td>
</tr>
<tr>
<td>10:00</td>
<td>62</td>
</tr>
<tr>
<td>11:00</td>
<td>64</td>
</tr>
<tr>
<td>12:00</td>
<td>66</td>
</tr>
</tbody>
</table>

2 °F per hour

3. Number of Students | Number of Magazines Sold
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>25</td>
<td>250</td>
</tr>
</tbody>
</table>

10 magazines per student

4. Number of Trees | Number of Apples
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>15</td>
<td>300</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
</tr>
</tbody>
</table>

20 apples per tree

Find the constant rate of change for each graph.

5. Temperature Change

-5 °F per hour

6. Meat Consumption

3 lb per person
Lesson 8 Reteach

Slope

Slope is the rate of change between any two points on a line.

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}
\]

Example

The table shows the length of a patio as blocks are added.

<table>
<thead>
<tr>
<th>Number of Patio Blocks</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.)</td>
<td>0</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

Graph the data. Then find the slope of the line.

Explain what the slope represents.

\[
slope = \frac{24 - 8}{3 - 1} = \frac{16}{2} = \frac{8}{1}
\]

So, for every 8 inches, there is 1 patio block.

Exercises

Graph the data. Then find the slope of the line. Explain what the slope represents.

1. The table shows the number of juice bottles per case.

<table>
<thead>
<tr>
<th>Cases</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juice Bottles</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
</tr>
</tbody>
</table>

12; 12 juice bottles per case

2. At 6 A.M., the retention pond had 28 inches of water in it. The water receded so that at 10 A.M. there were 16 inches of water left.

−3; the water went down 3 inches per hour
Lesson 8 Skills Practice

Slope

Graph the data. Then find the slope. Explain what the slope represents.

1.  

<table>
<thead>
<tr>
<th>Hours</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages ($)</td>
<td>11</td>
<td>22</td>
<td>33</td>
<td>44</td>
</tr>
</tbody>
</table>

11; $11 per hour

2.  

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>70</th>
<th>78</th>
<th>86</th>
<th>94</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of People on Beach</td>
<td>24</td>
<td>40</td>
<td>56</td>
<td>72</td>
</tr>
</tbody>
</table>

2; the number of people on the beach increases by 2 for each degree increase in temperature

3. SWIMMING Latonya swims 50 meters in \( \frac{1}{2} \) minute.

100; 100 meters per minute

4. SONGS Find the slope of the line on the graph showing the cost to download songs. 0.5
Lesson 9 Reteach

Direct Variation

When two variable quantities have a constant ratio, their relationship is called a direct variation. The constant ratio is called the constant of proportionality.

Example 1
The time it takes Lucia to pick pints of blackberries is shown in the graph. Determine the constant of proportionality.

Since the graph forms a line, the rate of change is constant. Use the graph to find the constant of proportionality.

\[
\frac{\text{minutes}}{\text{number of pints}} = \frac{15}{1} \quad \text{or} \quad \frac{30}{2} \quad \text{or} \quad \frac{45}{3} \quad \text{or} \quad \frac{15}{1}
\]

It takes 15 minutes for Lucia to pick 1 pint of blackberries.

Example 2
There are 12 trading cards in a package. Make a table and graph to show the number of cards in 1, 2, 3, and 4 packages. Is there a constant rate? a direct variation?

<table>
<thead>
<tr>
<th>Numbers of Packages</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cards</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
</tr>
</tbody>
</table>

Because there is a constant increase of 12 cards, there is a constant rate of change. The equation relating the variables is \( y = 12x \), where \( y \) is the number of cards and \( x \) is the number of packages. This is a direct variation. The constant of proportionality is 12.

Exercises

1. SOAP Wilhema bought 6 bars of soap for $12. The next day, Sophia bought 10 bars of the same kind of soap for $20. What is the cost of 1 bar of soap? $2

2. COOKING Franklin is cooking a 3-pound turkey breast for 6 people. If the number of pounds of turkey varies directly with the number of people, make a table to show the number of pounds of turkey for 2, 4, and 8 people.

<table>
<thead>
<tr>
<th>People</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey (lb)</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
Lesson 9 Skills Practice

Direct Variation

Determine whether each linear function is a direct variation. If so, state the constant of proportionality.

1. Speed, \(x\) 25 30 35 40 \(y\) 100 120 140 160
   - yes; 4

2. Price, \(x\) $5 $8 $11 $14
   Tax, \(y\) $0.50 $0.80 $1.10 $1.40
   - yes; \(\frac{1}{10}\)

3. Seconds, \(x\) 15 30 45 60
   Number of Sit-ups, \(y\) 5 10 15 20
   - yes; \(\frac{1}{3}\)

4. DISHES The number of place settings of dishes varies directly with the number of boxes. How many place settings are in each box? 4

5. FIGURINES Kentish is arranging figurines on shelves. The number of figurines varies directly with the number of shelves. What is the constant of proportionality? 10

6. CAT FOOD Loretta paid $6.70 for 5 cans of cat food and $10.72 for 8 cans of cat food. How much did 1 can of cat food cost? $1.34

7. PILLOWS You need 2 yards of fabric to cover 3 pillows and 6 yards to cover 9 pillows. How much fabric do you need to cover 15 pillows? 10 yards

8. READING The number of pages read varies directly as the number of minutes. If you can read 20 pages in 45 minutes, how many pages can you read in 18 minutes? 8
Lesson 1 Reteach

Percent of a Number

To find the percent of a number, you can write the percent as a fraction and then multiply or write the percent as a decimal and then multiply.

Example 1
Find 25% of 80.

\[
25\% = \frac{25}{100} = \frac{1}{4}
\]

Write 25% as a fraction, and reduce to lowest terms.

\[
\frac{1}{4} \text{ of } 80 = \frac{1}{4} \times 80 = 20
\]

Multiply.

So, 25% of 80 is 20.

Example 2
What number is 15% of 200?

\[
15\% \times 200 = 0.15 \times 200 = 30
\]

Write a multiplication expression.

\[
\text{Write 15% as a decimal.}
\]

\[
\text{Multiply.}
\]

So, 15% of 200 is 30.

Exercises

Find each number.

1. Find 20% of 50. 10
2. What is 55% of $400? $220
3. 5% of 1,500 is what number? 75
4. Find 190% of 20. 38
5. What is 24% of $500? $120
6. 8% of $300 is how much? $24
7. What is 12.5% of 60? 7.5
8. Find 0.2% of 40. 0.8
9. Find 3% of $800. $24
10. What is 0.5% of 180? 9
11. 0.25% of 42 is what number? 10.5
12. What is 0.02% of 280? 5.6

Course 2 • Chapter 2 Percents
Lesson 1 Skills Practice

Percent of a Number

Find each number.

1. Find 80% of 80. 64

2. What is 95% of 600? 570

3. 35% of 20 is what number? 7

4. Find 60% of $150. $90

5. What is 75% of 240? 180

6. 380% of 30 is what number? 114

7. Find 40% of 80. 32

8. What is 30% of $320? $96

9. 12% of 150 is what number? 18

10. Find 58% of 200. 116

11. What is 18% of $450? $81

12. What is 70% of 1,760? 1,232

13. Find 92% of 120. 110.4

14. 45% of 156 is what number? 70.2

15. What is 12% of 12? 1.44

16. Find 60% of 264. 158.4

17. 37.5% of 16 is what number? 6

18. What is 82.5% of 400? 330

19. What is 0.25% of 900? 225

20. Find 1.5% of 220. 3.3
Lesson 2 Reteach

Percent and Estimation

To estimate the percent of a number, you can use a fraction or a multiple of 10% or 1%.

Example 1
Estimate 77% of 800.

77% is about 75% or \( \frac{3}{4} \).

\[ 77\% \text{ of } 800 \approx \frac{3}{4} \cdot 800 \]

Use \( \frac{3}{4} \) to estimate.

\[ \approx 600 \]

Multiply.

So, 77% of 800 is about 600.

Example 2
Estimate 137% of 50.

137% is more than 100%, so 137% of 50 is greater than 50. 137% \( \approx 140\% \).

\[ 140\% \text{ of } 50 = (100\% \text{ of } 50) + (40\% \text{ of } 50) \]

\[ = (1 \cdot 50) + \left( \frac{2}{5} \cdot 50 \right) \]

\[ = 50 + 20 \text{ or } 70 \]

Simplify.

So, 137% of 50 is about 70.

Example 3
Estimate 0.5% of 692.

0.5% is half of 1%. 692 is about 700.

\[ 1\% \text{ of } 700 = 0.01 \cdot 700 \]

To multiply by 1%, move the decimal point two places to the left.

\[ = 7 \]

One half of 7 is \( \frac{1}{2} \cdot 7 \) or 3.5.

So, 0.5% of 697 is about 3.5.

Exercises 1–9. Sample answers are given.

Estimate.

1. 24% of 36
   \[ \frac{1}{4} \cdot 36 = 9 \]

2. 81% of 25
   \[ \frac{4}{5} \cdot 25 = 20 \]

3. 11% of 67
   \[ 0.1 \cdot 70 = 7 \]

4. 150% of 179
   \[ (1 \cdot 180) + \left( \frac{1}{2} \cdot 180 \right) = 270 \]

5. 67% of 450
   \[ \frac{2}{3} \cdot 450 = 300 \]

6. 79% of 590
   \[ \frac{3}{4} \cdot 600 = 450 \]

7. 0.4% of 200
   \[ 0.01 \cdot 200 = 2 \text{ and} \]

8. 42% of 61
   \[ 0.1 \cdot 60 = 6 \text{ and} \]

9. 19% of 41
   \[ 0.1 \cdot 40 = 4 \text{ and} \]

\[ \frac{1}{2} \cdot 2 = 1 \]

\[ 4 \cdot 6 = 24 \]

\[ 2 \cdot 4 = 8 \]
Lesson 2 Skills Practice

Percent and Estimation

Estimate by using fractions.

1. $51\%$ of $128$
   \[ \frac{1}{2} \cdot 128 = 64 \]

2. $76\%$ of $200$
   \[ \frac{3}{4} \cdot 200 = 150 \]

3. $32.9\%$ of $90$
   \[ \frac{1}{3} \cdot 90 = 30 \]

4. $23\%$ of $8$
   \[ \frac{1}{4} \cdot 8 = 2 \]

5. $19\%$ of $45$
   \[ \frac{4}{5} \cdot 15 = 12 \]

6. $81\%$ of $16$
   \[ \frac{1}{5} \cdot 45 = 9 \]

Estimate by using 10%.

7. $12\%$ of $98$
   \[ 0.1 \cdot 100 = 10 \]

8. $89\%$ of $300$
   \[ 0.1 \cdot 300 = 30 \text{ and } 9 \cdot 30 = 270 \]

9. $31\%$ of $80$
   \[ 0.1 \cdot 80 = 8 \text{ and } 3 \cdot 8 = 24 \]

10. $28\%$ of $49$
    \[ 0.1 \cdot 50 = 5 \text{ and } 3 \cdot 5 = 15 \]

11. $62\%$ of $13$
    \[ 0.1 \cdot 10 = 1 \text{ and } 6 \cdot 1 = 6 \]

12. $77\%$ of $28$
    \[ 0.1 \cdot 30 = 3 \text{ and } 8 \cdot 3 = 24 \]

Estimate.

13. $308\%$ of $500$
    \[ 3 \cdot 500 = 1,500 \]

14. $0.5\%$ of $87$
    \[ 0.01 \cdot 90 = 0.9 \text{ and } \frac{1}{2} \cdot 0.9 = 0.45 \]

15. $153\%$ of $20$
    \[ (1 \cdot 20) + \left( \frac{1}{2} \cdot 20 \right) = 30 \]

16. $0.6\%$ of $41$
    \[ 0.01 \cdot 40 = 0.4 \text{ and } \frac{1}{2} \cdot 0.4 = 0.2 \]

17. $231\%$ of $54$
    \[ (2 \cdot 54) + \left( \frac{1}{3} \cdot 54 \right) = 126 \]

18. $0.9\%$ of $116$
    \[ 0.01 \cdot 116 = 1.16 \]

19. $0.26\%$ of $36$
    \[ 0.01 \cdot 36 = 0.36 \text{ and } \frac{1}{4} \cdot 0.36 = 0.09 \]

20. $425\%$ of $119$
    \[ (4 \cdot 120) + \left( \frac{1}{4} \cdot 120 \right) = 510 \]
Lesson 3 Reteach

The Percent Proportion

A percent proportion compares part of a quantity to a whole quantity for one ratio and lists the percent as a number over 100 for the other ratio.

\[
\frac{p}{w} = \frac{n}{100}
\]

Example 1
What percent of 24 is 18?

\[
\frac{p}{w} = \frac{n}{100}
\]

Let \( n\% \) represent the percent.

\[
\frac{18}{24} = \frac{n}{100}
\]

Write the proportion.

\[18 \times 100 = 24 \times n\]

Find the cross products.

\[1,800 = 24n\]

Simplify.

\[
\frac{1,800}{24} = \frac{24n}{24}
\]

Divide each side by 24.

\[75 = n\]

So, 18 is 75% of 24.

Example 2
What number is 60% of 150?

\[
\frac{p}{w} = \frac{n}{100}
\]

Let \( n\% \) represent the percent.

\[
\frac{n}{150} = \frac{60}{100}
\]

Write the proportion.

\[n \times 100 = 150 \times 60\]

Find the cross products.

\[100n = 9,000\]

Simplify.

\[
\frac{100n}{100} = \frac{9,000}{100}
\]

Divide each side by 100.

\[n = 90\]

So, 90 is 60% of 150.

Exercises

Find each number. Round to the nearest tenth if necessary.

1. What number is 25% of 20? 5
2. What percent of 50 is 30? 60%
3. 30 is 75% of what number? 40
4. 40% of what number is 36? 90
5. What number is 20% of 625? 125
6. 12 is what percent of 30? 40%
Lesson 3 Skills Practice
The Percent Proportion

Find each number. Round to the nearest tenth if necessary.

1. 50 is 20% of what number? 250
2. What percent of 20 is 4? 20%

3. What number is 70% of 250? 175
4. 10 is 5% of what number? 200

5. What number is 45% of 180? 81
6. 40% of what number is 82? 205

7. What percent of 90 is 36? 40%
8. 60 is 25% of what number? 240

9. What number is 32% of 1,000? 320
10. What percent of 125 is 5? 4%

11. 73 is 20% of what number? 365
12. 57% of 109 is what number? 62.1

13. What percent of 185 is 35? 18.9%
14. 25 is what percent of 365? 6.8%

15. 85% of 190 is what number? 161.5
16. 12.5 is 25% of what number? 50

17. What percent of 128 is 24? 18.8%
18. 5.25% of 170 is what number? 8.9

19. What is 82% of 230? 188.6
20. What percent of 49 is 7? 14.3%
Lesson 4 Reteach

The Percent Equation

To solve any type of percent problem, you can use the percent equation, part = percent \cdot whole, where the percent is written as a decimal.

Example 1
600 is what percent of 750?

600 is the part and 750 is the whole. Let \( n \) represent the percent.

\[
\text{part} = \text{percent} \cdot \text{whole} \\
600 = n \cdot 750
\]

Write the percent equation.

\[
\frac{600}{750} = \frac{750n}{750}
\]

Divide each side by 750.

\[
0.8 = n
\]

Simplify.

\[
80\% = n
\]

Write 0.8 as a percent. So, 600 is 80\% of 750.

Example 2
45 is 90\% of what number?

45 is the part and 90\% or 0.9 is the percent. Let \( w \) represent the whole.

\[
\text{part} = \text{percent} \cdot \text{whole} \\
45 = 0.9 \cdot w
\]

Write the percent equation.

\[
\frac{45}{0.9} = \frac{0.9w}{0.9}
\]

Divide each side by 0.9.

\[
50 = w
\]

Simplify. So, 45 is 90\% of 50.

Exercises

Write an equation for each problem. Then solve. Round to the nearest tenth if necessary.

1. What percent of 56 is 14?
   \[
   14 = n \cdot 56; 25\%
   \]

2. 36 is what percent of 40?
   \[
   36 = n \cdot 40; 90\%
   \]

3. 80 is 40\% of what number?
   \[
   80 = 0.4 \cdot w; 200
   \]

4. 65\% of what number is 78?
   \[
   78 = 0.65 \cdot w; 120
   \]

5. What percent of 2,000 is 8?
   \[
   8 = n \cdot 2,000; 0.4\%
   \]

6. What is 110\% of 80?
   \[
   p = 1.1 \cdot 80; 88
   \]

7. 85 is what percent of 170?
   \[
   85 = n \cdot 170; 50\%
   \]

8. Find 30\% of 70.
   \[
   p = 0.3 \cdot 70; 21
   \]
Lesson 4 Skills Practice

The Percent Equation

Write an equation for each problem. Then solve. Round to the nearest tenth if necessary.

1. 25% of 176 is what number?
   \[ p = 0.25 \cdot 176; 44 \]

2. What is 90% of 20?
   \[ p = 0.9 \cdot 20; 18 \]

3. 24 is what percent of 30?
   \[ 24 = n \cdot 30; 80\% \]

4. 80% of what number is 94?
   \[ 94 = 0.8 \cdot w; 117.5 \]

5. What is 60% of 45?
   \[ p = 0.6 \cdot 45; 27 \]

6. 9 is what percent of 30?
   \[ 9 = n \cdot 30; 30\% \]

7. What percent of 125 is 25?
   \[ 25 = n \cdot 125; 20\% \]

8. What is 120% of 20?
   \[ p = 1.2 \cdot 20; 24 \]

9. 2% of what number is 5?
   \[ 5 = 0.02 \cdot w; 250 \]

10. 15% of 290 is what number?
    \[ p = 0.15 \cdot 290; 43.5 \]

11. 16 is what percent of 4,000?
    \[ 16 = n \cdot 4,000; 0.4\% \]

12. What is 140% of 60?
    \[ p = 1.4 \cdot 60; 84 \]

13. 344.8 is what percent of 862?
    \[ 215 = n \cdot 860; 25\% \]

14. 6% of what number is 21?
    \[ 21 = 0.06 \cdot w; 350 \]

15. What number is 60% of 605?
    \[ p = 0.6 \cdot 605; 363 \]

16. 32% of 250 is what number?
    \[ p = 0.32 \cdot 250; 80 \]

17. Find 30% of 70.
    \[ p = 0.3 \cdot 70; 21 \]

18. What is 80% of 65?
    \[ p = 0.8 \cdot 65; 52 \]
Reteach

**Problem-Solving Investigation:**
*Determine Reasonable Answers*

When solving problems, it is often helpful to determine reasonable answers by using rounding and estimation. Checking answers with a calculator is always helpful in determining if the answer found is, in fact, reasonable.

**Example**

**SURVEYS** On its blog, the student council reported that 4.8% of the 895 students have a pet hamster. Julian said that 45 students have a pet hamster. Is 45 students a reasonable estimate? Justify your answer.

**Understand**  The number of students is 895. The percent of students with pet hamsters is 4.8%. Julian’s estimate is 45 students.

**Plan**  Round 895 to 900 and 4.8% to 5%. Then use mental math to find 5% of 900.

**Solve**  Round 895 to 900.
Round 4.8% to 5%.

\[
10\% \text{ of } 900 = 0.1 \times 900, \text{ or } 90
\]
Use mental math. 10% = 0.1

5% is \(\frac{1}{2}\) of 10%.

So, \(\frac{1}{2}\) of 90 is 45.

So, 45 is a reasonable estimate for the number of students with pet hamsters.

**Check**  Use a calculator to check.

\[
0.048 \times 895 = 42.96
\]

Since 45 is close to 42.96, the answer is reasonable.

**Exercises**

1. **TELEVISION** A recent survey shows that 67% of students watch 3 or more hours of television a night. Suppose there are 892 students in your school. What would be a reasonable estimate of the number of students in your school who watch 3 or more hours of television a night? Explain your reasoning.  **630 students; 70% of 900 students is 630 students.**

2. **REUNIONS** The Hernandez family invited 150 relatives to a family reunion. Seventy-eight percent of the relatives attended the reunion. Is 110, 120, or 130 a reasonable estimate for the number of relatives that attended the reunion? Explain.  **120; Sample answer: 78% rounds to 80%; 80% of 150 = 120; Since 117 is close to 120, 120 is a reasonable answer.**
Skills Practice

Problem-Solving Investigation:
Determine Reasonable Answers

Determine reasonable answers for each.

1. **MONEY** Gillian, Roger, LaToya, and Ichiro had lunch at a restaurant. After sales tax and tip were added, the total bill was $23.80. They decided that everyone would pay 25% of the total bill. What is a reasonable amount for how much each person paid?  
   **Sample answer:** $6

2. **SPORTS** Of the 82,000 fans that attended a bowl game between Ohio State and Notre Dame, 60% were Ohio State fans. About how many fans at the game were for Notre Dame?  
   **32,000 (32,800 exactly)**

3. **ICE CREAM** A survey of 1,950 people found that 39% preferred chocolate ice cream to vanilla. About how many people preferred chocolate ice cream according to the survey?  
   **800 (760.5 exactly)**

4. **EARTH** The surface area of Earth is approximately 70% water. If the surface area is about 510,000,000 square kilometers, about how many square kilometers are water?  
   **350,000,000 km² (357,000,000 km² exactly)**

5. **COLLEGE** Of 7,450 first-year college students interviewed, 72% had changed their major area of study since the beginning of the academic year. About how many students had kept the same major?  
   **2,250 (2,086 exactly)**

6. **MONEY** While shopping, Hilary spent $149. If the amount she spent was 20% of her savings, how much savings did she have before she shopped?  
   **$750 ($745 exactly)**
Lesson 5 Reteach

Percent of Change

A percent of change is a ratio that compares the change in quantity to the original amount. If the original quantity is increased, it is a percent of increase. If the original quantity is decreased, it is a percent of decrease.

Example 1
Last year, 2,376 people attended the rodeo. This year, attendance was 2,950. What was the percent of change in attendance to the nearest whole percent?

Since this year’s attendance is greater than last year’s attendance, this is a percent of increase.

The amount of change is 2,950 – 2,376 or 574.

\[
\text{percent of change} = \frac{\text{amount of increase}}{\text{original amount}}
\]

\[
= \frac{574}{2,376}
\]

\[
\approx 0.24 \text{ or } 24\%
\]

The percent of increase is about 24%.

Example 2
Che’s grade on the first math exam was 94. His grade on the second math exam was 86. What was the percent of change in Che’s grade to the nearest whole percent?

Since the second grade is less than the first grade, this is a percent of decrease. The amount of change is 94 – 86 or 8.

\[
\text{percent of change} = \frac{\text{amount of decrease}}{\text{original amount}}
\]

\[
= \frac{8}{94}
\]

\[
\approx 0.09 \text{ or } 9\%
\]

The percent of decrease is 9%.

Exercises
Find each percent of change. Round to the nearest whole percent if necessary. State whether the percent of change is an increase or decrease.

1. original: 4  
   new: 5  
   25%; increase

2. original: 1.0  
   new: 1.3  
   30%; increase

3. original: 15  
   new: 12  
   20%; decrease

4. original: $30  
   new: $18  
   40%; decrease

5. original: 60  
   new: 63  
   5%; increase

6. original: 160  
   new: 136  
   15%; decrease

7. original: 7.7  
   new: 10.5  
   36%; increase

8. original: 9.6  
   new: 5.9  
   39%; decrease
### Lesson 5 Skills Practice

**Percent of Change**

Find each percent of change. Round to the nearest whole percent if necessary. State whether the percent of change is an *increase* or *decrease*.

<table>
<thead>
<tr>
<th>Original Value</th>
<th>New Value</th>
<th>Percentage Change</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. original: 35</td>
<td>new: 70</td>
<td>100%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>2. original: 8</td>
<td>new: 12</td>
<td>50%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>3. original: 45</td>
<td>new: 30</td>
<td>33%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>4. original: $350</td>
<td>new: $400</td>
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<td>5. original: $75</td>
<td>new: $60</td>
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<td>decrease</td>
</tr>
<tr>
<td>6. original: 250</td>
<td>new: 100</td>
<td>60%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>7. original: $30</td>
<td>new: $110</td>
<td>267%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>8. original: 35</td>
<td>new: 28</td>
<td>20%; decrease</td>
<td>decrease</td>
</tr>
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<td>9. original: $12.50</td>
<td>new: $15</td>
<td>20%; increase</td>
<td>increase</td>
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<td>10. original: 80</td>
<td>new: 52</td>
<td>35%; decrease</td>
<td>decrease</td>
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<tr>
<td>11. original: 45</td>
<td>new: 63</td>
<td>40%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>12. original: 120</td>
<td>new: 132</td>
<td>10%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>13. original: $210</td>
<td>new: $105</td>
<td>50%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>14. original: 84</td>
<td>new: 111</td>
<td>32%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>15. original: $84</td>
<td>new: $100</td>
<td>19%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>16. original: 6.8</td>
<td>new: 8.2</td>
<td>21%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>17. original: 1.5</td>
<td>new: 2.5</td>
<td>67%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>18. original: 91</td>
<td>new: 77</td>
<td>15%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>19. original: $465.50</td>
<td>new: $350</td>
<td>25%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>20. original: $87.05</td>
<td>new: $100</td>
<td>15%; increase</td>
<td>increase</td>
</tr>
<tr>
<td>21. original: 144</td>
<td>new: 108</td>
<td>25%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>22. original: 20.8</td>
<td>new: 12.2</td>
<td>41%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>23. original: $75</td>
<td>new: $15</td>
<td>80%; decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>24. original: 8.6</td>
<td>new: 7</td>
<td>19%; decrease</td>
<td>decrease</td>
</tr>
</tbody>
</table>


Lesson 6 Reteach
Sales Tax, Tips, and Markup

Sales Tax is a percent of the purchase price and is an amount paid in addition to the purchase price. Tip, or gratuity, is a small amount of money in return for service. The amount a store increases the price of an item by is called the markup.

Example 1

SOCCER  Find the total cost of a $17.75 soccer ball if the sales tax is 6%.

Method 1
First, find the sales tax.
6% of $17.75 = 0.06 \cdot 17.75
\approx 1.07
The sales tax is $1.07.
Next, add the sales tax to the regular price.
1.07 + 17.75 = 18.82
The total cost of the soccer ball is $18.82.

Method 2
100% + 6% = 106% Add the percent of tax to 100%.
The total cost is 106% of the regular price.
106% of $17.75 = 1.06 \cdot 17.75
\approx 18.82

Example 2

MEAL  A customer wants to leave a 15% tip on a bill for $18.50 at a restaurant.

Method 1 Add tip to regular price.
First, find the tip.
15% of $18.50 = 0.15 \cdot 18.50
= 2.78
Next, add the tip to the bill total.
$18.50 + $2.78 = $21.28
The total cost of the bill is $21.28.

Method 2 Add the percent of tip to 100%.
100% + 15% = 115% Add the percent of tip to 100%.
The total cost is 115% of the bill.
115% of $18.50 = 1.15 \cdot 18.50
= 21.28

Exercises
Find the total cost to the nearest cent.

1. $22.95 shirt, 6% tax
   $24.33

2. $24 lunch, 15% tip
   $27.60

3. $10.85 book, 4% tax
   $11.28

4. $97.55 business breakfast, 18% tip
   $115.11

5. $59.99 DVD box set, 6.5% tax
   $63.89

6. $37.65 dinner, 15% tip
   $43.30
Lesson 6 Skills Practice

Sales Tax, Tips, and Markup

Find the total cost to the nearest cent.

1. $49.95 CD player; 5% tax
   $52.45

2. $69 shoes; 6% tax
   $73.14

3. $37 dinner; 15% tip
   $42.55

4. $2.99 socks; 5.5% markup
   $3.15

5. $115 coat; 7% tax
   $123.05

6. $15 lunch; 20% tip
   $18.00

7. $299 DVD player; 7% tax
   $319.93

8. $43 shirt; 6% tax
   $45.58

9. $16 haircut; 15% tip
   $18.40

10. $8.75 breakfast; 15% tip
    $10.06

11. $47 tie; 4.5% markup
    $49.12

12. $40.80 dinner; 17% tip
    $47.74

13. $52 lunch; 20% tip
    $62.40

14. $18.99 CD; 6% markup
    $20.13

15. $22 haircut; 20% tip
    $26.40

16. $128 catered dinners; 18% tip
    $151.04
Lesson 7 Reteach

Discount

Discount is the amount by which the regular price of an item is reduced. The sale price is the regular price minus the discount.

Example

TENNIS Find the price of a $69.50 tennis racket that is on sale for 20% off the regular price.

Method 1: Subtract the discount from the regular price.
First, find the amount of the discount.

\[ 20\% \text{ of } $69.50 = 0.2 \cdot $69.50 \]
\[ = $13.90 \]

Write 20% as a decimal.  
The discount is $13.90.

Next, subtract the discount from the regular price.

\[ $69.50 - $13.90 = $55.60. \]

Method 2: Subtract the percent of discount from 100%.

\[ 100\% - 20\% = 80\% \]

Subtract the discount from 100%.

