Simplification of Boolean Expression

Karnaugh Maps
QUINE McCLUSKY METHOD
EPRESSO Algorithm
HAZARDS–Static and Dynamic

Karnaugh Maps

- Review
QUINE McCLUSKEY METHOD

- Form the PI chart
- Remove the EPI
- Re-form the PI chart (with non-essential PIs only)
- Apply the following 2 rules to reduce the PI chart
  - A row that is covered by another row may be eliminated
  - A column that covers another column may be eliminated

Note: If the PI chart is cyclic (no SPI and can’t be reduced using the above two rules), then arbitrarily select a PI for SPI and apply the rules for the remaining PI chart.

Learning by example

- \( F(A, B, C, D) = \Sigma m(2, 4, 6, 8, 9, 10, 12, 13, 15) \)
Step 1

- Step 1: Grouping Minterms based on number of 1’s

<table>
<thead>
<tr>
<th>Minterms</th>
<th>ABCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0010</td>
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<tr>
<td>4</td>
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<tr>
<td>8</td>
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<tr>
<td>6</td>
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</tr>
<tr>
<td>9</td>
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<td>10</td>
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<tr>
<td>13</td>
<td>1101</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
</tr>
</tbody>
</table>

Step 2

- Step 2: Make a list of minterms that can be combined (minterms differing by a single literal)

<table>
<thead>
<tr>
<th>List 1</th>
<th>List 2</th>
<th>List 3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8</td>
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<td>4, 6</td>
</tr>
<tr>
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<td>9</td>
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<td>8, 9</td>
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<tr>
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<tr>
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Step 3

Step 3: Determine the minimum number of PI's

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<th>15</th>
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<tbody>
<tr>
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<td></td>
<td>x</td>
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<td>x</td>
<td>x</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>PI 4</td>
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<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 5</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 6</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>PI 7</strong></td>
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<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Step 4

Step 4: Select additional PI's for example

\[ F(A,B,C,D) = \text{PI}1 + \text{PI}3 + \text{PI}4 + \text{PI}7 \]
Covering Procedure

- Rule 1: A row that is **covered** by another row may be eliminated from the chart. When identical rows are present, all but one of the rows may be eliminated.
- Rule 2: A column that **covers** another column may be eliminated. All but one column from a set of identical columns may be eliminated.

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Covering Procedure Cont.

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
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<td><strong>9</strong></td>
<td><strong>10</strong></td>
<td><strong>11</strong></td>
<td><strong>13</strong></td>
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<td>**** Pi 1</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Pi 2</td>
<td>X</td>
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<tr>
<td>Pi 3</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Pi 4</td>
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<td>X</td>
<td>X</td>
<td>X</td>
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<td>X</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Pi 6</td>
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<td>X</td>
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<td>X</td>
<td>X</td>
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<td>X</td>
</tr>
<tr>
<td>**** Pi 7</td>
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<td>X</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

F(A,B,C,D) = ?
**Cyclic PI**

- A cyclic PI chart is a chart that contains no essential PI and that cannot be reduced by rules 1 and 2.

\[
F(A,B,C,D) = \Sigma m(1,2,3,4,5,6)
\]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>* PI 1</td>
<td>X</td>
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<td></td>
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<td></td>
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<tr>
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<td>X</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>PI 3</td>
<td>X</td>
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<td></td>
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<td>X</td>
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<tr>
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<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 5</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 6</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

**Procedure**

- Step 1: Identify any minterms covered by only one PI in the chart. Select these PIs for the cover. Note that this step identifies essential PIs on the first pass and nonessential PIs on the subsequent passes (from step 4).
- Step 2: Remove rows corresponding to the identified essential and nonessential PIs. Remove columns corresponding to minterms covered by the removed rows.
- Step 3: If a cyclic chart results after completing step 2, go to step 5. Otherwise, apply the reduction procedure of rules 1 and 2.
- Step 4: If a cyclic chart results from step 3, go to step 5. Otherwise, return to step 1.
- Step 5: Apply the cyclic chart procedure. Repeat step 5 until a void chart occurs or until a noncyclic chart is produced. In the latter case, return to step 1.
### Incompletely Specified Function

**F(A,B,C,D) = Σ m(2,3,7,10,12,15,27) + d(5,18,19,21,23)**

<table>
<thead>
<tr>
<th>Minterm</th>
<th>ABCDE</th>
<th>Minterm</th>
<th>ABCDE</th>
<th>Minterm</th>
<th>ABCDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>00010</td>
<td>2,3</td>
<td>0001-</td>
<td>2,3,18,19</td>
<td>-001-</td>
</tr>
<tr>
<td>3</td>
<td>00011</td>
<td>2,10</td>
<td>0-010</td>
<td>0-11</td>
<td>Pi2</td>
</tr>
<tr>
<td>5</td>
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<td>2,18</td>
<td>-0010</td>
<td>5,7,21,23</td>
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<tr>
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<td>01010</td>
<td>3,7</td>
<td>00-11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>01100</td>
<td>Pi7</td>
<td>3,19</td>
<td>-0011</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>10010</td>
<td>5,7</td>
<td>001-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>00111</td>
<td>5,21</td>
<td>-0101</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>10011</td>
<td>18,19</td>
<td>1001-</td>
<td></td>
<td></td>
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<tr>
<td>21</td>
<td>10101</td>
<td>7,15</td>
<td>0-111</td>
<td>Pi5</td>
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</tr>
<tr>
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<td>01111</td>
<td>7,23</td>
<td>-0111</td>
<td></td>
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<td>11011</td>
<td>19,27</td>
<td>1-011</td>
<td>Pi6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>21,23</td>
<td>101-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Listing don’t cares as regular minterms
- Do not list don’t cares in the PI chart
- Don’t care terms do not need to be checked out

**F(A,B,C,D,E) = Pi1 + Pi4 + Pi5 + Pi6 + Pi7**
SYSTEMS WITH MULTIPLE OUTPUTS

Simplification of switching functions by exploiting potential gate sharing to obtain a simpler overall design.

