Lesson Plans for the week

Lesson 1:  Introduce probability (2 Dice)
Lesson 2:  Fairness
Lesson 3:  Battleship probability and more about fairness
Lesson 4:  Expected Value
Lesson 5:  More Expected Value

Assessment of lessons 1 through 5
Lesson 1  Two Dice

Goals:  Generate and define sample space
        Understand that intuition is often different from actual probability
        Generate, define, and discuss probability distribution

Materials:  A copy of the “Two Dice” worksheet, Dice (two per pair of students)

Steps:

1. Pair students into groups of 2.

2. Have students answer the first three questions on their worksheets, then explain the rules of the game.

3. After the game is over, have a representative from each group come to the board to record their results. Use this time to discuss and guide the students to complete the remainder of the worksheet.

   Many students might mistakenly believe that even has an advantage over odd, because there are more even numbers, \{2,4,6,8,10,12\} than there are odd \{3,5,7,9,11\}.

   During this time in the lesson take time to develop organized lists of possibilities, and compare these possibilities with the total number of possible outcomes the students generated in question 1.

   After developing the probability distribution, it is helpful to use a java applet, or graphing calculator to simulate rolling a large number of two-dice-sums. Discuss the distribution graph.

4. Next, have the students play the same game, except this time multiply the dice instead of adding them.

5. Discuss the results for the revised game.
Activities
With
Two Dice

1. If two dice are rolled, what are the possible sums of the two dice? List all possibilities below.

2. How many sums are odd? ___________ Even?____________

3. Choose one player to be odd, and one player to be even.

It’s almost time to play! Here are the rules:
• You will roll two dice and find the sum.
• If the sum is even, the even player gets a point.
• If the sum is odd, the odd player gets a point.
• Roll the dice 40 times and record points for each player

4. Before you play, who do you think has the advantage? Odd, or even?___________

5. Play the game and record your results below.

6. Now, who do you think has the advantage? Odd, or even?__________________

7. Compare your results with the other groups. What do you think about advantage now?
Activities
With
Two Dice

8. In the space below, list all the different ways to get each sum? For example, to get a sum of 3, the possibilities are: 1 + 2, 2+1. Make a table to organize your data.

9. Now list the probabilities of getting each sum.

10. Examine the probabilities for odd and even. Who has the advantage? Why?

Multiplication game.
Play the game again, but instead of adding the dice, multiply the dice together. Before you play, who do you think has the advantage? Odd or Even?

After playing the game, who has the advantage? Explain your answer below.
Lesson 2: Dice Differences (From: Navigating Probability in Grades 6-8, NCTM, page 41-42)

Goals:
• Understand sample space and the numeric outcomes associated with it.
• Organize the results of a random experiment
• Understand the pooling of results
• Calculate the relative frequencies of outcomes and interpret them as probabilities

Materials:
• A copy of the worksheet, “Dice Differences” for each pair of students.
• A pair of dice for each pair of students.

Activity:
The students should play the game in pairs. Each pair should assign one student to be player A and the other to be player B. Each player rolls a pair of dice. If the numbers indicated on the top faces are different, the player subtracts the lesser value from the greater. For example, if the numbers are 6 and 3, the difference is 6 – 3 = 3. If the numbers are the same, the difference is zero.

Before play begins, be sure the students understand that the difference will be either 0 or positive. If they are familiar with the term absolute difference, you can explain that they are to compute the absolute difference of the numbers on the two dice.

Distribute the activity sheets to each pair of students, and go over the rules, which are explained on the first page. On each roll, player A scores a point if the difference is 0, 1, or 2, and player B scores a point if the difference is 3, 4, or 5. You may want to play one or two demonstration rounds. Have the students record the outcomes of their games and tally their points on the activity sheet. Then have them record on the chart the number of times each difference occurred. Each game is played until one player has accumulated 10 points.

Discussion:
After each pair of students has completed playing at least three games, bring the class together. Pool the data on the number of games won by players A and B. Ask the students if the game seems fair. They will probably agree that it is not a fair game. Ask them how these data show that the game is not fair. They will probably agree that since A and B do not win about equally often, the game is not fair.