The sale price is 80% of the regular price.

\[ 80\% \text{ of } $69.50 = 0.80 \cdot 69.50 \]
\[ = 55.60 \]

The sale price of the tennis racket is $55.60.

Exercises

Find the sale price to the nearest cent.

1. $32.45 shirt; 15% discount  
   $27.58

2. $128.79 watch; 30% discount  
   $90.15

3. $40.00 jeans; 20% discount  
   $32.00

4. $74.00 sweatshirt; 25% discount  
   $55.50

5. $28.00 basketball; 50% discount  
   $14.00

6. $98.00 tent; 40% discount  
   $58.80
Lesson 7 Skills Practice

Discount

Find the sale price to the nearest cent.

1. $89.95 DVD player; 5% discount
   $85.45

2. $75 dress shirt; 20% discount
   $60.00

3. $14 socks; 15% discount
   $11.90

4. $2.99 toy; 30% discount
   $2.09

5. $140 coat; 10% discount
   $126.00

6. $65 dress pants; 20% discount
   $52.00

7. $325 tent; 15% discount
   $276.25

8. $80 boots; 25% discount
   $60.00

9. $45.50 book; 30% discount
   $31.85

10. $52 tie; 50% discount
    $26.00

11. $35 volleyball; 20% discount
    $28.00

12. $490 stove; 15% discount
    $416.50

13. $299 bicycle; 10% discount
    $269.10

14. $32 shorts; 50% discount
    $16.00

15. $5 box of cereal; 40% discount
    $3.00

16. $45 shelf; 35% discount
    $29.25
Lesson 8 Reteach

Financial Literacy

Simple interest is the amount of money paid or earned for the use of money. To find simple interest I, use the formula \( I = prt \). Principal \( p \) is the amount of money deposited or invested. Rate \( r \) is the annual interest rate written as a decimal. Time \( t \) is the amount of time the money is invested in years.

Example 1
Find the simple interest earned in a savings account where $136 is deposited for 2 years if the interest rate is 7.5% per year.

\[
I = prt
\]

\[
I = 136 \cdot 0.075 \cdot 2
\]

\[
I = 20.40
\]

The simple interest earned is $20.40.

Example 2
Find the simple interest for $600 invested at 8.5% for 6 months.

6 months = \( \frac{6}{12} \) or 0.5 year

\[
I = prt
\]

\[
I = 600 \cdot 0.085 \cdot 0.5
\]

\[
I = 25.50
\]

The simple interest is $25.50.

Exercises
Find the simple interest earned to the nearest cent for each principal, interest rate, and time.

1. $300, 5%, 2 years
   \[
   I = 30
   \]

2. $650, 8%, 3 years
   \[
   I = 156
   \]

3. $575, 4.5%, 4 years
   \[
   I = 103.50
   \]

4. $735, 7%, \( 2\frac{1}{2} \) years
   \[
   I = 128.63
   \]

5. $1,665, 6.75%, 3 years
   \[
   I = 337.16
   \]

6. $2,105, 11%, \( 1\frac{3}{4} \) years
   \[
   I = 405.21
   \]

7. $903, 8.75%, 18 months
   \[
   I = 118.52
   \]

8. $4,275, 19%, 3 months
   \[
   I = 203.06
   \]
Lesson 8 Skills Practice

Financial Literacy

Find the simple interest earned to the nearest cent for each principal, interest rate, and time.

1. $500, 4%, 2 years  
   $40

2. $350, 6.2%, 3 years  
   $65.10

3. $740, 3.25%, 2 years  
   $48.10

4. $725, 4.3%, 2 $\frac{1}{2}$ years  
   $77.94

5. $955, 6.75%, 3 $\frac{1}{4}$ years  
   $209.50

6. $1,540, 8.25%, 2 years  
   $254.10

7. $3,500, 4.2%, 1 $\frac{3}{4}$ years  
   $257.25

8. $568, 16%, 8 months  
   $60.59

Find the simple interest paid to the nearest cent for each loan, interest rate, and time.

9. $800, 9%, 4 years  
   $288

10. $280, 5.5%, 4 years  
    $61.60

11. $1,150, 7.6%, 5 years  
    $437

12. $266, 5.2%, 3 years  
    $41.50

13. $450, 22%, 1 year  
    $99

14. $2,180, 7.7%, 2 $\frac{1}{2}$ years  
    $419.65

15. $2,650, 3.65%, 4 $\frac{1}{2}$ years  
    $435.26

16. $1,245, 5.4%, 6 months  
    $33.62
Lesson 1 Reteach

Integers and Absolute Value

Integers less than zero are **negative integers**. Integers greater than zero are **positive integers**.

The **absolute value** of an integer is the distance the number is from zero on a number line. Two vertical bars are used to represent absolute value. The symbol for absolute value of 3 is |3|.

**Example 1**
Write an integer that represents 160 feet below sea level.
Because it represents **below** sea level, the integer is −160.

**Example 2**
Evaluate |−2|.
On the number line, the point −2 is 2 units away from 0. So, |−2| = 2.

**Exercises**
Write an integer for each situation.
1. 12°C above zero 12
2. a loss of $24 −24
3. a gain of 20 pounds 20
4. falling 6 feet −6

Evaluate each expression.
5. |12| 12
6. |−150| 150
7. |−8| + 2 10
8. |6| + |5| 11
9. |−19| − 17 2
10. |84| − |−62| 22
Lesson 1 Skills Practice

Integers and Absolute Value

Write an integer for each situation.

1. 15°C below zero  $-15$
2. a profit of $27  $27$
3. 2010 A.D.  $2010$
4. average attendance is down 38 people  $-38$
5. 376 feet above sea level  $376$
6. a withdrawal of $200  $-200$
7. 3 points lost  $-3$
8. a bonus of $150  $150$
9. a deposit of $41  $41$
10. 240 B.C.  $-240$
11. a wage increase of $120  $120$
12. 60 feet below sea level  $-60$

Evaluate each expression.

13. $|−1| 1$
14. $|9| 9$
15. $|23| 23$
16. $|−107| 107$
17. $|−45| 45$
18. $|19| 19$
19. $|0| 0$
20. $|6|−|−2| 4$
21. $|−8| + |4| 12$
22. $|−12|−|12| 0$

Graph each set of integers on a number line.

23. $\{0, 2, −3\}$
24. $\{−4, −1, 3\}$
Lesson 2 Reteach

Add Integers

To add integers with the same sign, add their absolute values. The sum is:
- positive if both integers are positive.
- negative if both integers are negative.

To add integers with different signs, subtract their absolute values. The sum is:
- positive if the positive integer’s absolute value is greater.
- negative if the negative integer’s absolute value is greater.

To add integers, it is helpful to use a number line.

Example 1
Find \(4 + (-6)\).

Use a number line.
- Start at 0.
- Move 4 units right.
- Then move 6 units left.

\[4 + (-6) = -2\]

Example 2
Find \(-2 + (-3)\).

Use a number line.
- Start at 0.
- Move 2 units left.
- Move another 3 units left.

\[-2 + (-3) = -5\]

Exercises
Add.
1. \(-5 + (-2) = -7\)
2. \(8 + 1 = 9\)
3. \(-7 + 10 = 3\)
4. \(16 + (-11) = 5\)
5. \(-22 + (-7) = -29\)
6. \(-50 + 50 = 0\)
7. \(-10 + (-10) = -20\)
8. \(100 + (-25) = 75\)
9. \(-35 + (-20) = -55\)
10. \(-7 + (-3) + 10 = 0\)
11. \(-42 + 36 + (-36) = -42\)
12. \(-17 + 17 + 9 = 9\)

Write an addition expression to describe each situation. Then find each sum.
13. HAWK A hawk is in a tree 100 feet above the ground. It flies down to the ground. \(100 + (-100) = 0\)
14. RUNNING Leah ran 6 blocks north then back 4 blocks south. \(6 + (-4) = 2\)
Lesson 2 Skills Practice

Add Integers

Add.

1. $5 + (-8) = -3$
2. $-3 + 3 = 0$
3. $-3 + (-8) = -11$
4. $-7 + (-7) = -14$
5. $-8 + 10 = 2$
6. $-7 + 13 = 6$
7. $15 + (-10) = 5$
8. $-11 + (-12) = -23$
9. $25 + (-12) = 13$
10. $-14 + (-13) = -27$
11. $14 + (-27) = -13$
12. $-28 + 16 = -12$
13. $5 + 11 + (-5) = 11$
14. $7 + (-5) + 5 = 7$
15. $9 + (-9) + 10 = 10$
16. $-2 + 19 + 2 = 19$

17. FOOTBALL The Dolphins football team gained 16 yards on their first play then lost 11 yards on the next play. Write an addition expression to represent this situation. Find the sum and explain its meaning.

$16 + (-11); 5; \text{They gained 5 yards over the 2 plays.}$

18. SAVINGS ACCOUNT Demetrius deposits $120 into his account. One week later, he withdraws $36. Write an addition expression to represent this situation. How much higher or lower is the amount in his account after these two transactions?

$120 + (-36); \$84 \text{ higher}$
Lesson 3 Reteach

Subtract Integers

To subtract an integer, add its opposite.

Example 1
Find $6 - 9$.

$6 - 9 = 6 + (-9)$  
To subtract 9, add $-9$.

$= -3$  
Simplify.

Example 2
Find $-10 - (-12)$.

$-10 - (-12) = -10 + 12$  
To subtract $-12$, add 12.

$= 2$  
Simplify.

Example 3
Evaluate $a - b$ if $a = -3$ and $b = 7$.

$a - b = -3 - 7$  
Replace $a$ with $-3$ and $b$ with 7.

$= -3 + (-7)$  
To subtract 7, add $-7$.

$= -10$  
Simplify.

Exercises

Subtract.

1. $7 - 9$  
2. $20 - (-6)$  
3. $-10 - 4$  
4. $0 - 12$  
5. $-7 - 8$  
6. $13 - 18$  
7. $-20 - (-5)$  
8. $-8 - (-6)$  
9. $25 - (-14)$  
10. $-75 - 50$  
11. $15 - 65$  
12. $19 - (-10)$

Evaluate each expression if $m = -2$, $n = 10$, and $p = 5$.

13. $m - 6$  
14. $9 - n$  
15. $p - (-8)$  
16. $p - m$  
17. $m - n$  
18. $-25 - p$
Lesson 3 Skills Practice

Subtract Integers

Subtract.

1. $5 - 2$  **3**
2. $6 - (-7)$  **13**

3. $-3 - 2$  **-5**
4. $8 - 13$  **-5**

5. $-7 - (-7)$  **0**
6. $6 - 12$  **-6**

7. $15 - (-7)$  **22**
8. $-15 - 6$  **-21**

9. $-3 - 8$  **-11**
10. $-10 - 12$  **-22**

11. $13 - (-12)$  **25**
12. $14 - (-22)$  **36**

13. $10 - (-20)$  **30**
14. $-16 - 14$  **-30**

15. $-25 - 25$  **-50**
16. $6 - (-31)$  **37**

17. $-18 - (-40)$  **22**
18. $15 - (-61)$  **76**

Evaluate each expression if $r = -4$, $s = 10$, and $t = -7$.

19. $r - 7$  **-11**
20. $t - s$  **-17**

21. $s - (-8)$  **18**
22. $t - r$  **-3**

23. $s - t$  **17**
24. $r - s$  **-14**
Looking for a pattern is one strategy that can help you when solving problems. You can use the four-step problem-solving plan along with looking for a pattern to solve problems.

### Understand
- Determine what information is given in the problem and what you need to find.

### Plan
- Select a strategy, including a possible estimate.

### Solve
- Solve the problem by carrying out your plan.

### Check
- Examine your answer to see if it seems reasonable.

#### Example 1
**MEMBERSHIP**
The local tennis club started the year with 675 members. In one month, they had 690 members. After two months, they had 705 members. After three months, they had 720 members. When the tennis club reaches 750 members, they will close their enrollment. How many months will it take the club to reach their maximum enrollment if they continue adding new members at the same rate?

**Understand**
The club began with 675 members and is adding new members every month. It needs to find out when it reaches its maximum enrollment of 750 members.

**Plan**
Look for a pattern or rule that increases the membership each month. Then use the rule to extend the pattern to find the solution.

**Solve**
After the initial 675 members, 15 new members joined each month. Extend the pattern to find the solution.

\[
675, \quad 690, \quad 705, \quad 720, \quad 735, \quad 750
\]

They will have reached their maximum enrollment in 5 months.

**Check**
They increased by \(5 \cdot 15\) or 75 members in 5 months, which, when added to the original 675 members, is \(675 + 75 = 750\). So, 5 months is a reasonable answer.

#### Exercises
1. **PRODUCE**
   A farmer has 42 apples on his front porch. The next day, there are only 36 apples left on the porch. After 2 days, there are only 30 apples left on the porch, and in 3 days, 24 apples remain on the porch. After how many days will there be no more apples on the porch if the same amount continues to disappear each day? **7 days**

2. **TELEPHONE**
   A hotel charges a standard rate of $3 per international phone call. After one minute, the charge is $4.50. In two minutes, the charge is $6.00. If Susan only has $10.00, how long can her phone conversation be if the charges per minute stay constant? **4 minutes**
Skills Practice

Problem-Solving Investigation: Look for a Pattern

Use the look for a pattern strategy to solve the problem.

1. NUMBERS What are the next two numbers in the pattern listed below?  
   7, 21, 63, 189, …  
   567, 1701

2. POPULATION The Springfield Zoo is breeding gorillas. They have  
   3 gorillas which can mate and give birth. After the first year there are  
   7 gorillas. After the second year there are 11 gorillas. If the gorillas  
   continue to increase at the same rate, how long will it take for the  
   Springfield Zoo to have 35 gorillas? 8 years

3. ALGEBRA Read the table below to find a pattern relating x and y.  
   y is two more than three times x.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>

4. SAVINGS Maria receives $50 for her birthday. She decides to put the  
   money into a bank account and start saving her money from babysitting  
   in order to buy a television that costs $200. After the first week she  
   has $74. After the second week, she has $98. After the third week she  
   has $122. How many weeks will she have to save at the same rate in  
   order to buy the television? 7 weeks
Lesson 4 Reteach

Multiply Integers

The product of two integers with different signs is negative. The product of two integers with the same sign is positive.

Example 1
Find $5(-2)$.

$5(-2) = -10$  The integers have different signs. The product is negative.

Example 2
Find $-3(7)$.

$-3(7) = -21$  The integers have different signs. The product is negative.

Example 3
Find $-6(-9)$.

$-6(-9) = 54$  The integers have the same sign. The product is positive.

Example 4
Find $(-7)^2$.

$(-7)^2 = (-7)(-7)$  There are 2 factors of $-7$. The product is positive.

$= 49$

Example 5
Find $-2(-3)(4)$.

$-2(-3)(4)$  Multiply $-2$ and $-3$.

$= 6(4)$  Multiply 6 and 4.

$= 24$

Exercises

Multiply.

1. $-5(8)$  $-40$  
2. $-3(-7)$  $21$  
3. $10(-8)$  $-80$

4. $-8(3)$  $-24$  
5. $-12(-12)$  $144$  
6. $(-8)^2$  $64$

7. $-5(7)$  $-35$  
8. $3(-2)$  $-6$  
9. $-6(-3)$  $18$

10. $5(-4)(5)$  $-100$  
11. $-4(-4)$  $16$  
12. $2(-3)(5)$  $-30$

13. $-2(-3)$  $6$  
14. $9(-4)$  $-36$  
15. $(-3)(-4)$  $12$

16. $-3(-3)(5)$  $45$  
17. $-2(5)^2$  $-50$  
18. $(-3)(-4)(5)$  $60$
Lesson 4 Skills Practice

Multiply Integers

Multiply.

1. \(-4(6)\) \(-24\)
2. \(-2(-8)\) \(16\)
3. \(12(-4)\) \(-48\)
4. \(-6(5)\) \(-30\)
5. \(-10(-9)\) \(90\)
6. \(-5^2\) \(-25\)
7. \((-5)^2\) \(25\)
8. \(-30(5)\) \(-150\)
9. \(20(-6)\) \(-120\)
10. \(-14(-6)\) \(84\)
11. \((-13)^2\) \(169\)
12. \(-7(15)\) \(-105\)
13. \(-3(4)\) \(-12\)
14. \(7(-3)\) \(-21\)
15. \(3(-3)\) \(-9\)
16. \(-2(-10)\) \(20\)
17. \((-5)(-3)(4)\) \(60\)
18. \(-3(-3)(4)\) \(36\)
19. \(-3(-5)\) \(15\)
20. \(5(-3)\) \(-15\)
21. \(7(-5)(4)\) \(-140\)
22. \(-2(-5)(-3)\) \(-30\)
23. \(-10(-3)\) \(30\)
24. \(-2(-3)^2\) \(-18\)
Lesson 5 Reteach

Divide Integers

The quotient of two integers with different signs is negative.
The quotient of two integers with the same sign is positive.

Example 1
Find $30 \div (-5)$.

$30 \div (-5)$  The integers have different signs.
$30 \div (-5) = -6$  The quotient is negative.

Example 2
Find $-100 \div (-5)$.

$-100 \div (-5)$  The integers have the same sign.
$-100 \div (-5) = 20$  The quotient is positive.

Exercises

Divide.

1. $-12 \div 4 \quad -3$
2. $-14 \div (-7) \quad 2$
3. $\frac{18}{-2} \quad -9$
4. $-6 \div (-3) \quad 2$
5. $-10 \div 10 \quad -1$
6. $\frac{-80}{-20} \quad 4$
7. $350 \div (-25) \quad -14$
8. $-420 \div (-3) \quad 140$
9. $\frac{540}{45} \quad 12$
10. $\frac{-256}{16} \quad -16$

ALGEBRA  Evaluate each expression if $d = -24$, $e = -4$, and $f = 8$.

11. $12 \div e \quad -3$
12. $40 \div f \quad 5$
13. $d \div 6 \quad -4$
14. $d \div e \quad 6$
15. $f \div e \quad -2$
16. $e^2 \div f \quad 2$
17. $-\frac{d}{e} \quad -6$
18. $ef \div 2 \quad -16$
19. $\frac{f+8}{-4} \quad -4$
20. $\frac{d-e}{5} \quad -4$
Lesson 5 Skills Practice

Divide Integers

Divide.

1. \(-15 \div 3\) \(-5\)  
2. \(-24 \div (-8)\) \(3\)

3. \(22 \div (-2)\) \(-11\)  
4. \(-49 \div (-7)\) \(7\)

5. \(-8 \div (-8)\) \(1\)  
6. \(
\frac{-4}{\frac{-4}{36}}\) \(-9\)

7. \(225 \div (-15)\) \(-15\)  
8. \(
\frac{0}{\frac{-9}{0}}\) \(0\)

9. \(-38 \div 2\) \(-19\)  
10. \(
\frac{64}{\frac{4}{16}}\) \(16\)

11. \(-500 \div (-50)\) \(10\)  
12. \(-189 \div (-21)\) \(9\)

ALGEBRA Evaluate each expression if \(m = -32\), \(n = 2\), and \(p = -8\).

13. \(m \div n\) \(-16\)  
14. \(p \div 4\) \(-2\)

15. \(p^2 \div m\) \(-2\)  
16. \(m \div p\) \(4\)

17. \(
\frac{-p}{n}\) \(4\)  
18. \(p \div (-n^2)\) \(2\)

19. \(
\frac{p}{4n}\) \(-1\)  
20. \(
\frac{18 - n}{-4}\) \(-4\)

21. \(
\frac{m + 8}{-4}\) \(6\)  
22. \(
\frac{m + n}{6}\) \(-5\)
Lesson 1 Reteach
Terminating and Repeating Decimals

To write a fraction as a decimal, divide the numerator by the denominator. Division ends when the remainder is zero.
You can use bar notation to indicate that a number pattern repeats indefinitely. A bar is written over the digits that repeat.

Example 1
Write \( \frac{3}{20} \) as a decimal.

\[
\begin{align*}
20 \div 3.00 & = 0.15 \\
20 & \\
100 & \\
100 & \\
0 & \\
\end{align*}
\]

Divide 3 by 20.

The remainder is 0.

So, \( \frac{3}{20} = 0.15 \).

Example 2
Write \( \frac{5}{9} \) as a decimal.

\[
\begin{align*}
9 \div 5.000 & = 0.555... \\
9 & \\
45 & \\
50 & \\
45 & \\
5 & \\
\end{align*}
\]

The remainder after each step is 5.

You can use bar notation \( 0.\overline{5} \) to indicate that 5 repeats forever. So, \( \frac{5}{9} = 0.\overline{5} \).

Example 3
Write \(-0.32\) as a fraction in simplest form.

\[
\begin{align*}
-0.32 & = -\frac{32}{100} \\
& = -\frac{8}{25} \\
& \text{The 2 is in the hundredths place.} \\
& \text{Simplify.}
\end{align*}
\]

Exercises
Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

1. \( \frac{8}{10} \) - 0.8
2. \( -\frac{3}{5} \) -0.6
3. \( \frac{7}{11} \) 0.6\overline{3}
4. \( 4\frac{7}{8} \) 4.875
5. \( -\frac{13}{15} \) -0.86
6. \( 3\frac{47}{99} \) 3.47

Write each decimal as a fraction in simplest form.

7. \( -0.14 \) -\( \frac{7}{50} \)
8. 0.3 \( \frac{3}{10} \)
9. 0.94 \( \frac{47}{50} \)
Lesson 1 Skills Practice

Terminating and Repeating Decimals

Write each repeating decimal using bar notation.

1. $0.7\overline{3}5$  
2. $0.4\overline{2}$  
3. $5.1\overline{2}6126126\ldots$  

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

4. $-\frac{3}{5} -0.6$  
5. $\frac{19}{20} 0.9\overline{5}$  
6. $3\frac{4}{5} 3.8$  

7. $\frac{23}{50} 0.4\overline{6}$  
8. $-1\frac{5}{8} -1.6\overline{2}5$  
9. $\frac{19}{25} 0.7\overline{6}$  

10. $4\frac{17}{37} 4.4\overline{5}9$  
11. $-5\frac{3}{11} -5.2\overline{7}$  
12. $\frac{17}{24} 0.708\overline{3}$  

13. $6\frac{7}{32} 6.2\overline{1}875$  
14. $7\frac{9}{22} 7.4\overline{0}9$  
15. $-1\frac{17}{48} -1.3\overline{5}41\overline{6}$  

Write each decimal as a fraction in simplest form.

16. $0.8 \frac{4}{5}$  
17. $0.52 \frac{13}{25}$  
18. $-0.92 \frac{-23}{25}$  

19. $-0.48 \frac{-12}{25}$  
20. $0.86 \frac{43}{50}$  
21. $0.76 \frac{19}{25}$  

Course 2 • Chapter 4 Rational Numbers
Lesson 2 Reteach

Compare and Order Rational Numbers

To compare fractions, rewrite them so they have the same denominator. The least common denominator (LCD) of two fractions is the LCM of their denominators.

Another way to compare fractions is to express them as decimals. Then compare the decimals.

Example 1
Which fraction is greater, $\frac{3}{4}$ or $\frac{4}{5}$?

Method 1 Rename using the LCD.

\[
\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20} \\
\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}
\]

The LCD is 20.

Because the denominators are the same, compare numerators.

Since $\frac{16}{20} > \frac{15}{20}$ then $\frac{4}{5} > \frac{3}{4}$.

Exercises

Replace each $\bigcirc$ with $<$, $>$, or $=$ to make a true sentence. Use a number line if necessary.

1. $\frac{1}{2} \bigcirc \frac{3}{8} >$

2. $\frac{4}{5} \bigcirc \frac{8}{10} =$

3. $\frac{3}{4} \bigcirc \frac{7}{8} <$

4. $\frac{1}{2} \bigcirc \frac{5}{9} <$

5. $\frac{9}{14} \bigcirc \frac{3}{7} >$

6. $-\frac{5}{7} \bigcirc -\frac{6}{11} <$

7. $-3\frac{1}{3} \bigcirc -3\frac{2}{6} =$

8. $\frac{9}{10} \bigcirc \frac{3}{5} >$

Course 2 • Chapter 4 Rational Numbers
Lesson 2 Skills Practice

Compare and Order Rational Numbers

Replace each \(\bullet\) with \(<\), \(>\), or \(=\) to make a true sentence.

1. \(\frac{4}{7} \, \bullet \, \frac{3}{5}\) < 
2. \(\frac{5}{12} \, \bullet \, \frac{7}{24}\) > 
3. \(\frac{6}{28} \, \bullet \, \frac{3}{7}\) < 

4. \(\frac{7}{15} \, \bullet \, \frac{1}{4}\) > 
5. \(\frac{7}{11} \, \bullet \, \frac{3}{5}\) > 
6. \(\frac{5}{17} \, \bullet \, \frac{7}{8}\) < 

7. \(\frac{5}{12} \, \bullet \, \frac{7}{10}\) < 
8. \(\frac{15}{16} \, \bullet \, \frac{1}{4}\) > 
9. \(\frac{5}{8} \, \bullet \, \frac{3}{5}\) > 

10. \(\frac{3}{10} \, \bullet \, \frac{2}{9}\) > 
11. \(-\frac{3}{7} \, \bullet \, -\frac{5}{7}\) > 
12. \(\frac{9}{12} \, \bullet \, \frac{3}{4}\) = 

13. \(-\frac{4}{5} \, \bullet \, -\frac{2}{3}\) < 
14. \(\frac{4}{5} \, \bullet \, \frac{5}{4}\) < 
15. \(\frac{1}{3} \, \bullet \, \frac{1}{2}\) < 

16. \(\frac{8}{7}\) = 
17. \(\frac{3}{4} \, \bullet \, \frac{7}{8}\) < 
18. \(\frac{2}{3} \, \bullet \, \frac{3}{4}\) < 

Order each set of numbers from least to greatest.

19. 0.48, 0.46, \(\frac{9}{20}\) 
20. 0.99, 0.89, \(\frac{7}{8}\) 
21. \(\frac{1}{4}\), 0.2, 0.4 

\(\frac{9}{20}\), 0.46, 0.48 
\(\frac{7}{8}\), 0.89, 0.99 
0.2, \(\frac{1}{4}\), 0.4
Lesson 3 Reteach

Add and Subtract Like Fractions

Like fractions are fractions that have the same denominator. To add or subtract like fractions, add or subtract the numerators and write the result over the denominator. Simplify if necessary.

Example 1
Find $\frac{3}{4} + \frac{1}{4}$. Write in simplest form.

$\frac{3}{4} + \frac{1}{4} = \frac{3 + 1}{4}$ Add the numerators.

$= \frac{4}{4}$ Write the sum over the denominator.

$= 1$ Simplify.

Example 2
Find $\frac{2}{3} - \frac{1}{3}$. Write in simplest form.

$\frac{2}{3} - \frac{1}{3} = \frac{2 - 1}{3}$ Subtract the numerators.

$= \frac{1}{3}$ Write the difference over the denominator.

Exercises

Add or subtract. Write in simplest form.

1. $\frac{5}{8} + \frac{1}{8}$

2. $\frac{7}{9} - \frac{2}{9}$

3. $-\frac{1}{4} + \frac{3}{4}$

4. $\frac{7}{8} - \frac{5}{8}$

5. $\frac{5}{9} + \frac{5}{9}$

6. $-\frac{3}{8} - \frac{1}{8}$

7. $\frac{3}{10} + \frac{1}{10}$

8. $\frac{2}{5} - \frac{1}{5}$

9. $\frac{7}{15} + \frac{4}{15}$

10. $\frac{7}{9} - \frac{8}{9}$
Lesson 3 Skills Practice

Add and Subtract Like Fractions

Add or subtract. Write in simplest form.

1. $\frac{3}{8} + \frac{3}{8} = \frac{3}{4}$
2. $\frac{7}{10} - \frac{5}{10} = \frac{1}{5}$
3. $\frac{9}{10} + \frac{3}{10} = 1\frac{1}{5}$

4. $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$
5. $\frac{2}{3} + \frac{2}{3} = 1\frac{1}{3}$
6. $\frac{5}{9} - \frac{2}{9} = \frac{1}{3}$

7. $\frac{8}{15} - \frac{1}{15} = \frac{7}{15}$
8. $\frac{5}{12} + \frac{5}{12} = \frac{5}{6}$
9. $\frac{7}{10} - \frac{3}{10} = \frac{2}{5}$

10. $\frac{7}{16} + \frac{5}{16} = \frac{3}{4}$
11. $\frac{19}{20} - \frac{3}{20} = \frac{4}{5}$
12. $-\frac{5}{9} + \frac{7}{9} = \frac{2}{9}$

13. $-\frac{4}{9} - \frac{1}{9} = -\frac{5}{9}$
14. $\frac{2}{3} + \frac{1}{3} = 1$
15. $-\frac{3}{4} - \frac{2}{4} = -1\frac{1}{4}$

16. $\frac{7}{8} - \frac{5}{8} = \frac{1}{4}$
17. $\frac{8}{9} - \frac{5}{9} = \frac{1}{3}$
18. $-\frac{5}{12} - \left(-\frac{3}{12}\right) = -\frac{1}{6}$

19. $\frac{7}{9} + \frac{2}{9} = 1$
20. $\frac{3}{5} + \frac{4}{5} = 1\frac{2}{5}$
21. $-\frac{11}{12} - \frac{5}{12} = -1\frac{1}{3}$

22. $\frac{5}{6} + \frac{4}{6} = 1\frac{1}{2}$
23. $\frac{3}{8} + \frac{5}{8} = 1$
24. $-\frac{7}{16} - \left(-\frac{3}{16}\right) = -\frac{1}{4}$
Lesson 4 Reteach
Add and Subtract Unlike Fractions

To add or subtract fractions with different denominators,
- Rename the fractions using the least common denominator (LCD).
- Add or subtract as with like fractions.
- If necessary, simplify the sum or difference.