- Introduction of “Flags” column
- Minterms can be combined only if they possess one or more common flags and the term that results from the combination carries only flags that are common to both minterms
- Minterm(s) are checked off only if all flags are included in the resulting combination

Example:

\[ F_\alpha(A,B,C,D) = \Sigma m(0,2,7,10) + d(12,15) \]
\[ F_\beta(A,B,C,D) = \Sigma m(2,4,5) + d(6,7,8,10) \]
\[ F_\gamma(A,B,C,D) = \Sigma m(2,7,8) + d(0,5,13) \]
SYSTEMS WITH MULTIPLE OUTPUTS

<table>
<thead>
<tr>
<th>Minterm</th>
<th>ABCD</th>
<th>Flags</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
<td>αβγ</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
<td>αβγ  P110</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
<td>β γ</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>βγ   P111</td>
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<tr>
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<tr>
<td>6</td>
<td>0110</td>
<td>β γ</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
<td>αβ</td>
</tr>
<tr>
<td>12</td>
<td>1100</td>
<td>α P112</td>
</tr>
<tr>
<td>7</td>
<td>0111</td>
<td>αβγ  P113</td>
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<td>γ P11</td>
</tr>
<tr>
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</table>

**List 1**

<table>
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<tr>
<th>Minterm</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>4</td>
<td>0100</td>
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<td>1000</td>
<td>βγ   P111</td>
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<td>α P112</td>
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<td>γ P11</td>
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**List 2**

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<td>αβγ</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
<td>αβγ  P110</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
<td>β γ</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>βγ   P111</td>
</tr>
<tr>
<td>5</td>
<td>0101</td>
<td>βγ</td>
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<td>6</td>
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<td>β γ</td>
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<tr>
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<td>αβ</td>
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<td>γ P11</td>
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<tr>
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<td>α γ</td>
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**List 3**

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<td>0010</td>
<td>αβγ  P110</td>
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<tr>
<td>4</td>
<td>0100</td>
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<td>α P112</td>
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<tr>
<td>13</td>
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<td>γ P11</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
<td>α γ</td>
</tr>
</tbody>
</table>
SYSTEMS WITH MULTIPLE OUTPUTS

\[ F_\alpha = PI2 + PI5 + PI13 \]
\[ F_\beta = PI1 + PI5 \]
\[ F_\gamma = PI2 + PI3 + PI13 \]

<table>
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<tr>
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<th>( F_\gamma )</th>
</tr>
</thead>
<tbody>
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<td>✓</td>
</tr>
<tr>
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<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>8</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>*PI3</td>
<td>( \gamma )</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>PI7</td>
<td>( \beta )</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>PI9</td>
<td>( \alpha )</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>PI11</td>
<td>( \beta )</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>*PI13</td>
<td>( \alpha \beta )</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

PETRICK’S ALGORITHM

Use an algebraic approach to generate all possible covers of a function

- Write a POS expression representing all possible covers, \( C \)
- Convert \( C \) into SOP form
- Select the product that represents a minimum cost
PETRICK’S ALGORITHM

\[ C = (P1 + P6)(P1 + P2)(P2 + P3)(P3 + P4)(P4 + P5)(P5 + P6) \]
\[ = P1 P3 P5 + P1 P2 P3 P5 P6 + P1 P2 P4 P5 + P1 P2 P4 P6 + P2 P3 P5 P6 \]

PI1 PI3 PI5 is a minimum solution

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>* P1</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1 2</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1 3</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>P1 4</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>P1 5</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1 6</td>
<td>X</td>
<td></td>
<td></td>
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<td>X</td>
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</tr>
</tbody>
</table>

HAZARDS—Static and Dynamic

- A Static Hazard exists in a combinational circuit realizing a Boolean function if the function assumes momentarily a wrong value during a single change in its variables.

- A Dynamic Hazard exists in combinational circuit realizing a Boolean function, whose value must change from 0/1 to 1/0 due to a change in one of its variables, does not change in a single step.
HAZARDS–Static and Dynamic

X1 = X3 = 1 => D = 1, regardless of X2 in steady state

X2 = ___ : X2 = ___

D = X1 X2' + X2 X3
HAZARDS–Static and Dynamic

- **Theorem:** Static 1 hazard exists in the realization of a Boolean function in the form of a sum of cubes if there exists two disjoint and adjacent cubes.
- **Theorem:** Static 1 hazard can be eliminated in a Boolean function expressed as a sum of cubes by providing the bridging cube for each pair of disjoint and adjacent cubes.

Example of Static 0 Hazard

- \( F = \sum (0,2,6,7) \) using OR-AND network => static 0 hazard
Example of Static 0 Hazard

\[ F = (X_1' + X_2) (X_1 + X_3') (X_2 + X_3') \]

To eliminate static 0 hazard:
- AND – OR circuit \( \Rightarrow \) Static 1 Hazard
- OR – AND circuit \( \Rightarrow \) Static 0 Hazard