Pool the data about the number of times each difference occurred. Compute the relative frequency of each difference. For the pooled data, the relative frequencies should be close to the theoretical probabilities. Develop the theoretical probabilities with the students and ask them to add up the probabilities for 0, 1, 2, and for 3, 4, 5.

Enter into discussion about fairness, and about how the game might be changed to make it fair.

Additional Activity:
Use the worksheet: Develop a Dice Game and have groups of three or four develop a game of dice that is fair. Allow the students to teach their games to other groups as time permits.
Dice Differences

Names _______________________

This game is for two players.

Rules
1. Decide who will be player A and who will be player B.
2. Decide who goes first, and then alternate turns.
3. On your turn, do the following:
   • Roll two dice.
   • If the values are different, subtract the lesser value from the greater value.
   • Record the difference in the spaces below. (See the sample game.)
   • If the values are the same, enter a zero.
4. Play at least three games.

Scoring
• Player A scores a point if the difference is 0, 1, or 2.
• Player B scores a point if the difference is 3, 4, or 5.
• The winner of a game is the first player to score 10 points.

Sample game. 0 2 1 2 3 3 1 0 1 4
          1 2 1 3

Player A’s tally of points: __________
Player B’s tally of points: __________

Game 1. ________________________________

Player A’s tally of points: __________
Player B’s tally of points: __________

Game 2. ________________________________

Player A’s tally of points: __________
Player B’s tally of points: __________

Game 3. ________________________________

Player A’s tally of points: __________
Player B’s tally of points: __________
Dice Differences (continued)

Name ______________________________


Player A’s tally of points: ____________
Player B’s tally of points: ____________

Game 5.  ____  ____  ____  ____  ____  ____  ____  ____  ____

Player A’s tally of points: ____________
Player B’s tally of points: ____________

For each game, record the number of times each difference occurred:

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1. How many games did player A win? ______

2. How many games did player B win? ______

3. Is this a fair game? ____________________ Explain your answer. Use the data above to help you.

________________________________________

________________________________________
Make a Dice Game That Would Be Fair So That Either Player Has An Equal Chance Of Winning.

Write the rules for your game on the back of this page.
Lesson 3
Battleship Probability:

Materials:  Battleship game boards (at least 2 per student)
           Battleship playing pieces (cut up and put in envelope). 1 set per student.

Steps:

1. Pair students up in groups of two. If there is an extra, two students can be on one side.

2. Give each student a game board and explain the rules of play.
   • Ships can be placed only horizontally or vertically
   • Each player takes a turn “firing” at the opponent
   • Hits and misses can be marked on the game board
   • The first player to sink his/her opponents ships wins

3. Depending on familiarity with the game, you may want to allow the students to play one game “freestyle” before the
   next step. While the students play, have them record the number of moves they took. After the game is done, record
   the number of moves on the board.

4. Talk about probability in the game of Battleship. Ask the students to discuss how the rules of probability might help
   them win the game.

5. As the students start to play the game. Ask them to record their moves and the probability that their move will be a
   hit. For example, on the first move, there is a 18/100 chance of getting a hit. The next shot might be 18/99, 18/98, …
   Until they make a hit. Once a hit is scored, the probability goes down drastically, to 1/3, ¼, etc.

6. Assess the students as they play…

7. Have the students record the number of moves taken to win on the board.

8. Have the students determine the minimum and maximum number of moves to win the game.

Additional Activity: Is Fairness in a game important?

1. Ask students if they think it is important that a game is fair. Most will answer yes.

2. Tell them that you are going to make the game of battleship fair. Hand out the envelopes with the “move squares” in
   them.

3. Tell the students that when it is their turn, they are to randomly draw a move card, and play only that card.

4. After the game, discuss whether we always want games to be fair.
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Lesson 4: Expected Value

Goals:
- Introduce students to the idea of expected value.
- Show the relationship between probability and expected value.

Materials:
- *Exploring Expected Value* activity sheets, dice, and decks of cards.

Steps:
- Put students in groups of three or four.
- Hand out the *Exploring Expected Value* two-page worksheet.
- Introduce the topic of expected value using simple examples with dice, coin flips, or cards.
- Allow the students to work their way through the activities on the handouts.
- Assess the students as they work.
- Have each group present their game of marbles at the end of page 2.
Exploring Expected Value:

Think about the following simple dice games. Use dice, and simulate the games to determine whether you think you would win money, lose money, or break even. Explain your reasoning.