Example
Find \( \frac{2}{3} + \frac{1}{4} \).

Method 1 Use a model.
\[
\begin{array}{c}
\frac{2}{3} \\
+ \frac{1}{4} \\
\hline
\frac{11}{12}
\end{array}
\]

Method 2 Use the LCD.
\[
\frac{2}{3} + \frac{1}{4} = \frac{2}{3} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{3}{3} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}
\]

Exercises
Add or subtract. Write in simplest form.

1. \( \frac{1}{2} + \frac{3}{4} \) \( 1 \frac{1}{4} \)
2. \( \frac{3}{8} - \frac{1}{2} \) \( -\frac{1}{8} \)
3. \( \frac{7}{15} + \left( -\frac{5}{6} \right) \) \( -\frac{11}{30} \)
4. \( \frac{2}{5} - \frac{1}{3} \) \( \frac{1}{15} \)
5. \( \frac{5}{9} + \left( -\frac{5}{12} \right) \) \( \frac{5}{36} \)
6. \( \frac{11}{12} - \frac{3}{4} \) \( \frac{1}{6} \)
7. \( \frac{7}{8} - \left( -\frac{1}{3} \right) \) \( 1 \frac{5}{24} \)
8. \( \frac{7}{9} - \frac{1}{2} \) \( \frac{5}{18} \)
9. \( \frac{3}{10} + \frac{7}{12} \) \( \frac{53}{60} \)
10. \( \frac{3}{5} + \frac{2}{3} \) \( 1 \frac{4}{15} \)
Lesson 4 Skills Practice

Add and Subtract Unlike Fractions

Add or subtract. Write in simplest form.

1. $\frac{8}{15} - \frac{1}{5} = \frac{3}{3}$
2. $\frac{5}{6} + \frac{5}{12} = \frac{11}{4}$

3. $\frac{3}{5} - \frac{3}{10} = \frac{3}{10}$
4. $\frac{7}{16} + \frac{3}{8} = \frac{13}{16}$

5. $\frac{19}{20} - \frac{3}{10} = \frac{13}{20}$
6. $\frac{4}{9} - \frac{1}{12} = \frac{13}{36}$

7. $\frac{2}{3} + \frac{3}{7} = \frac{1}{21}$
8. $\frac{3}{4} + \frac{1}{7} = \frac{25}{28}$

9. $\frac{7}{8} - \frac{2}{3} = \frac{5}{24}$
10. $\frac{8}{9} - \frac{5}{6} = \frac{1}{18}$

11. $\frac{5}{12} - \frac{3}{10} = \frac{7}{60}$
12. $\frac{7}{9} + \frac{2}{3} = \frac{4}{9}$

13. $\frac{3}{5} + \frac{4}{7} = \frac{6}{35}$
14. $\frac{11}{12} - \frac{1}{2} = \frac{5}{12}$

15. $\frac{3}{4} - \left( -\frac{1}{2} \right) = \frac{1}{4}$
16. $\frac{5}{6} + \frac{1}{4} = \frac{7}{12}$

17. $-\frac{2}{3} - \left( -\frac{3}{4} \right) = \frac{1}{12}$
18. $\frac{7}{8} + \frac{1}{12} = \frac{23}{24}$

19. $-\frac{3}{10} + \frac{5}{20} - \frac{1}{20}$
20. $\frac{7}{12} - \left( -\frac{1}{3} \right) = \frac{11}{12}$
Lesson 5 Reteach

Add and Subtract Mixed Numbers

To add or subtract mixed numbers:
- Add or subtract the fractions. Rename using the LCD if necessary.
- Then, add or subtract the whole numbers.
- Simplify if necessary.

Example 1
Find \(6 \frac{1}{10} + 2 \frac{3}{10}\). Write in simplest form.

\[
\begin{align*}
6 \frac{1}{10} & \quad + \quad 2 \frac{3}{10} \\
\hline
8 \frac{4}{10} & \quad \text{or} \quad 8 \frac{2}{5}
\end{align*}
\]

Add the whole numbers and the fractions separately.

Example 2
Find \(8 \frac{2}{3} - 6 \frac{1}{2}\).

\[
\begin{align*}
8 \frac{2}{3} & \quad \rightarrow \quad 8 \frac{4}{6} \\
-6 \frac{1}{2} & \quad \rightarrow \quad 6 \frac{3}{6}
\end{align*}
\]

Rename the fractions using the LCD.

\[
\begin{align*}
\hline
2 \frac{1}{6}
\end{align*}
\]

Subtract.

Example 3
Find \(4 \frac{1}{4} - 2 \frac{3}{5}\).

\[
\begin{align*}
4 \frac{1}{4} & \quad \rightarrow \quad 4 \frac{5}{20} \quad \rightarrow \quad 3 \frac{25}{20} \\
-2 \frac{3}{5} & \quad \rightarrow \quad 2 \frac{12}{20} \quad \rightarrow \quad 2 \frac{12}{20}
\end{align*}
\]

Rename \(4 \frac{5}{20}\) as \(3 \frac{25}{20}\).

\[
\begin{align*}
\hline
1 \frac{13}{20}
\end{align*}
\]

Subtract the whole numbers and then the fractions.

Exercises
Add or subtract. Write in simplest form.

1. \(\frac{3}{5} + \frac{4}{5} \quad \frac{5}{5}\)
2. \(\frac{2}{6} - \frac{1}{6} \quad \frac{1}{3}\)
3. \(\frac{2}{3} + \frac{1}{2} \quad \frac{6}{6}\)
4. \(\frac{3}{4} - \frac{1}{6} \quad \frac{2}{7}\)
5. \(8 - \frac{7}{8} \quad \frac{1}{8}\)
6. \(\frac{4}{5} + \frac{3}{10} \quad \frac{2}{10}\)
Lesson 5 Skills Practice

Add and Subtract Mixed Numbers

Add or subtract. Write in simplest form.

1. \(3\frac{2}{5} + 1\frac{1}{5}\)  \(= 4\frac{3}{5}\)

2. \(6\frac{7}{10} + 12\frac{1}{10}\)  \(= 18\frac{4}{5}\)

3. \(5\frac{3}{8} - 4\frac{1}{8}\)  \(= 1\frac{1}{4}\)

4. \(3\frac{1}{2} - 2\frac{1}{2}\)  \(= 1\)

5. \(7\frac{1}{4} - 5\frac{3}{4}\)  \(= 1\frac{1}{2}\)

6. \(8\frac{5}{6} + 9\frac{5}{6}\)  \(= 18\frac{2}{3}\)

7. \(2\frac{1}{2} - 1\frac{1}{4}\)  \(= 1\frac{1}{4}\)

8. \(3\frac{7}{8} + 5\frac{3}{4}\)  \(= 9\frac{5}{8}\)

9. \(2\frac{5}{6} - 7\frac{7}{8}\)  \(= 1\frac{23}{24}\)

10. \(8\frac{1}{5} + 3\frac{7}{10}\)  \(= 11\frac{9}{10}\)

11. \(8\frac{4}{5} - 2\frac{9}{10}\)  \(= 5\frac{9}{10}\)

12. \(3\frac{1}{4} - 2\frac{5}{6}\)  \(= \frac{5}{12}\)

13. \(4\frac{3}{5} + 5\frac{1}{2}\)  \(= 10\frac{1}{10}\)

14. \(10 - 7\frac{7}{8}\)  \(= 2\frac{1}{8}\)
Reteach

**Problem-Solving Investigation: Draw a Diagram**

When solving problems, draw a diagram to show what you have and what you need to find.

**Example 1**

**CARNIVAL** Jim has to reach a target at a carnival game to win a prize. After 3 throws he has gone 75 feet, which is \(\frac{3}{4}\) of the way to the target. How far away is the target?

**Understand**

We know that 75 feet is \(\frac{3}{4}\) of the way to the target.

**Plan**

Draw a diagram to show the distance already thrown and the fraction it represents.

**Solve**

\[
\begin{array}{c}
\text{Begin} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad 75 \text{ feet} \quad \text{Target}
\end{array}
\]

If \(\frac{3}{4}\) of the distance is 75 feet, then \(\frac{1}{4}\) of the distance is 25 feet. So, the missing \(\frac{1}{4}\) must be another 25 feet.

\[
\begin{array}{c}
\text{Begin} \quad +25 \quad +25 \quad +25 \quad 75 \text{ feet} \quad +25 \quad \text{Target}
\end{array}
\]

The total distance that Jim must throw to hit the target is 100 feet.

**Check**

Since \(\frac{3}{4} \times 100 = 75\), the solution checks.

**Exercises**

**Draw a diagram to solve.**

1. **SALES** Sharon wants to buy a used car. She has saved $1,500, which is approximately \(\frac{1}{5}\) of the price of the car. What is the price of the car? **$7,500; See students’ work for diagram.**

2. **TRAVEL** The Jones family has traveled 360 miles. They are \(\frac{4}{5}\) of the way to their destination. How far away is their destination from where they started? **450 miles; See students’ work for diagram.**
Skills Practice

Problem-Solving Investigation: Draw a Diagram

Draw a diagram to solve. 1–6. See students’ work for diagram.

1. HOMEWORK Shantel is studying for her history test. After 20 minutes, she is $\frac{1}{4}$ of the way done. How much longer will she study? 60 minutes

2. RECIPES Damon is making muffins. He has added $\frac{3}{4}$ of the ingredients. If he has added 6 ingredients, how many more does he have to add to be finished? 2 ingredients

3. TRAVEL The Smithsons are going to Dallas on vacation. They have traveled $\frac{1}{3}$ of the total distance. If they have traveled 126 miles, how far is it from their house to Dallas? 378 miles

4. PHYSICS A ball is dropped from 256 feet above the ground. It bounces up $\frac{1}{4}$ as high as it fell. This is true for each successive bounce. What height will the ball reach on the third bounce? 4 feet

5. SCHOOL Mrs. Wright says that $\frac{2}{3}$ of her class has arrived for the day. If 10 students have arrived, how many students are in her class? 15 students

6. TRAVEL Jeremy walked $\frac{1}{4}$ of the way to school, ran $\frac{1}{4}$ of the way to school, then rode with his best friend the rest of the way. If he walked 1.5 miles, how far did he ride with his friend? 3 miles
Lesson 6 Reteach

Multiply Fractions

To multiply fractions, multiply the numerators and multiply the denominators.

\[
\frac{5}{6} \times \frac{3}{5} = \frac{5 \times 3}{6 \times 5} = \frac{15}{30} = \frac{1}{2}
\]

To multiply mixed numbers, rename each mixed number as an improper fraction. Then multiply the fractions.

\[
2\frac{2}{3} \times 1\frac{1}{4} = \frac{8}{3} \times \frac{5}{4} = \frac{40}{12} = 3\frac{1}{3}
\]

Example 1

Find \(\frac{2}{3} \times \frac{4}{5}\). Write in simplest form.

\[
\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} \quad \text{← Multiply the numerators.}
\]

\[
= \frac{8}{15} \quad \text{← Multiply the denominators.}
\]

\[
= \frac{8}{15} \quad \text{Simplify.}
\]

Example 2

Find \(\frac{1}{3} \times 2\frac{1}{2}\). Write in simplest form.

\[
\frac{1}{3} \times 2\frac{1}{2} = \frac{1}{3} \times \frac{5}{2} \quad \text{Rename } 2\frac{1}{2} \text{ as an improper fraction, } \frac{5}{2}.
\]

\[
= \frac{1 \times 5}{3 \times 2} \quad \text{Multiply.}
\]

\[
= \frac{5}{6} \quad \text{Simplify.}
\]

Exercises

Multiply. Write in simplest form.

1. \(\frac{2}{3} \times \frac{2}{3} \quad \frac{4}{9}\)
2. \(\frac{1}{2} \times \frac{7}{8} \quad \frac{7}{16}\)
3. \(-\frac{1}{3} \times \frac{3}{5} \quad -\frac{1}{5}\)

4. \(\frac{5}{9} \times 4 \quad 2\frac{2}{9}\)
5. \(\frac{2}{3} \times \left(-\frac{3}{5}\right) \quad -1\)
6. \(3\frac{3}{4} \times 1\frac{1}{6} \quad 4\frac{3}{8}\)

7. \(\frac{3}{4} \times 1\frac{2}{3} \quad 1\frac{1}{4}\)
8. \(-3\frac{1}{3} \times \left(-2\frac{1}{2}\right) \quad 8\frac{1}{3}\)
9. \(4\frac{1}{5} \times \frac{1}{7} \quad \frac{3}{5}\)

10. \(\frac{7}{5} \times 8 \quad 11\frac{1}{5}\)
11. \(-2\frac{1}{3} \times \frac{4}{6} \quad -1\frac{5}{9}\)
12. \(\frac{1}{8} \times 2\frac{3}{4} \quad \frac{11}{32}\)
Lesson 6 Skills Practice

Multiply Fractions

Multiply. Write in simplest form.

1. \(\frac{1}{2} \times \frac{4}{5} = \frac{2}{5}\)
2. \(\frac{1}{9} \times \frac{3}{5} = \frac{1}{15}\)
3. \(\frac{15}{24} \times \frac{3}{20} = \frac{3}{32}\)

4. \(-\frac{1}{7} \times \frac{1}{5} = -\frac{1}{35}\)
5. \(\frac{5}{7} \times \frac{14}{15} = \frac{2}{3}\)
6. \(\frac{9}{10} \times \frac{5}{9} = \frac{1}{2}\)

7. \(\frac{4}{11} \times \frac{3}{8} = \frac{3}{22}\)
8. \(\frac{2}{3} \times \frac{7}{9} = \frac{14}{27}\)
9. \(-\frac{9}{13} \times \frac{26}{27} = -\frac{2}{3}\)

10. \(-\frac{4}{9} \times (-5) = \frac{22}{9}\)
11. \(7 \times \frac{2}{7} = 2\)
12. \(\frac{24}{5} \times \frac{1}{3} = \frac{14}{15}\)

13. \(\frac{4}{2} \times \frac{1}{3} = \frac{1}{2}\)
14. \(\frac{5}{4} \times 12 = 69\)
15. \(14 \times \frac{2}{7} = 34\)

16. \(\frac{2}{5} \times \frac{1}{7} = \frac{35}{7}\)
17. \(\frac{1}{9} \times \frac{2}{7} = \frac{35}{7}\)
18. \(-\frac{5}{6} \times (-\frac{3}{8}) = 37\frac{3}{16}\)

19. \(\frac{10}{9} \times \frac{4}{1} = 45\frac{29}{36}\)
20. \(\frac{9}{7} \times \left(-\frac{3}{4}\right) = -75\frac{7}{9}\)
21. \(\frac{3}{4} \times \frac{2}{7} = \frac{9}{14}\)
Lesson 7 Reteach

Convert Between Systems

Use unit ratios to convert between systems. Here are some relationships.

<table>
<thead>
<tr>
<th>Type of Measure</th>
<th>Customary</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1 inch</td>
<td>(\approx 2.54) centimeters</td>
</tr>
<tr>
<td></td>
<td>1 foot</td>
<td>(\approx 0.30) meter</td>
</tr>
<tr>
<td></td>
<td>1 yard</td>
<td>(\approx 0.91) meter</td>
</tr>
<tr>
<td></td>
<td>1 mile</td>
<td>(\approx 1.61) kilometers</td>
</tr>
<tr>
<td>Weight/Mass</td>
<td>1 pound</td>
<td>(\approx 453.6) grams</td>
</tr>
<tr>
<td></td>
<td>1 pound</td>
<td>(\approx 0.4536) kilogram</td>
</tr>
<tr>
<td></td>
<td>1 ton</td>
<td>(\approx 907.2) kilograms</td>
</tr>
<tr>
<td>Capacity</td>
<td>1 cup</td>
<td>(\approx 236.59) milliliters</td>
</tr>
<tr>
<td></td>
<td>1 pint</td>
<td>(\approx 473.18) milliliters</td>
</tr>
<tr>
<td></td>
<td>1 quart</td>
<td>(\approx 946.35) milliliters</td>
</tr>
<tr>
<td></td>
<td>1 gallon</td>
<td>(\approx 3.79) liters</td>
</tr>
</tbody>
</table>

Example

Convert 36.5 inches to centimeters. Round to the nearest hundredth if necessary.

Since 1 inch \(\approx 2.54\) centimeters, use the ratio \(\frac{2.54 \text{ cm}}{1 \text{ in.}}\).

\[
36.5 \text{ in.} \approx 36.5 \text{ in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \quad \text{Multiply.}
\]

\[
\approx 36.5 \text{ in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} \quad \text{Divide out common units.}
\]

\[
\approx 36.5 \times 2.54 \text{ cm or } 92.71 \text{ cm}
\]

So, 36.5 inches is approximately 92.71 centimeters.

Exercises

Complete. Round to the nearest hundredth if necessary.

1. \(2 \text{ c} \approx \underline{473.18}\) mL
2. \(4.8 \text{ pt} \approx \underline{2271.26}\) mL
3. \(16 \text{ mi} \approx \underline{25.76}\) km
4. \(5 \text{ gal} \approx \underline{18.95}\) L
5. \(4.2 \text{ yd} \approx \underline{3.82}\) m
6. \(2 \text{ T} \approx \underline{1814.4}\) kg
Lesson 7 Skills Practice

Convert Between Systems

Complete. Round to the nearest hundredth if necessary.

1. \(4 \text{ c} \approx 946.36 \text{ mL}\)
2. \(2.7 \text{ lb} \approx 1.22 \text{ kg}\)

3. \(9 \text{ ft} \approx 2.7 \text{ m}\)
4. \(3 \text{ qt} \approx 2839.05 \text{ mL}\)

5. \(7 \text{ in.} \approx 17.78 \text{ cm}\)
6. \(7 \text{ mi} \approx 11.27 \text{ km}\)

7. \(16 \text{ yd} \approx 14.56 \text{ m}\)
8. \(3 \text{ T} \approx 2721.6 \text{ kg}\)

9. \(453.6 \text{ g} \approx 1 \text{ lb}\)
10. \(5.08 \text{ cm} \approx 2 \text{ in.}\)

11. \(41 \text{ kg} \approx 90.39 \text{ lb}\)
12. \(25 \text{ mi} \approx 40.25 \text{ km}\)

13. \(28 \text{ qt} \approx 26.50 \text{ L}\)
14. \(14 \text{ in.} \approx 35.56 \text{ cm}\)

15. \(32 \text{ cm} \approx 12.60 \text{ in.}\)
16. \(950 \text{ mL} \approx 4.02 \text{ c}\)

17. \(6.5 \text{ gal} \approx 24.64 \text{ L}\)
18. \(2.8 \text{ T} \approx 2540.16 \text{ kg}\)

19. \(500 \text{ mL} \approx 1.06 \text{ pt}\)
20. \(65 \text{ in.} \approx 1.65 \text{ m}\)

21. **RACE** Sterling just completed the 100-meter dash at his track meet. About how many yards did he run? **109.89 yards**
Lesson 8 Reteach

Divide Fractions

To divide by a fraction, multiply by its multiplicative inverse or reciprocal. To divide by a mixed number, rename the mixed number as an improper fraction.

Example
Find \( \frac{3\frac{1}{3}}{\frac{2}{9}} \). Write in simplest form.

\[
\frac{3\frac{1}{3}}{\frac{2}{9}} = \frac{\frac{10}{3}}{\frac{2}{9}}
\]

Rename \( \frac{3\frac{1}{3}}{3} \) as an improper fraction.

\[
\frac{10}{3} \cdot \frac{9}{2} = \frac{45}{6}
\]

Multiply by the reciprocal of \( \frac{2}{9} \), which is \( \frac{9}{2} \).

\[
\frac{45}{6} \cdot \frac{2}{9} = \frac{15}{1}
\]

Divide out common factors.

\[
= 15
\]

Multiply.

Exercises
Divide. Write in simplest form.

1. \( \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \)
2. \( \frac{2}{5} \div \frac{5}{6} = \frac{12}{25} \)
3. \( -\frac{1}{2} \div \frac{1}{5} = -2\frac{1}{2} \)

4. \( 5 \div \left(-\frac{1}{2}\right) = -10 \)
5. \( \frac{5}{8} \div 10 = \frac{1}{16} \)
6. \( \frac{7}{3} \div 2 = 3\frac{2}{3} \)

7. \( \frac{5}{6} \div \frac{3\frac{1}{2}}{2} = \frac{5}{21} \)
8. \( 36 \div 1\frac{1}{2} = 24 \)
9. \( -2\frac{1}{2} \div (-10) = \frac{1}{4} \)

10. \( \frac{5\frac{2}{5}}{1\frac{4}{5}} = 3 \)
11. \( \frac{6\frac{2}{3}}{\frac{3}{9}} = 2\frac{1}{7} \)
12. \( 4\frac{1}{4} \div \frac{2}{8} = 17 \)

13. \( \frac{4\frac{6}{7}}{2\frac{3}{7}} = 2 \)
14. \( 12 \div \left(-2\frac{1}{2}\right) = -4\frac{4}{5} \)
15. \( \frac{4\frac{1}{6}}{3\frac{1}{6}} = 1\frac{6}{19} \)
Lesson 8 Skills Practice

Divide Fractions

Divide. Write in simplest form.

1. \(-\frac{1}{6} ÷ \frac{1}{5} = -\frac{5}{6}\)
2. \(5 ÷ \frac{3}{5} = 8\frac{1}{3}\)
3. \(\frac{6}{7} ÷ \frac{1}{7} = 6\)

4. \(\frac{3}{4} ÷ \frac{1}{2} = 1\frac{1}{2}\)
5. \(8 ÷ \frac{1}{3} = 24\)
6. \(-\frac{1}{5} ÷ \left(-\frac{1}{4}\right) = \frac{4}{5}\)

7. \(7 ÷ \frac{3}{7} = 16\frac{1}{3}\)
8. \(\frac{4}{7} ÷ \frac{8}{9} = \frac{9}{14}\)
9. \(8\frac{1}{3} ÷ 5 = 1\frac{2}{3}\)

10. \(\frac{9}{7} ÷ \frac{3}{14} = 6\)
11. \(\frac{12}{5} ÷ \left(-\frac{3}{10}\right) = -8\)
12. \(5 ÷ 3\frac{3}{4} = 1\frac{1}{3}\)

13. \(6\frac{4}{5} ÷ 17 = \frac{2}{5}\)
14. \(7\frac{1}{3} ÷ 4 = 1\frac{5}{6}\)
15. \(\frac{3}{4} ÷ 5\frac{1}{2} = \frac{3}{22}\)

16. \(\frac{2}{7} ÷ 1\frac{13}{14} = \frac{4}{27}\)
17. \(\frac{3}{8} ÷ 6\frac{1}{4} = \frac{3}{50}\)
18. \(7\frac{1}{2} ÷ \left(-2\frac{5}{6}\right) = -2\frac{11}{17}\)

19. \(-3\frac{4}{9} ÷ \left(-2\frac{1}{3}\right) = 1\frac{10}{21}\)
20. \(2\frac{2}{3} ÷ 1\frac{1}{6} = 2\frac{2}{7}\)
21. \(4\frac{3}{4} ÷ 2\frac{1}{2} = 1\frac{9}{10}\)
Lesson 1 Reteach

Algebraic Expressions

To evaluate an algebraic expression you replace each variable with its numerical value, then use the order of operations to simplify.

Example 1
Evaluate $6x - 7$ if $x = 8$.

$6x - 7 = 6(8) - 7$
Replace $x$ with 8.

$= 48 - 7$
Use the order of operations.

$= 41$
Subtract 7 from 48.

Example 2
Evaluate $5m - 3n$ if $m = 6$ and $n = 5$.

$5m - 3n = 5(6) - 3(5)$
Replace $m$ with 6 and $n$ with 5.

$= 30 - 15$
Use the order of operations.

$= 15$
Subtract 15 from 30.

Example 3
Evaluate $\frac{ab}{3}$ if $a = 7$ and $b = 6$.

$\frac{ab}{3} = \frac{(7)(6)}{3}$
Replace $a$ with 7 and $b$ with 6.

$= \frac{42}{3}$
The fraction bar is like a grouping symbol.

$= 14$
Divide.

Example 4
Evaluate $x^3 + 4$ if $x = 3$.

$x^3 + 4 = 3^3 + 4$
Replace $x$ with 3.

$= 27 + 4$
Use the order of operations.

$= 31$
Add 27 and 4.

Exercises
Evaluate each expression if $a = 4$, $b = 2$, and $c = 7$.

1. $3ac$ 84
2. $5b^3$ 40
3. $abc$ 56
4. $5 + 6c$ 47
5. $\frac{ab}{8}$ 1
6. $2a - 3b$ 2
7. $\frac{b^4}{4}$ 4
8. $c - a$ 3
9. $20 - bc$ 6
10. $2bc$ 28
11. $ac - 3b$ 22
12. $6a^2$ 96
13. $7c$ 49
14. $6a - b$ 22
15. $ab - c$ 96
Lesson 1 Skills Practice

Algebraic Expressions

Evaluate each expression if \( w = 2, x = 3, y = 5, \) and \( z = 6. \)

1. \( 2w \) 4
2. \( y + 5 \) 10
3. \( 9 - z \) 3
4. \( x + w \) 5
5. \( 3 + 4z \) 27
6. \( 6y - 5 \) 25
7. \( y^2 \) 25
8. \( y - x \) 2
9. \( \frac{z}{2} \) 3

Evaluate each expression if \( m = 3, n = 7, \) and \( p = 9. \)

10. \( m + n \) 10
11. \( 12 - 3m \) 3
12. \( 5p \) 45
13. \( 3.3p \) 29.7
14. \( 3.3p + 2 \) 31.7
15. \( 2p + 3.3 \) 21.3
16. \( 20 + 2n \) 34
17. \( 20 - 2n \) 6
18. \( \frac{n}{7} \) 1
19. \( n^2 \) 49
20. \( 6m^2 \) 54
21. \( \frac{p^2}{3} \) 27
22. \( 1.1 + n \) 8.1
23. \( p - 8.1 \) 0.9
24. \( 3.6m \) 10.8
25. \( 3n - 2m \) 15
26. \( 3m - n \) 2
27. \( 2.1n + p \) 23.7
28. \( \frac{m^2}{p} \) 1
29. \( \frac{2.5m + 2.5}{5} \) 2
30. \( \frac{(n + 2)^2}{3} \) 27
Lesson 2 Reteach

Sequences

An arithmetic sequence is a list in which each term is found by adding the same number to the previous term.

\[ 1, 3, 5, 7, 9, \ldots \]

\[ +2 \quad +2 \quad +2 \quad +2 \]

Example 1
Describe the relationship between terms in the arithmetic sequence 17, 23, 29, 35, \ldots Then write the next three terms in the sequence.

Each term is found by adding 6 to the previous term.

\[ 35 + 6 = 41 \]
\[ 41 + 6 = 47 \]
\[ 47 + 6 = 53 \]

The next three terms are 41, 47, and 53.

Example 2
MONEY Brian's parents have decided to start giving him a monthly allowance for one year. Each month they will increase his allowance by $10. Suppose this pattern continues. What algebraic expression can be used to find Brian's allowance after any given number of months? How much money will Brian receive for allowance for the 10th month?

Make a table to display the sequence.

<table>
<thead>
<tr>
<th>Position</th>
<th>Operation</th>
<th>Value of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1 \cdot 10)</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>(2 \cdot 10)</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>(3 \cdot 10)</td>
<td>30</td>
</tr>
<tr>
<td>(n)</td>
<td>(n \cdot 10)</td>
<td>10(n)</td>
</tr>
</tbody>
</table>

Each term is 20 times its position number. So, the expression is 10\(n\).

How much money will Brian receive after 10 months?

\[ 10n \]

Write the expression.

\[ 10(10) = 100 \]

Replace \(n\) with 10

So, Brian will receive \$100 after 10 months.

Exercises
Describe the relationship between terms in the arithmetic sequences. Write the next three terms in the sequence.

1. 2, 4, 6, 8, \ldots \quad 2. 4, 7, 10, 13, \ldots \quad 3. 0.3, 0.6, 0.9, 1.2, \ldots \\
\quad +2; 10, 12, 14 \quad +3; 16, 19, 22 \quad +0.3; 1.5, 1.8, 2.1 \\

4. 200, 212, 224, 236, \ldots \quad 5. 1.5, 2.0, 2.5, 3.0, \ldots \quad 6. 12, 19, 26, 33, \ldots \\
\quad +12; 248, 260, 272 \quad +0.5; 3.5, 4.0, 4.5 \quad +7; 40, 47, 54 \\

7. SALES Mama’s bakery just opened and is currently selling only two types of pastry. Each month, Mama’s bakery will add two more types of pastry to their menu. Suppose this pattern continues. What algebraic expression can be used to find the number of pastries offered after any given number of months? How many pastries will be offered in one year?

\[ 2n; 24 \]
Lesson 2 Skills Practice

Sequences

Describe the relationship between the terms in each arithmetic sequence.

1. 3, 6, 9, 12... +3
2. 1, 3, 5, 7, ... +2
3. 1, 2, 3, 4... +1
4. 0, 7, 14, 21, ... +7
5. 2, 5, 8, 11, ... +3
6. 5, 10, 15, 20, ... +5
7. 0.3, 0.6, 0.9, 1.2, ... +0.3
8. 1, 10, 19, 28, ... +9
9. 6, 18, 24, 30, ... +6
10. 0, 7, 14, 21, ... +2
11. 3, 7, 11, 15, ... +4
12. 0, 4.5, 9, 13.5, ... +4.5
13. 11, 22, 33, 44, ... +11
14. 16, 21, 26, 31, ... +5

Give the next three terms in each sequence.

15. 3, 6, 9, 12... 15, 18, 21
16. 18, 21, 24, 27, ... 30, 33, 36
17. 7, 10, 13, 16, ... 19, 22, 25
18. 4, 8, 12, 16, ... 20, 24, 28
19. 0, 7, 14, 21, ... 25, 35, 42
20. 7, 12, 17, 22, ... 27, 32, 37
21. 5, 7, 9, 11, ... 13, 15, 17
22. 5, 15, 25, 35, ... 45, 55, 65
23. 21, 42, 63, 84, ... 105, 126, 147
24. 1.1, 2.2, 3.3, 4.4, ... 5.5, 6.6, 7.7
25. 0.5, 1.0, 1.5, 2.0, ... 2.5, 3.0, 3.5
26. 1.7, 1.9, 2.1, 2.3, ... 2.5, 2.7, 2.9
27. 0.5, 1.5, 2.5, 3.5, ... 4.5, 5.5, 6.5
28. 0.1, 0.2, 0.3, 0.4, ... 0.5, 0.6, 0.7
Lesson 3 Reteach

Properties of Operations

Example 1
Name the property shown by the statement \( u + v = v + u \).
The order in which the variables are being added changed. This is the Commutative Property of Addition.

Example 2
State whether the following conjecture is true or false. If false, provide a counterexample.

*Subtraction of integers is commutative.*

Write two subtraction expressions using the Commutative Property.

\[
17 - 9 \overset{?}{=} 9 - 17 \quad \text{State the conjecture.}
\]

\[
8 \neq -8 \quad \text{Subtract.}
\]

We found a counterexample. That is, \( 17 - 9 \neq 9 - 17 \). So, subtraction is *not* commutative. The conjecture is false.

Example 3
Simplify the expression. Justify each step.

\[
9 + (3x + 4)
\]

\[
9 + (3x + 4) = 9 + (4 + 3x) \quad \text{Commutative Property of Addition}
\]

\[
= (9 + 4) + 3x \quad \text{Associative Property of Addition}
\]

\[
= 13 + 3x \quad \text{Simplify.}
\]

Exercises
Name the property shown by each statement.

1. \( 7 \cdot 1 = 7 \)
2. \( 4 + (3y + 2) = (4 + 3y) + 2 \)

- **Multiplicative Identity**
- **Associative Property of Addition**

State whether the following conjectures are true or false. If false, provide a counterexample.

3. The product of two even numbers is odd. **false;** \( 4 \cdot 6 = 24 \)

4. The difference of two odd numbers is even. **true**

5. Simplify \( 4 + (5x + 2) \). Justify each step.

\[
4 + (5x + 2) = 4 + (2 + 5x) \quad \text{Commutative Property of Addition}
\]

\[
= (4 + 2) + 5x \quad \text{Associative Property of Addition}
\]

\[
= 6 + 5x \quad \text{Simplify}
\]
Lesson 3 Skills Practice

Properties of Operations

Name the property shown by each statement.

1. \(9 \cdot 6 = 6 \cdot 9\)  
   - Commutative Property of Multiplication

2. \(m + 0 = m\)  
   - Additive Identity

3. \(14 \cdot 1 = 14\)  
   - Multiplicative Identity

4. \(2 + (8 + 3) = (2 + 8) + 3\)  
   - Associative Property of Addition

5. \(x + y = y + x\)  
   - Commutative Property of Addition

6. \((m + 2) + n = n + (m + 2)\)  
   - Commutative Property of Addition

State whether the following conjectures are true or false. If false, provide a counterexample.

7. The sum of an even whole number and an odd whole number is always odd.  
   - true

8. Division of whole numbers is always commutative.  
   - false; \(8 \div 4 \neq 4 \div 8\)

Simplify each expression. Justify each step.

9. \(5 + (b + 2)\)  
   - \(5 + (b + 2) = 5 + (2 + b)\) Commutative Property of Addition
   - \((5 + 2) + b\) Associative Property of Addition
   - \(7 + b\) Simplify.

10. \(8(2q)\)  
    - \((8 \cdot 2)q\) Associative Property of Multiplication
    - \(16q\) Simplify.

11. RAIN Piper recorded the amount of rain that fell for four nights in the table below. Use mental math to find the total amount of rain. Explain your reasoning.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain (in.)</td>
<td>2.6</td>
<td>1.5</td>
<td>1.4</td>
<td>2.5</td>
</tr>
</tbody>
</table>

8 in.; Sample answer: \(2.6 + 1.4 = 4\) and \(1.5 + 2.5 = 4\), \(4 + 4 = 8\)
Lesson 4 Reteach

The Distributive Property

<table>
<thead>
<tr>
<th>Words</th>
<th>To multiply a sum or difference by a number, multiply each term inside the parentheses by the number outside the parentheses.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>$a(b + c) = ab + ac$  \hspace{1cm}  $a(b - c) = ab - ac$</td>
</tr>
<tr>
<td>Examples</td>
<td>$3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$  \hspace{1cm}  $6(8 - 3) = 6 \cdot 8 - 6 \cdot 3$</td>
</tr>
</tbody>
</table>

Examples
Use the Distributive Property to evaluate each expression.

1. $5(x + 3)$
   
   $5(x + 3) = 5 \cdot x + 5 \cdot 3$ \hspace{1cm} Expand using the Distributive Property
   
   $= 5x + 15$ \hspace{1cm} Simplify.

2. $(4x - y)9$
   
   $(4x - y)9 = [4x + (-y)]9$ \hspace{1cm} Rewrite $4x - y$ as $4x + (-y)$.
   
   $= (4x)9 + (-y)9$ \hspace{1cm} Expand using the Distributive Property.
   
   $= 36x + (-9y)$ \hspace{1cm} Simplify.
   
   $= 36x - 9y$ \hspace{1cm} Definition of subtraction.

Example 3

MOVIES Alwyn is taking three of his friends to the movies. Tickets cost $8.90 per person. Find Alwyn’s total cost.

You can use the Distributive Property to find the total cost mentally.

$4(9 - 0.10) = 4(9) - 4(0.10)$ \hspace{1cm} Distributive Property

$= 36 - 0.40$ \hspace{1cm} Multiply.

$= 35.60$ \hspace{1cm} Subtract.

Alwyn will pay $35.60 for himself and three friends to go to the movies.

Exercises
Use the Distributive Property to evaluate or rewrite each expression.

1. $5(w + 4)$
   
   $5w + 20$

2. $(x - 5)(-2)$
   
   $-2x + 10$

3. $7(6x - 2y)$
   
   $42x - 14y$

4. $-6(4 + 2m)$
   
   $-24 - 12m$

5. $8(2n + 7)$
   
   $16n + 56$

6. $(3v + 6w)2$
   
   $6v + 12w$

7. BOOKS Mariah bought 7 books costing $11.20 each. Find the total cost of the 7 books. Justify your answer by using the Distributive Property.

$78.40; 7(11 + 0.20) = 7 \cdot 11 + 7 \cdot 0.20 = 77 + 1.40$
Lesson 4 Skills Practice

The Distributive Property

Use the Distributive Property to evaluate each expression.

1. \(3(2 + 8)\)
   \[30\]

2. \((-3 + 4)2\)
   \[2\]

3. \(-5(4 - 2)\)
   \[-10\]

4. \((12 + 13)(-2)\)
   \[-50\]

5. \(8(10 - 4)\)
   \[48\]

6. \((-4 + -7)(-3)\)
   \[33\]

7. \((-7 + 3)4\)
   \[-16\]

8. \(-1(18 - 11)\)
   \[-7\]

Use the Distributive Property to rewrite each expression.

9. \(6(t + 2)\)
   \[6t + 12\]

10. \(-5(4 + x)\)
    \[-20 - 5x\]

11. \((5 + v)(-3)\)
    \[-15 - 3v\]

12. \((w - 2)4\)
    \[4w - 8\]

13. \(-7(8n - m)\)
    \[-56n + 7m\]

14. \((6 + d)(-6)\)
    \[-36 - 6d\]

15. \((4c + 2d)(-2)\)
    \[-8c - 4d\]

16. \(-2(3f - 5g)\)
    \[-6f + 10g\]

17. TRAIN RIDE Mr. and Mrs. Caputo are taking their family into the city on the train. The cost per person is $5.80. If there are 4 members in their family, how much does the train trip cost? Justify your answer by using the Distributive Property.
   \[23.20; 4($6 - 0.20) = 4 \cdot $6 - 4 \cdot $0.20 = $24 - $0.80\]

18. CAMPING Chantee went camping over the weekend. The cost for the site was $16.95 a night for three nights. How much did it cost her to camp? Justify your answer by using the Distributive Property.
   \[50.85; 3($17 - 0.05) = 3 \cdot $17 - 3 \cdot $0.05 = $51 - $0.15\]
Example
Kylee is training for the marathon she will run in a few months. She will begin by running 3 miles the first day, 5 miles the next day, and 7 miles the next day.
Make a table to find the number of miles Kylee will run on her tenth day of training.

Understand
Kylee will run 3 miles the first day, 5 miles the next day, and 7 miles the next day. You need to find the number of miles she will run on day 10 of training.

Plan
Make a table and find a pattern. Then extend the pattern to find the solution.

Solve
The first three days, Kylee will run 3 miles, 5 miles, and 7 miles. Extend the pattern.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td>21</td>
</tr>
</tbody>
</table>

Kylee will run 21 miles on day 10 of her training.

Check
Use counters or cubes to model the daily pattern of miles ran.
Count the number of objects used to represent the number of miles ran on day 10.

Exercises
1. RUNNING Suppose Kylee’s friend Derrick is also training for the marathon. On his first four days of training, he runs 1 mile, 1 mile, 2 miles, and 2 miles. How many miles will Derrick run on day 10 of his training? 5
2. RUNNING If Kylee and Derrick continue training in this pattern, how many days will they train before each one is running at least 26 miles? Kylee: 13 days, Derrick: 51 days
Skills Practice

Problem-Solving Investigation: Make a Table

Use the make a table strategy to solve Exercises 1–4.

1. VOLUNTEERING Isabel volunteers during the summer at the hospital. The first four weeks she worked 6 hours, 8 hours, 10 hours, and 12 hours each week. If she continues working in this pattern, how many weeks will she need to work in order to accumulate 100 total volunteer hours?  **8 weeks**

2. DOGS The table shows the number of minutes Jamal walks his dog each day. If the pattern continues, how long will he walk his dog on day 7?  **75 minutes, or 1 hour and 15 minutes**

<table>
<thead>
<tr>
<th>Day</th>
<th>Walk (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
</tr>
</tbody>
</table>

3. MONEY Charlotte is saving money for a new computer. Her first deposits were $10, $20, and $40. If she continues saving in this pattern, how many deposits will she make before she has the $625 she needs to buy the computer?  **6**

4. GEOMETRY Draw the next two figures in the pattern shown below.

![Pattern diagram]

Use any strategy to solve Exercises 5–8.

5. ROSES Mr. Wong planted a new rose bush and recorded the number of blooms each day. The table shows his data. If the rose bush continues blooming in this pattern, how many blooms can Mr. Wong expect on Sunday?  **29**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Blooms</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>21</td>
</tr>
</tbody>
</table>

6. FRIENDS Lauren, Mark, Juan, and Sasha are sitting on a bench. The two girls are sitting in the middle. Juan is next to Lauren, and Mark is not the last one on the right. In what order are the friends sitting?  **Mark, Sasha, Lauren, Juan**

7. LUNCH Will bought three items for lunch from the menu shown. What did he have for lunch if it cost him $4.50?  **soup, salad, and fruit**

<table>
<thead>
<tr>
<th>Lunch Menu</th>
<th></th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandwich</td>
<td></td>
<td>3.25</td>
</tr>
<tr>
<td>Fries</td>
<td></td>
<td>1.50</td>
</tr>
<tr>
<td>Salad</td>
<td></td>
<td>2.25</td>
</tr>
<tr>
<td>Soup</td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>Fruit</td>
<td></td>
<td>1.25</td>
</tr>
</tbody>
</table>

8. NUMBERS Caleb is thinking of two numbers between 1 and 20 whose sum is 17 and whose difference is 7. Find the numbers.  **5 and 12**
Lesson 5 Reteach

Simplify Algebraic Expressions

When a plus or minus sign separates an algebraic expression into parts, each part is called a term. The numerical factor of a term that contains a variable is called the coefficient of the variable. A term without a variable is called a constant. Like terms contain the same variables to the same powers, such as $3x^2$ and $2x^2$.

Example
1. Identify the terms, like terms, coefficients, and constants in the expression $7x - 5 + x - 3x$.

$7x - 5 + x - 3x = 7x + (-5) + x + (-3x)$  \quad \text{Definition of subtraction}

$= 7x + (-5) + 1x + (-3x)$  \quad \text{Identity Property; } x = 1x$

The terms are $7x$, $-5$, $x$, and $-3x$. The like terms are $7x$, $x$, and $-3x$. The coefficients are $7$, $1$, and $-3$. The constant is $-5$.

An algebraic expression is in simplest form if it has no like terms and no parentheses.

Examples
Write each expression in simplest form.

2. $5x + 3x$

$5x + 3x = (5 + 3)x$ or $8x$  \quad \text{Distributive Property; simplify.}$

3. $-2m + 5 + 6m - 3$

$-2m$ and $6m$ are like terms. $5$ and $-3$ are also like terms.

$-2m + 5 + 6m - 3 = -2m + 5 + 6m + (-3)$  \quad \text{Definition of subtraction}

$= -2m + 6m + 5 + (-3)$  \quad \text{Commutative Property}

$= (-2 + 6)m + 5 + (-3)$  \quad \text{Distributive Property}

$= 4m + 2$  \quad \text{Simplify.}$

Exercises
Identify the terms, like terms, coefficients, and constants in each expression.

1. $-4y - 3 + 2y$

   - terms: $-4y$, $-3$, $2y$;
   - like terms: $-4y$, $2y$;
   - coefficients: $-4$, $2$;
   - constant: $-3$

2. $-5g + 3 + 2g - g$

   - terms: $-5g$, $3$, $2g$, $-g$;
   - like terms: $-5g$, $2g$, $-g$;
   - coefficients: $-5$, $2$, $-1$;
   - constant: $3$

3. $5 + 3a - 4 - a$

   - terms: $5$, $3a$, $-4$, $-a$;
   - like terms: $3a$, $-a$;
   - coefficients: $5$, $-4$;
   - constants: $5$, $-4$

Write each expression in simplest form.

4. $3d + 6d$  \quad $9d$

5. $2 + 5s - 4$  \quad $5s - 2$

6. $2z + 3 - 9z - 8$  \quad $-7z - 5$
Lesson 5 Skills Practice

Simplify Algebraic Expressions

Identify the terms, like terms, coefficients, and constants in each expression.

1. $4e + 7e + 5$
   - terms: $4e, 7e, 5$; like terms: $4e, 7e$
   - coefficients: $4, 7$; constant: $5$

2. $5a + 2 - 7$
   - terms: $5a, 2, -7$; like terms: $2, -7$
   - coefficients: $5$; constants: $2, -7$

3. $-3h - 2h + 6h + 9$
   - terms: $-3h, -2h, 6h, 9$; like terms: $-3h, -2h, 6h$
   - coefficients: $-3, -2, 6$; constant: $9$

4. $4 - 4y + y - 3$
   - terms: $4, -4y, y, -3$; like terms: $-4y, y$
   - coefficients: $-4, 1$; constants: $4, -3$

5. $7 - 5y + 2 + 1$
   - terms: $7, -5y, 2, 1$; like terms: $7, 2, 1$
   - coefficient: $-5$; constants: $7, 2, 1$

6. $2m + 3m - m$
   - terms: $2m, 3m, -m$; like terms: $2m, 3m, -m$
   - coefficients: $2, 3, -1$; constants: none

7. $9k + 7 - k + 4$
   - terms: $9k, 7, -k, 4$; like terms: $9k, -k$
   - coefficients: $9, -1$; constants: $7, 4$

8. $-8p + 6p - 2$
   - terms: $-8p, 6p, -2$; like terms: $-8p, 6p$
   - coefficients: $-8, 6$; constant: $-2$

Write each expression in simplest form.

9. $3t + 6t$
   - $9t$

10. $4r + r$
    - $5r$

11. $7f - 2f$
    - $5f$

12. $9a - 8a$
    - $a$

13. $5c + 8c$
    - $13c$

14. $2g - 5g$
    - $-3g$

15. $8k + 3 + 4k$
    - $12k + 3$

16. $7m - 5m - 6$
    - $2m - 6$

17. $9 - 6x + 5$
    - $-6x + 14$

18. $7p - 1 - 9p + 5$
    - $-2p + 4$

19. $-b - 3b + 8b + 4$
    - $4b + 4$

20. $5h - 6 - 8 + 7h$
    - $12h - 14$

21. $8b + 6 - 8b + 1$
    - $7$

22. $t - 5 - 2t + 5$
    - $-t$

23. $4w + 5w + w$
    - $10w$

24. $6m - 7 + 2m + 7$
    - $8m$

25. $5f - 7f + f$
    - $-f$

26. $12y - 8 + 4y + y$
    - $17y - 8$

Write an expression in simplest form that represents the total amount in each situation.

27. **RUNNING** You run $m$ miles on Friday, the same amount on Saturday, and 4 miles on Sunday. $2m + 4$

28. **READING** Hendrick read $b$ books in January, twice that amount in February, and 1 book in March. $3b + 1$
Lesson 6 Reteach

Add Linear Expressions

You can use models to add linear expressions.

Example 1
Add \((3x + 5) + (2x + 3)\).

Step 1 Model each expression.

\[
\begin{align*}
\text{Model 1} & : 3x + 5 \\
\text{Model 2} & : 2x + 3
\end{align*}
\]

Step 2 Combine like tiles and write an expression for the combined tiles.

\[
\begin{align*}
\text{Combined} & : 5x + 8
\end{align*}
\]

So, \((3x + 5) + (2x + 3) = 5x + 8\).

Example 2
Add \((x - 2) + (-2x + 4)\).

Step 1 Model each expression.

\[
\begin{align*}
\text{Model 1} & : x - 2 \\
\text{Model 2} & : -2x + 4
\end{align*}
\]

Step 2 Combine like tiles and write an expression for the combined tiles.

\[
\begin{align*}
\text{Combined} & : -x + 2
\end{align*}
\]

Step 3 Remove all zero pairs and write an expression for the remaining tiles.

\[
\begin{align*}
\text{Remaining} & : -x + 2
\end{align*}
\]

So, \((x - 2) + (-2x + 4) = -x + 2\).

Exercises

Add. Use models if needed.

1. \((5x + 2) + (3x + 1)\) \(8x + 3\)
2. \((-8x + 1) + (-2x + 6)\) \(-10x + 7\)
3. \((-7x + 4) + (x - 5)\) \(-6x - 1\)
4. \((-6x + 1) + (4x - 1)\) \(-2x\)
Lesson 6 Skills Practice

Add Linear Expressions

Add. Use models if needed.

1. $(5x + 7) + (x + 2) \quad 6x + 9$
2. $(-6x + 3) + (x - 7) \quad -5x - 4$

3. $(-x + 12) + (-4x + 2) \quad -5x + 14$
4. $(-5x + 3) + (-7x - 1) \quad -12x + 2$

5. $(-x + 3) + (4x - 10) \quad 3x - 7$
6. $(5x + 4) + (-8x - 2) \quad -3x + 2$

7. $(-7x + 1) + (4x - 5) \quad -3x - 4$
8. $(6x - 2) + (-x + 5) \quad 5x + 3$

9. $(-9x + 1) + (-7x + 8) \quad -16x + 9$
10. $(-3x - 9) + (4x + 8) \quad x - 1$

11. $(-9x - 12) + (x - 8) \quad -8x - 20$
12. $(14x + 7) + (-3x + 2) \quad 11x + 9$

13. $(2x - 1) + (-3x + 7) \quad -x + 6$
14. $(-5x + 4) + (-9x - 2) \quad -14x + 2$

15. $(11x + 2) + (-8x - 2) \quad 3x$
16. $(-9x - 10) + (-5x - 4) \quad -14x - 14$

17. Find the sum of $(10x + 3)$ and $(-4x - 2)$. $6x + 1$

18. Find the sum of $(x + 3)$ and $(-x - 4)$. $-1$

19. GEOMETRY Write and simplify an expression to represent the perimeter of the triangle shown. Then find the value of $x$ if the perimeter is 45 feet. $6x + 3; 7$

\[\begin{align*}
(x + 4) \text{ ft} & \quad (2x + 1) \text{ ft} \\
(3x - 2) \text{ ft} &
\end{align*}\]
Lesson 7 Reteach

Subtract Linear Expressions

When subtracting expressions, subtract like terms. You can use models or the additive inverse.

Example 1

Find \((-3x - 2) - (4x)\).

Step 1 Model the expression \(-3x - 2\).

\[-x - x - x \quad \boxed{-1} \quad (\text{-3x}) \quad + \boxed{-1} \quad (\text{-2})\]

Step 2 Since there are no positive x-tiles to remove, add four zero pairs of x-tiles. Remove four positive x-tiles.

\[-x\quad -x\quad -x\quad -x\quad \boxed{-1} \quad \boxed{-1}\]

So, \((-3x - 2) - (4x) = -7x - 2\).

Example 2

Subtract \((4x + 6) - (-7x + 1)\).

The additive inverse of \(-7x + 1\) is \(7x - 1\).

\[
\begin{align*}
4x + 6 & \quad \text{Arrange like terms in columns.} \\
+ 7x - 1 & \quad \text{Add.} \\
11x + 5 & \quad \text{Add.}
\end{align*}
\]

So, \((4x + 6) - (-7x + 1) = 11x + 5\).

Exercises

Subtract. Use models if needed.

1. \((9x + 10) - (2x + 4)\) \(7x + 6\)

2. \((3x + 4) - (2x - 5)\) \(x + 9\)

3. \((6x + 3) - (-x - 2)\) \(7x + 5\)

4. \((4x - 1) - (x + 3)\) \(3x - 4\)

5. \((3x - 1) - (2x - 6)\) \(x + 5\)
Lesson 7 Skills Practice

Subtract Linear Expressions

Subtract. Use models if needed.

1. \((5x + 7) - (x + 2)\) \(4x + 5\)

2. \((2x - 6) - (x - 7)\) \(x + 1\)

3. \((-x + 12) - (-4x + 2)\) \(3x + 10\)

4. \((-5x + 3) - (-7x - 1)\) \(2x + 4\)

5. \((-x + 3) - (4x - 10)\) \(-5x + 13\)

6. \((5x + 4) - (-8x - 2)\) \(13x + 6\)

7. \((-7x + 1) - (4x - 5)\) \(-11x + 6\)

8. \((6x - 2) - (-x + 5)\) \(7x - 7\)

9. \((-9x + 1) - (-7x + 8)\) \(-2x - 7\)

10. \((-3x - 9) - (4x + 8)\) \(-7x - 17\)

11. \((-9x - 12) - (x - 8)\) \(-10x - 4\)

12. \((14x + 7) - (-3x + 2)\) \(17x + 5\)

13. \((5x - 1) - (-3x + 7)\) \(8x - 8\)

14. \((-5x + 4) - (-9x - 2)\) \(4x + 6\)

15. \((11x + 2) - (-8x - 2)\) \(19x + 4\)

16. \((-9x - 10) - (-5x - 4)\) \(-4x - 6\)

17. \((x - 2) - (x - 6)\) \(4\)

18. \((-6x + 1) - (-3x + 1)\) \(-3x\)

19. \((2x + 4) - (5x - 2)\) \(-3x + 6\)

20. \((-12x - 6) - (-4x + 3)\) \(-8x - 9\)

21. **GEOMETRY** The perimeter of the triangle shown is \((10x + 1)\) feet. Find the length of the missing side. \((2x - 3)\) ft
Lesson 8 Reteach

Factor Linear Expressions

A linear expression is in factored form when it is expressed as the product of its factors.

Example 1
Factor \(5x + 10\).

Use the GCF to factor the linear expression.

\[
5x = 5 \cdot x
\]

Write the prime factorization of \(5x\) and \(10\).

\[
10 = 5 \cdot 2
\]

Circle the common factors.

The GCF of \(5x\) and \(10\) is \(5\). Write each term as a product of the GCF and its remaining factors.

\[
5x + 10 = 5(x) + 5(2) = 5(x + 2)
\]

Distributive Property

So, \(5x + 10 = 5(x + 2)\).

Example 2
Factor \(3x + 8\).

\[
3x = 3 \cdot x
\]

\[
8 = 2 \cdot 2 \cdot 2
\]

There are no common factors, so \(3x + 8\) cannot be factored.

Exercises
Factor each expression. If the expression cannot be factored, write cannot be factored.

1. \(15x + 10\) \(5(3x + 2)\)
2. \(7x - 3\) cannot be factored

3. \(6x + 9\) \(3(x + 2)\)
4. \(30x - 25\) \(5(6x - 5)\)

5. \(13x + 14\) cannot be factored
6. \(50x - 75\) \(25(2x - 3)\)

7. \(24x - 18\) \(6(4x - 3)\)
8. \(18x + 13\) cannot be factored

9. \(16x - 12\) \(4(4x - 3)\)
10. \(36x + 45\) \(9(4x + 5)\)
Lesson 8 Skills Practice

Factor Linear Expressions

Factor each expression. If the expression cannot be factored, write cannot be factored.

1. $17x + 34$  $17(x + 2)$
2. $10x + 25$  $5(2x + 5)$
3. $30x + 18$  $6(5x + 3)$
4. $45x - 18$  $9(5x - 2)$
5. $38x - 12$  $2(17x - 6)$
6. $28x + 15$  cannot be factored
7. $3x - 27$  $3(x - 9)$
8. $6x + 24$  $6(x + 4)$
9. $26x - 5$  cannot be factored
10. $48x + 56$  $8(6x + 7)$
11. $15x - 14$  cannot be factored
12. $20x - 100$  $20(x - 5)$
13. $7x + 35$  $7(x + 5)$
14. $7x + 17$  cannot be factored
15. $9x - 63$  $9(x - 7)$
16. $39x + 13$  $13(3x + 1)$
17. $8x + 15$  cannot be factored
18. $18x - 12$  $6(3x - 2)$
19. $24x + 48$  $24(x + 2)$
20. $45x - 81$  $9(5x - 9)$

21. The area of a rectangular sandbox is $(5x + 40)$ feet. Factor $5x + 40$ to find possible dimensions of the sandbox. $5(x + 8)$ ft
Lesson 1 Reteach

Solve One-Step Addition and Subtraction Equations

Remember, equations must always remain balanced. If you subtract the same number from each side of an equation, the two sides remain equal. Also, if you add the same number to each side of an equation, the two sides remain equal.

Example 1
Solve $x + 5 = 11$. Check your solution.

\[
x + 5 = 11 \quad \text{Write the equation.}
\]
\[
-5 = -5 \quad \text{Subtract 5 from each side.}
\]
\[
x = 6 \quad \text{Simplify.}
\]

Check $x + 5 = 11$ Write the original equation.
\[
6 + 5 \not= 11 \quad \text{Replace } x \text{ with 6.}
\]
\[
11 = 11 \checkmark \quad \text{This sentence is true.}
\]

The solution is 6.

Example 2
Solve $15 = t - 12$. Check your solution.

\[
15 = t - 12 \quad \text{Write the equation.}
\]
\[
+12 = +12 \quad \text{Add 12 to each side.}
\]
\[
27 = t \quad \text{Simplify.}
\]

Check $15 = t - 12$ Write the original equation.
\[
15 \not= 27 - 12 \quad \text{Replace } t \text{ with 27.}
\]
\[
15 = 15 \checkmark \quad \text{This sentence is true.}
\]

The solution is 27.

Exercises
Solve each equation. Check your solution.