1. You pay $1 to play. Roll one die. If the die shows an odd number you win $1. If the die shows an even number, you lose.

2. You pay $1 to play. Roll one die. If the die shows an odd number, you win $2. If the die shows an even number, you win $0.50.

3. You pay $1 to play. Roll two dice. If the sum of the two dice is odd, you win $2. If the sum of the dice is even, you lose.

Now use a deck of playing cards to determine if the following games will allow you to make money, lose money, or break even. Each game costs $1 to play. Explain your answers.

1. If you draw a black card, you win $2.

2. If you draw a king, you win $13.

3. If you draw a heart, you win $3.

4. If you draw the Ace of Hearts, you win $49.
Consider the following games. If you were designing a game that you wanted to be fair for the player, how much money would you have to pay out if the game costs $1 to play?

1. The player rolls one die. 1, 2, or 3 wins.

2. The player draws a card from the deck. Any Heart wins.

3. The player draws a card from the deck. The Ace of Spades wins.

4. The player rolls one die. 5 or 6 wins.

There are 10 red marbles, and 90 blue marbles in a jar. The game costs $1 to play. Drawing a red marble wins. If this was your game, how much money would you pay the winner? Why?
Lesson 5: More Expected Value

Goals:
• Understand how the idea of expected value helps charities design raffles.
• Explore the different variables involved in a charitable raffle.

Materials:
• Charity Raffles Activity Sheet

Steps:
1. Allow students to work in groups of 3 or 4.
2. Pass out Charity Raffle worksheet
3. Walk around to each group and guide them through the first question.
4. Allow the students to discuss and share their answers with the rest of the class.
You see it at church, school, sporting events, cheerleading, etc. People are selling raffle tickets for a chance to win a **BIG** prize. Sometimes the tickets cost $1.00, and sometimes they cost as much as $100. Sometimes there is only one big prize, and other times there are several prizes.

Why do charities sell raffle tickets and hold raffles? ................. **TO MAKE MONEY!!!**

When charities develop raffles, what information do they need to consider before they begin?

If a charity is trying to raise $5000 to send kids to camp by raffling off a motorcycle that cost $14,000, how many tickets do they need to sell? Does the price of the ticket change this number?

A local deer hunter’s association is raffling off a bag of decoys and a canoe that cost $2,100. If they wanted to sell $5 tickets, what is the minimum number of tickets they would need to sell?

How many would they need to sell if they wanted to make a $2,000 profit?
PERFORMANCE PACKAGE TASK 1
(Expected Value and Probability)

**Content Standard:** NCTM Data Analysis and Probability Standard  
**Level:** Grades 6–8

**Specific Statement(s) from the Standard:**

- Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.
- Compute and interpret the expected value of random variables in simple cases;

**Product(s):**

Students will develop three different raffles.

**Task Description:**

1. Develop a raffle that is fair (expected value=1)
   - You must determine the prize or prizes you will be raffling, as well as their cost.
   - You must determine how many tickets will be sold, and at what price.
   - This raffle must be fair.

2. Develop a raffle that will raise a minimum profit of $500 for the school library.
   - You must determine the prize or prizes you will be raffling, as well as their cost.
   - Determine how many tickets will be sold, and at what price.
   - Determine the probability of winning the raffle, assuming all the tickets are sold.
   - Explain, mathematically, how your raffle will raise the desired amount of money.

3. Develop a raffle that will raise a minimum profit of $9000 for new computers for the school.
   - You must determine the prize or prizes you will be raffling, as well as their cost.
   - Determine how many tickets will be sold, and at what price.
   - Determine the probability of winning the raffle, assuming all the tickets are sold.
   - Explain, mathematically, how your raffle will raise the desired amount of money.

**Special Notes:**
PERFORMANCE PACKAGE TASK 1  
(Title of Package)  

FEEDBACK CHECKLIST FOR TASK 1

The purpose of the checklist is to provide feedback to the student about his/her work relative to the content standard. Have the standard available for reference.

Y=Yes  
N=Needs Improvement

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Overall Comments  (information about student progress, quality of the work, next steps for teacher and student, needed adjustments in the teaching and learning processes, and problems to be addressed):