1. $h + 3 = 14$ 11 2. $m + 8 = 22$ 14 3. $p + 5 = 15$ 10 4. $17 = y + 8$ 9
5. $w + 4 = -1$ -5 6. $k + 5 = -3$ -8 7. $25 = 14 + r$ 11 8. $57 + z = 97$ 40
9. $b - 3 = 6$ 9 10. $7 = c - 5$ 12 11. $j - 12 = 18$ 30 12. $v - 4 = 18$ 22
13. $-9 = w - 12$ 3 14. $y - 8 = -12$ -4 15. $14 = f - 2$ 16 16. $23 = n - 12$ 35
Lesson 1 Skills Practice

Solve One-Step Addition and Subtraction Equations

Solve each equation. Check your solution.

1. \(x + 2 = 8\) 6  
2. \(y + 7 = 9\) 2  
3. \(a + 5 = 12\) 7  

4. \(16 = n + 6\) 10  
5. \(q + 10 = 22\) 12  
6. \(m + 9 = 17\) 8  

7. \(b - 4 = 9\) 13  
8. \(8 = c - 4\) 12  
9. \(11 = t - 7\) 18  

10. \(d - 10 = 8\) 18  
11. \(x - 11 = 9\) 20  
12. \(2 = z - 14\) 16  

13. \(72 = 24 + w\) 48  
14. \(86 + y = 99\) 13  
15. \(6 + y = -8\) -14  

16. \(-5 = m + 11\) -16  
17. \(n + 3.5 = 6.7\) 3.2  
18. \(x + 1.6 = 0.8\) -0.8  

19. \(98 = t - 18\) 116  
20. \(12 = g - 56\) 68  
21. \(x - 18 = -2\) 16  

22. \(p - 11 = -5\) 6  
23. \(a - 1.5 = 4.2\) 5.7  
24. \(7.4 = n - 2.6\) 10
Lesson 2 Reteach

Multiplication and Division Equations

Use the Division Property of Equality to solve multiplication equations and the Multiplication Property of Equality to solve division equations.

The Division Property of Equality states that if you divide each side of an equation by the same nonzero number, the two sides remain equal.

The Multiplication Property of Equality states that if you multiply each side of an equation by the same number, the two sides remain equal.

Example 1
Solve $30 = 6x$.

$30 = 6x$ Write the equation.

$\frac{30}{6} = \frac{6x}{6}$ Divide each side of the equation by 6.

$5 = x$ $30 \div 6 = 5$.

The solution is 5.

Example 2
Solve $\frac{x}{-5} = -2$.

$\frac{x}{-5} = -2$ Write the equation.

$\frac{x}{-5} (-5) = -2(-5)$ Multiply each side of the equation by $-5$.

$x = 10$ $-2(-5) = 10$.

The solution is 10.

Exercises
Solve each equation. Check your solution.

1. $3x = 12$ $4$
2. $9k = -360$ $-40$
3. $-15a = -45$ $3$
4. $14 = 2b$ $7$
5. $\frac{x}{5} = 12$ $60$
6. $16 = \frac{a}{3}$ $48$
7. $\frac{c}{-2} = 7$ $-14$
8. $-7y = 42$ $-6$
9. $\frac{m}{6} = -4$ $-24$
10. $-2 = \frac{b}{-9}$ $18$
Lesson 2 Skills Practice

Multiplication and Division Equations

Solve each equation. Check your solution.

1. \(7a = 56\) \hspace{1cm} 2. \(-5b = -20\)

3. \(14 = 14c\) \hspace{1cm} 4. \(\frac{e}{-9} = -6\)

5. \(\frac{k}{12} = 2\) \hspace{1cm} 6. \(\frac{m}{6} = -10\)

7. \(66 = -11y\) \hspace{1cm} 8. \(\frac{x}{19} = 4\)

9. \(-15 = \frac{z}{-8}\) \hspace{1cm} 10. \(-3z = 93\)

11. \(5 = \frac{g}{4}\) \hspace{1cm} 12. \(\frac{a}{3} = -12\)

13. \(-8 = \frac{t}{9}\) \hspace{1cm} 14. \(3c = 15\)

15. \(-7 = \frac{w}{6}\) \hspace{1cm} 16. \(-6y = -6\)

17. \(18 = -9b\) \hspace{1cm} 18. \(-13c = -52\)

19. \(4h = -44\) \hspace{1cm} 20. \(-7x = -63\)
Lesson 3 Reteach

Solve Equations with Rational Coefficients

Multiplicative inverses, or reciprocals, are two numbers whose product is 1. To solve an equation in which the coefficient is a fraction, multiply each side of the equation by the reciprocal of the coefficient.

Example 1
Solve $15 = 0.5n$. Check the solution.

\[
15 = 0.5n \quad \text{Write the equation.}
\]

\[
\frac{15}{0.5} = \frac{0.5n}{0.5} \quad \text{Division Property of Equality}
\]

\[
30 = n \quad \text{Simplify.}
\]

Example 2
Solve $\frac{4}{5}x = 8$. Check your solution.

\[
\frac{4}{5}x = 8 \quad \text{Write the equation.}
\]

\[
\left(\frac{5}{4}\right)\frac{4}{5}x = \left(\frac{5}{4}\right)8 \quad \text{Multiply each side by the reciprocal of } \frac{4}{5}, \frac{5}{4}.
\]

\[
x = 10 \quad \text{Simplify.}
\]

The solution is 10.

Exercises

Solve each equation. Check your solution.

1. $4.9 = 0.7m$  
2. $-\frac{1}{2} = -\frac{6}{18}h$  
3. $-2.8 = 4b$  
4. $\frac{3}{5}x = 12$  
5. $16 = \frac{10}{3}a$  
6. $9 = 0.3n$  
7. $\frac{15}{7}y = 3$  
8. $21 = 0.75a$  
9. $\frac{14}{3} = -\frac{7}{9}b$
Lesson 3 Skills Practice

Solve Equations with Rational Coefficients

Solve each equation. Check your solution.

1. $3.4a = 57.8$  
   $a = 17$

2. $-2 = 0.8n$  
   $n = -2.5$

3. $\frac{5}{6}k = -20$  
   $k = -24$

4. $12 = 0.9a$  
   $a = 13\frac{1}{3}$

5. $\frac{3}{4}c = -12$  
   $c = -16$

6. $0.36y = 18$  
   $y = 50$

7. $\frac{3}{5}y = 6$  
   $y = 10$

8. $-15 = \frac{3}{7}b$  
   $b = -35$

9. $\frac{6}{7}c = 18$  
   $c = 21$

10. $\frac{7}{3}x = \frac{2}{3}$  
    $x = \frac{2}{7}$

11. $\frac{11}{12} = \frac{3}{4}h$  
    $h = 1\frac{2}{9}$

12. $\frac{9}{14}y = \frac{3}{7}$  
    $y = 2\frac{2}{3}$

13. $\frac{m}{26} = -\frac{1}{2}$  
    $m = -13$

14. $0.6 = \frac{n}{5}$  
    $n = 3$

15. $1.5r = -5.07$  
    $r = -3.38$

16. $-4.3 = 0.5n$  
    $n = -8.6$

17. $1.5x = 9$  
    $x = 6$

18. $\frac{3}{8}x = 21$  
    $x = 56$

19. $-14 = \frac{7}{9}m$  
    $m = -18$

20. $3.2 = \frac{t}{8}$  
    $t = 25.6$

21. $\frac{m}{18} = \frac{1}{9}$  
    $m = -2$
Lesson 4 Reteach

Solve Two-Step Equations

To solve a two-step equation, undo the addition or subtraction first. Then undo the multiplication or division.

Example 1
Solve $7v - 3 = 25$. Check your solution.

$7v - 3 = 25$  \hspace{1cm} Write the equation.

$+3 = +3$  \hspace{1cm} Undo the subtraction by adding 3 to each side.

$7v = 28$  \hspace{1cm} Simplify.

$\frac{7v}{7} = \frac{28}{7}$  \hspace{1cm} Undo the multiplication by dividing each side by 7.

$v = 4$  \hspace{1cm} Simplify.

Check  \hspace{1cm} Write the original equation.

$7v - 3 = 25$  \hspace{1cm} Replace v with 4.

$7(4) - 3 \div 25$  \hspace{1cm} Multiply.

$28 - 3 \div 25$  \hspace{1cm} The solution checks.

$25 = 25 \checkmark$

The solution is 4.

Example 2
Solve $-10 = 8 + 3x$. Check your solution.

$-10 = 8 + 3x$  \hspace{1cm} Write the equation.

$-8 = -8$  \hspace{1cm} Undo the addition by subtracting 8 from each side.

$-18 = 3x$  \hspace{1cm} Simplify.

$\frac{-18}{3} = \frac{3x}{3}$  \hspace{1cm} Undo the multiplication by dividing each side by 3.

$-6 = x$  \hspace{1cm} Simplify.

Check  \hspace{1cm} Write the original equation.

$-10 = 8 + 3x$  \hspace{1cm} Replace x with $-6$.

$-10 \div 8 + 3(-6)$  \hspace{1cm} Multiply.

$-10 \div 8 + (-18)$  \hspace{1cm} The solution checks.

$-10 = -10 \checkmark$

The solution is $-6$.

Exercises
Solve each equation. Check your solution.

1. $4y + 1 = 13$  \hspace{1cm} 2. $6x + 2 = 26$  \hspace{1cm} 3. $-3 = 5k + 7$  \hspace{1cm} 4. $\frac{2}{3}n + 4 = -26$

\hspace{1cm} $3$  \hspace{1cm} $4$  \hspace{1cm} $-2$  \hspace{1cm} $-45$

5. $7 = -3c - 2$  \hspace{1cm} 6. $-8p + 3 = -29$  \hspace{1cm} 7. $-5 = -5t - 5$  \hspace{1cm} 8. $-9r + 12 = -24$

\hspace{1cm} $-3$  \hspace{1cm} $4$  \hspace{1cm} $0$  \hspace{1cm} $4$

9. $11 + \frac{7}{9}n = 4$  \hspace{1cm} 10. $35 = 7 + 4b$  \hspace{1cm} 11. $-15 + \frac{4}{5}p = 9$  \hspace{1cm} 12. $49 = 16 + 3y$

\hspace{1cm} $-9$  \hspace{1cm} $7$  \hspace{1cm} $30$  \hspace{1cm} $11$

13. $2 = 4t - 14$  \hspace{1cm} 14. $-9x - 10 = 62$  \hspace{1cm} 15. $30 = 12z - 18$  \hspace{1cm} 16. $7 + 4g = 7$

\hspace{1cm} $4$  \hspace{1cm} $-8$  \hspace{1cm} $4$  \hspace{1cm} $0$

Course 2 • Chapter 6 Equations and Inequalities
Lesson 4 Skills Practice

Solve Two-Step Equations

Solve each equation. Check your solution.

1. \(2x + 1 = 9\)
   \(4\)

2. \(5b + 2 = 17\)
   \(3\)

3. \(3w + 5 = 23\)
   \(6\)

4. \(\frac{3}{8}n + 1 = -25\)
   \(-69\frac{1}{3}\)

5. \(4t - 2 = 14\)
   \(4\)

6. \(7k - 3 = 32\)
   \(5\)

7. \(8x - 1 = 63\)
   \(8\)

8. \(2x - 5 = 15\)
   \(10\)

9. \(2 + \frac{1}{6}a = -4\)
   \(-36\)

10. \(9 + 4b = 17\)
    \(2\)

11. \(2p + 14 = 0\)
    \(-7\)

12. \(3y + \frac{2}{5} = -\frac{1}{5}\)
    \(-0.2\) or \(-\frac{1}{5}\)

13. \(-\frac{2}{3}w + 5 = 4\)
    \(1.5\)

14. \(8x + 7 = -9\)
    \(-2\)

15. \(5d - 1 = -11\)
    \(-2\)

16. \(4d - 35 = -3\)
    \(8\)

17. \(11x - 24 = -2\)
    \(2\)

18. \(15a - 54 = -9\)
    \(3\)

19. \(3g - 49 = -7\)
    \(14\)

20. \(-\frac{1}{2}x - 7 = 18\)
    \(-50\)

21. \(-9d - 1 = 17\)
    \(-2\)

22. \(-\frac{4}{5}f + 1 = -13\)
    \(17.5\)

23. \(-5b + 24 = -1\)
    \(5\)

24. \(-6x + 4 = -2\)
    \(1\)
Lesson 5 Reteach

More Two-Step Equations

An equation in the form \( p(x + q) = r \) contains two factors, \( p \) and \( x + q \) and is considered a two-step equation.

Example 1
Solve \( 6(x + 2) = 42 \). Check your solution.

\[
\begin{align*}
6(x + 2) &= 42 \\
6(x + 2) &= 42 \\
6 &= 6 \\
x + 2 &= 7 \\
x &= 5 \\
\end{align*}
\]

Check
\[
\begin{align*}
6(x + 2) &= 42 \\
6(5 + 2) &= 42 \\
6(7) &= 42 \\
42 &= 42 \checkmark
\end{align*}
\]

The solution is 5.

Example 2
Solve \( \frac{4}{5}(x - 5) = 4 \). Check your solution.

\[
\begin{align*}
\frac{4}{5}(x - 5) &= 4 \\
\frac{4}{5}(x - 5) &= \frac{4}{5} \cdot 4 \\
(x - 5) &= \frac{4}{5} \cdot 4 \\
(x - 5) &= \frac{4}{5} \cdot 4 \\
x - 5 &= \frac{4}{5} \cdot 4 \\
x &= 10 \\
\end{align*}
\]

Check
\[
\begin{align*}
\frac{4}{5}(x - 5) &= 4 \\
\frac{4}{5}(10 - 5) &= 4 \\
\frac{4}{5}(5) &= 4 \checkmark
\end{align*}
\]

The solution is 10.

Exercises
Solve each equation.

1. \( 7(x + 4) = 49 \)  
2. \( 2(x - 8) = -22 \)  
3. \( 10(x + 3) = -20 \)  
4. \( 25(x - 3) = 175 \)

\[
\begin{align*}
3 &\quad -3 &\quad -5 &\quad 10 \\
\end{align*}
\]

5. \( \frac{3}{4}(x - 12) = 3 \)  
6. \( \frac{2}{3}(x + 4) = 14 \)  
7. \( \frac{7}{9}(x + 5) = 21 \)  
8. \( \frac{1}{8}(x - 15) = 4 \)

\[
\begin{align*}
16 &\quad 17 &\quad 22 &\quad 47 \\
\end{align*}
\]
Lesson 5 Skills Practice

More Two-Step Equations

Solve each equation. Check your solution.

1. $3(x + 5) = 39$
   
   $x = 8$

2. $7(x + 8) = 49$
   
   $x = -1$

3. $-5(x - 6) = 15$
   
   $x = 3$

4. $10(x - 5) = -80$
   
   $x = -3$

5. $4(x + 9) = 20$
   
   $x = -4$

6. $6(x + 12) = -42$
   
   $x = -19$

7. $\frac{4}{9}(x + 13) = 8$
   
   $x = 5$

8. $\frac{9}{10}(x + 8) = 18$
   
   $x = 12$

9. $\frac{2}{7}(x - 9) = -4$
   
   $x = -5$

10. $\frac{3}{7}(x - 2) = 15$
    
    $x = 37$

11. $1.5(x + 7) = 11.25$
    
    $x = 0.5$

12. $4.5(x - 9) = -13.5$
    
    $x = 6$

13. $8.3(x - 3.1) = -37.35$
    
    $x = -1.4$

14. $0.4(x + 2.4) = 2.96$
    
    $x = 5$

15. $\frac{4}{5}(x + 7) = 20$
    
    $x = 18$

16. $\frac{6}{11}(x + 5) = 6$
    
    $x = 6$

17. $-\frac{1}{8}(x - 4) = -4$
    
    $x = 36$

18. $\frac{2}{5}(x - 16) = -6$
    
    $x = 1$

19. $9.2(x + 6.4) = 132.48$
    
    $x = 8$

20. $8.2(x - 7) = -24.6$
    
    $x = 4$

21. $\frac{2}{5}(x - 19) = -15$
    
    $x = -6$

22. $0.1(x + 7) = 3.5$
    
    $x = 28$

23. $-2.8(x + 4.9) = 18.2$
    
    $x = 11.4$

24. $6.5(x - 4) = 19.5$
    
    $x = 7$
By working backward from where you end to where you began, you can solve problems. Use the four step problem solving model to stay organized when working backward.

Example 1
Jonah put half of his birthday money into his savings account. Then he paid back the $10 that he owed his brother for dance tickets. Lastly, he spent $3 on lunch at school. At the end of the day he was left with $12. How much money did Jonah receive for his birthday?

Understand
You know that he had $12 left and the amounts he spent throughout the day. You need to find out how much money he received for his birthday.

Plan
Start with the amount of money he was left with and work backward.

Solve
He had $12 left.
Undo the $3 he spent on lunch.
Undo the $10 he gave back to his brother
Undo the half put into his savings account
So, Jonah received $50 for his birthday.

Check
Assume that Jonah receive $50 for his birthday. After putting half into his savings account he had $50 ÷ 2 or $25. Then he gave $10 to his brother for dance tickets, so he had $25 – $10 or $15. Lastly, he spent $3 on lunch at school, so he had $15 – $3, or $12. So, our answer of $50 is correct.

Exercises
Solve each problem by using the work backward strategy.

1. On Monday everyone was present in Mr. Miller’s class. At 12:00, 5 students left early for doctors’ appointments. At 1:15, half of the remaining students went to an assembly. Finally, at 2:00, 6 more students left for a student council meeting. At the end of the day, there were only 5 students in the room. Assuming that no students returned after having left, how many students are in Mr. Miller’s class? 27 students

2. Jordan was trading baseball cards with some friends. He gave 15 cards to Tommy and got 3 back. He gave two thirds of his remaining cards to Elaine and kept the rest for himself. When he got home he counted that he had 25 cards. How many baseball cards did Jordan start with? 87 baseball cards
Skills Practice

Problem-Solving Investigation: Work Backward

Solve. Use the work backward strategy

1. GOVERNMENT There are 99 members in the Ohio House of Representatives. All of them were present when a vote was taken on a piece of legislation. If 6 of them did not vote, and 13 more voted “yes” than voted “no,” how many “no” votes were there?
   There were 40 “no” votes.

2. MONEY Jessie and Amar eat lunch at a restaurant and their bill is $21.65. Amar gives the cashier a coupon for $6 off their bill, and also hands the cashier two bills. If he receives $4.35 in change, what were the denominations of the two bills he gave the cashier?
   They were both ten dollar bills.

3. AGE Justine is 13 years younger than her uncle Stewart. Stewart is 18 years older than Justine’s sister, Hana. Hana’s mother is 8 years older than Stewart, and 28 years older than her youngest child, Jared. If Jared is 12 years old, how old is Justine?
   Justine is 19 years old.

4. NUMBER THEORY A number is divided by 6. Then 7 is added to the divisor. After dividing by 4, the result is 4. What is the number?
   54

5. COMPACT DISCS Carmella borrowed half as many CDs from the library as her friend Ariel. Ariel borrowed 2 more than Juan, but four less than Sierra. Sierra borrowed 12 CDs. How many did each person borrow?
   Ariel 8 CDs
   Juan 6 CDs
   Carmella 4 CDs

6. TIME Ashish needs to leave for the bus stop 15 minutes earlier than his friend Rami. Rami leaves five minutes later than Leann, but 10 minutes earlier than Raphael. If Raphael leaves for the bus stop at 8:15, what time does Ashish need to leave?
   Ashish needs to leave at 7:50.
Lesson 6 Reteach

Solve Inequalities by Addition or Subtraction

Solving an inequality means finding values for the variable that make the inequality true. You can use the Addition and Subtraction Properties of Inequality to help solve an inequality. When you add or subtract the same number from each side of an inequality, the inequality remains true.

Examples
Solve each inequality.

1  \( x + 4 > 9 \)
   \( x + 4 - 4 > 9 - 4 \)
   \( x > 5 \)

Any number greater than 5 will make the statement true. Therefore, the solution is \( x > 5 \).

2  \(-12 \geq n - 9 \)
   \(-12 + 9 \geq n - 9 + 9 \)
   \(-3 \geq n \)

The solution is \(-3 \geq n \) or \( n \leq -3 \).

3  Solve \( a + \frac{1}{3} < 1 \). Graph the solution set on a number line.

\[
\begin{align*}
  a + \frac{1}{3} &< 1 \\
  a + \frac{1}{3} - \frac{1}{3} &< 1 - \frac{1}{3} \\
  a &< \frac{2}{3}
\end{align*}
\]

Exercises
Solve each inequality.

1. \( t - 6 > 3 \quad t > 9 \)

2. \( b + 9 \leq 2 \quad b \leq -7 \)

3. \( 8 < r - 9 \quad r > 17 \)

4. \(-4 < p + 4 \quad p > -8 \)

Solve each inequality. Graph the solution set on a number line.

5. \( s + 8 < 9 \)

6. \(-3 \leq d - 2 \)

\( s < 1 \)

\( d \geq -1 \)
Lesson 6 Skills Practice

Solve Inequalities by Addition or Subtraction

Solve each inequality.
1. \(a + 4 < 9\) \(a < 5\)  
2. \(e - 7 > 1\) \(e > 8\)  
3. \(-4 \geq k - 2\) \(k \leq -2\)  
4. \(y + 6 > 9\) \(y > 3\)  
5. \(n - 9 \geq 5\) \(n \geq 14\)  
6. \(-4 > h - 2\) \(h < -2\)  
7. \(-19 > x - 11\) \(x < -8\)  
8. \(5 \leq q + 12\) \(q \geq -7\)

Solve each inequality. Graph the solution set on a number line.
9. \(8 < p - 1\) \(p > 9\);  
10. \(w + 5 \geq -6\) \(w \geq -11\);  
11. \(1 > x + 6\) \(x < -5\);  
12. \(4 \leq v - 7\) \(v \geq 11\);  
13. \(b - 3 \leq -8\) \(b \leq -5\);  
14. \(m + 9 < -8\) \(m < -17\);  

Write an inequality and solve each problem.
15. Two less than a number is less than 9. \(n - 2 < 9\); \(n < 11\)  
16. The difference between a number and 3 is no more than 2. \(n - 3 \leq 2\); \(n \leq 5\)  
17. The sum of a number and 8 is more than 4. \(n + 8 > 4\); \(n > -4\)  
18. Two more than a number is less than 13. \(2 + n < 13\); \(n < 11\)
Lesson 7 Reteach

Solve Inequalities by Multiplication or Division

When you multiply or divide each side of an inequality by a positive number, the inequality remains true. However, when you multiply or divide each side of an inequality by a negative number, the direction of the inequality must be reversed for the inequality to remain true.

Example 1
Solve $\frac{t}{-6} \leq -4$. Then graph the solution set on a number line.

$$\frac{t}{-6} \leq -4$$
Write the inequality.

$$\frac{t}{-6}(-6) \geq -4(-6)$$
Multiply each side by $-6$ and reverse the inequality symbol.

$$t \geq 24$$
Simplify.

To graph the solution, place a closed circle at 24 and draw a line and arrow to the right.

Example 2
Solve $\frac{4}{5}x - 5 < 23$.

$$\frac{4}{5}x - 5 < 23$$
Write the inequality.

$$\frac{4}{5}x - 5 + 5 < 23 + 5$$
Add 5 to each side.

$$\frac{4}{5}x < 28$$
Simplify.

$$\left(\frac{5}{4}\right) \left(\frac{4}{5}x\right) < \left(\frac{5}{4}\right)28$$
Multiply each side by $\frac{5}{4}$.

$$x < 35$$
Simplify.

Exercises
Solve each inequality. Then graph the solution on a number line.

1. $3a > 12 \quad a > 4$;  

2. $6 \geq \frac{r}{-2} \quad r \geq -12$;

3. $-3.1c + 2 \geq 2 \quad c \leq 0$  

4. $13 > -\frac{2}{3}y - 3 \quad y > -24$

5. $-\frac{h}{5} - 6 < -10 \quad h > 20$  

6. $6a + 13 \leq 31 \quad a \leq 3$
Lesson 7 Skills Practice

Solve Inequalities by Multiplication or Division

Solve each inequality. Graph the solution set on a number line.

1. \(3v > 12 \quad v > 4\);
2. \(\frac{p}{4} < -15 \quad p < -60\);

3. \(-12 \leq -3g \quad g \leq 4\);
4. \(60 \geq 12c \quad c \leq 5\);

5. \(\frac{a}{2} > -4 \quad a > -8\);
6. \(1 \leq \frac{u}{5} \quad u \geq 5\);

7. \(-14 \geq 7n \quad n \leq -2\);
8. \(-4d \geq -36 \quad d \leq 9\);

Solve each inequality. Check your solution.

9. \(3a + 6 < -10 \quad a < -\frac{16}{3}\)
10. \(\frac{b}{5} - 4 \geq -29 \quad b \geq -125\)

11. \(\frac{m}{2} + 6 < 10 \quad m < 8\)
12. \(\frac{2}{3} + \frac{1}{6}r > -1 \quad r > -10\)

13. \(-6d + 7 \leq 1 \quad d \geq 1\)
14. \(\frac{z}{-8} - 5 < -3 \quad z > -16\)

15. \(-2y - 5 \leq 31 \quad y \geq -18\)
16. \(2.1n \leq -4.6n + 13.4 \quad n \leq 2\)

17. \(3x + 2 < x - 6 \quad x < -4\)
18. \(y - 3 > 2y - 7 \quad y < 4\)

19. \(\frac{a}{4} + 5 < a - 4 \quad a > 12\)
20. \(1.5g - 12 > \frac{3g}{4} \quad g > 16\)
Lesson 8 Reteach
Solve Two-Step Inequalities

A two-step inequality is an inequality that contains two operations. To solve a two-step inequality, use inverse operations to undo each operation in reverse order of the order of operations.

Example 1
Solve \(4x - 2 \leq 18\). Graph the solution set on a number line.

\[
\begin{align*}
4x - 2 & \leq 18 \quad \text{Write the inequality.} \\
+2 & +2 \quad \text{Addition Property of Inequality} \\
4x & \leq 20 \quad \text{Simplify.} \\
\frac{4x}{4} & \leq \frac{20}{4} \quad \text{Division Property of Inequality} \\
x & \leq 5 \quad \text{Simplify.}
\end{align*}
\]

The solution is \(x \leq 5\).

Graph the solution set.

\[
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \text{Draw a closed dot at 5 with an arrow to the left.}
\end{array}
\]

Check \(4x - 2 \leq 18\) Write the inequality.

\[
\begin{align*}
4(3) - 2 & \leq 18 \quad \text{Replace } x \text{ with a number less than or equal to } 5. \\
10 & \leq 18 \quad \text{This statement is true.}
\end{align*}
\]

Exercises
Solve each inequality. Graph the solution set on a number line.

1. \(3x - 4 < 17 \quad x < 7\)

2. \(-2 - x \leq 3 \quad x \geq -5\)

3. \(12 < 2x + 6 \quad x > 3\)

4. \(\frac{x}{2} - 3 \leq -2 \quad x \leq 2\)

5. \(7 > x - 2 \quad x < 9\)

6. \(1 \geq -\frac{x}{3} + 1 \quad x \geq 0\)
Lesson 8 Skills Practice
Solve Two-Step Inequalities

Solve each inequality. Graph the solution set on a number line.

1. \( \frac{x}{2} - 1 < 5 \) \( x < 12 \)

\[ \begin{array}{c}
\text{8} & \text{9} & \text{10} & \text{11} & \text{12} & \text{13} & \text{14} & \text{15} & \text{16} \\
\hline
\end{array} \]

2. \( 13 \geq -x + 7 \) \( x \geq -6 \)

\[ \begin{array}{c}
\text{-10} & \text{-9} & \text{-8} & \text{-7} & \text{-6} & \text{-5} & \text{-4} & \text{-3} & \text{-2} \\
\hline
\end{array} \]

3. \( -2 + 3x > -23 \) \( x > -7 \)

\[ \begin{array}{c}
\text{-11} & \text{-10} & \text{-9} & \text{-8} & \text{-7} & \text{-6} & \text{-5} & \text{-4} & \text{-3} \\
\hline
\end{array} \]

4. \( 3x - 4 \leq 23 \) \( x \leq 9 \)

\[ \begin{array}{c}
\text{5} & \text{6} & \text{7} & \text{8} & \text{9} & \text{10} & \text{11} & \text{12} & \text{13} \\
\hline
\end{array} \]

5. \( 10 \geq \frac{x}{3} + 6 \) \( x \leq 12 \)

\[ \begin{array}{c}
\text{8} & \text{9} & \text{10} & \text{11} & \text{12} & \text{13} & \text{14} & \text{15} & \text{16} \\
\hline
\end{array} \]

6. \( -2x + 4 < 16 \) \( x > -6 \)

\[ \begin{array}{c}
\text{-10} & \text{-9} & \text{-8} & \text{-7} & \text{-6} & \text{-5} & \text{-4} & \text{-3} & \text{-2} \\
\hline
\end{array} \]

7. \( 13 > 3 + 2x \) \( x < 5 \)

\[ \begin{array}{c}
\text{1} & \text{2} & \text{3} & \text{4} & \text{5} & \text{6} & \text{7} & \text{8} & \text{9} \\
\hline
\end{array} \]

8. \( 11x - 1 \leq 120 \) \( x \leq 11 \)

\[ \begin{array}{c}
\text{7} & \text{8} & \text{9} & \text{10} & \text{11} & \text{12} & \text{13} & \text{14} & \text{15} \\
\hline
\end{array} \]

9. \( -4 + \frac{x}{5} < -4 \) \( x < 0 \)

\[ \begin{array}{c}
\text{-4} & \text{-3} & \text{-2} & \text{-1} & \text{0} & \text{1} & \text{2} & \text{3} & \text{4} \\
\hline
\end{array} \]

10. \( 6x - 4 \geq -28 \) \( x \geq -4 \)

\[ \begin{array}{c}
\text{-8} & \text{-7} & \text{-6} & \text{-5} & \text{-4} & \text{-3} & \text{-2} & \text{-1} & \text{0} \\
\hline
\end{array} \]

11. \( -6 \leq -4 - 2x \) \( x \leq 1 \)

\[ \begin{array}{c}
\text{-3} & \text{-2} & \text{-1} & \text{0} & \text{1} & \text{2} & \text{3} & \text{4} & \text{5} \\
\hline
\end{array} \]

12. \( 5 > 2 + \frac{x}{3} \) \( x < 9 \)

\[ \begin{array}{c}
\text{4} & \text{5} & \text{6} & \text{7} & \text{8} & \text{9} & \text{10} & \text{11} & \text{12} \\
\hline
\end{array} \]
Lesson 1 Reteach

Classify Angles

- An angle is formed by two rays that share a common endpoint called the **vertex**.
- An angle can be named in several ways. The symbol for angle is \( \angle \).
- Angles are classified according to their measures. Two angles that have the same measure are said to be **congruent**.
- Two angles are **vertical** if they are opposite angles formed by the intersection of two lines. Vertical angles are congruent.
- Two angles are **adjacent** if they share a common vertex, a common side, and do not overlap.

### Example

**Name each angle below. Then classify the angle as acute, right, obtuse, or straight.**

1. \( \angle ABC \), \( \angle CBA \), or \( \angle BAC \).

Use the vertex as the middle letter and a point from each side, \( \angle ABC \), \( \angle CBA \), or use the vertex or the number only, \( \angle B \) or \( \angle 1 \). The angle is less than 90°, so it is an acute angle.

2. \( \angle D \) or \( \angle 2 \).

Use the vertex or the number only, \( \angle D \) or \( \angle 2 \). The angle is less than 90°, so it is a right angle.

3. **What is the value of \( x \) in the figure at the right?**

The angle labeled 5x° and the angle labeled 55° are vertical angles. Since vertical angles are congruent, the value of \( x \) is 11.

### Exercises

**Name each angle. Then classify the angle as acute, right, obtuse, or straight.**

1. \( \angle H \) or \( \angle 3 \); acute

2. \( \angle MNO \), \( \angle ONM \), or \( \angle N \); obtuse

3. \( \angle Q \); right

4. Find the value of \( x \) in the figure at the right.

50
Lesson 1 Skills Practice

Classify Angles

Name each angle in four ways. Then classify the angle as acute, right, obtuse, or straight.

1. \(\angle ABC, \angle CBA, \angle B, \angle D; \) obtuse
2. \(\angle DEF, \angle FED, \angle E, \angle F; \) right
3. \(\angle GHI, \angle IHG, \angle H, \angle I; \) acute
4. \(\angle JKL, \angle LKJ, \angle K, \angle M; \) straight
5. \(\angle MNO, \angle ONM, \angle N, \angle O; \) obtuse
6. \(\angle PQR, \angle RQP, \angle Q, \angle P; \) acute

Refer to the diagram at the right. Identify each angle pair as adjacent, vertical, or neither.

7. \(\angle 7 \) and \(\angle 12 \) adjacent
8. \(\angle 8 \) and \(\angle 11 \) vertical
9. \(\angle 7 \) and \(\angle 10 \) vertical
10. \(\angle 9 \) and \(\angle 11 \) neither
11. \(\angle 8 \) and \(\angle 9 \) adjacent
12. \(\angle 10 \) and \(\angle 12 \) neither

Refer to the figure at the right to determine the measure of each given angle.

13. \(\angle SYX \) 76°
14. \(\angle XYW \) 40°
15. \(\angle WYV \) 64°
16. \(\angle SYW \) 116°
17. \(\angle TYX \) 140°
18. \(\angle VYX \) 104°
Lesson 2 Reteach

Complementary and Supplementary Angles

- Two angles are **complementary** if the sum of their measures is 90°.
- Two angles are **supplementary** if the sum of their measures is 180°.

**Examples**

Identify each pair of angles as complementary, supplementary, or neither.

1. \[30° + 150° = 180°\]
   The angles are supplementary.

2. \[16° + 74° = 90°\]
   The angles are complementary.

**Example 3**

ALGEBRA Find the value of \(x\).

Since the two angles form a straight line, they are supplementary. The sum of their measures is 180°.

\[5x + 35 = 180\]

Write the equation.

\[-35 = -35\]

Subtract 35 from each side.

\[\frac{5x}{5} = \frac{145}{5}\]

Divide each side by 5

\[x = 29\]

Simplify.

**Exercises**

Identify each pair of angles as complementary, supplementary, or neither.

1. supplementary

2. complementary

3. neither

ALGEBRA Find the value of \(x\) in each figure.

4. \[36° + 6x° = 180°\]

5. \[4x° + 56° = 180°\]

6. \[22° + 2x° = 180°\]
Lesson 2 Skills Practice

Complementary and Supplementary Angles

Identify each pair of angles as complementary, supplementary, or neither.

1. supplementary
2. complementary
3. neither
4. neither
5. supplementary
6. complementary
7. supplementary
8. neither
9. complementary

ALGEBRA Find the value of $x$ in each figure.

10. 54
11. 128
12. 12
13. 43
14. 21
15. 3
16. 163
17. 45
18. 11
Lesson 3 Reteach

Triangles

Every triangle has at least two acute angles. One way you can classify a triangle is by using the third angle. Another way to classify triangles is by their sides. Sides with the same length are congruent segments.

**Classify Triangles Using Angles**

<table>
<thead>
<tr>
<th>All acute angles</th>
<th>1 right angle</th>
<th>1 obtuse angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>acute triangle</td>
<td>right triangle</td>
<td>obtuse triangle</td>
</tr>
</tbody>
</table>

**Classify Triangles Using Sides**

<table>
<thead>
<tr>
<th>No congruent sides</th>
<th>At least 2 congruent sides</th>
<th>3 congruent sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>scalene triangle</td>
<td>isosceles triangle</td>
<td>equilateral triangle</td>
</tr>
</tbody>
</table>

**Example**
The figure shows a triangular pennant tied to a pole. Classify the marked triangle by its angles and by its sides.

The triangle has three acute angles and two sides the same length. So, it is an acute, isosceles triangle.

**Exercises**

Draw a triangle that satisfies each set of conditions. Then classify each triangle.

1. A triangle with three acute angles and three congruent sides
   - acute, equilateral triangle

2. A triangle with one right angle and no congruent sides
   - right, scalene triangle
Lesson 3 Skills Practice

Triangles

Find the value of $x$. Then classify the triangle by its angles.

1. $\text{Obtuse, Scalene Triangle}$
2. $\text{Obtuse, Scalene Triangle}$
3. $\text{Right, Isosceles Triangle}$

4. $\text{Obtuse, Scalene Triangle}$
5. $\text{Right, Isosceles Triangle}$
6. $\text{Acute, Equilateral Triangle}$

7. $\text{Acute, Scalene Triangle}$
8. $\text{Acute, Scalene Triangle}$
9. $\text{Right, Isosceles Triangle}$

10. a triangle with one obtuse angle and no congruent sides
    
11. a triangle with three acute angles and three congruent sides
    
12. a triangle with one right angle and two congruent sides
Reteach

Problem-Solving Investigation: Make a Model

You may need to use the make a model strategy to solve some problems.

You can always use the four-step plan to solve a problem.

Understand • Determine what information is given in the problem and what you need to find.
Plan • Select a strategy including a possible estimate.
Solve • Solve the problem by carrying out your plan.
Check • Examine your answer to see if it seems reasonable.

Example
Kisha is trying to make a box out of a piece of cardboard by cutting a square out of each corner. She will then fold up the sides and tape them together. The cardboard measures 4 feet 6 inches by 6 feet 6 inches. She wants the box to measure 3 feet wide by 5 feet long. What size squares should Kisha cut out of the corners to make the box?

Understand She wants to know what size squares to cut out of each corner to make a box which measures 3 feet by 5 feet.

Plan Start by making a model of the cardboard. Label the sides of the cardboard in feet. Draw lines to show the squares that will be cut out of the corners.

Solve Subtract 5 feet from 6 feet 6 inches and divide by 2.

1 ft 6 in. = 18 in.

18 in. ÷ 2 = 9 in.

The square must have sides that are 9 inches long.

Check Check that width of the box meets the specifications. Subtracting 18 inches or 1 foot 6 inches from 4 feet 6 inches yields 3 feet, which is the width required.

Exercises

Make a model to solve each problem.

1. CONSTRUCTION A chicken coop will be 20 feet long and 16 feet wide. One side that is 20 feet long will be formed by the barn. The other three sides will be made of wire fencing with posts at every corner and every 4 feet between each corner. How many feet of fencing and how many posts are needed to build the chicken coop?

52 feet of fencing and 14 posts

2. GEOMETRY What is the fewest number of one-inch cubes needed to make a rectangular prism that measures 4 inches by 5 inches by 6 inches? (Hint: The prism can be hollow inside.) 96 cubes
Skills Practice

Problem-Solving Investigation: Make a Model

Make a model to solve each problem.

1. SHIPPING A spice distributor is making boxes in which to pack cylindrical spice containers. The diameter of each container is 2 inches. The height of each container is 4 inches. If they place 4 rows with 3 containers in each row in a box, what is the volume of the box? 192 cubic inches

2. SEWING Kacey has a bread basket in the shape of a rectangular prism that measures 12 inches high, 18 inches long, and 16 inches wide. She wants to cover the inside of the basket with a 50-inch by 20-inch piece of fabric. Does Kacey have enough fabric to cover the inside of the basket? Explain your answer. No. She only has 1,000 square inches of fabric. She needs at least 1,104 square inches.

3. BEADS Elsa is making a wooden box for sorting and storing her bead collection. The outer dimensions of the box are 10 inches by 10 inches. She wants to make 100 compartments that are approximately 1-inch squares. How many horizontal and vertical dividers will Elsa need to make the compartments? 9 horizontal dividers and 9 vertical dividers

4. ARRANGING TABLES Donna is arranging four tables to make seating for her party guests. Standing alone, each table will seat 4 people on each side and 2 people at each end. She can either place the tables end-to-end to make one long table or she can separate the tables into four individual tables. How many more guests can she seat if she separates the tables than if she places them end-to-end? 12 more guests

5. MAKING FRAMES Hamish is making picture frames by gluing square tiles onto the wooden sides. The wooden sides measure 8 inches wide by 10 inches long by 1 inch wide. If he glues a 1-inch square tile at every corner and covers the remainder of the wood sides with \(\frac{1}{2}\)-inch square tiles, how many of each size tile does Hamish need to make 4 frames? 16 1-inch square tiles and 448 \(\frac{1}{2}\)-inch square tiles

Use any strategy to solve Exercises 6–8.

6. QUIZ SCORES Mandy answered 10 questions out of 12 correctly on her math quiz. How many questions must she answer correctly to get the same score on a quiz with 30 questions? 25 questions

7. NUMBER THEORY There are two single digit numbers. One number is 4 less than the other number. The sum of the digits is 12. Find the two numbers. 4, 8

8. GARDENING Justin helped his dad in the yard 3 times as long as Paula. Paula helped her dad 2 hours less than Carly. Carly helped her dad in the yard 4 hours. How many hours did Justin help his dad? 6 hours
Lesson 4 Reteach

Scale Drawings

A scale drawing represents something that is too large or too small to be drawn or built at actual size. Similarly, a scale model can be used to represent something that is too large or built too small for an actual-size model. The scale gives the relationship between the drawing/model measure and the actual measure.

Example
On this map, each grid unit represents 50 yards. Find the horizontal distance from Patrick’s Point to Agate Beach.

Scale
map → 1 unit
actual → 50 yards

Patrick’s Point
to Agate Beach
map → 8 units
actual → x yards

\[ 1 \times x = 50 \times 8 \]
\[ x = 400 \]

It is 400 yards from Patrick’s Point to Agate Beach.

Exercises
Find the actual distance between each pair of cities. Round to the nearest tenth if necessary.

<table>
<thead>
<tr>
<th>Cities</th>
<th>Map Distance</th>
<th>Scale</th>
<th>Actual Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Los Angeles and San Diego, CA</td>
<td>6.35 cm</td>
<td>1 cm = 20 mi</td>
<td>127 mi</td>
</tr>
<tr>
<td>2. Lexington and Louisville, KY</td>
<td>15.6 cm</td>
<td>1 cm = 5 mi</td>
<td>78 mi</td>
</tr>
<tr>
<td>3. Des Moines and Cedar Rapids, IA</td>
<td>16.27 cm</td>
<td>2 cm = 15 mi</td>
<td>122.0 mi</td>
</tr>
<tr>
<td>4. Miami and Jacksonville, FL</td>
<td>11.73 cm</td>
<td>( \frac{1}{2} ) cm = 20 mi</td>
<td>469.2 mi</td>
</tr>
</tbody>
</table>

Find the length of each object on the scale drawing with the given scale. Then find the scale factor.

5. an automobile 16 feet long; 1 inch:6 inches \( 32 \text{ in.}; \frac{1}{6} \)
6. a pond 85 feet across; 1 inch = 4 feet \( 21 \frac{1}{4} \text{ in.}; \frac{1}{48} \)
7. a parking lot 200 meters wide; 1 centimeter:25 meters \( 8 \text{ cm}; \frac{1}{2,500} \)
8. a flag 5 feet wide; 2 inches = 1 foot \( 10 \text{ in.}; \frac{1}{6} \)
Lesson 4 Skills Practice

Scale Drawings

ARCHITECTURE The scale on a set of architectural drawings for a house is \(\frac{1}{2}\) inch = 1\(\frac{1}{2}\) feet. Find the length of each part of the house.

<table>
<thead>
<tr>
<th>Room</th>
<th>Drawing Length</th>
<th>Actual Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Living Room</td>
<td>5 inches</td>
<td>15 ft</td>
</tr>
<tr>
<td>2. Dining Room</td>
<td>4 inches</td>
<td>12 ft</td>
</tr>
<tr>
<td>3. Kitchen</td>
<td>5(\frac{1}{2}) inches</td>
<td>16(\frac{1}{2}) ft</td>
</tr>
<tr>
<td>4. Laundry Room</td>
<td>3(\frac{1}{4}) inches</td>
<td>9(\frac{3}{4}) ft</td>
</tr>
<tr>
<td>5. Basement</td>
<td>10 inches</td>
<td>30 ft</td>
</tr>
<tr>
<td>6. Garage</td>
<td>8(\frac{1}{3}) inches</td>
<td>25 ft</td>
</tr>
</tbody>
</table>

ARCHITECTURE As part of a city building refurbishment project, architects have constructed a scale model of several city buildings to present to the city commission for approval. The scale of the model is 1 inch = 9 feet.

7. The courthouse is the tallest building in the city. If it is 7\(\frac{1}{2}\) inches tall in the model, how tall is the actual building? 67\(\frac{1}{2}\) ft

8. The city commission would like to install new flagpoles that are each 45 feet tall. How tall are the flagpoles in the model? 5 in.

9. In the model, two of the flagpoles are 4 inches apart. How far apart will they be when they are installed? 36 ft

10. The model includes a new park in the center of the city. If the dimensions of the park in the model are 9 inches by 17 inches, what are the actual dimensions of the park? 81 ft by 153 ft

11. Find the scale factor. \(\frac{1}{108}\)
Lesson 5 Reteach

Draw Three-Dimensional Figures

A solid is a three-dimensional figure.

Example 1

Draw a top, a side, and a front view of the solid at the right.

The top view is a triangle. The side and front views are rectangles.

Example 2

Draw the corner view using the top, side, and front views shown below.

Step 1 Use the top view to draw the base of the figure, a 1-by-3 rectangle.
Step 2 Add edges to make the base a solid figure.
Step 3 Use the side and front views to complete the figure.

Exercises

1. Draw a top, a side, and a front view of the solid.

2. Draw a corner view of the three-dimensional figure whose top, side, and front views are shown. Use isometric dot paper.

Sample answer:
Lesson 5 Skills Practice

Draw Three-Dimensional Figures

Draw a top, a side, and a front view of each solid.

1.

2.

3.

Draw a corner view of each three-dimensional figure whose top, side, and front views are shown. Use isometric dot paper.

4.

5.
Lesson 6 Reteach

Cross Sections

A polyhedron is a three-dimensional figure with flat surfaces that are polygons. A prism is a polyhedron with two parallel, congruent faces called bases. A pyramid is a polyhedron with one base that is a polygon and faces that are triangles. Prisms and pyramids are named by the shape of their bases.

Example
Identify the figure. Then name the bases, faces, edges, and vertices.

The figure is a pentagonal prism.
The bases are $ABCDE$ and $FGHIJ$.
The faces are $ABCDE$, $FGHIJ$, $ABGF$, $BCHG$, $CDIH$, $DEJI$, and $EAFJ$.
The edges are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, $\overline{AF}$, $\overline{BG}$, $\overline{CH}$, $\overline{DI}$, $\overline{EJ}$, $\overline{FG}$, $\overline{GH}$, $\overline{HI}$, $\overline{IJ}$, $\overline{JF}$.
The vertices are $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $I$, $J$.

Exercises
Identify each figure. Then name the bases, faces, edges, and vertices.

1. The figure is a triangular prism.
The bases are $RST$ and $UVW$.
The faces are $RST$, $UVW$, $RTWU$, $TSVW$, and $SRUV$.
The edges are $\overline{RT}$, $\overline{TS}$, $\overline{SR}$, $\overline{RU}$, $\overline{TW}$, $\overline{SV}$, $\overline{UW}$, $\overline{WV}$, $\overline{VU}$.
The vertices are $R$, $S$, $T$, $U$, $V$, $W$.

2. The figure is a rectangular pyramid.
The base is $GHIJ$.
The faces are $FGJ$, $FGH$, $FHI$, $FIJ$, $GHIJ$.
The edges are $\overline{FG}$, $\overline{FH}$, $\overline{FI}$, $\overline{FJ}$, $\overline{GH}$, $\overline{HI}$, $\overline{IJ}$, $\overline{JG}$.
The vertices are $F$, $G$, $H$, $I$, $J$. 

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Lesson 6 Skills Practice

Cross Sections

Identify each figure. Then name the bases, faces, edges, and vertices.

1. The figure is a triangular pyramid. The base is $FGH$.
The faces are $FGH$, $GHE$, $EFG$, and $EHF$.
The edges are $EG$, $EF$, $EH$, $GH$, $HF$, $GF$.
The vertices are $E$, $F$, $G$, and $H$.

2. The figure is a rectangular pyramid. The base is $MNOP$.
The faces are $MNOP$, $NOQ$, $OPQ$, $MPQ$, and $MNQ$.
The edges are $MN$, $NO$, $OP$, $PM$, $MQ$, $NQ$, $OQ$, $PQ$.
The vertices are $M$, $N$, $O$, $P$, and $Q$.

Describe the shape resulting from each cross section.

3. rectangle

4. rectangle

5. oval

6. parallelogram
Lesson 1 Reteach

Circumference

The diameter, \( d \), is the distance across a circle through its center.
The radius, \( r \), is the distance from the center to any point on a circle.
The circumference, \( C \), is the distance around a circle.

The diameter of a circle is twice its radius.
The radius is half the diameter.
The circumference of a circle is equal to \( \pi \) times its diameter or \( \pi \) times twice its radius.

\[
d = 2r
\]
\[
r = \frac{d}{2}
\]
\[
C = \pi d
\]
\[
C = 2\pi r
\]

Example 1

The radius of a circle is 7 meters. Find the diameter.

\[
d = 2r
\]
\[
d = 2 \cdot 7 \quad \text{Replace } r \text{ with 7.}
\]
\[
d = 14 \quad \text{Multiply.}
\]
The diameter is 14 meters.

Example 2

Find the circumference of a circle with a radius that is 13 inches. Use 3.14 for \( \pi \). Round to the nearest tenth.

\[
C = 2\pi r
\]
Write the formula.
\[
C \approx 2 \times 3.14 \times 13 \quad \text{Replace } r \text{ with 13 and } \pi \text{ with 3.14.}
\]
\[
C \approx 81.64 \quad \text{Multiply.}
\]
Rounded to the nearest tenth, the circumference is about 81.6 inches.

Exercises

Find the circumference of each circle. Use 3.14 or \( \frac{22}{7} \) for \( \pi \). Round to the nearest tenth if necessary.

1. \[
3.14 \times 5 = 15.7 \text{ m}
\]
2. \[
3.14 \times 16 = 50.2 \text{ in.}
\]
3. \[
\frac{22}{7} \times 21 = 66 \text{ ft}
\]
Lesson 1 Skills Practice

Circumference

Find the radius or diameter of each circle with the given dimensions.

1. \( r = 13 \text{ cm} \)  
   \[ \text{26 cm} \]

2. \( d = 4 \text{ ft} \)  
   \[ \text{2 ft} \]

3. \( r = 10 \text{ mm} \)  
   \[ \text{20 mm} \]

4. \( d = 16 \text{ in.} \)  
   \[ \text{8 in.} \]

5. \( r = 7 \text{ mi} \)  
   \[ \text{14 mi} \]

6. \( d = 22 \text{ yd} \)  
   \[ \text{11 yd} \]

Find the circumference of each circle. Use \( 3.14 \) or \( \frac{22}{7} \) for \( \pi \). Round to the nearest tenth if necessary.

7. \[ \text{9 cm} \]
   \[ 3.14 \times 9 = 28.3 \text{ cm} \]

8. \[ \text{3 in.} \]
   \[ 3.14 \times 3 = 9.4 \text{ in.} \]

9. \[ \text{11 m} \]
   \[ 3.14 \times 22 = 69.1 \text{ m} \]

10. \[ \text{21 mi} \]
    \[ \frac{22}{7} \times 42 = 132 \text{ mi} \]

11. \[ \text{70 yd} \]
    \[ \frac{22}{7} \times 70 = 220 \text{ yd} \]

12. \[ \text{18 mm} \]
    \[ 3.14 \times 36 = 113.0 \text{ mm} \]

13. \[ \text{5 ft} \]
    \[ 3.14 \times 10 = 31.4 \text{ ft} \]

14. \[ \text{12 cm} \]
    \[ 3.14 \times 12 = 37.7 \text{ cm} \]

15. \[ \text{14 m} \]
    \[ \frac{22}{7} \times 14 = 44 \text{ m} \]

16. \[ \text{17.5 km} \]
    \[ \frac{22}{7} \times 35 = 110 \text{ km} \]

17. \[ \text{9 yd} \]
    \[ 3.14 \times 18 = 56.5 \text{ yd} \]

18. \[ \text{25 ft} \]
    \[ 3.14 \times 25 = 78.5 \text{ ft} \]
Lesson 2 Reteach

Area of Circles

The area $A$ of a circle equals the product of pi ($\pi$) and the square of its radius $r$.

$$A = \pi r^2$$

Example 1
Find the area of the circle. Use 3.14 for $\pi$.

$$A = \pi r^2 \quad \text{Area of circle}$$

$$A \approx 3.14 \cdot 5^2 \quad \text{Replace } \pi \text{ with 3.14 and } r \text{ with 5.}$$

$$A \approx 3.14 \cdot 25 \quad 5^2 = 5 \cdot 5 = 25$$

$$A \approx 78.5$$

The area of the circle is approximately 78.5 square centimeters.

Example 2
Find the area of a semicircle that has a diameter of 9.4 millimeters. Use 3.14 for $\pi$. Round to the nearest tenth.

$$A = \frac{1}{2} \pi r^2 \quad \text{Area of semicircle}$$

$$A \approx \frac{1}{2} \cdot 3.14 \cdot 4.7^2 \quad \text{Replace } \pi \text{ with 3.14 and } r \text{ with } 9.4 \div 2 \text{ or } 4.7.$$ 

$$A \approx 34.7$$

Multiply.

The area of the semicircle is approximately 34.7 square millimeters.

Exercises
Find the area of each circle. Round to the nearest tenth. Use 3.14 or $\frac{22}{7}$ for $\pi$.

1. $\frac{22}{7} \times 7 \times 7 = 154 \text{ in}^2$

2. $3.14 \times 12.5 \times 12.5 = 490.6 \text{ mm}^2$

3. $3.14 \times 12 \times 12 = 452.2 \text{ ft}^2$

Find the area of each semicircle. Round to the nearest tenth. Use 3.14 or $\frac{22}{7}$ for $\pi$.

4. $\frac{1}{2} \times \frac{22}{7} \times 14 \times 14 = 308 \text{ m}^2$

5. $\frac{1}{2} \times 3.14 \times 3 \times 3 = 14.1 \text{ ft}^2$
Lesson 2 Skills Practice

Area of Circles

Find the area of each circle. Round to the nearest tenth.
Use 3.14 or $\frac{22}{7}$ for $\pi$.

1. \[3.14 \times 1 \times 1 = 3.1 \text{ cm}^2\]
2. \[3.14 \times 2 \times 2 = 12.6 \text{ yd}^2\]
3. \[\frac{22}{7} \times 35 \times 35 = 3,850 \text{ mm}^2\]
4. \[\frac{22}{7} \times 7 \times 7 = 154 \text{ in}^2\]
5. \[3.14 \times 2.15 \times 2.15 = 14.5 \text{ ft}^2\]
6. \[3.14 \times 4 \times 4 = 50.2 \text{ cm}^2\]
7. radius = 5.7 mm \[3.14 \times 5.7 \times 5.7 = 102.0 \text{ mm}^2\]
8. radius = 8.2 ft \[3.14 \times 8.2 \times 8.2 = 211.1 \text{ ft}^2\]
9. diameter = 3 in. \[3.14 \times 1.5 \times 1.5 = 7.1 \text{ in}^2\]
10. diameter = 15.6 cm \[3.14 \times 7.8 \times 7.8 = 191.0 \text{ cm}^2\]

Find the area of each semicircle. Round to the nearest tenth.
Use 3.14 for $\pi$.

11. \[\approx 34.7 \text{ yd}^2\]
12. \[\approx 794.8 \text{ in}^2\]
Lesson 3 Reteach
Area of Composite Figures

To find the area of a composite figure, decompose the figure into shapes whose areas you know how to find. Then find the sum of these areas.

Example
Find the area of the composite figure.

The figure can be separated into a semicircle and trapezoid.

Area of semicircle

\[ A = \frac{1}{2} \pi r^2 \]

\[ A = \frac{1}{2} \cdot \pi \cdot (7)^2 \]

\[ A \approx 77.0 \]

Area of trapezoid

\[ A = \frac{1}{2} h(b_1 + b_2) \]

\[ A = \frac{1}{2} \cdot 10 \cdot (14 + 18) \]

\[ A = 160 \]

The area of the figure is about 77.0 + 160 or 237 square inches.

Exercises

Find the area of each figure. Round to the nearest tenth if necessary.

1. \(54.1 \text{ mm}^2\)

2. \(108 \text{ ft}^2\)

3. \(79 \text{ mi}^2\)

4. \(192 \text{ m}^2\)

5. \(89.1 \text{ yd}^2\)

6. \(54.5 \text{ in}^2\)
Lesson 3 Skills Practice

Area of Composite Figures

Find the area of each figure. Round to the nearest tenth if necessary.

1. \(6 \text{ m} \times 10 \text{ m} + 7 \text{ m} \times 6 \text{ m} = 81 \text{ m}^2\)

2. \(\frac{1}{2} \times 12 \text{ yd} \times 12 \text{ yd} + \pi \times (\frac{12 \text{ yd}}{2})^2 = 200.5 \text{ yd}^2\)

3. \(\frac{1}{2} \times 14 \text{ cm} \times 10 \text{ cm} + \frac{1}{2} \times 10 \text{ cm} \times 7 \text{ cm} = 113 \text{ cm}^2\)

4. \(\frac{1}{2} \times 6 \text{ ft} \times 4 \text{ ft} + \frac{1}{2} \times 3 \text{ ft} \times 5 \text{ ft} = 21 \text{ ft}^2\)

5. \(\frac{1}{2} \times 5 \text{ cm} \times 6 \text{ cm} + \frac{1}{2} \times 5 \text{ cm} \times 6 \text{ cm} = 24.0 \text{ cm}^2\)

6. \(\frac{1}{2} \times 9 \text{ in} \times 4 \text{ in} + \frac{1}{2} \times 10 \text{ in} \times 18 \text{ in} = 261.5 \text{ in}^2\)

7. \(6 \text{ m} \times 8 \text{ m} + 7 \text{ m} \times 14 \text{ m} + 6 \text{ m} \times 14 \text{ m} + 6 \text{ m} \times 14 \text{ m} = 292 \text{ m}^2\)

8. \(\pi \times (\frac{13 \text{ m}}{2})^2 + \pi \times (\frac{13 \text{ m}}{2})^2 + \pi \times (\frac{12 \text{ m}}{2})^2 = 232.0 \text{ m}^2\)

9. \(\frac{1}{2} \times 4 \text{ km} \times 5 \text{ km} + \frac{1}{2} \times 4 \text{ km} \times 5 \text{ km} + \frac{1}{2} \times 12 \text{ km} \times 5 \text{ km} = 81.8 \text{ km}^2\)

Find the area of the shaded region.

10. \(\frac{1}{2} \times 20 \text{ cm} \times 16 \text{ cm} = 160 \text{ cm}^2\)

11. \(8 \text{ ft} \times 32 \text{ ft} + 8 \text{ ft} \times 8 \text{ ft} + 8 \text{ ft} \times 8 \text{ ft} = 512 \text{ ft}^2\)
Lesson 4 Reteach

Volume of Prisms

The volume of a three-dimensional shape is the measure of space occupied by it. It is measured in cubic units such as cubic centimeters (cm$^3$) or cubic inches (in$^3$). The volume of the shape at the right can be shown using cubes.

The bottom layer, or base, has $4 \cdot 3$ or 12 cubes.

There are two layers.

It takes $12 \cdot 2$ or 24 cubes to fill the box. So, the volume of the box is 24 cubic meters.

A rectangular prism is a three-dimensional shape that has two parallel and congruent sides, or bases, that are rectangles. To find the volume of a rectangular prism, multiply the area of the base times the height, or find the product of the length $\ell$, the width $w$, and the height $h$.

$$V = Bh \text{ or } V = \ell wh$$

Example

Find the volume of the rectangular prism.

$$V = \ell wh \quad \text{Volume of a rectangular prism}$$

$$V = 5 \cdot 6 \cdot 8 \quad \text{Replace } \ell \text{ with } 5, w \text{ with } 6, \text{ and } h \text{ with } 8.$$  

$$V = 240 \quad \text{Multiply.}$$

The volume is 240 cubic inches.

Exercises

Find the volume of each prism. Round to the nearest tenth if necessary.

1. $84 \text{ m}^3$

2. $16.2 \text{ ft}^3$
Lesson 4 Skills Practice

Volume of Prisms

Find the volume of each prism. Round to the nearest tenth if necessary.

1. \( \text{Volume} = 63 \text{ cm}^3 \)
2. \( \text{Volume} = 300 \text{ in}^3 \)
3. \( \text{Volume} = 96 \text{ m}^3 \)
4. \( \text{Volume} = 90 \text{ mm}^3 \)
5. \( \text{Volume} = 186.2 \text{ in}^3 \)
6. \( \text{Volume} = 97.2 \text{ m}^3 \)
7. \( \text{Volume} = 47\frac{1}{2} \text{ ft}^3 \)
8. \( \text{Volume} = 345.6 \text{ in}^3 \)
9. \( \text{Volume} = 4.1 \text{ cm}^3 \)
Reteach

Problem-Solving Investigation: Solve a Simpler Problem

When problem solving, sometimes it is easier to solve a simpler problem first to find the correct strategy for solving a more difficult problem.

Example

SPORTS West High School wants to paint the football field blue, but not the center. The diagram shows the dimensions of the field and center circle. How much area will they need to paint blue?

Understand

You know that the field is one large rectangle and the center symbol is a large circle.

Plan

You can find the area of the rectangle and the area of the circle and subtract.

Solve

Area of rectangle: \( A = \ell w \)
\[ A = 100 \times 75 \text{ or } 7,500 \]

Area of circle: \( A = \pi r^2 \)
\[ A = \pi \times 15^2 \text{ or } 706.9 \]

Subtract:
\[ 7,500 - 706.9 \text{ or } 6,793.1 \text{ ft}^2 \]

So, they would need to paint about 6,793.1 square feet of field.

Check

Use estimation to check. The area of the entire field is 7,500 feet and the circle is approximately 700 feet, so the area should be less than 6,800 feet. Since 6,793.1 is less than 6,800 feet, the answer is reasonable.

Exercises

1. FRAMES Joan wants to paint her favorite picture frame. How much paint would she need to use in order to cover just the frame? \(41 \text{ in}^2\)

2. WALLPAPER Calbert wants to wallpaper one wall of his bathroom. He has two semi-circular windows along the wall. How much wallpaper must he purchase? Round to the nearest tenth. \(83.4 \text{ ft}^2\)
Skills Practice

Problem-Solving Investigation: Solve a Simpler Problem

Solve a simpler problem to solve.

1. POOL  Find the area of the sidewalk around the pool shown below.  \(376 \text{ ft}^2\)

   ![Diagram of a pool with a sidewalk]

2. GEOMETRY  Find the area of the shape shown.  \(34 \text{ cm}^2\)

   ![Diagram of a shape]

3. POPULATION  The population of Ghostown, USA, is decreasing at a rate of 3 people per year. If there are currently 831 people living in the town, when will the town be deserted?  \(277 \text{ years}\)

4. STAINED GLASS  Find the area of the stained glass window shown below. Round to the nearest tenth.  \(6.6 \text{ in}^2\)

   ![Diagram of a stained glass window]

5. STOVETOPS  What is the area of the stovetop shown, not including the burners? Round to the nearest tenth.  \(4.2 \text{ ft}^2\)

   ![Diagram of a stovetop]

6. POOLS  Water is being added at a rate of 50 gallons per minute to a pool. How long will it take until the 10,000 gallon pool is full?  \(200 \text{ minutes or 3 hours and 20 minutes}\)
Lesson 5 Reteach

Volume of Pyramids

A pyramid is a three-dimensional shape with one base and triangular lateral faces. The volume $V$ of a pyramid is one third the area of the base $B$ times the height $h$.

$$V = \frac{1}{3} Bh$$

Example

Find the volume of the pyramid. Round to the nearest tenth.

$$V = \frac{1}{3} Bh$$  Volume of a pyramid

$$V = \frac{1}{3}(\ell w)h$$  The base is a rectangle, so $B = \ell w$.

$$V = \frac{1}{3} (4.3 \cdot 3.2) \cdot 11$$  $\ell = 4.3$, $w = 3.2$, $h = 11$

$$V \approx 50.5$$  Simplify.

The volume is about 50.5 cubic meters.

Exercises

Find the volume of each pyramid. Round to the nearest tenth if necessary.

1. 112 cm$^3$

2. 135 ft$^3$

3. 53.7 in$^3$

4. 696.2 m$^3$

5. 1040 in$^3$

6. 27.3 ft$^3$
Lesson 5 Skills Practice

Volume of Pyramids

Find the volume of each pyramid. Round to the nearest tenth if necessary.

1. \[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]
   \[ \text{Volume} = \frac{1}{3} \times 6.4 \times 4 \times 12 = 51.2 \text{ km}^3 \]

2. \[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]
   \[ \text{Volume} = \frac{1}{3} \times 9 \times 8 \times 6 = 144 \text{ yd}^3 \]

3. \[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]
   \[ \text{Volume} = \frac{1}{3} \times 10 \times 10 \times 62 = 206.7 \text{ ft}^3 \]

4. \[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]
   \[ \text{Volume} = \frac{1}{3} \times 8 \times 19 \times 8 = 405.3 \text{ in}^3 \]

5. \[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]
   \[ \text{Volume} = \frac{1}{3} \times 14 \times 4.9 \times 17 = 388.7 \text{ in}^3 \]

6. \[ \text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height} \]
   \[ \text{Volume} = \frac{1}{3} \times 9.5 \times 12 \times 13 = 494 \text{ m}^3 \]

Find the height of each pyramid.

7. square pyramid: volume 225 cubic inches, base edge 5 inches  \[ \text{height} = \frac{225}{\frac{1}{3} \times 5^2} = 27 \text{ in.} \]

8. triangular pyramid: volume 56 cubic centimeters, base edge 8 centimeters, base height 7 centimeters  \[ \text{height} = \frac{56}{\frac{1}{3} \times \frac{1}{2} \times 8 \times 7} = 6 \text{ cm} \]
Lesson 6 Reteach

Surface Area of Prisms

The sum of the areas of all the surfaces, or faces, of a three-dimensional shape is the **surface area**. The surface area S.A. of a rectangular prism with length \( \ell \), width \( w \), and height \( h \) is the sum of the areas of its faces.

\[
S.A. = 2\ell w + 2\ell h + 2wh
\]

**Example**

Find the surface area of the rectangular prism.

<table>
<thead>
<tr>
<th>Faces</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>top and bottom</td>
<td>( 2 (4 \cdot 3) = 24 )</td>
</tr>
<tr>
<td>front and back</td>
<td>( 2 (4 \cdot 2) = 16 )</td>
</tr>
<tr>
<td>two sides</td>
<td>( 2 (2 \cdot 3) = 12 )</td>
</tr>
<tr>
<td>sum of the areas</td>
<td>( 24 + 16 + 12 = 52 )</td>
</tr>
</tbody>
</table>

Alternatively, replace \( \ell \) with 4, \( w \) with 3, and \( h \) with 2 in the formula for surface area.

\[
S.A. = 2\ell w + 2\ell h + 2wh
\]

\[
= 2 (4 \cdot 3) + 2 (4 \cdot 2) + 2 (3 \cdot 2)
\]

\[
= 24 + 16 + 12
\]

\[
= 52
\]

So, the surface area of the rectangular prism is 52 square meters.

**Exercises**

Find the surface area of each prism.

1. 

   ![Rectangular Prism](image1)

   \( 102 \text{ cm}^2 \)

2. 

   ![Rectangular Prism](image2)

   \( 232 \text{ in}^2 \)

3. 

   ![Rectangular Prism](image3)

   \( 40.46 \text{ ft}^2 \)

4. 

   ![Rectangular Prism](image4)

   \( 480 \text{ mm}^2 \)
Lesson 6 Skills Practice

Surface Area of Prisms

Find the surface area of each prism. Round to the nearest tenth if necessary.

1. 396 cm²

2. 78 ft²

3. 606 in²

4. 522 mm²

5. 143 cm²

6. 127.8 in²

7. 400 in²

8. 73.0 cm²

9. 201.6 mm²

10. Find the surface area of a rectangular prism that has a length of 8 inches, a width of 3 inches, and a height of 6 inches. **180 in²**

11. Find the surface area of a triangular prism. The sides of the right triangular base measure 9 centimeters, 12 centimeters and 15 centimeters. The height of the prism is 20 centimeters. **828 cm²**
Lesson 7 Reteach

Surface Area of Pyramids

The total surface area S.A. of a regular pyramid is the lateral area L.A. plus the area of the base.

\[ S.A. = B + L.A. \quad \text{or} \quad S.A. = B + \frac{1}{2} P\ell \]

**Example 1**

Find the total surface area of the pyramid.

\[ S.A. = B + \frac{1}{2} P\ell \]

Surface area of a pyramid

\[ S.A. = 49 + \frac{1}{2} (28 \cdot 6) \quad P = 4(7) \text{ or } 28, \quad \ell = 6, \quad B = 7 \cdot 7 \text{ or } 49 \]

\[ S.A. = 133 \quad \text{Simplify.} \]

The surface area of the pyramid is 133 square inches.

**Example 2**

Find the total surface area of the pyramid.

\[ S.A. = B + \frac{1}{2} P\ell \]

Surface area of a pyramid

\[ S.A. = 15.6 + \frac{1}{2} (18 \cdot 5) \quad P = 3(6) \text{ or } 18, \quad \ell = 5, \quad B = 15.6 \]

\[ S.A. = 60.6 \quad \text{Simplify.} \]

The surface area of the pyramid is 60.6 square meters.

**Exercises**

Find the total surface area of each pyramid. Round to the nearest tenth.

1. \[ 16 \text{ ft}^2 \]

2. \[ 48.9 \text{ cm}^2 \]
Lesson 7 Skills Practice

Surface Area of Pyramids

Find the total surface area of each pyramid. Round to the nearest tenth if necessary.

1.  
   ![Pyramid Diagram]
   
   12 in.
   
   15 in.
   
   15 in.
   
   585 in\(^2\)

2.  
   ![Pyramid Diagram]
   
   14 ft
   
   20 ft
   
   20 ft
   
   960 ft\(^2\)

3.  
   ![Pyramid Diagram]
   
   9 cm
   
   6 cm
   
   6 cm
   
   6 cm
   
   Area of base 15.6 cm\(^2\)
   
   96.6 cm\(^2\)

4.  
   ![Pyramid Diagram]
   
   16 m
   
   10 m
   
   10 m
   
   Area of base 43.3 m\(^2\)
   
   283.3 m\(^2\)

5. The base of a square pyramid has a side length of 50 centimeters. The slant height is 32 centimeters. Find the surface area. 5,700 cm\(^2\)

6. An equilateral triangular pyramid has a slant height of 8.3 inches. The triangular base has a perimeter of 4.8 inches and an area of 1.1 square inches. Find the surface area of the pyramid. 21.02 in\(^2\)
Lesson 8 Reteach

Volume and Surface Area of Composite Figures

Example 1
Find the surface area of the composite figure.

To find the surface area, find the sum of the areas of exposed surfaces. The lateral area of the prism is $50 + 10 + 50 + 10 = 120 \text{ m}^2$. The area of the bottom of the prism is $10 \times 2 = 20 \text{ m}^2$. The surface area of the triangular prism is $2 + 2 + 28 + 20 = 52 \text{ m}^2$. So, the surface area is $120 + 20 + 52 = 192 \text{ m}^2$.

Example 2
Find the volume of the composite figure.

The figure is made up of two rectangular prisms.
$V = \ell wh + \ell wh$

$V = 2 \cdot 1 \cdot 1 + 2 \cdot 0.5 \cdot 0.5$

$V = 2 + 0.5$ or $2.5$

The volume of the composite figure is $2.5$ cubic meters.

Exercises

1. Find the volume of the composite figure. 2. Find the surface area of the composite figure.

100 in$^2$ 512 ft$^2$
Lesson 8 Skills Practice

Volume and Surface Area of Composite Figures

Find the volume and surface area of each composite figure.

1.  
\[ \text{Volume: } 560 \text{ m}^3; \text{ Surface Area: } 444 \text{ m}^2 \]

2.  
\[ \text{Volume: } 2,268 \text{ mm}^3; \text{ Surface Area: } 1,182 \text{ mm}^2 \]

3.  
\[ \text{Volume: } 17.25 \text{ cm}^3; \text{ Surface Area: } 45.9 \text{ cm}^2 \]

Find the volume of each composite figure.

4.  
\[ 2,400 \text{ ft}^3 \]

5.  
\[ 39 \text{ in}^3 \]

6.  
\[ 1,012 \text{ mm}^3 \]

7. **MULCH** Marcus is putting a border of mulch around a tree. The figure shows the top view of the mulch. The mulch will be 3 inches deep. Find the volume of mulch.  
\[ 528 \text{ in}^3 \]
Lesson 1 Reteach

Probability of Simple Events

When tossing a coin, there are two possible outcomes, heads and tails. Suppose you are looking for heads. If the coin lands on heads, this would be a favorable outcome. The chance that some event will happen (in this case, getting heads) is called probability. You can use a ratio to find probability.

The probability of an event is a number from 0 to 1, including 0 and 1. The closer a probability is to 1, the more likely it is to happen.

Example 1
There are four equally likely outcomes on the spinner. Find the probability of spinning green or blue.

\[ P(\text{green or blue}) = \frac{\text{number of favorable outcomes}}{\text{number of total outcomes}} = \frac{2}{4} = \frac{1}{2} \]

The probability of landing on green or blue is \( \frac{1}{2}, 0.50, \) or \( 50\% \).

Complementary events are two events in which either one or the other must happen, but both cannot happen at the same time. The sum of the probabilities of complementary events is 1.

Example 2
There is a 25\% chance that Sam will win a prize. What is the probability that Sam will not win a prize?

\[ P(\text{win}) + P(\text{not win}) = 1 \\
0.25 + P(\text{not win}) = 1 \\
-0.25 = -0.25 \\
P(\text{not win}) = 0.75 \]

So, the probability that Sam won’t win a prize is 0.75, 75\%, or \( \frac{3}{4} \).

Exercises

1. There is a 90\% chance that it will rain. What is the probability that it will not rain? \( \frac{1}{10}, 0.10, \) or 10\%

One pen is chosen without looking from a bag that has 3 blue pens, 6 red, and 3 green. Find the probability of each event. Write each answer as a fraction, a decimal, and a percent.

2. \( P(\text{green}) \) \( \frac{1}{4}, 0.25, \) or 25\%

3. \( P(\text{blue or red}) \) \( \frac{3}{4}, 0.75, \) or 75\%

4. \( P(\text{not red}) \) \( \frac{1}{2}, 0.5, \) or 50\%
Lesson 1 Skills Practice

Probability of Simple Events

A card is randomly chosen. Find each probability. Write each answer as a fraction, a decimal, and a percent.

1. \( P(B) \frac{1}{8}, 0.125, \) or 12.5%

2. \( P(Q \text{ or } R) \frac{1}{4}, 0.25, \) or 25%

3. \( P(\text{vowel}) \frac{3}{8}, 0.375, \) or 37.5%

4. \( P(\text{consonant or vowel}) \frac{8}{8}, 1, \) or 100%

5. \( P(\text{consonant or A}) \frac{3}{4}, 0.75, \) or 75%

6. \( P(T) \frac{0}{8}, 0.0, \) or 0%

The spinner shown is spun once. Write a sentence explaining how likely it is for each event to occur.

7. \( P(\text{dog}) \) Since the probability of spinning a dog or not spinning a dog is 50%, spinning a dog is equally likely to occur.

8. \( P(\text{hamster}) \) Since the probability of spinning a hamster is 16.6%, spinning a hamster is less likely to occur.

9. \( P(\text{dog or cat}) \) Since the probability of spinning either a dog or a cat is 83.3%, spinning a dog or cat is likely to occur.

10. \( P(\text{bird}) \) Since the probability of spinning a bird is 0%, spinning a bird is impossible to occur.

11. \( P(\text{mammal}) \) Since the probability of spinning a mammal is 100%, spinning a mammal is certain to occur.

WEATHER The weather reporter says that there is a 12% chance that it will be moderately windy tomorrow.

12. What is the probability that it will not be windy? \( \frac{22}{25}, 0.88, \) or 88%

13. Will tomorrow be a good day to fly a kite? Explain. No; a 12% chance means that it is unlikely to be windy.
Lesson 2 Reteach

Theoretical and Experimental Probability

Experimental probability is found using frequencies obtained in an experiment or game. Theoretical probability is the expected probability of an event occurring.

Example 1
The graph shows the results of an experiment in which a number cube was rolled 100 times. Find the experimental probability of rolling a 3 for this experiment. Then compare it to the theoretical probability.

\[ P(3) = \frac{\text{number of times 3 occurs}}{\text{number of possible outcomes}} = \frac{16}{100} \text{ or } \frac{4}{25} \]

The experimental probability of rolling a 3 is \( \frac{4}{25} \), which is close to its theoretical probability of \( \frac{1}{6} \).

Example 2
In a telephone poll, 225 people were asked for whom they planned to vote in the race for mayor. What is the experimental probability of Juarez getting a vote from a person selected at random?

Of the 225 people polled, 75 planned to vote for Juarez. So, the experimental probability is \( \frac{75}{225} \text{ or } \frac{1}{3} \).

Example 3
Suppose 5,700 people vote in the election. How many can be expected to vote for Juarez?

\( \frac{1}{3} \cdot 5,700 = 1,900 \)

About 1,900 will vote for Juarez.

Exercises

1. Pets
   Use the graph of a survey of 150 students asked whether they prefer cats or dogs.
   a. What is the experimental probability of a student preferring dogs? \( \frac{22}{25} \)
   b. Suppose 100 students were surveyed. How many can be expected to prefer dogs? 88
   c. Suppose 300 students were surveyed. How many can be expected to prefer cats? 36
Lesson 2 Skills Practice

Theoretical and Experimental Probability

1. A number cube is rolled 50 times and the results are shown in the graph below.

   ![Number Cube Experiment Graph]

   a. Find the experimental probability of rolling a 2. \( \frac{4}{25} \)
   
   b. What is the theoretical probability of rolling a 2? \( \frac{1}{6} \)
   
   c. Find the experimental probability of not rolling a 2. \( \frac{21}{25} \)
   
   d. What is the theoretical probability of not rolling a 2? \( \frac{5}{6} \)
   
   e. Find the experimental probability of rolling a 1. \( \frac{1}{5} \)

2. SEASONS Use the results of the survey at the right.

   a. What is the experimental probability that a person’s favorite season is fall? Write the probability as a fraction. \( \frac{1}{4} \)
   
   b. Out of 300 people, how many would you expect to say that fall is their favorite season? 75
   
   c. Out of 20 people, how many would you expect to say that they like all the seasons? 2
   
   d. Out of 650 people, how many more would you expect to say that they like summer more than they like winter? 169
Lesson 3 Reteach

Probability of Compound Events

A tree diagram or table is used to show all of the possible outcomes, or sample space, in a probability experiment.

Example 1
WATCHES A certain type of watch comes in brown or black and in a small or large size. Find the number of color-size combinations that are possible.

Make a tree diagram to show the sample space. Then give the total number of outcomes.

<table>
<thead>
<tr>
<th>Color</th>
<th>Size</th>
<th>Sample Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown</td>
<td>Small</td>
<td>Brown, Small</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>Brown, Large</td>
</tr>
<tr>
<td>Black</td>
<td>Small</td>
<td>Black, Small</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>Black, Large</td>
</tr>
</tbody>
</table>

There are four different color and size combinations.

Example 2
CHILDREN The chance of having either a boy or a girl is 50%. What is the probability of the Smiths having two girls?

Make a tree diagram to show the sample space. Then find the probability of having two girls.

<table>
<thead>
<tr>
<th>Child 1</th>
<th>Child 2</th>
<th>Sample Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>boy</td>
<td>boy</td>
<td>boy, boy</td>
</tr>
<tr>
<td>girl</td>
<td>boy</td>
<td>boy, girl</td>
</tr>
<tr>
<td>girl</td>
<td>boy</td>
<td>girl, boy</td>
</tr>
<tr>
<td>girl</td>
<td>girl</td>
<td>girl, girl</td>
</tr>
</tbody>
</table>

The sample space contains 4 possible outcomes. Only 1 outcome has both children being girls. So, the probability of the Smiths having two girls is $\frac{1}{4}$.

Exercises
For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

1. choosing an outfit from a green shirt, blue shirt, or a red shirt, and black pants or blue pants  See students' work. There are 6 outcomes.

2. choosing a vowel from the word COUNTING and a consonant from the word PRIME  See students' work. There are 9 outcomes.
Lesson 3 Skills Practice

Probability of Compound Events

The spinner at the right is spun twice.

1. Draw a tree diagram to represent the situation.

2. What is the probability of getting at least one A? \( \frac{5}{9} \)

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

3. choosing a hamburger or hot dog and potato salad or macaroni salad

<table>
<thead>
<tr>
<th>Entree</th>
<th>Salad</th>
<th>Sample Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>hamburger</td>
<td>potato</td>
<td>hamburger, potato salad</td>
</tr>
<tr>
<td>hot dog</td>
<td>macaroni</td>
<td>hamburger, macaroni salad</td>
</tr>
<tr>
<td></td>
<td>potato</td>
<td>hot dog, potato salad</td>
</tr>
<tr>
<td></td>
<td>macaroni</td>
<td>hot dog, macaroni salad</td>
</tr>
</tbody>
</table>

There are 4 possible outcomes.

4. choosing a vowel from the word COMPUTER and a consonant from the word BOOK

<table>
<thead>
<tr>
<th>Vowel from COMPUTER</th>
<th>Consonant from BOOK</th>
<th>Sample Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>B</td>
<td>OB</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>OK</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>UB</td>
</tr>
<tr>
<td>U</td>
<td>K</td>
<td>UK</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>EB</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>EK</td>
</tr>
</tbody>
</table>

There are 6 possible outcomes.

5. choosing between the numbers 1, 2 or 3, and the colors blue, red, or green

<table>
<thead>
<tr>
<th>Number</th>
<th>Color</th>
<th>Sample Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>blue</td>
<td>1, blue</td>
</tr>
<tr>
<td></td>
<td>red</td>
<td>1, red</td>
</tr>
<tr>
<td></td>
<td>green</td>
<td>1, green</td>
</tr>
<tr>
<td>2</td>
<td>blue</td>
<td>2, blue</td>
</tr>
<tr>
<td></td>
<td>red</td>
<td>2, red</td>
</tr>
<tr>
<td></td>
<td>green</td>
<td>2, green</td>
</tr>
<tr>
<td>3</td>
<td>blue</td>
<td>3, blue</td>
</tr>
<tr>
<td></td>
<td>red</td>
<td>3, red</td>
</tr>
<tr>
<td></td>
<td>green</td>
<td>3, green</td>
</tr>
</tbody>
</table>

There are 9 possible outcomes.
Lesson 4 Reteach

Simulations

A simulation is an experiment that is designed to act out a given situation. Simulations often use models to act out events that would be difficult to perform.

Example 1
The weather forecast states that there is a 75% chance of snow tomorrow. Describe a method you could use to simulate this situation.

\[ 75\% = \frac{75}{100} = \frac{3}{4} \]

Write 75% as a fraction in simplest form.

Since there is a 75% chance of snow, there is a 25% or \( \frac{1}{4} \) chance that it will not snow. Place 3 red marbles to represent snow, and 1 blue marble to represent no snow in a bag and randomly pick one marble. Repeat this selection several times to find the experimental probability that it will snow tomorrow.

Example 2
A fresh-squeezed juice store offers 5 different types of fruit juice in a small or large size. If each type of juice and size is equally likely to be chosen, describe a model that could be used to simulate the orders of the next 6 customers.

Since there are 5 types of juices and 2 sizes, choose a method that has 10 possible outcomes, such as tossing a coin and spinning a spinner that has 5 equal sections. Let each specific outcome represent a different choice.

Toss the coin and spin the spinner 6 times to simulate the choices made by 6 customers.

Exercises
For Exercises 1–4, describe a model that can be used to simulate the given situation.

1. GAMES A game requires drawing balls numbered 1 through 6 for each of 5 digits to determine the winning number. Describe a model that could be used to simulate the selection of the number. Sample answer: Rolling a number cube 5 times and recording the results. Repeating this several times to find the experiment probability.

2. TESTING The questions on a multiple-choice test each have 3 answer choices. Describe a model that you could use to simulate the outcome of guessing the answers to a 25-question test.
   Sample answer: Spinning a spinner with 3 equal sections 25 times and recording the results.

3. WEATHER The weather forecast states that there is a 35% chance of rain tomorrow. Describe a method you could use to model this situation. Sample answer:
   \[ 35\% = \frac{35}{100} = \frac{7}{20} \]
   Place 7 red marbles to represent rain and 13 blue marbles to represent no rain in a bag and randomly pick one marble. Repeat this selection several times to find the experimental probability of rain tomorrow.

4. TEAMS Mr. Jenkins needs to choose 3 captains randomly for teams for a game. If there are 15 students in his classroom, describe a model that he could use to simulate choosing these 3 captains. Sample answer: Use a spinner with 5 equal sections and a spinner with 3 equal sections to represent the 15 students. Spin each 3 times to find the 3 captains.
Lesson 4 Skills Practice

Simulations

1. **QUIZZES** Describe a situation that you could use to answer a 15-question quiz, if five questions are true or false questions. **Sample answer:** Toss a coin for the true or false questions 5 times and assign true for heads and false for tails.

2. **PRIZES** During the grand opening of a fast food restaurant, every person that comes to the restaurant receives a prize. There are 6 different prizes. Describe a model that could be used to simulate which prizes the first 75 customers will receive. **Sample answer:** A number cube numbered 1 through 6 can be used to represent each prize. Roll the cube 75 times and record the results to represent the 75 customers that receive a prize.

3. **STUDENT COUNCIL** Mrs. Corley wants to randomly choose 3 students to represent her homeroom on student council. There are 30 students in the class. Describe a model that could be used to simulate this situation. **Sample answer:** Spinning a spinner split into 5 equal sections and rolling a number cube would result in 30 outcomes. Assign each outcome to represent a student in the class. Repeat three times to choose three students.

4. **SALES** A music store has determined that 65% of customers who buy a compact disc buy a pop music compact disc. Describe a model that you could use to simulate a CD purchase. **Sample answer:** $65\% = \frac{65}{100} = \frac{13}{20}$. Place 13 red marbles to represent pop music compact discs and 7 blue marbles to represent other music compact discs in a bag and randomly pick one marble. Repeat this selection several times to find the experimental probability that a customer will purchase a pop music compact disc.

5. **SANDWICHES** A sandwich shop offers 6 different types of sandwiches on either white or wheat bread. If each type of sandwich and bread is equally likely to be chosen by a customer, describe a model that could be used to simulate the orders of the next 10 customers. **Sample answer:** Since there are 6 types of sandwiches and two types of bread, choose a method that has 12 possible outcomes, such as rolling a number cube and tossing a coin. Let each specific outcome represent a different choice.
Reteach

Problem-Solving Investigation: Act It Out

By acting out a problem, you are able to see all possible solutions to the problem being posed.

Example

CLOTHING Will has two shirts and three pairs of pants to choose from for his outfit to wear on the first day of school. How many different outfits can he make by wearing one shirt and one pair of pants?

Understand We know that he has two shirts and three pairs of pants from which to choose. We can use a coin for the shirts and an equally divided spinner labeled 1, 2, and 3 for the pants.

Plan Let’s make a list showing all possible outcomes of tossing a coin and then spinning a spinner.

<table>
<thead>
<tr>
<th>Flip a Coin</th>
<th>Spin a Spinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
</tr>
<tr>
<td>T</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>3</td>
</tr>
</tbody>
</table>

There are six possible outcomes of tossing a coin and spinning a spinner. So, there are 6 different possible outfits that Will can wear for the first day of school.

Check Tossing a coin has two outcomes and there are two shirts. Spinning a three-section spinner has three outcomes and there are three pairs of pants. Therefore, the solution of 6 different outcomes with a coin and spinner represent the 6 possible outfit outcomes for Will.

Exercises

1. SCIENCE FAIR There are 4 students with projects to present at the school science fair. How many different ways can these 4 projects be displayed on four tables in a row? 16 ways

2. GENDER Determine whether tossing a coin is a good way to predict the gender of the next 5 babies born at General Hospital. Justify your answer. No; there is only one scenario in which the prediction is correct.

3. OLYMPICS Four runners are entered in the first hurdles heat of twelve heats at the Olympics. The first two move on to the next round. Assuming no ties, how many different ways can the four runners come in first and second place? 12 ways
Skills Practice

Problem-Solving Investigation: Act It Out

Use the act it out strategy to solve.

1. SCHOOL  Determine whether rolling a 6-sided number cube is a good way to answer a 20-question multiple-choice test if there are six choices for each question. Justify your answer.  **No, you only have a 1 in 6 chance of getting the correct answer.**

2. GYMNASTICS  Five gymnasts are entered in a competition. Assuming that there are no ties, how many ways can first, second, and third places be awarded?  **60**

3. LUNCH  How many ways can 3 friends sit together in three seats at lunch?  **6**

4. SCHEDULE  How many different schedules can Charla create if she has to take English, math, science, social studies, and art next semester? Assume that there is only one lunch period available and there are five periods for classes.  **120**

5. BAND CONCERTS  The band is having a holiday concert. In the first row, the first trumpet is always furthest to the right and the first trombone is always the furthest to the left. How many ways are there to arrange the other 4 people who need to sit in the front?  **24**

6. TEAMS  Mr. D is picking teams for volleyball by having the students count off by 2s. The 1s will be on one team and the 2s on the other. Would flipping a coin work just as well to pick the teams? Justify your answer.  **Yes; because the probability of getting either heads or tails is \( \frac{1}{2} \), the same as getting a one or a two.**
Lesson 5 Reteach

**Fundamental Counting Principle**

If event $M$ can occur in $m$ ways and is followed by event $N$ that can occur in $n$ ways, then the event $M$ followed by $N$ can occur in $m \times n$ ways. This is called the **Fundamental Counting Principle**.

**Example**

**CLOTHING** Andy has 5 shirts, 3 pairs of pants, and 6 pairs of socks. How many different outfits can Andy choose with a shirt, pair of pants, and pair of socks?

<table>
<thead>
<tr>
<th>number of shirts</th>
<th>number of pants</th>
<th>number of pairs of socks</th>
<th>total number of outfits</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>6</td>
<td>90</td>
</tr>
</tbody>
</table>

Andy can choose 90 different outfits.

**Exercises**

Use the **Fundamental Counting Principle** to find the total number of outcomes in each situation.

1. rolling two number cubes 36

2. tossing 3 coins 8

3. picking one consonant and one vowel 105

4. choosing one of 3 processor speeds, 2 sizes of memory, and 4 sizes of hard drive 24

5. choosing a 4-, 6-, or 8-cylinder engine and 2- or 4-wheel drive 6

6. rolling 2 number cubes and tossing 2 coins 144

7. choosing a color from 4 colors and a whole number from 4 to 10 28
Lesson 5 Skills Practice

Fundamental Counting Principle

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. rolling two number cubes and tossing one coin  \(72\)

2. choosing rye or Bermuda grass and 3 different mixtures of fertilizer  \(6\)

3. making a sandwich with ham, turkey, or roast beef; Swiss or provolone cheese; and mustard or mayonnaise  \(12\)

4. tossing 4 coins  \(16\)

5. choosing from 3 sizes of bottled water and from distilled, filtered, or spring water  \(9\)

6. choosing from 3 flavors and 3 sizes of juice  \(9\)

7. choosing from 35 flavors of ice cream; one, two, or three scoops; and sugar or waffle cone  \(210\)

8. picking a day of the week and a date in the month of April  \(210\)

9. rolling 3 number cubes and tossing 2 coins  \(864\)

10. choosing a 4-letter password using only 5 letters that may each be used more than once  \(625\)

11. choosing a bicycle with or without shock absorbers; with or without lights; and 5 color choices  \(20\)

12. a license plate that has 3 numbers from 0 to 9 and 2 letters where each number and a letter may be used more than once  \(676,000\)
Lesson 6 Reteach

Permutations

A permutation is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

Example 1

BOOKS How many ways can 4 different books be arranged on a bookshelf?

This is a permutation. Suppose the books are placed on the shelf from left to right.

There are 4 choices for the first book.

There are 3 choices that remain for the second book.

There are 2 choices that remain for the third book.

There is 1 choice that remains for the fourth book.

\[ 4 \cdot 3 \cdot 2 \cdot 1 = 24 \]

Simplify.

So, there are 24 ways to arrange 4 different books on a bookshelf.

Example 1

Find \( P(5,4) \).

\[ 4 \cdot 3 \cdot 2 \cdot 1 = 24 \]

Simplify.

Exercises

Find each value. Use a calculator if needed.

1. \( P(3,2) \) 6
2. \( P(7,6) \) 5,040

3. \( P(6,3) \) 120
4. \( P(9,3) \) 504

5. How many ways can you arrange the letters in the word group? 120

6. How many different 4-digit numbers can be created if no digit can be repeated? Remember, a number cannot begin with 0. 4,536
Lesson 6 Skills Practice

Permutations

Find each value. Use a calculator if needed.

1. \( P(2,2) \) \( \quad \) 2
2. \( P(4,3) \) \( \quad \) 24
3. \( P(5,4) \) \( \quad \) 120
4. \( P(9,5) \) \( \quad \) 15,120
5. \( P(8,7) \) \( \quad \) 40,320
6. \( P(12,13) \) \( \quad \) 1,320
7. \( P(11,3) \) \( \quad \) 990
8. \( P(10,4) \) \( \quad \) 5,040
9. \( P(6,5) \) \( \quad \) 720
10. \( P(5,3) \) \( \quad \) 60
11. \( P(7,4) \) \( \quad \) 840
12. \( P(6,4) \) \( \quad \) 360

13. How many ways can you arrange the letters in the word prime? \( \quad \) 120

14. How many ways can you arrange 8 different crates on a shelf if they are placed from left to right? \( \quad \) 40,320
Lesson 7 Reteach
Independent and Dependent Events

The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

Example 1
Two number cubes, one red and one blue, are rolled. What is the probability that the outcome of the red number cube is even and the outcome of the blue number cube is a 5?

\[ P(\text{red number cube is even}) = \frac{1}{2} \]
\[ P(\text{blue number cube is a 5}) = \frac{1}{6} \]
\[ P(\text{red number cube is even and blue number cube is a 5}) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \]

The probability that the two events will occur is \( \frac{1}{12} \).

Example 2
There are 6 black socks and 4 white socks in a drawer. If one sock is taken out without looking and then a second is taken out, what is the probability that they both will be black?

\[ P(\text{first sock is black}) = \frac{6}{10} = \frac{3}{5} \]
6 is the number of black socks; 10 is the total number of socks.

\[ P(\text{second sock is black}) = \frac{5}{9} \]
5 is the number of black socks after one black sock is removed; 9 is the total number of socks after one black sock is removed.

\[ P(\text{two black socks}) = \frac{3}{5} \cdot \frac{5}{9} = \frac{1}{3} \]

The probability of choosing two black socks is \( \frac{1}{3} \).

Exercises
A card is drawn from a deck of 10 cards numbered 1 through 10 and a number cube is rolled. Find each probability.

1. \( P(10 \text{ and 3}) = \frac{1}{60} \)
2. \( P(\text{two even numbers}) = \frac{1}{4} \)
3. \( P(\text{two prime numbers}) = \frac{1}{5} \)
4. \( P(9 \text{ and an odd number}) = \frac{1}{20} \)
5. \( P(\text{two numbers less than 4}) = \frac{3}{20} \)
6. \( P(\text{two numbers greater than 5}) = \frac{1}{12} \)

There are 4 red, 6 green, and 5 yellow pencils in a jar. Once a pencil is selected, it is not replaced. Find each probability.

7. \( P(\text{red and then yellow}) = \frac{2}{21} \)
8. \( P(\text{two green}) = \frac{1}{7} \)
9. \( P(\text{green and then yellow}) = \frac{1}{7} \)
10. \( P(\text{red and then green}) = \frac{4}{35} \)
Lesson 7 Skills Practice

Independent and Dependent Events

For Exercises 1–6, a number cube is rolled and the spinner at the right is spun. Find each probability.

1. \( P(1 \text{ and } A) \) \( \frac{1}{24} \)
2. \( P(\text{odd and } B) \) \( \frac{1}{8} \)
3. \( P(\text{prime and } D) \) \( \frac{1}{8} \)
4. \( P(\text{greater than } 4 \text{ and } C) \) \( \frac{1}{12} \)
5. \( P(\text{less than } 3 \text{ and consonant}) \) \( \frac{1}{4} \)
6. \( P(\text{prime and consonant}) \) \( \frac{3}{8} \)

7. What is the probability of spinning the spinner above 3 times and getting a vowel each time? \( \frac{1}{64} \)

8. What is the probability of rolling a number cube 3 times and getting a number less than 3 each time? \( \frac{1}{27} \)

Each spinner at the right is spun. Find each probability.

9. \( P(A \text{ and } 2) \) \( \frac{1}{15} \)
10. \( P(\text{vowel and even}) \) \( \frac{1}{5} \)
11. \( P(\text{consonant and } 1) \) \( \frac{1}{10} \)
12. \( P(D \text{ and greater than } 1) \) \( \frac{1}{6} \)

There are 3 red, 1 blue, and 2 yellow marbles in a bag. Once a marble is selected, it is not replaced. Find each probability.

13. \( P(\text{red and then yellow}) \) \( \frac{1}{5} \)
14. \( P(\text{blue and then yellow}) \) \( \frac{1}{15} \)
15. \( P(\text{red and then blue}) \) \( \frac{1}{10} \)
16. \( P(\text{two yellow marbles}) \) \( \frac{1}{15} \)
17. \( P(\text{two red marbles in a row}) \) \( \frac{1}{5} \)
18. \( P(\text{three red marbles}) \) \( \frac{1}{20} \)

GAMES There are 13 yellow cards, 6 blue, 10 red, and 8 green cards in a stack of cards turned face down. Once a card is selected, it is not replaced. Find each probability.

19. \( P(\text{2 blue cards}) \) \( \frac{5}{222} \)
20. \( P(\text{2 red cards}) \) \( \frac{5}{74} \)
21. \( P(\text{a yellow card and then a green card}) \) \( \frac{26}{333} \)
22. \( P(\text{a blue card and then a red card}) \) \( \frac{5}{111} \)
23. \( P(\text{two cards that are not red}) \) \( \frac{39}{74} \)
24. \( P(\text{two cards that are neither red or green}) \) \( \frac{19}{74} \)
Lesson 1 Reteach

Make Predictions

A survey is a method of collecting information. The group being studied is the population. To save time and money, part of the group, called a sample, is surveyed.

A good sample is:

• selected at random, or without preference,
• representative of the population, and
• large enough to provide accurate data.

Examples

Every sixth student who walked into the school was asked how he or she got to school.

1. What is the probability that a student at the school rode a bike to school?

\[ P(\text{ride bike}) = \frac{\text{number of students that rode a bike}}{\text{number of students surveyed}} \]

\[ = \frac{10}{40} = \frac{1}{4} \]

So, \( P(\text{ride bike}) = \frac{1}{4} \), 0.25, or 25%.

2. There are 360 students at the school. Predict how many bike to school.

Write an equivalent ratio. Let \( s \) = number of students who will ride a bike.

\[ \frac{10}{40} = \frac{s}{360} \]

You can solve the equivalent ratio to find that of the 360 students, 90 will ride a bike to school.

Exercises

SCHOOL Use the following information and the table shown. Every tenth student entering the school was asked which one of the four subjects was his or her favorite.

1. Find the probability that any student attending school prefers science.

\[ \frac{3}{8}, 0.375, \text{ or } 37.5\% \]

2. There are 400 students at the school. Predict how many students would prefer science. 150 students
Lesson 1 Skills Practice

Make Predictions

For Exercise 1–4, use the table and the following information. A survey of students’ favorite sports was taken from a random sample of students in a school. The results are shown in the table.

<table>
<thead>
<tr>
<th>Students’ Favorite Sports</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer</td>
<td>8</td>
</tr>
<tr>
<td>Baseball/Softball</td>
<td>3</td>
</tr>
<tr>
<td>Volleyball</td>
<td>5</td>
</tr>
<tr>
<td>Track &amp; Field</td>
<td>4</td>
</tr>
</tbody>
</table>

1. What is the size of the sample? 20

2. What is the probability that a student will prefer soccer? \( \frac{2}{5} \), 0.4, or 40%

3. What is the probability that a student will prefer volleyball? \( \frac{1}{4} \), 0.25, or 25%

4. There are 550 students in the school. Predict how many students at the school prefer track and field. **110 students**

Use the percent equation to help you solve.

5. **GARDENING** A survey showed that 74% of a nursery’s mail-order customers spent more than $100 on plants each spring. Predict how many of 125,000 mail-order customers will spend less than $100 on plants next spring. **32,500 customers**

6. **SAVING MONEY** A survey of high school students with jobs asked whether the students saved some of the money they earned. 82% of the students said they saved some money. Out of 340 students, predict how many would save some of their earnings. **about 279 students**

7. **TRAVEL COMPANY CUSTOMERS** A survey showed that 55% of a travel company’s customers were planning an overseas vacation the following year. Predict how many of the travel company’s 12,400 travelers will vacation overseas the following year. **6,820 customers**
Lesson 2 Reteach

Unbiased and Biased Samples

Data gathered from a representative sample can be used to make predictions about a population. An unbiased sample is selected so that it is representative of the entire population. In a biased sample, one or more parts of the population are favored over others.

Examples

Determine whether each sample is valid. Justify your answer.

1. To determine the favorite dog breed of people who enter dog shows, every fifth person entering a dog show is surveyed.
   Since the people are selected according to a specific pattern, the sample is a systematic random sample. It is a valid unbiased sample.

2. To determine what type of pet people prefer, the spectators at a dog show are surveyed.
   The spectators at a dog show probably prefer dogs. This is a biased sample that is not valid. The sample is a convenience sample since the people surveyed are in one location.

Examples

COOKIES  Students in the eighth grade surveyed 50 students at random about their favorite cookies. The results are in the table at the right.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oatmeal</td>
<td>15</td>
</tr>
<tr>
<td>Peanut butter</td>
<td>11</td>
</tr>
<tr>
<td>Chocolate chip</td>
<td>16</td>
</tr>
<tr>
<td>Sugar</td>
<td>8</td>
</tr>
</tbody>
</table>

3. What percent of students prefer chocolate chip cookies?
   16 out of 50 students prefer chocolate chip cookies.
   \[16 \div 50 = 0.32\]
   32% of the students prefer chocolate chip cookies.

4. If the students order 500 boxes of cookie dough, how many boxes should be chocolate chip?
   Find 32% of 500.
   \[0.32 \times 500 = 160\]
   About 160 boxes of cookie dough should be chocolate chip.

Exercises

Determine whether each sample is valid. Justify your answer.

1. To determine if the tomatoes in 5 boxes stacked on a pallet are not spoiled, the restaurant manager checks 3 tomatoes from the top box.
   This is a biased sample that is not valid, since only tomatoes in the top box are represented. This is a convenience sample.

A random survey of the students in eighth grade shows that 7 prefer hamburgers, 5 prefer chicken, and 3 prefer hot dogs.

2. Is the sample valid? What percent prefer hot dogs?  yes; 20%

3. If 120 students will attend the eighth grade picnic, how many hot dogs should be ordered for each student to get one?  about 24 hot dogs
Lesson 2 Skills Practice

Unbiased and Biased Samples

Determine whether each conclusion is valid. Justify your answer.

1. To evaluate the defect rate of its memory chips, an integrated circuit manufacturer tests every 100th chip off the production line. Out of 10 chips tested, one chip is found to be defective. The manufacturer concludes that 3 chips out of 3,000 will be defective.

   This is an unbiased, systematic random sample. The conclusion is valid.

2. Students who wish to represent the school at a school board meeting are asked to stop by the office after lunch. After lunch, 5 students wish to represent the school.

   This is a biased sample since only students with strong opinions are likely to volunteer. This is a voluntary response sample. The conclusion is not valid.

3. To determine if the class understood the homework assignment, the math teacher checks the top 3 papers in the pile of collected homework. The teacher finds that all students understood the homework assignment.

   This is a biased sample, since only papers turned in and on top of the pile are represented. This is a convenience sample. The conclusion is not valid.

4. A member of the cafeteria staff asks every fifth student leaving the cafeteria to rank 5 vegetables from most favorite to least favorite. She finds that corn is one of the favorite vegetables.

   This is an unbiased, systematic random sample. The conclusion is valid.

5. One bead for every member of the school orchestra is placed in a bag. All but 2 of the beads are white. Each member draws a bead from the bag, and the members who pick the non-white beads will represent the orchestra. It is predicted that two different instrument players will choose the white beads.

   This is an unbiased, simple random sample. The conclusion is valid.

6. A real estate agent surveys people about their housing preferences at an open house for a luxury townhouse. He finds that most people prefer townhomes.

   This is a biased sample since the people surveyed probably prefer townhouses. This is a convenience sample. The conclusion is not valid.

7. To determine the most popular children’s programs, a television station asks parents to call in and complete a phone survey. The television station finds that the children’s programs that are animated are the most popular.

   This is a biased sample, since only parents with strong opinions are likely to call. This is a voluntary response sample. The conclusion is not valid.
Lesson 3 Reteach

Misleading Graphs and Statistics

Graphs can be misleading for many reasons: there is no title, the scale does not include 0; there are no labels on either axis; the intervals on a scale are not equal; or the size of the graphics misrepresents the data.

Example

WEEKLY CHORES The line graphs below show the total hours Salomon spent doing his chores one month. Which graph would be best to use to convince his parents he deserves a raise in his allowance? Explain.

He should use Graph A because it makes the total hours seem much larger.

Exercises

PROFITS For Exercises 1 and 2, use the graphs below. It shows a company’s profits over a four-month period.

1. Which graph would be best to use to convince potential investors to invest in this company? Graph A

2. Why might the graph be misleading? There is no vertical scale.
Lesson 3 Skills Practice

Misleading Graphs and Statistics

1. LUNCH Which graph could be used to indicate a greater increase in yearly lunch prices? Explain.

Sample answer: Graph B; The ratio of the areas of the bars in Graph A are proportional to the cost of the lunches. The ratio of the area of the sandwiches in Graph B are not proportional to the cost of the lunches.

GEOGRAPHY For Exercises 2–4, use the table that shows the miles of shoreline for five states.

<table>
<thead>
<tr>
<th>State</th>
<th>Length of Shoreline (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virginia</td>
<td>3,315</td>
</tr>
<tr>
<td>Maryland</td>
<td>3,190</td>
</tr>
<tr>
<td>Washington</td>
<td>3,026</td>
</tr>
<tr>
<td>North Carolina</td>
<td>3,375</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>89</td>
</tr>
</tbody>
</table>

2. Find the mean, median, and mode of the data. 2,599; 3,190; none

3. Which measure of center is misleading in describing the miles of shoreline for the states? Explain. The mean is misleading. Sample answer: All states but one have over 3,000 miles of shoreline. The outlier of 89 causes the mean to be a poor choice to describe the data.

4. Which measure of center most accurately describes the data? median
Reteach

Problem-Solving Investigation: Use a Graph

When solving problems, a graph can show a visual representation of the situation and help you make conclusions about the particular set of data.

**Example**

**POPULATION** The table shows the enrollment of Mill High School students over five years. Estimate the enrollment for the 2010–2011 school year.

<table>
<thead>
<tr>
<th>School Year</th>
<th>05–06</th>
<th>06–07</th>
<th>07–08</th>
<th>08–09</th>
<th>09–10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>115</td>
<td>134</td>
<td>168</td>
<td>160</td>
<td>185</td>
</tr>
</tbody>
</table>

**Understand**

You know the enrollment of students for five years. You need to estimate the enrollment for the 2010–2011 school year.

**Plan**

Organize the data in a graph so that you can see a trend in the enrollment levels.

**Solve**

![Graph of Mill High School Enrollment]

The graph shows that the enrollment increases over the years. By using the graph, you can conclude that Mill High School had about 225 students enrolled for the 2010–2011 school year.

**Check**

Draw a line through as close to as many points as possible. The estimate is close to the line, so the answer is reasonable.

**Exercises 1–2. Sample answers are given.**

1. **TEMPERATURE** The chart to the right shows the average December temperatures in degrees Fahrenheit over four years. Predict the average temperature for the next year.

   **Sample answer:** 14°F

2. **POPULATION** Every five years the population of your neighborhood is recorded. What do you predict the population will be in 2015?

   **Sample answer:** 2,550 people

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Skills Practice

Problem-Solving Investigation: Use a Graph

Use a graph to solve the problem. For Exercises 1–3, refer to the graph.

1. Estimate the temperature at which the rate of chirping is 130 per minute.  **72 degrees**

2. Predict the number of cricket chirps per minute at 86 degrees.  **180 chirps/minute**

3. Predict the number of chirps per minute at 90 degrees.  **194 chirps/minute**

4. **CROCHETING** The table shows the number of dishcloths that Desiree can crochet.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

   a. Create a graph of the data.

   b. Use the graph to find the number of dishcloths Desiree could crochet in 6 hours.  **3**

5. **ADVERTISEMENTS** The school athletic booster organization charges $10 for a 2-inch advertisement and $20 for a 4-inch advertisement in the football program. Predict the cost of a 6-inch advertisement.  **$30**
Lesson 4 Reteach

Compare Populations

Example

The double box plot shows the ages of people at two different movies. Compare their centers and variations. Write an inference you can draw about the two populations.

Neither box plot is symmetric. Use the median to compare the centers and the interquartile range to compare the variations.

<table>
<thead>
<tr>
<th></th>
<th>Movie A</th>
<th>Movie B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Median</strong></td>
<td>33.5</td>
<td>35.5</td>
</tr>
<tr>
<td><strong>Interquartile Range</strong></td>
<td>35 – 32, or 3</td>
<td>38 – 34, or 4</td>
</tr>
</tbody>
</table>

The median age for people attending Movie B is 2 years older than those attending Movie A. There is a greater spread of data around the median for the ages of people in Movie B. Overall, the people who attended Movie B are older than those who attended Movie A.

Exercise

The double dot plot shows the cost of backpacks at two different stores. Compare the centers and variations of the two populations. Write an inference you can draw about the two populations.

Sample answer: Store 1 has a median price of $20 and an interquartile range of $7. Store 2 has a median price of $22 and an interquartile range of $5. The prices are more varied at Store 1, yet overall their prices are less than those of Store 2.
Lesson 4 Skills Practice

Compare Populations

1. The double box plot shows the heights in inches for the players on two professional basketball teams. Compare their centers and variations. Write an inference you can draw about the two populations.

Sample answer: The NY Knicks’ data centers around 80 inches with an interquartile range of 5 inches. The NJ Nets’ data centers around 79 inches with an interquartile range of 7 inches. While the median heights are about the same, the variation in heights is greater for the NJ Nets than the NY Knicks.

2. The double dot plot shows the number of minutes two students spent practicing the piano. Compare their centers and variations. Round to the nearest tenth. Write an inference you can draw about the two populations.

Sample answer: Lily’s data has a mean of 60 minutes with a mean absolute deviation of about 4.4 minutes. Alessandra’s data has a mean of 50 minutes with a mean absolute deviation of about 4.4 minutes. Both data sets have the same variation, but Lily’s data centers around a higher value. Generally, Lily practiced more minutes than Alessandra.

3. The double box plot shows the daily number of customers for two ice cream parlors. Compare the centers and variations of the two populations. Which ice cream parlor has the greater number of daily customers?

Sample answer: The Corner Creamery’s data centers around 70 customers with an interquartile range of 35 customers. Sue’s Ice Cream data centers around 50 customers with an interquartile range of 20 customers. Overall, Corner Creamery has a greater number of daily customers, but Sue’s Ice Cream has a greater consistency in its distribution.
Lesson 5 Reteach

Select an Appropriate Display

There are many different ways to display data. Some of these displays and their uses are listed below.

<table>
<thead>
<tr>
<th>Type of Display</th>
<th>Best Used to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar Graph</td>
<td>show the number of items in specific categories</td>
</tr>
<tr>
<td>Box Plot</td>
<td>show measures of variation for a set of data</td>
</tr>
<tr>
<td>Circle Graph</td>
<td>compare parts of the data to the whole</td>
</tr>
<tr>
<td>Histogram</td>
<td>show frequency of data divided into equal intervals</td>
</tr>
<tr>
<td>Line Graph</td>
<td>show change over a period of time</td>
</tr>
<tr>
<td>Double Bar Graph</td>
<td>compares two sets of categorical data</td>
</tr>
<tr>
<td>Line Plot</td>
<td>show frequency of data with a number line</td>
</tr>
</tbody>
</table>

When deciding what type of display to use, ask the following questions.

- What type of information is given?
- What do you want your graph or display to show?
- How will the graph or display be analyzed?

Remember, all data sets can be displayed in more than one way. And there is often more than one appropriate way to display a given set of data.

Examples

Select an appropriate type of display for each situation. Justify your reasoning.

1. the change in the winning times for the Kentucky Derby for the last 15 years
   This data does not deal with categories or intervals. It deals with the change of a value over time. A line graph is a good way to show changes over time.

2. energy usage in the U.S., categorized by the type of user
   In this case, there are specific categories. If you want to show the specific amount of energy used in each category, use a bar graph. If you want to show how each category is related to the whole, use a circle graph.

Exercises

Select an appropriate type of display for each situation. Justify your reasoning.

1. the cost of automobile insurance over the past 12 years
   line graph or bar graph; data changes over time

2. the amount of national park land in each state, arranged in square miles
   histogram; arrange data in intervals
Lesson 5 Skills Practice

Select an Appropriate Display

Select an appropriate type of display for each situation. Justify your reasoning.

1. sales of a leading brand of cereal for the last 12 years
   line or bar graph; data changes over time

2. test grades for a class, arranged in intervals
   histogram; data in intervals

3. average weight of wildcats, categorized by species
   bar graph; data in categories

4. ages of all students at a summer camp
   histogram or bar graph; data separated by ages

5. points scored by members of a basketball team as compared to the team total
   circle graph; part compared to whole

6. energy usage in your home for the past year, categorized by month
   histogram; data in intervals

Select an appropriate type of display for each situation. Justify your reasoning. Then construct the display. What can you conclude from your display?

7. Dwyane Wade’s Points per Game

<table>
<thead>
<tr>
<th>Season</th>
<th>Points per Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003–2004</td>
<td>16.2</td>
</tr>
<tr>
<td>2004–2005</td>
<td>24.1</td>
</tr>
<tr>
<td>2005–2006</td>
<td>27.2</td>
</tr>
<tr>
<td>2006–2007</td>
<td>27.4</td>
</tr>
<tr>
<td>2007–2008</td>
<td>24.6</td>
</tr>
</tbody>
</table>

line or bar graph; data changes over time

Dwayne scored the most number of points in the 2010–2011 season.

8. Time to Walk to School

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Percent of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fewer than 10</td>
<td>20</td>
</tr>
<tr>
<td>10–20</td>
<td>46</td>
</tr>
<tr>
<td>21–30</td>
<td>18</td>
</tr>
<tr>
<td>31–40</td>
<td>10</td>
</tr>
<tr>
<td>More than 40</td>
<td>6</td>
</tr>
</tbody>
</table>

circle graph; comparing part to total

It takes 10–20 minutes for about 50% of the students to walk to school.